

# Calculating Wave Reflection From FUNWAVE-TVD Output Using Python

Steven SHI

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## 1 Objective

waveref.py is a Python module containing functions, including functions that produce wave reflection statistics from two gauges (Goda 76) and help determine ideal gauge placement (Wenneker and Hofland 14).

## 2 Implementation

### 2.1 Prerequisites

The module is dependent on NumPy, and tested in Python 3.10.5.

### 2.2 wn\_shallow()

wn\_shallow() is a function that returns a wave number approximation using shallow water reductions. Suppose  $\tanh kh \approx 2\pi h/L$ . Thus:

$$\begin{aligned}c &= ghT/L = \sqrt{gh} \\L &= \sqrt{gh}T \\ \frac{2\pi}{L} &= \frac{2\pi}{\sqrt{gh}T} \\ k &= \frac{\omega}{\sqrt{gh}}\end{aligned}\tag{1}$$

Where  $k$  is the wave number,  $T$  is the period,  $L$  is wave length,  $\omega$  is the angular wave frequency,  $g$  is the gravitational acceleration constant,  $k$  is the wave number, and  $h$  water depth.

### 2.3 wn()

wn() is a recursive function that returns a wave number approximation using Newton-Rhapson method. Observe the wave dispersion relation:

$$\omega^2 = gk \tanh kh\tag{2}$$

Construct  $F : (\omega, k, h) \rightarrow \mathbb{R}$  such that:

$$F(\omega, k, h) = gk \tanh kh - \omega^2\tag{3}$$

Suppose  $\omega$  and  $h$  are constant. Suppose initial guess  $k_i$ . Then,

$$k_{i+1} = k_0 - \frac{F(k_i)}{F'(k_i)} \quad (4)$$

Given (2):

$$k_{i+1} = k_i - \frac{gk_i \tanh k_i h - \omega^2}{g \tanh k_i h - gk_i h \sec^2 k_i h} \quad (5)$$

Define  $\text{wn}()$  by the following pseudocode:

```

wn( $k_i, \omega, h, \epsilon$ ) = {
  (1)  $k_{i+1} \leftarrow k_i - \frac{F(k_i)}{F'(k_i)}$ 

  (2) If  $|k_{i+1} - k_i| \leq \epsilon$  then return  $k_i$ .

  (3) Else, wn( $k_{i+1}, \omega, h, \epsilon$ )
}
```

## 2.4 reflection()

reflection() returns wave reflection statistics for flat-bottom, 1-D cases. Let:

- $x_i$  and  $x_r$  respectively be the coordinates for the incident wave gauge and reflection wave gauge
- $\eta_i$  and  $\eta_r$  respectively be the surface elevation at  $x_i$  and  $x_r$
- $\Delta l$  be the distance between  $x_i$  and  $x_r$
- $\Delta t$  be the time step
- $h$  be the water depth at  $x_r$

The amplitude of incident and reflected waves can be approximated using a combination of FFT on the time series for  $\eta_i, \eta_r$  and estimates taken from Goda 76.