Name - Shipra Saini

Student ID - 1001662152

Assignment - 8

Task - 1

Let M be the sensor in Maine

S be the sensor in Sahara

T be the temperature daily high more than 80 degree.

Given,

$P(M) = 0.05$

$P(S) = 0.95$

$P(T/M) = 0.2$

$P(T/S) = 0.9$

$P(\neg T/M) = 0.8$

$P(\neg T/S) = 0.1$

(a) $P(M/\neg T) = \dfrac{P(M \cap \neg T)}{P(\neg T)}$

$$= \frac{P(\neg T/M)\, P(M)}{P(\neg T/M) P(M) + P(\neg T/S)\, P(S)}$$

$$= \frac{0.8 \times 0.05}{0.8 \times 0.05 + 0.1 \times 0.95}$$

$$= \frac{0.04}{0.135} = 0.29629$$

So, there is 29.629 % chance.

(b) Let

E_1 be the first email recieved

E_2 be the second email recieved

$$P(E_2/E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)}$$

$$P(E_2 \cap E_1) = P(E_2 \cap E_1/M)\, P(M) + P(E_2 \cap E_1/S)\, P(S)$$

Since, it is given that it is conditionally independent problem

$$P(E_2 \cap E_1) = P(E_2/M)\, P(E_1/M)\, P(M) + P(E_2/S)\, P(E_1/S)\, P(S)$$

$$= 0.8 \times 0.8 \times 0.05 + 0.1 \times 0.1 \times 0.95$$

$$= 0.0415$$

$$P(E_1) = P(E_1 \cap E_2 \cap M) + P(E_1 \cap E_2 \cap S) + P(E_1 \cap 7E_2 \cap M) + P(E_1 \cap 7E_2 \cap S)$$

Since, they are conditionally independent

$$P(E1) = P(E1/M)\, P(E2/M)\, P(M) + P(E1/S)\, P(E2/S)\, P(S) + P(E1/M)\, P(7E2/M)\, P(M) + P(E1/S)\, P(7E2/S)\, P(S)$$

$$= P(E1/M)\, P(M)\left[P(E2/M) + P(7E2/M)\right] + P(E1/S)\, P(S)\left[P(E2/S) + P(7E2/S)\right]$$

Now, $P(A/B) + P(7A/B) = 1$

$\therefore P(E_1) = P(E_1/M)\,P(M) + P(E_1/S)\,P(S)$

$P(E_1) = 0.8 \times 0.05 + 0.1 \times 0.95$

$= 0.135$

$\Rightarrow P(E_2/E_1) = \dfrac{P(E_2 \cap E_1)}{P(E_1)}$

$= \dfrac{0.0415}{0.135}$

$= 0.30740$

(C) Let E_1, E_2, E_3 be 3 emails

$P(E_1 \cap E_2 \cap E_3) = P(E_1 \cap E_2 \cap E_3/M)\,P(S) + P(E_1 \cap E_2 \cap E_3/S)\,P(S)$

They are conditionally independent

$= P(E_1|M)\,P(E_2|M)\,P(E_3|M)\,P(M) + P(E_1|S)\,P(E_2|S)\,P(E_3|S)\,P(S)$

$= 0.8 \times 0.8 \times 0.8 \times 0.05 + 0.1 \times 0.1 \times 0.1 \times 0.95$

$= 0.0256 + 0.00095$

$= 0.02655$

Task-2

Given

11 variables $A, B_1, B_2, \ldots, B_{10}$

Variable A has 5 values

Each Variable $B_1, B_2, B_3, \ldots, B_{10}$ has 7 values

Each B_i is conditionally independent of all other 9 B_j variables (with $j != i$) given A

(a) Joint distribution table of 11 variables

A	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
5	7	7	7	7	7	7	7	7	7	7

$$= 5^1 \times 7^{10} = 1,412,376,245$$

(b) Joint Probability distribution

$$P(A, B_1, B_2, \ldots, B_{10}) = P\left(\frac{B_1}{A}\right), P\left(\frac{B_2}{A}\right), P\left(\frac{B_3}{A}\right), \ldots, P\left(\frac{B_{10}}{A}\right)$$

Each $\left(\frac{B_i}{A}\right)$ needs $= 5 \times (7-1) = 5 \times 6 = 30$ values

Total 10 $\left(\frac{B_i}{A}\right)$ needs $= 30 \times 10 = 300$ values

$P(A)$ needs $= 5-1 = 4$ value

Total we need $= 300 + 4 = 304$ values

Task-3)

(a) $P(A|B=T) = \alpha \Big[\langle P(A=T, B=T, C=T)\ P(A=F, B=T, C=T) \rangle$
$+ \langle P(A=T, B=T, C=F), P(A=F, B=T, C=F) \rangle \Big]$

$= \alpha \big[\langle 0.048, 0.012 \rangle + \langle 0.196, 0.294 \rangle \big]$

$= \alpha \big[\langle 0.244, 0.306 \rangle \big]$

$= \langle 0.444, 0.556 \rangle$

$P(A|B=F) = \alpha \Big[\langle P(A=T, B=F, C=T), P(A=F, B=F, C=T) \rangle$
$+ \Big[\langle P(A=T, B=F, C=F), P(A=F, B=F, C=F) \rangle \Big]$

$= \alpha \big[\langle 0.192, 0.048 \rangle + \langle 0.084, 0.126 \rangle \big]$

$= \alpha \big[\langle 0.276, 0.174 \rangle \big]$

$= \langle 0.613, 0.387 \rangle$

(b)

$$P(A \mid B=T, C=T) = \alpha\left[\langle P(A=T, B=T, C=T), \right.$$
$$\left. P(A=F, B=T, C=T)\rangle\right]$$

$$= \alpha \langle 0.048, 0.012 \rangle$$

$$= \langle 0.8, 0.2 \rangle$$

$$P(A \mid B=T, C=F) = \alpha\left[\langle P(A=T, B=T, C=F\rangle, \right.$$
$$\left. \langle P(A=F, B=T, C=F\rangle\right]$$

$$= \alpha \langle 0.196, 0.294 \rangle$$

$$= \langle 0.4, 0.6 \rangle$$

$$P(A \mid B=F, C=T) = \alpha\left[\langle P(A=T, B=F, C=T), \right.$$
$$\left. P(A=F, B=F, C=T)\rangle\right]$$

$$= \alpha \langle 0.192, 0.048 \rangle$$

$$= \langle 0.8, 0.2 \rangle$$

$$P(A \mid B=F, C=F) = \alpha\left[\langle P(A=T, B=F, C=F), \right.$$
$$\left. P(A=F, B=F, C=F)\rangle\right]$$

$$= \alpha \langle 0.084, 0.126 \rangle$$

$$= \langle 0.4, 0.6 \rangle$$

$$P(A \mid B, C) = \begin{matrix} \langle 0.8, 0.2 \rangle & \langle 0.4, 0.6 \rangle \\ \langle 0.8, 0.2 \rangle & \langle 0.4, 0.6 \rangle \end{matrix}$$

(C)

$$P(A,C \mid B=T) = \alpha\left[\langle P(A=T, C=T, B=T),\ P(A=T, C=F, B=T),\ P(A=F, C=T, B=T),\ P(A=F, C=F, B=T)\rangle\right]$$

$$= \alpha\langle 0.048, 0.196, 0.012, 0.294\rangle$$

$$= \langle 0.087, 0.356, 0.022, 0.534\rangle$$

$$P(A,C \mid B=F) = \alpha\left[\langle P(A=T, C=T, B=F),\ P(A=T), C=F, B=F),\ P(A=F, C=T, B=F),\ P(A=F, C=F, B=F)\rangle\right]$$

$$= \alpha\langle 0.192, 0.084, 0.048, 0.126\rangle$$

$$= \langle 0.427, 0.187, 0.107, 0.28\rangle$$

$$P(A,C \mid B) = \langle 0.087, 0.356, 0.022, 0.534\rangle$$

$$\langle 0.427, 0.187, 0.107, 0.28\rangle$$

(d) Given B, A to be conditionally independent of C we have to prove either

1) $P(A|B,C) = P(A|C)$

2) $P(B|A,C) = P(B|C)$

3) $P(A,B|C) = P(A|C)\,P(B|C)$

As we know,

$$P(A|B,C) = \langle 0.8, 0.2\rangle \quad \langle 0.4, 0.6\rangle$$
$$\langle 0.8, 0.2\rangle \quad \langle 0.4, 0.6\rangle$$

$P(A|C)$:-

$$P(A|C=T) = \alpha[\langle P(A=T, B=T, C=T), P(A=F, B=T, C=T)\rangle + \langle P(A=T, B=F, C=T), P(A=F, B=F, C=T)\rangle]$$

$$= \alpha[\langle 0.048, 0.012\rangle + \langle 0.192, 0.048\rangle]$$

$$= \alpha\langle 0.240, 0.060\rangle$$

$$= \langle 0.8, 0.2\rangle$$

$$P(A|C=F) = \alpha[<P(A=T, B=T, C=F), P(A=F, B=T, C=F)> + <P(A=T, B=F, C=F), P(A=F, B=F, C=F)>]$$

$$= \alpha <0.280, 0.420>$$

$$= <0.4, 0.6>$$

$\therefore$, $P(A|B,C) = P(A|C)$

Hence, we can say that A is conditionally independent of C when B is given