

Students: Group 11 (Shawn Hillyer, Jesse Thoren, Jason Goldfine-Middleton)
Assignment: Project 3
Due date: 11/13/2016

Note: Our zip file contains all of our code, but we tried to include the code in the file as well for most problems without being overly obtuse.

Problem 1

Part A

Determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost.

- Formulate the problem as a linear program with an objective function and all constraints.
- Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.
- What are the optimal shipping routes and minimum cost.

Part A-i

Define Variables:

Let p be the Plants {P1, P2, P3, P4}
Let w be the Warehouses {W1, W2, W3}
Let r be the Retailers {R1, R2, R3, R4, R5, R6, R7}
Let X_{pw} be the units to ship from p to w
Let Y_{wr} be the units to ship from w to r
Let S_p be the supply available at each plant p
Let D_r be the demand at each retailer r .

Objective: Minimize the cost of shipping the products from the plants to the warehouses and warehouses to the retailers.

MINIMIZE $10X_{11} + 15X_{12} + 11X_{21} + 8X_{22} + 13X_{31} + 8X_{32} + 9X_{33} + 14X_{42} + 8X_{43} + 5Y_{11} + 6Y_{12} + 7Y_{13} + 10Y_{14} + 12Y_{23} + 8Y_{24} + 10Y_{25} + 14Y_{26} + 14Y_{34} + 12Y_{35} + 12Y_{36} + 6Y_{37}$

Constraints:

s.t.:

// Each Plant cannot ship more than it has
 $X_{11} + X_{12} \leq 150$ //p1 supply
 $X_{21} + X_{22} \leq 450$ //p2 supply
 $X_{31} + X_{32} + X_{33} \leq 250$ //p3 supply
 $X_{42} + X_{43} \leq 150$ //p4 supply

// Each retailer receives exactly what its demand is
 $Y_{11} = 100$ // Retailer 1 demand
 $Y_{12} = 150$ // Retailer 2 demand
 $Y_{13} + Y_{23} = 100$ // Retailer 3 demand
 $Y_{14} + Y_{24} + Y_{34} = 200$ // Retailer 4 demand
 $Y_{25} + Y_{35} = 200$ // Retailer 5 demand

$$Y_{26} + Y_{36} = 150 \quad // \text{Retailer 6 demand}$$

$$Y_{37} = 100 \quad // \text{Retailer 7 demand}$$

// Units shipped from each warehouse must be \leq amount shipped to the warehouse

$$Y_{11} + Y_{12} + Y_{13} + Y_{14} - X_{11} - X_{21} - X_{31} \leq 0$$

$$Y_{23} + Y_{24} + Y_{25} + Y_{26} - X_{12} - X_{22} - X_{32} - X_{42} \leq 0$$

$$Y_{34} + Y_{35} + Y_{36} + Y_{37} - X_{33} - X_{43} \leq 0$$

Non-negativity constraints:

$$X_{pw} \text{ and } Y_{wr} \geq 0$$

Part A-ii

The optimal solution was generated using the above formulas translated into a LINDO program.

Code is in file CS325Project3Problem1a.ltx as well as below. The output from LINDO follows the code.

```
MINIMIZE 10X11 + 15X12 + 11X21 + 8X22 + 13X31 + 8X32 + 9X33 + 14X42 + 8X43 + 5Y11 +
6Y12 + 7Y13 + 10Y14 + 12Y23 + 8Y24 + 10Y25 + 14Y26 + 14Y34 + 12Y35 + 12Y36 + 6Y37
ST
    X11 + X12 <= 150
    X21 + X22 <= 450
    X31 + X32 + X33 <= 250
    X42 + X43 <= 150
    Y11 = 100
    Y12 = 150
    Y13 + Y23 = 100
    Y14 + Y24 + Y34 = 200
    Y25 + Y35 = 200
    Y26 + Y36 = 150
    Y37 = 100
    Y11 + Y12 + Y13 + Y14 - X11 - X21 - X31 <= 0
    Y23 + Y24 + Y25 + Y26 - X12 - X22 - X32 - X42 <= 0
    Y34 + Y35 + Y36 + Y37 - X33 - X43 <= 0
```

Results:

LP OPTIMUM FOUND AT STEP 10		
OBJECTIVE FUNCTION VALUE		
1)	17100.00	
VARIABLE	VALUE	REDUCED COST
X11	150.000000	0.000000
X12	0.000000	8.000000
X21	200.000000	0.000000
X22	250.000000	0.000000
X31	0.000000	2.000000
X32	150.000000	0.000000
X33	100.000000	0.000000
X42	0.000000	7.000000

X43	150.000000	0.000000
Y11	100.000000	0.000000
Y12	150.000000	0.000000
Y13	100.000000	0.000000
Y14	0.000000	5.000000
Y23	0.000000	2.000000
Y24	200.000000	0.000000
Y25	200.000000	0.000000
Y26	0.000000	1.000000
Y34	0.000000	7.000000
Y35	0.000000	3.000000
Y36	150.000000	0.000000
Y37	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.000000
4)	0.000000	0.000000
5)	0.000000	1.000000
6)	0.000000	-16.000000
7)	0.000000	-17.000000
8)	0.000000	-18.000000
9)	0.000000	-16.000000
10)	0.000000	-18.000000
11)	0.000000	-21.000000
12)	0.000000	-15.000000
13)	0.000000	11.000000
14)	0.000000	8.000000
15)	0.000000	9.000000

NO. ITERATIONS= 10

Part A-iii:

Optimal shipping routes and minimum cost can be determined by examining the results from the program:

The minimum cost is 17,100.

The shipping routes are as follows:

- P1 ships 150 units to W1
- P2 ships 200 units to W1, 250 units to W2
- P3 ships 150 units to W2 and 100 units to W3
- P4 ships 150 units to W3
- (W1 has 150+200=350 units. W2 has 250+150=400 units. W3 has 100+150=250 units)
- W1 ships 100 to R1, 150 to R2, 100 to R3. (350 units out - OK)
- W2 ships 200 to R4 and 200 R5 (400 total units out - OK)
- W3 ships 150 to R6 and 100 to R7 (250 units out OK).

Part B

We can solve this question with some simple logic. By eliminating warehouse two, we see the following simple facts:

- Warehouse 3 can receive product only from P3 and P4. That's a maximum of 400 units of supply coming in to warehouse 3 (250+150).
- The demand from retailers 5, 6, and 7 to receive from Warehouse3 is a total of 450. But there is no way for warehouse 3 to get 450 units. Those three retailers have no other warehouse if warehouse 1 is shut down.

There is no *feasible solution* to this problem, therefore it is "unfeasible".

We can verify this by running a modified version of the Linear programming that removes warehouse 2:

MINIMIZE $10X_{11} + 11X_{21} + 13X_{31} + 9X_{33} + 8X_{43} + 5Y_{11} + 6Y_{12} + 7Y_{13} + 10Y_{14} + 14Y_{34} + 12Y_{35} + 12Y_{36} + 6Y_{37}$

Constraints:

s.t.:

// Each Plant cannot ship more than it has

$$X_{11} \leq 150 \quad //p1 \text{ supply}$$

$$X_{21} \leq 450 \quad //p2 \text{ supply}$$

$$X_{31} + X_{33} \leq 250 \quad //p3 \text{ supply}$$

$$X_{43} \leq 150 \quad //p4 \text{ supply}$$

// Each retailer receives exactly what its demand is

$$Y_{11} = 100 \quad // \text{Retailer 1 demand}$$

$$Y_{12} = 150 \quad // \text{Retailer 2 demand}$$

$$Y_{13} = 100 \quad // \text{Retailer 3 demand}$$

$$Y_{14} + Y_{34} = 200 \quad // \text{Retailer 4 demand}$$

$$Y_{35} = 200 \quad // \text{Retailer 5 demand}$$

$$Y_{36} = 150 \quad // \text{Retailer 6 demand}$$

$$Y_{37} = 100 \quad // \text{Retailer 7 demand}$$

// Units shipped from each warehouse must be \leq amount shipped to the warehouse

$$Y_{11} + Y_{12} + Y_{13} + Y_{14} - X_{11} - X_{21} - X_{31} \leq 0$$

$$Y_{34} + Y_{35} + Y_{36} + Y_{37} - X_{33} - X_{43} \leq 0$$

Non-negativity constraints:

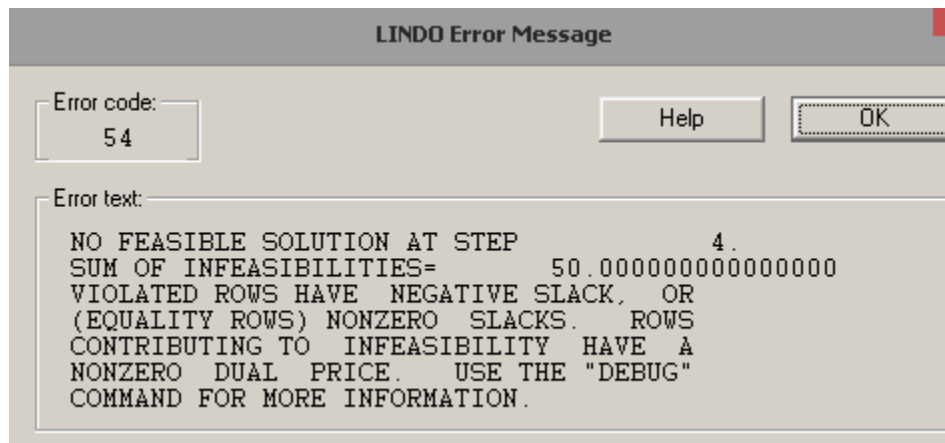
$$X_{pw} \text{ and } Y_{wr} \geq 0$$

The LINDO code: CS325Project3Problem1b.ltx

```
MINIMIZE 10X11 + 11X21 + 13X31 + 9X33 + 8X43 + 5Y11 + 6Y12 + 7Y13 + 10Y14 + 14Y34 +
12Y35 + 12Y36 + 6Y37
ST
X11 <= 150
X21 <= 450
X31 + X33 <= 250
```

```
X43 <= 150
Y11 = 100
Y12 = 150
Y13 = 100
Y14 + Y34 = 200
Y35 = 200
Y36 = 150
Y37 = 100
Y11 + Y12 + Y13 + Y14 - X11 - X21 - X31 <= 0
Y34 + Y35 + Y36 + Y37 - X33 - X43 <= 0
```

Results:



Part C

Yes, it is possible if we limit warehouse 2 to having only 100 refrigerators per week. The optimal solution is cost 18300.00. A single constant (blue below) can be added to solve this problem. We just limit the number of units that can be shipped from each plant to warehouse 2 to be no more than 100.

MINIMIZE $10X_{11} + 15X_{12} + 11X_{21} + 8X_{22} + 13X_{31} + 8X_{32} + 9X_{33} + 14X_{42} + 8X_{43} + 5Y_{11} + 6Y_{12} + 7Y_{13} + 10Y_{14} + 12Y_{23} + 8Y_{24} + 10Y_{25} + 14Y_{26} + 14Y_{34} + 12Y_{35} + 12Y_{36} + 6Y_{37}$

Constants:

s.t.:

```
// New added constraint: Amount shipped to Warehouse 2 must be no greater than 100 units
 $X_{12} + X_{22} + X_{32} + X_{42} \leq 100$ 
```

```
// Each Plant cannot ship more than it has
```

```
 $X_{11} + X_{12} \leq 150$  //p1 supply
```

```
 $X_{21} + X_{22} \leq 450$  //p2 supply
```

```
 $X_{31} + X_{32} + X_{33} \leq 250$  //p3 supply
```

```
 $X_{42} + X_{43} \leq 150$  //p4 supply
```

```
// Each retailer receives exactly what its demand is
```

```
 $Y_{11} = 100$  // Retailer 1 demand
```

```
 $Y_{12} = 150$  // Retailer 2 demand
```

```
 $Y_{13} + Y_{23} = 100$  // Retailer 3 demand
```

```
 $Y_{14} + Y_{24} + Y_{34} = 200$  // Retailer 4 demand
```

```
 $Y_{25} + Y_{35} = 200$  // Retailer 5 demand
```

```
 $Y_{26} + Y_{36} = 150$  // Retailer 6 demand
```

```
 $Y_{37} = 100$  // Retailer 7 demand
```

```
// Units shipped from each warehouse must be <= amount shipped to the warehouse
```

```
 $Y_{11} + Y_{12} + Y_{13} + Y_{14} - X_{11} - X_{21} - X_{31} \leq 0$ 
```

```
 $Y_{23} + Y_{24} + Y_{25} + Y_{26} - X_{12} - X_{22} - X_{32} - X_{42} \leq 0$ 
```

```
 $Y_{34} + Y_{35} + Y_{36} + Y_{37} - X_{33} - X_{43} \leq 0$ 
```

Non-negativity constraints:

X_{pw} and $Y_{wr} \geq 0$

LINDO CODE (adds one constraint to Part A code) is in CS325Project3Problem1c.ltx

```
MINIMIZE  $10X_{11} + 15X_{12} + 11X_{21} + 8X_{22} + 13X_{31} + 8X_{32} + 9X_{33} + 14X_{42} + 8X_{43} + 5Y_{11} + 6Y_{12} + 7Y_{13} + 10Y_{14} + 12Y_{23} + 8Y_{24} + 10Y_{25} + 14Y_{26} + 14Y_{34} + 12Y_{35} + 12Y_{36} + 6Y_{37}$ 
```

```
ST
```

```
 $X_{12} + X_{22} + X_{32} + X_{42} \leq 100$ 
```

```
 $X_{11} + X_{12} \leq 150$ 
```

```
 $X_{21} + X_{22} \leq 450$ 
```

```
 $X_{31} + X_{32} + X_{33} \leq 250$ 
```

```
 $X_{42} + X_{43} \leq 150$ 
```

```
 $Y_{11} = 100$ 
```

```

Y12=150
Y13 + Y23=100
Y14 + Y24 + Y34=200
Y25 + Y35=200
Y26 + Y36=150
Y37= 100
Y11 + Y12 + Y13 + Y14-X11-X21-X31 <= 0
Y23 + Y24 + Y25 + Y26-X12-X22-X32-X42 <= 0
Y34 + Y35 + Y36 + Y37-X33-X43 <= 0

```

Results

LP OPTIMUM FOUND AT STEP 11

OBJECTIVE FUNCTION VALUE

1) 18300.00

VARIABLE	VALUE	REDUCED COST
X11	150.000000	0.000000
X12	0.000000	8.000000
X21	350.000000	0.000000
X22	100.000000	0.000000
X31	0.000000	4.000000
X32	0.000000	2.000000
X33	250.000000	0.000000
X42	0.000000	9.000000
X43	150.000000	0.000000
Y11	100.000000	0.000000
Y12	150.000000	0.000000
Y13	100.000000	0.000000
Y14	150.000000	0.000000
Y23	0.000000	7.000000
Y24	50.000000	0.000000
Y25	50.000000	0.000000
Y26	0.000000	4.000000
Y34	0.000000	4.000000
Y35	150.000000	0.000000
Y36	150.000000	0.000000
Y37	100.000000	0.000000

Part D

Write out a generalized linear programming model. Give the objective function and constraints as mathematical formula.

For a given Plant-Warehouse-Retailer distribution model with n plants, m warehouses, and q retailers:

Let p be the subscript index for plants $\{P_1, P_2, \dots, P_n\}$

Let w be the subscript index for Warehouses $\{W_1, W_2, \dots, W_m\}$

Let r be the subscript index for Retailers $\{R_1, R_2, \dots, R_q\}$

Let X_{pw} be the units to ship from P_p to W_w

Let Y_{wr} be the units to ship from W_w to R_r

Let S_p be the supply available at P_p

Let D_r be the demand at each R_r .

Let C_{pw} be the cost of shipping from P_p to W_w .

Let K_{wr} be the cost of shipping from W_w to R_r

Objective:

Minimize the cost function, f :

$$f(n, m, q) = \sum_{w=1}^m \sum_{p=1}^n C_{pw} \cdot X_{pw} + \sum_{r=1}^q \sum_{w=1}^m K_{wr} \cdot Y_{wr}$$

Constraints:

//The sum of shipments from plant p can be no more than the supply available at plant p (S_p). Note, where there are 0 shipments from plant p to warehouse w , X_{pw} evaluates to 0.

For a given plant p , where $1 \leq p \leq n$, this is given by the inequality:

$$\sum_{w=1}^m X_{pw} \leq S_p$$

//The sum of shipments to retailer r must be exactly equal to the demand at Retailer r . Note, where there are 0 shipments from warehouse w to retailer r , Y_{wr} evaluates to 0.

For a given retailer r , where $1 \leq r \leq q$, this is given by the equality:

$$\sum_{w=1}^m Y_{wr} = D_r$$

//The number of units shipped from each warehouse w must be less than or equal to the amount of units shipped to the warehouse. That is, for each warehouse, the sum of outgoing shipments, Y_{wr} , minus the sum of incoming shipments, X_{pw} , must be non-positive.

For a given warehouse w , where $1 \leq w \leq m$, this is given by the inequality:

$$\sum_{r=1}^q Y_{wr} - \sum_{p=1}^n X_{pw} \leq 0$$

//The amount shipped from any plant to any warehouse must be non-negative.

$X_{pw} \geq 0$ for all values $1 \leq p \leq n$, and $1 \leq w \leq m$

//The amount shipped from any warehouse to any retailer must also be non-negative.

$Y_{wr} \geq 0$ for all values $1 \leq w \leq m$, and $1 \leq r \leq q$.

Problem 2

****Note that these variables correspond to 100 grams of a given ingredient.****

****That is, $t_m = .5$ corresponds to 50 grams of tomatoes.****

Tomato -	t_m
Lettuce -	l_e
Spinach -	s_p
Carrot -	c_a
Sunflower Seeds -	s_s
Smoked Tofu -	s_t
Chickpeas -	c_h
Oil -	o_i

At least 15 grams of protein:

$$\begin{aligned} &****0.85*t_m + 1.62*l_e + 2.86*s_p + 0.93*c_a + 23.4*s_s + 16*s_t + 9*c_h + 0*o_i \geq 15**** \\ &-0.85*t_m - 1.62*l_e - 2.86*s_p - 0.93*c_a - 23.4*s_s - 16*s_t - 9*c_h - 0*o_i \leq -15 \end{aligned}$$

At least 2 and at most 8 grams of fat:

$$\begin{aligned} &****0.33*t_m + 0.20*l_e + 0.39*s_p + 0.24*c_a + 48.7*s_s + 5*s_t + 2.6*c_h + 100*o_i \geq 2**** \\ &-0.33*t_m - 0.20*l_e - 0.39*s_p - 0.24*c_a - 48.7*s_s - 5*s_t - 2.6*c_h - 100*o_i \leq -2 \\ &0.33*t_m + 0.20*l_e + 0.39*s_p + 0.24*c_a + 48.7*s_s + 5*s_t + 2.6*c_h + 100*o_i \leq 8 \end{aligned}$$

At least 4 grams of carbohydrates:

$$\begin{aligned} &****4.64*t_m + 2.37*l_e + 3.63*s_p + 9.58*c_a + 15*s_s + 3*s_t + 27*c_h + 0*o_i \geq 4**** \\ &-4.64*t_m - 2.37*l_e - 3.63*s_p - 9.58*c_a - 15*s_s - 3*s_t - 27*c_h - 0*o_i \leq -4 \end{aligned}$$

At most 200 mg of sodium:

$$9*t_m + 28*l_e + 65*s_p + 69*c_a + 3.8*s_s + 120*s_t + 78*c_h + 0*o_i \leq 200$$

At least 40% leafy greens by mass:

$$\begin{aligned} &****l_e + s_p \geq .4(t_m + l_e + s_p + c_a + s_s + s_t + c_h + o_i)**** \\ &****-.4*t_m + .6*l_e + .6*s_p - .4*c_a - .4*s_s - .4*s_t - .4*c_h - .4*o_i \geq 0**** \\ &.4*t_m - .6*l_e - .6*s_p + .4*c_a + .4*s_s + .4*s_t + .4*c_h + .4*o_i \leq 0 \end{aligned}$$

Cost:

$$f = 1*t_m + .75*l_e + .5*s_p + .5*c_a + .45*s_s + 2.15*s_t + .95*c_h + 2*o_i$$

Energy:

$$f = 21*t_m + 16*l_e + 40*s_p + 41*c_a + 585*s_s + 120*s_t + 164*c_h + 884*o_i$$

Part A:

Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.

i. Formulate the problem as a linear program with an objective function and all constraints.

System of inequalities:

$-0.85*tm - 1.62*le - 2.86*sp - 0.93*ca - 23.4*ss - 16*st - 9*ch - 0*oi \leq -15$
 $-0.33*tm - 0.20*le - 0.39*sp - 0.24*ca - 48.7*ss - 5*st - 2.6*ch - 100*oi \leq -2$
 $0.33*tm + 0.20*le + 0.39*sp + 0.24*ca + 48.7*ss + 5*st + 2.6*ch + 100*oi \leq 8$
 $-4.64*tm - 2.37*le - 3.63*sp - 9.58*ca - 15*ss - 3*st - 27*ch - 0*oi \leq -4$
 $9*tm + 28*le + 65*sp + 69*ca + 3.8*ss + 120*st + 78*ch + 0*oi \leq 200$
 $.4*tm - .6*le - .6*sp + .4*ca + .4*ss + .4*st + .4*ch + .4*oi \leq 0$
 $tm, le, sp, ca, ss, st, ch, oi \geq 0$ // Each of these variables may not be negative.

Objective function:

$f = 21*tm + 16*le + 40*sp + 41*ca + 585*ss + 120*st + 164*ch + 884*oi$

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

MATLAB: CS325Project3Problem2a.m

```

A = [-0.85 -1.62 -2.86 -0.93 -23.4 -16 -9 0
      -0.33 -0.20 -0.39 -0.24 -48.7 -5 -2.6 -100
      0.33 0.20 0.39 0.24 48.7 5 2.6 100
      -4.64 -2.37 -3.63 -9.58 -15 -3 -27 0
      9 28 65 69 3.8 120 78 0
      0.4 -0.6 -0.6 0.4 0.4 0.4 0.4 0.4];

b = [-15 -2 8 -4 200 0];

f = [21 16 40 41 585 120 164 884];

Aeq = [];
beq = [];

lb = [0, 0, 0, 0, 0, 0, 0, 0];
ub = [];

x = linprog(f,A,b,Aeq,beq,lb,ub);
  
```

Part A-ii Solution:

This returns $x = [1.5616e-11, 0.5855, 2.7476e-11, 7.1982e-12, 1.0126e-12, 0.8782, 2.8351e-12, 4.9367e-13]$

Which corresponds to 58.55 grams of Lettuce and 87.82 grams of Smoked Tofu (Appetizing?!)

This has a total of $16* .5855 + 120*.8782 = 114.752$ calories, and is the minimum calorie salad that meets the nutritional requirements.

Part A-iii Solution: **What is the cost of the low calorie salad?**

Cost = $0.75 \cdot .5855 + 2.15 \cdot .8782 = \2.33 per low calorie "salad".

Part B: Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost.

i. Formulate the problem as a linear program with an objective function and all constraints.

System of inequalities:

$-0.85 \cdot tm - 1.62 \cdot le - 2.86 \cdot sp - 0.93 \cdot ca - 23.4 \cdot ss - 16 \cdot st - 9 \cdot ch - 0 \cdot oi \leq -15$
 $-0.33 \cdot tm - 0.20 \cdot le - 0.39 \cdot sp - 0.24 \cdot ca - 48.7 \cdot ss - 5 \cdot st - 2.6 \cdot ch - 100 \cdot oi \leq -2$
 $0.33 \cdot tm + 0.20 \cdot le + 0.39 \cdot sp + 0.24 \cdot ca + 48.7 \cdot ss + 5 \cdot st + 2.6 \cdot ch + 100 \cdot oi \leq 8$
 $-4.64 \cdot tm - 2.37 \cdot le - 3.63 \cdot sp - 9.58 \cdot ca - 15 \cdot ss - 3 \cdot st - 27 \cdot ch - 0 \cdot oi \leq -4$
 $9 \cdot tm + 28 \cdot le + 65 \cdot sp + 69 \cdot ca + 3.8 \cdot ss + 120 \cdot st + 78 \cdot ch + 0 \cdot oi \leq 200$
 $.4 \cdot tm - .6 \cdot le - .6 \cdot sp + .4 \cdot ca + .4 \cdot ss + .4 \cdot st + .4 \cdot ch + .4 \cdot oi \leq 0$
 $tm, le, sp, ca, ss, st, ch, oi \geq 0$ // Each of these variables may not be negative.

Objective function:

$f = 1 \cdot tm + .75 \cdot le + .5 \cdot sp + .5 \cdot ca + .45 \cdot ss + 2.15 \cdot st + .95 \cdot ch + 2 \cdot oi$

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

MATLAB: CS325Project3Problem2b.m

```
A = [-0.85 -1.62 -2.86 -0.93 -23.4 -16 -9 0  
     -0.33 -0.20 -0.39 -0.24 -48.7 -5 -2.6 -100  
      0.33 0.20 0.39 0.24 48.7 5 2.6 100  
     -4.64 -2.37 -3.63 -9.58 -15 -3 -27 0  
      9 28 65 69 3.8 120 78 0  
      0.4 -0.6 -0.6 0.4 0.4 0.4 0.4 0.4];
```

```
b = [-15 -2 8 -4 200 0];
```

```
f = [1 .75 .5 .5 .45 2.15 .95 2];
```

```
Aeq = [];
```

```
beq = [];
```

```
lb = [0, 0, 0, 0, 0, 0, 0, 0];
```

```
ub = [];
```

```
x = linprog(f,A,b,Aeq,beq,lb,ub);
```

This returns $x = [3.8585e-14, 1.3023e-13, 0.8323, 6.3868e-14, 0.0961, 1.2666e-13, 1.1524, 3.9520e-15]$

Which corresponds to 83.23 grams of spinach, 9.61 grams of sunflower seeds, and 115.24 grams of chickpeas.

This has a total cost of $.5*.8323 + .45*.0961 + .95*1.1524 = \1.56 .

iii. How many calories are in the low cost salad?

Energy = $40*.8323 + 585*.0961 + 164*1.1524 = 278.5041$ calories.

Part C: Compare the results from part A and B. Veronica's goal is to create a Very Veggie Salad that is both low calorie and low cost. She would like to sell the salad for \$5.00 and still have a profit of at least \$3.00. However if she can advertise that the salad has under 250 calories then she may be able to sell more.

i. Suggest some possible ways that she select a combination of ingredients that is "near optimal" for both objective. This is a type of multi-objective optimization.

Note that the minimum calorie salad found in part A has 114.752 calories and costs \$2.33.

Note that the minimum cost salad found in part B has 278.5041 calories and costs \$1.56.

Considering the constraints added in part C, we have two viable options for creating an optimized salad:

Option A: Minimize the cost for a salad that has less than 250 calories.

Modifying our code in part B by adding an additional inequality to ensure that the amount of calories is at most 250, as below:

MATLAB: **CS325Project3Problem2COptionA.m**

```
A = [-0.85 -1.62 -2.86 -0.93 -23.4 -16 -9 0
      -0.33 -0.20 -0.39 -0.24 -48.7 -5 -2.6 -100
      0.33 0.20 0.39 0.24 48.7 5 2.6 100
      -4.64 -2.37 -3.63 -9.58 -15 -3 -27 0
      9 28 65 69 3.8 120 78 0
      0.4 -0.6 -0.6 0.4 0.4 0.4 0.4 0.4
      21 16 40 41 585 120 164 884];
```

```

b = [-15 -2 8 -4 200 0 250];

f = [1 .75 .5 .5 .45 2.15 .95 2];

Aeq = [];
beq = [];

lb = [0, 0, 0, 0, 0, 0, 0, 0];
ub = [];

x = linprog(f,A,b,Aeq,beq,lb,ub);

```

We get that $x = [4.2813\text{e-}14, 1.2122\text{e-}13, 0.7620, 7.3806\text{e-}14, 0.0938, 0.1689, 0.8802, 4.8830\text{e-}15]$

Which corresponds to a salad with 76.20 grams of spinach, 9.38 grams of sunflower seeds, 16.89 grams of smoked tofu, and 88.02 grams of chickpeas.

This salad has 249.9738 calories, and costs \$1.63

Option B: Minimize the amount of calories for a salad that costs at most \$2.00 to make (so that when she sells for \$5, she still makes a profit of \$3)

Modifying our code in part A by adding an additional inequality to ensure that the cost is at most \$2.00, as below:

MATLAB: **CS325Project3Problem2COptionB.m**

```

A = [-0.85 -1.62 -2.86 -0.93 -23.4 -16 -9 0
      -0.33 -0.20 -0.39 -0.24 -48.7 -5 -2.6 -100
      0.33 0.20 0.39 0.24 48.7 5 2.6 100
      -4.64 -2.37 -3.63 -9.58 -15 -3 -27 0
      9 28 65 69 3.8 120 78 0
      0.4 -0.6 -0.6 0.4 0.4 0.4 0.4 0.4
      1 .75 .5 .5 .45 2.15 .95 2];

b = [-15 -2 8 -4 200 0 2];

f = [21 16 40 41 585 120 164 884];

Aeq = [];
beq = [];

lb = [0, 0, 0, 0, 0, 0, 0, 0];
ub = [];

x = linprog(f,A,b,Aeq,beq,lb,ub);

```

We get that $x = [4.2810e-15, 1.3774e-14, 0.5503, 5.4151e-15, 0.0294, 0.7961, 5.6316e-14, 5.6112e-16]$

Which corresponds to a salad with 55.03 grams of spinach, 2.94 grams of sunflower seeds, and 79.61 grams of smoked tofu.

This salad has 134.743 calories and costs \$2.00.

ii. What combination of ingredient would you suggest and what is the associated cost and calorie?

We would suggest a salad with 76.20 grams of spinach, 9.38 grams of sunflower seeds, 16.89 grams of smoked tofu, and 88.02 grams of chickpeas.

This has 249.9738 calories and costs \$1.63. If sold at \$5, it has a profit of \$3.37 per salad sold.

iii. From a business standpoint, we can expect that her advertisement is going to have the same effect if the salad has 210 calories as it would if it had 249.99, because she is specifically advertising that her salad "has under 250 calories". Given this, it is far better to go with the salad in option A, because it is able to leverage additional profit per salad over B, even though the two salads would attract the exact same amount of business. Hence, we can safely conclude that our salad is optimal for maximizing profit with the additional constraints given in Part C.

Furthermore, this salad is almost assuredly better from a business standpoint than the minimum cost salad because of the extra boost from advertisement. The minimum cost salad (which cost \$1.56, for a profit per salad of \$3.44 per salad sold) only leverages an additional 7 cents per salad over the optimal profit salad found in part C. Since the profit ratio of the minimum cost salad (part B) to the optimal profit salad (part C) is $3.44/3.37 = 1.02$, we can conclude that the optimal profit salad in part C will always produce more total profit than the minimum cost salad in part B, assuming that the advertising campaign described in C produces more than a 2.08% increase in salads sold.

Problem 3

Note: LINDO was used to solve all four parts of Problem 3.

a)

As found in lindo_p3a_sol.txt:

```
da = 0
db = 2
dc = 3
dd = 3
de = 9
df = 6
dg = 8
dh = 9
di = 8
dj = 10
dk = 14
dl = 15
dm = 17
```

Code:

```
MAX da + db + dc + dd + de + df + dg + dh + di + dj + dk + dl + dm
ST
da = 0
db - da <= 2
dc - da <= 3
dd - da <= 8
dh - da <= 9
da - db <= 4
dc - db <= 5
de - db <= 7
df - db <= 4
dd - dc <= 10
db - dc <= 5
dg - dc <= 9
di - dc <= 11
df - dc <= 4
da - dd <= 8
dg - dd <= 2
dj - dd <= 5
df - dd <= 1
dh - de <= 5
dc - de <= 4
di - de <= 10
di - df <= 2
dg - df <= 2
dd - dg <= 2
dj - dg <= 8
dk - dg <= 12
di - dh <= 5
dk - dh <= 10
da - di <= 20
dk - di <= 6
```



```

dj - di <= 2
dm - di <= 12
di - dj <= 2
dk - dj <= 4
dl - dj <= 5
dh - dk <= 10
dm - dk <= 10
dm - dl <= 2
END

```

b) If a vertex z is added to the graph for which there is no path from vertex a to vertex z , what will be the result when you attempt to find the lengths of shortest paths as in part a).

The result will be the same, except that dz will be unbounded. This is shown in `lindo_p3b_sol.txt`. It was unnecessary and undesirable to add a constraint for dz , because since there was a path from a to all other vertices, there could be no path from any vertex to z .

Source code:

```

MAX da + db + dc + dd + de + df + dg + dh + di + dj + dk + dl + dm + dz
ST
da = 0
db - da <= 2
dc - da <= 3
dd - da <= 8
dh - da <= 9
da - db <= 4
dc - db <= 5
de - db <= 7
df - db <= 4
dd - dc <= 10
db - dc <= 5
dg - dc <= 9
di - dc <= 11
df - dc <= 4
da - dd <= 8
dg - dd <= 2
dj - dd <= 5
df - dd <= 1
dh - de <= 5
dc - de <= 4
di - de <= 10
di - df <= 2
dg - df <= 2
dd - dg <= 2
dj - dg <= 8
dk - dg <= 12
di - dh <= 5
dk - dh <= 10
da - di <= 20
dk - di <= 6
dj - di <= 2
dm - di <= 12
di - dj <= 2
dk - dj <= 4

```

```
d1 - dj <= 5
dh - dk <= 10
dm - dk <= 10
dm - d1 <= 2
END
```

c) What are the lengths of the shortest paths from each vertex to vertex m. How can you solve this problem with just one linear program?

This problem is essentially a single-source shortest path problem if we simply reverse the direction of each edge but leave its weight the same. `lindo_p3c.ltx` shows how this could be represented as a linear program. In this case, we let $d_m = 0$. The results were as follows, from `lindo_p3c_sol.txt`:

```
da = 17
db = 15
dc = 15
dd = 12
de = 19
df = 11
dg = 14
dh = 14
di = 9
dj = 7
dk = 10
dl = 2
dm = 0
```

d) Suppose that all paths must pass through vertex i. How can you calculate the length of the shortest path from any vertex x to vertex y that pass through vertex i (for all $x, y \in V$)? Calculate the lengths of these paths for the given graph. (Note for some vertices x and y it may be impossible to pass through vertex i).

This problem could be solved by finding the shortest path to vertex i from each vertex, finding the shortest path from vertex i to each vertex, and then adding each pair together. To find the shortest path through vertex i from vertex a to vertex d, for example: add the shortest path length from a to i to the shortest path length from i to d.

The results of finding the shortest path from each vertex to i are in the file `lindo_p3d2_sol.txt`. The results of finding the shortest path to each vertex from i are in the file `lindo_p3d1_sol.txt`.

The Python script `p3d_sol.py` was used to generate the table of sums that can be found in `p3d_vertex_to_vertex_table.pdf`. The table is on the next page.

[illegible]