Calculating Wave Reflection From FUNWAVE-TVD Output Using Python

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August 12, 2022

1 Objective

waveref.py is a Python module containing functions, including functions that produce wave reflection statistics from two gauges (Goda 76) and help determine ideal gauge placement (Wenneker and Hofland 14).

2 Implementation

2.1 Prerequisites

The module is dependent on NumPy, and tested in Python 3.10.5.

2.2 wn_shallow()

wn_shallow() is a function that returns a wave number approximation using shallow water reductions. Suppose $\tanh kh \approx 2\pi h/L$. Thus:

$$
\begin{aligned}
c &= ghT/L = \sqrt{gh} \\
L &= \sqrt{gh}T \\
\frac{2\pi}{L} &= \frac{2\pi}{\sqrt{gh}T} \\
k &= \frac{\omega}{\sqrt{gh}} \qquad (1)
\end{aligned}
$$

Where k is the wave number, T is the period, L is wave length, ω is the angular wave frequency, g is the gravitational acceleration constant, k is the wave number, and h water depth.

2.3 wn()

wn() is a recursive function that returns a wave number approximation using Newton-Rhapson method. Observe the wave dispersion relation:

$$\omega^2 = gk \tanh kh \qquad (2)$$

Construct $F : (\omega, k, h) \to \mathbb{R}$ such that:

$$F(\omega, k, h) = gk \tanh kh - \omega^2 \qquad (3)$$

Suppose ω and h are constant. Suppose initial guess k_i. Then,

$$k_{i+1} = k_0 - \frac{F(k_i)}{F'(k_i)} \tag{4}$$

Given (2):

$$k_{i+1} = k_i - \frac{gk_i \tanh k_i h - \omega^2}{g \tanh k_i h - gk_i h \sec^2 k_i h} \tag{5}$$

Define wn() by the following pseudocode:
wn(k_i, ω, h, ϵ) = {

(1) $k_{i+1} \leftarrow k_i - \frac{F(k_i)}{F'(k_i)}$

(2) If $|k_{i+1} - k_i| \leq \epsilon$ then return k_i.

(3) Else, wn(k_{i+1}, ω, h, ϵ)

}

2.4 reflection()

reflection() returns wave reflection statistics for flat-bottom, 1-D cases. Let:

- x_i and x_r respectively be the coordinates for the incident wave gauge and reflection wave gauge
- η_i and η_r respectively be the surface elevation at x_i and x_r
- Δl be the distance between x_i and x_r
- Δt be the time step
- h be the water depth at x_r

The amplitude of incident and reflected waves can be approximated using a combination of FFT on the time series for η_i, η_r and estimates taken from Goda 76.

$$\eta = \frac{H}{2}\cos(kx - \omega t)$$