Circuit Impedance Analysis

Complete Study Guide

Input & Output Impedance, Matching, and Measurement Techniques

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Contents

Preface

This comprehensive study guide covers the fundamental concepts of circuit impedance analysis, including input and output impedance characteristics, impedance matching theory, and practical measurement techniques. The material is presented in a systematic manner, progressing from basic concepts to advanced applications.

What You Will Learn

Fundamental Concepts:

- Understanding input and output impedance in electronic circuits
- The critical differences between impedance matching and voltage transfer
- Frequency-dependent behavior of circuit impedances
- Practical implications for circuit design and performance

Practical Skills:

- Measurement techniques for both input and output impedance
- Equipment selection and safety considerations
- Troubleshooting measurement problems
- Interpreting frequency response data

Advanced Topics:

- Transmission line theory and characteristic impedance
- Maximum power transfer theorem and applications
- RF system impedance matching techniques
- High-speed digital circuit considerations

How to Use This Guide

Each chapter follows a consistent structure:

- **TL;DR Section:** Quick summary for rapid review
- **Key Concepts:** Fundamental principles and definitions

- **Detailed Explanation:** Comprehensive coverage with examples
- **Important Notes:** Common mistakes and critical considerations
- **Summary:** Formulas, design examples, and quick reference

Prerequisites

This guide assumes familiarity with:

- Basic circuit analysis (Ohm's law, Kirchhoff's laws)
- AC circuit theory (phasors, reactance, impedance)
- Electronic devices (amplifiers, op-amps, transistors)
- Basic measurement techniques and laboratory equipment

Chapter 1

Input Impedance of Circuits

TL;DR - Quick Summary

Input impedance is the equivalent resistance seen by a signal source when connected to a circuit's input. It combines all resistances, capacitances, and inductances on the input side. **Rule:** Input impedance should be HIGH (at least 10× the source impedance) to prevent signal loss due to voltage divider effect. Low input impedance "overloads" the source, reducing signal strength before amplification. For microphones (mV signals), this is critical!

Key Concepts

What is Input Impedance?

Definition: The equivalent impedance seen looking into the input terminals of a circuit
Physical meaning:

- Represents combined effect of ALL components on input side
- Includes resistors, capacitors, inductors inside the circuit
- Not a physical resistor you can touch - it's a *concept*
- Measured in Ohms (Ω)

Circuit Model:
Signal Source → Input Impedance (appears as resistor to ground) → Circuit
Alternative representation:

- Can show as resistor Z_{in} connected across input terminals
- Even though drawn outside, it represents what's INSIDE the circuit
- Just a convenient way to analyze circuit behavior

Frequency Dependency

Resistive Components:

- Pure resistors: Constant with frequency

- Can use term "input resistance" for resistor-only circuits

Reactive Components:
Capacitive Reactance:

$$X_C = \frac{1}{2\pi f C} \tag{1.1}$$

- Decreases with frequency
- High frequency → Low X_C (capacitor acts like short)
- Low frequency → High X_C (capacitor acts like open)

Inductive Reactance:

$$X_L = 2\pi f L \tag{1.2}$$

- Increases with frequency
- High frequency → High X_L (inductor acts like open)
- Low frequency → Low X_L (inductor acts like short)

Total Input Impedance:

$$Z_{in} = R + jX = R + j(X_L - X_C) \tag{1.3}$$

Where $j = \sqrt{-1}$ (imaginary unit)

Loading Effect

The Problem: Source and input impedance form voltage divider
Circuit Model:
$V_{source} \rightarrow R_{source} \rightarrow Z_{in} \rightarrow$ Circuit Input
Voltage Division:

$$V_{input} = V_{source} \times \frac{Z_{in}}{R_{source} + Z_{in}} \tag{1.4}$$

For maximum signal transfer:

$$Z_{in} >> R_{source} \quad \text{(at least } 10\times\text{)} \tag{1.5}$$

When $Z_{in} >> R_{source}$:

$$V_{input} \approx V_{source} \quad \text{(minimal loss)} \tag{1.6}$$

When $Z_{in} \approx R_{source}$:

$$V_{input} = \frac{V_{source}}{2} \quad \text{(50\% loss!)} \tag{1.7}$$

Design Guidelines

High Input Impedance Applications:

- Amplifier inputs (especially first stage)
- Oscilloscope probes

- Voltmeters
- Buffer circuits
- Instrumentation amplifiers

Typical Input Impedance Values:

- Audio amplifiers: 10k - 1M
- Op-amp inputs: ¿1M (ideally)
- Oscilloscope: 1M —— 10-20pF
- Digital multimeter: ¿10M
- MOSFET gates: ¿10^{12} (essentially infinite)

Detailed Explanation

Amplifier Input Impedance Example

Scenario: Microphone connected to amplifier
Circuit Components:

- Microphone: AC signal source (0.001V to 0.1V typical)
- Source impedance: $R_s = 1k$ (microphone internal resistance)
- Amplifier input impedance: Z_{in} (to be determined)

Goal: Preserve signal strength for amplification
Analysis:
The microphone-amplifier connection forms a voltage divider:

$$V_{amp} = V_{mic} \times \frac{Z_{in}}{R_s + Z_{in}} \tag{1.8}$$

Case 1: High Input Impedance
$Z_{in} = 1M$, $R_s = 1k$, $V_{mic} = 10mV$

$$V_{amp} = 10mV \times \frac{1M}{1k + 1M} = 10mV \times \frac{1000}{1001} = 9.99mV \tag{1.9}$$

Loss: 0.01mV (0.1
Case 2: Moderate Input Impedance
$Z_{in} = 10k$, $R_s = 1k$, $V_{mic} = 10mV$

$$V_{amp} = 10mV \times \frac{10k}{1k + 10k} = 10mV \times \frac{10}{11} = 9.09mV \tag{1.10}$$

Loss: 0.91mV (9.1
Case 3: Low Input Impedance
$Z_{in} = 1k$, $R_s = 1k$, $V_{mic} = 10mV$

$$V_{amp} = 10mV \times \frac{1k}{1k + 1k} = 10mV \times \frac{1}{2} = 5mV \tag{1.11}$$

Loss: 5mV (50

Case 4: Very Low Input Impedance
$Z_{in} = 100$, $R_s = 1k$, $V_{mic} = 10mV$

$$V_{amp} = 10mV \times \frac{100}{1k + 100} = 10mV \times \frac{100}{1100} = 0.91mV \tag{1.12}$$

Loss: 9.09mV (91

Why High Input Impedance Matters

Signal Preservation:
For weak signals (sensors, microphones):

- Original signal may be only microvolts or millivolts
- Any loss reduces signal-to-noise ratio
- Lost signal cannot be recovered by amplification
- Prevention is better than trying to amplify a weakened signal

Mathematical Proof:
For $Z_{in} = n \times R_s$ where n is the ratio:

$$V_{amp} = V_{source} \times \frac{n}{n+1} \tag{1.13}$$

Percentage of signal preserved:

$$\%_{preserved} = \frac{n}{n+1} \times 100\% \tag{1.14}$$

Examples:

- $n = 1$ (equal impedances): 50% preserved
- $n = 10$ (10× rule): 91% preserved
- $n = 100$: 99% preserved
- $n = 1000$: 99.9% preserved

The 10× Rule:
Standard design guideline: $Z_{in} \geq 10 \times R_{source}$
This ensures ¿90% signal preservation, which is acceptable for most applications.

Frequency Effects Example

Circuit: Amplifier with input capacitor

Input impedance includes:

- Input resistor: $R_{in} = 1M$
- Input capacitor: $C_{in} = 10pF$ (parasitic)

At Low Frequencies (f = 1kHz):

Capacitive reactance:

$$X_C = \frac{1}{2\pi \times 1kHz \times 10pF} = 15.9M \tag{1.15}$$

Total impedance (parallel combination):

$$Z_{in} = R_{in}||X_C = \frac{1M \times 15.9M}{1M + 15.9M} = 0.94M \tag{1.16}$$

Input impedance R_{in} (capacitor negligible)

At High Frequencies (f = 100MHz):

Capacitive reactance:

$$X_C = \frac{1}{2\pi \times 100MHz \times 10pF} = 159 \tag{1.17}$$

Total impedance:

$$Z_{in} = \frac{1M \times 159}{1M + 159} 159 \tag{1.18}$$

Input impedance X_C (capacitor dominates!)

Conclusion: At high frequencies, parasitic capacitances reduce input impedance significantly.

Practical Design Considerations

Buffer Amplifier (Voltage Follower):

Purpose: Provide high input impedance interface

Op-Amp Buffer:

- Input impedance: ¿1M (differential), ¿10^{12} (CMOS)
- Output impedance: ¡1
- Gain: 1 (unity)
- Function: Impedance conversion, not amplification

MOSFET Input Stage:

Advantages:

- Gate input impedance: ¿10^{12} (essentially infinite)
- No gate current (except leakage)
- Excellent for high-impedance sources

Disadvantages:

- Sensitive to static electricity
- Gate-source capacitance affects high-frequency response

Source Loading Calculation

Given:

- Source voltage: $V_s = 5V$ (open circuit)
- Source resistance: $R_s = 600$ (audio standard)
- Load (input impedance): Z_{in}

Find: Actual voltage delivered to load
Solution:

$$V_{load} = V_s \times \frac{Z_{in}}{R_s + Z_{in}} \tag{1.19}$$

Power delivered to load:

$$P_{load} = \frac{V_{load}^2}{|Z_{in}|} \tag{1.20}$$

Power lost in source resistance:

$$P_{loss} = I^2 R_s = \left(\frac{V_s}{R_s + Z_{in}}\right)^2 R_s \tag{1.21}$$

Efficiency:

$$\eta = \frac{P_{load}}{P_{load} + P_{loss}} = \frac{|Z_{in}|}{R_s + |Z_{in}|} \tag{1.22}$$

For high efficiency: $Z_{in} >> R_s$

Important Notes & Caveats

- **Input Impedance is NOT a Physical Component:**
  - It's an electrical characteristic, not a resistor you can replace
  - Determined by the circuit design (all internal components)
  - Cannot be "removed" or "bypassed"
  - Changes with frequency due to reactive components
- **The 10× Rule is a Guideline:**
  - For 90% signal preservation (acceptable for most apps)
  - Precision applications may need 100× or 1000×
  - Audio applications often use 10×
  - Test equipment typically uses much higher ratios
- **Frequency Dependency is Critical:**
  - DC analysis only considers resistive components
  - AC analysis must include reactive components
  - Parasitic capacitances dominate at high frequencies
  - Input impedance can vary dramatically with frequency
- **Loading Effects are Cumulative:**
  - Each stage loads the previous stage
  - Multiple loads in parallel reduce effective impedance
  - Probe loading can affect measurements
  - Always consider the complete signal chain
- **Source Impedance Matters:**
  - Must know source impedance to design input impedance
  - Microphone: 150-10k
  - Function generator: 50 (standard)
  - Audio line: 600 (professional) or 10k (consumer)
  - Sensor outputs: Highly variable
- **Common Mistakes:**
  - Ignoring source impedance in calculations
  - Assuming input impedance is constant with frequency
  - Not considering parasitic capacitances
  - Using impedance magnitude when phase matters
  - Forgetting that parallel loads reduce impedance
- **Measurement Considerations:**
  - Measuring input impedance changes the circuit
  - Use appropriate test equipment
  - Consider frequency range of measurement
  - Beware of probe loading during measurement
- **Buffer Amplifiers are Your Friend:**

Summary - Quick Revision

Input Impedance Essentials:

Definition & Concepts:

- Equivalent impedance seen at input
- Combines R, L, C inside circuit
- Not a physical component
- Varies with frequency
- Measured in Ohms ()

Loading Effect:

- Forms voltage divider with source
- Low $Z_{in} \rightarrow$ Signal loss
- High $Z_{in} \rightarrow$ Signal preserved
- $10\times$ rule: $Z_{in} \geq 10R_{source}$

Applications:

- Amplifier inputs
- Measurement equipment
- Buffer circuits
- Sensor interfaces
- Audio equipment

Key Formulas:
Voltage Division:

$$V_{in} = V_s \times \frac{Z_{in}}{R_s + Z_{in}}$$

Signal Preservation:

$$\%_{preserved} = \frac{Z_{in}}{R_s + Z_{in}} \times 100\%$$

Capacitive Reactance:

$$X_C = \frac{1}{2\pi f C}$$

Inductive Reactance:

$$X_L = 2\pi f L$$

Complex Impedance:

$$Z = R + j(X_L - X_C)$$

Design Example - Audio Amplifier Input:
Specification: Interface microphone to amplifier

- Microphone source impedance: 600
- Signal level: 1mV to 10mV
- Requirement: ¡5% signal loss

Design Calculation:
For ¡5% loss: $\frac{Z_{in}}{R_s+Z_{in}} > 0.95$
Solving: $Z_{in} > 19 \times R_s = 19 \times 600 = 11.4k$
Design Choice: Use $Z_{in} = 47k$ (standard value)
Verification:

$$\%_{preserved} = \frac{47k}{600 + 47k} \times 100\% = 98.7\%$$

Loss = 1.3

Quick Reference - Typical Values:

Application	**Source Impedance**	**Required Z_{in}**
Microphone (dynamic)	150-600	¿6k
Microphone (condenser)	1k-5k	¿50k
Audio line (pro)	600	¿6k
Audio line (consumer)	10k	¿100k
Function generator	50	¿500
Sensor (high-Z)	¿1M	Buffer required

Exam Tip: Always identify the source impedance first, then apply the $10\times$ rule as a starting point. Remember that input impedance decreases at high frequencies due to parasitic capacitances, so check your design across the entire frequency range of interest!

Chapter 2

Output Impedance of Circuits

TL;DR - Quick Summary

Output impedance is the equivalent resistance seen looking back into a circuit's output terminals. It represents the combined effect of all resistances, capacitances, and inductances on the output side. **Rule:** Output impedance should be LOW (less than 1/10 of load impedance) to deliver maximum signal to the load. High output impedance causes voltage loss due to voltage divider effect, weakening the signal before it reaches the load (speaker, next circuit, etc.).

Key Concepts

What is Output Impedance?

Definition: The equivalent impedance seen looking back into the output terminals of a circuit
Physical meaning:

- Represents combined effect of ALL components on output side
- Includes resistors, capacitors, inductors inside the circuit
- Acts like internal resistance of a battery or voltage source
- Measured in Ohms (Ω)

Circuit Model:
Circuit → Output Impedance (in series) → Load
Thévenin Equivalent:

- Any circuit can be modeled as: V_{th} in series with Z_{out}
- V_{th} = Thévenin voltage (open-circuit voltage)
- Z_{out} = Thévenin impedance = Output impedance
- Load sees this equivalent circuit

The Voltage Divider Problem

Circuit Model:
$V_{source} \rightarrow Z_{out} \rightarrow Z_{load} \rightarrow$ Ground
Voltage delivered to load:

$$V_{load} = V_{source} \times \frac{Z_{load}}{Z_{out} + Z_{load}} \tag{2.1}$$

For maximum voltage transfer:

$$Z_{out} << Z_{load} \quad \text{(at least } 10\times \text{ smaller)} \tag{2.2}$$

When $Z_{out} << Z_{load}$:

$$V_{load} \approx V_{source} \quad \text{(minimal loss)} \tag{2.3}$$

When $Z_{out} = Z_{load}$:

$$V_{load} = \frac{V_{source}}{2} \quad \text{(50\% loss!)} \tag{2.4}$$

Power Transfer Considerations

Maximum Power Transfer Theorem:
Maximum power is transferred when $Z_{out} = Z_{load}$ (impedance matching)
Power delivered to load:

$$P_{load} = \frac{V_{source}^2 Z_{load}}{(Z_{out} + Z_{load})^2} \tag{2.5}$$

At maximum power transfer ($Z_{out} = Z_{load}$):

$$P_{max} = \frac{V_{source}^2}{4Z_{out}} \quad \text{(but only 50\% efficient)} \tag{2.6}$$

Voltage vs. Power Transfer:

- **Voltage transfer:** Want $Z_{out} << Z_{load}$ (high efficiency)
- **Power transfer:** Want $Z_{out} = Z_{load}$ (maximum power, 50% efficient)
- **Most applications:** Want voltage transfer (amplifiers, digital circuits)
- **Some RF applications:** Want power transfer (antennas, transmission lines)

Low Output Impedance Applications

Amplifier Outputs:

- Must drive various loads (speakers, headphones)
- Load impedance varies widely
- Low output impedance ensures consistent performance

Power Supplies:

- Must maintain voltage regardless of load current

- Low output impedance = good regulation
- High output impedance = voltage drops under load

Buffer Circuits:

- Convert high impedance to low impedance
- Enable driving multiple loads
- Isolate source from load variations

Typical Output Impedance Values:

- Op-amp outputs: ¡1 to 100
- Audio power amplifiers: 0.01 to 1
- Function generators: 50 (standard)
- DC power supplies: ¡0.01 (milliohms)
- Digital logic outputs: 10 to 100

Detailed Explanation

Amplifier-Speaker System Example

Scenario: Audio amplifier driving a loudspeaker
System Components:

- Amplifier output: 10V signal
- Amplifier output impedance: Z_{out} (to be analyzed)
- Speaker impedance: $Z_{speaker} = 8$ (standard)

Goal: Deliver maximum voltage to speaker for best audio quality
Analysis:
The amplifier-speaker connection forms a voltage divider:

$$V_{speaker} = V_{amp} \times \frac{Z_{speaker}}{Z_{out} + Z_{speaker}} \tag{2.7}$$

Case 1: Very Low Output Impedance
$Z_{out} = 0.1$, $Z_{speaker} = 8$, $V_{amp} = 10V$

$$V_{speaker} = 10V \times \frac{8}{0.1 + 8} = 10V \times \frac{8}{8.1} = 9.88V \tag{2.8}$$

Loss: 0.12V (1.2
Case 2: Low Output Impedance
$Z_{out} = 1$, $Z_{speaker} = 8$, $V_{amp} = 10V$

$$V_{speaker} = 10V \times \frac{8}{1 + 8} = 10V \times \frac{8}{9} = 8.89V \tag{2.9}$$

Loss: 1.11V (11.1

Case 3: Moderate Output Impedance

$Z_{out} = 2$, $Z_{speaker} = 8$, $V_{amp} = 10V$

$$V_{speaker} = 10V \times \frac{8}{2+8} = 10V \times \frac{8}{10} = 8V \tag{2.10}$$

Loss: 2V (20

Case 4: High Output Impedance

$Z_{out} = 8$, $Z_{speaker} = 8$, $V_{amp} = 10V$

$$V_{speaker} = 10V \times \frac{8}{8+8} = 10V \times \frac{1}{2} = 5V \tag{2.11}$$

Loss: 5V (50

Load Dependence Problem

Multiple Speaker Loads:

If amplifier drives two 8 speakers in parallel:

$$Z_{load} = \frac{8 \times 8}{8+8} = 4 \tag{2.12}$$

With $Z_{out} = 2$:

$$V_{speakers} = 10V \times \frac{4}{2+4} = 10V \times \frac{2}{3} = 6.67V \tag{2.13}$$

Different load → Different voltage! This is why low output impedance is crucial.

Load Regulation:

Change in output voltage due to load variation:

$$\text{Regulation} = \frac{V_{no-load} - V_{full-load}}{V_{full-load}} \times 100\% \tag{2.14}$$

Good regulation requires $Z_{out} << Z_{load}$

Power Supply Output Impedance

Ideal Power Supply: $Z_{out} = 0$ (perfect voltage source)

Real Power Supply: $Z_{out} > 0$ (causes voltage drop under load)

Example: 12V power supply with $Z_{out} = 0.1$

No load: $V_{out} = 12V$

Load = 1A:

$$V_{out} = 12V - (1A \times 0.1) = 12V - 0.1V = 11.9V \tag{2.15}$$

Load = 10A:

$$V_{out} = 12V - (10A \times 0.1) = 12V - 1V = 11V \tag{2.16}$$

Load regulation = $\frac{12V - 11V}{11V} \times 100\% = 9.1\%$

For good regulation, want ¡1

Frequency Effects on Output Impedance

Capacitive Loading:
Output impedance includes:

- Resistive components: R_{out}
- Output capacitance: C_{out} (parasitic)

At Low Frequencies:

$$Z_{out} R_{out} \quad \text{(capacitor open circuit)} \tag{2.17}$$

At High Frequencies:

$$Z_{out} \frac{1}{2\pi f C_{out}} \quad \text{(capacitor dominates)} \tag{2.18}$$

Result: Output impedance may decrease at high frequencies
Inductive Output Impedance:
For circuits with output inductance:

$$Z_{out} = R_{out} + j2\pi f L_{out} \tag{2.19}$$

Result: Output impedance increases with frequency

Op-Amp Output Impedance Example

Typical Op-Amp:

- Open-loop output impedance: 50-100
- Closed-loop output impedance: Much lower due to feedback

Feedback Effect:
For negative feedback with gain A and loop gain T:

$$Z_{out,closed} = \frac{Z_{out,open}}{1 + T} \tag{2.20}$$

Where $T = A \times \beta$ (loop gain)
Example:

- Open-loop gain: $A = 100,000$
- Feedback factor: $\beta = 0.1$ (gain = 10)
- Loop gain: $T = 10,000$
- Open-loop output impedance: 75

$$Z_{out,closed} = \frac{75}{1 + 10,000} = \frac{75}{10,001} = 0.0075 = 7.5m \tag{2.21}$$

Feedback reduces output impedance by factor of loop gain!

Buffer Amplifier (Voltage Follower)

Purpose: Convert high output impedance to low output impedance
Characteristics:

- High input impedance: ¿1M
- Low output impedance: ¡1
- Unity gain: $A_v = 1$
- No voltage amplification, just impedance conversion

Applications:

- Drive multiple loads from single source
- Isolate sensitive circuits from load variations
- Interface high-impedance sensors to low-impedance inputs
- Prevent loading of signal sources

Implementation:

- Op-amp voltage follower
- Emitter follower (BJT)
- Source follower (FET)

Important Notes & Caveats

- **Output Impedance vs. Maximum Power Transfer:**
  - Low output impedance: Good for voltage transfer (most applications)
  - Matched impedance: Good for power transfer (RF, some audio)
  - Don't confuse these concepts!
  - Most circuits want voltage transfer, not maximum power transfer
- **Load Dependence is the Enemy:**
  - High output impedance makes performance load-dependent
  - Output voltage changes with different loads
  - Multiple loads compound the problem
  - Low output impedance provides consistent performance
- **Frequency Dependency Matters:**
  - Output impedance varies with frequency
  - Parasitic inductances/capacitances affect high-frequency response
  - Feedback reduces output impedance but may be frequency-dependent
  - Always consider frequency range of operation
- **Current Capability is Separate:**
  - Low output impedance doesn't guarantee high current capability
  - Current is limited by power supply and active devices
  - Output impedance affects voltage regulation, not current limit
  - Both low impedance AND adequate current capability are needed
- **Measurement Challenges:**
  - Output impedance changes with signal level (nonlinear devices)
  - AC vs. DC output impedance may differ
  - Loading effects during measurement
  - Frequency-dependent measurements require special techniques
- **Feedback Reduces Output Impedance:**
  - Negative feedback dramatically reduces output impedance
  - Reduction factor equals (1 + loop gain)
  - This is why op-amps have very low output impedance
  - Feedback also improves linearity and reduces distortion
- **Common Mistakes:**
  - Confusing input and output impedance requirements
  - Ignoring frequency effects on impedance
  - Not considering multiple/variable loads
  - Assuming constant output impedance across all conditions
  - Forgetting that impedance magnitude and phase both matter
- **Real-World Considerations:**
  - Cable impedance adds to output impedance

Summary - Quick Revision

Output Impedance Essentials:

Definition & Concepts:

- Equivalent impedance at output terminals
- Combines R, L, C on output side
- Acts like internal series resistance
- Should be LOW for voltage transfer
- Measured in Ohms ()

Design Requirements:

- $Z_{out} < Z_{load}/10$ (voltage transfer)
- $Z_{out} = Z_{load}$ (maximum power transfer)
- Low impedance → Load independence
- High impedance → Load dependence

Applications:

- Audio amplifiers
- Power supplies
- Signal generators
- Buffer circuits
- Op-amp outputs

Key Formulas:
Voltage Division:

$$V_{load} = V_s \times \frac{Z_{load}}{Z_{out} + Z_{load}}$$

Power Delivered:

$$P_{load} = \frac{V_s^2 Z_{load}}{(Z_{out} + Z_{load})^2}$$

Load Regulation:

$$\text{Reg} = \frac{V_{no-load} - V_{load}}{V_{load}} \times 100\%$$

Feedback Effect:

$$Z_{out,FB} = \frac{Z_{out}}{1+T}$$

Design Example - Audio Power Amplifier:
Specification: Drive 8 speaker with ¡5

- Speaker impedance: 8 (nominal)
- Output power: 50W
- Requirement: ¡5

Design Calculation:
For ¡5
Solving: $Z_{out} < \frac{Z_{load}}{19} = \frac{8}{19} = 0.42$
Design Choice: Use $Z_{out} = 0.2$
Verification:

$$\%_{delivered} = \frac{8}{0.2+8} \times 100\% = 97.6\%$$

Loss = 2.4

Voltage Transfer vs. Power Transfer:

Objective	Impedance Ratio	Efficiency	Applications
Voltage Transfer	$Z_{out} << Z_{load}$	¿90%	Amplifiers, Logic
Maximum Power	$Z_{out} = Z_{load}$	50%	RF, Some Audio

Quick Reference - Typical Output Impedances:

Application	Output Impedance	Purpose
Op-amp (FB)	¡1	Drive various loads
Audio power amp	0.01-1	Drive speakers
Function generator	50	Standard test equipment

Chapter 3

Impedance Matching & Transmission Lines

TL;DR - Quick Summary

Impedance matching means making source, transmission line, and load impedances equal for maximum power transfer (not voltage transfer). Key concept: When $Z_{source} = Z_{line} = Z_{load}$, maximum power is delivered and reflections are eliminated. Mismatched impedances cause signal reflections, power loss, and potential source damage. Common in RF systems (50 standard), antennas, and high-speed digital circuits. Remember: Match for power, don't match for voltage!

Key Concepts

Impedance Matching Fundamentals

Definition: Making source, transmission line, and load impedances equal
Purpose:

- Maximize power transfer (not voltage transfer)
- Eliminate signal reflections
- Prevent standing waves on transmission lines
- Protect source from reflected power
- Optimize signal integrity in RF systems

Matching Condition:

$$Z_{source} = Z_{line} = Z_{load} \tag{3.1}$$

Standard Impedances:

- 50: Most common (coax cables, antennas, RF systems)
- 75: Television/video systems
- 100: Differential digital signals (Ethernet)
- 120: Audio balanced lines

Maximum Power Transfer Theorem

Theorem Statement: Maximum power is transferred when load impedance equals source impedance

Mathematical Proof:

For a source with voltage V_s and impedance Z_s driving load Z_L:

Current:

$$I = \frac{V_s}{Z_s + Z_L} \tag{3.2}$$

Power in load:

$$P_L = I^2 \cdot Z_L = \frac{V_s^2 Z_L}{(Z_s + Z_L)^2} \tag{3.3}$$

To find maximum, differentiate and set to zero:

$$\frac{dP_L}{dZ_L} = V_s^2 \frac{(Z_s + Z_L)^2 - 2Z_L(Z_s + Z_L)}{(Z_s + Z_L)^4} = 0 \tag{3.4}$$

Solving:

$$(Z_s + Z_L) - 2Z_L = 0 \quad \Rightarrow \quad Z_L = Z_s \tag{3.5}$$

Maximum power:

$$P_{max} = \frac{V_s^2}{4Z_s} \quad \text{(at 50\% efficiency)} \tag{3.6}$$

Transmission Line Basics

What is a Transmission Line?

- Two or more conductors used to transmit electrical energy
- Has distributed inductance and capacitance
- Exhibits characteristic impedance Z_0
- Examples: Coaxial cable, twisted pair, stripline, microstrip

Characteristic Impedance Z_0:

For coaxial cable:

$$Z_0 = \frac{138}{\sqrt{\epsilon_r}} \log\left(\frac{D}{d}\right) \quad \Omega \tag{3.7}$$

Where:

- D = outer conductor inner diameter
- d = inner conductor diameter
- ϵ_r = dielectric constant

For parallel wire:

$$Z_0 = \frac{276}{\sqrt{\epsilon_r}} \log\left(\frac{2h}{d}\right) \quad \Omega \tag{3.8}$$

Where:

- h = spacing between wire centers
- d = wire diameter

Key Properties:

- Z_0 is independent of line length
- Z_0 depends only on geometry and dielectric
- Real part dominates at RF frequencies (resistive)
- Typical values: 50, 75, 100, 300

Signal Reflections

What Causes Reflections?
Impedance discontinuities:

- Source impedance line impedance
- Line impedance load impedance
- Connectors, bends, stubs
- Changes in dielectric constant

Reflection Coefficient:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{3.9}$$

Special Cases:

- $\Gamma = 0$: No reflection (matched load, $Z_L = Z_0$)
- $\Gamma = +1$: Total positive reflection (open circuit, $Z_L = \infty$)
- $\Gamma = -1$: Total negative reflection (short circuit, $Z_L = 0$)
- $|\Gamma| < 1$: Partial reflection (mismatched load)

Standing Wave Ratio (SWR):

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \tag{3.10}$$

- $SWR = 1$: Perfect match (no reflections)
- $SWR = \infty$: Total mismatch (open or short)
- $SWR < 2$: Good match (¡ 11
- $SWR > 3$: Poor match (¿ 25

Power Considerations

Forward and Reflected Power:

Forward power:

$$P_f = \frac{|V_f|^2}{2Z_0} \tag{3.11}$$

Reflected power:

$$P_r = \frac{|V_r|^2}{2Z_0} = |\Gamma|^2 P_f \tag{3.12}$$

Delivered power:

$$P_L = P_f - P_r = P_f(1 - |\Gamma|^2) \tag{3.13}$$

Efficiency:

$$\eta = \frac{P_L}{P_f} = 1 - |\Gamma|^2 \tag{3.14}$$

Return Loss:

$$RL = -20\log_{10}(|\Gamma|) \quad \text{dB} \tag{3.15}$$

- $RL = \infty$ dB: Perfect match
- $RL = 20$ dB: 1% reflected power ($\Gamma = 0.1$)
- $RL = 10$ dB: 10% reflected power ($\Gamma = 0.316$)
- $RL = 0$ dB: 100% reflected power ($\Gamma = 1$)

Detailed Explanation

RF System Example: Transmitter to Antenna

System Components:

- RF transmitter with 50 output impedance
- 50 coaxial cable (transmission line)
- 50 antenna (load)
- Operating frequency: 1 GHz
- Transmitter power: 100W

Case 1: Perfect Match (50 - 50 - 50)

All impedances equal:

$$Z_{source} = Z_{line} = Z_{load} = 50 \tag{3.16}$$

Reflection coefficient:

$$\Gamma = \frac{50 - 50}{50 + 50} = 0 \tag{3.17}$$

Results:

- No reflections ($\Gamma = 0$)
- SWR = 1 (perfect)

- 100% power delivered to antenna
- No standing waves on transmission line
- Maximum efficiency (50% system efficiency at impedance match)

Case 2: High Impedance Load (50 - 50 - 75)
Antenna impedance higher than system impedance:
Reflection coefficient:

$$\Gamma = \frac{75 - 50}{75 + 50} = \frac{25}{125} = 0.2 \tag{3.18}$$

Standing wave ratio:

$$SWR = \frac{1 + 0.2}{1 - 0.2} = \frac{1.2}{0.8} = 1.5 \tag{3.19}$$

Power analysis:

- Reflected power: $P_r = (0.2)^2 \times 100W = 4W$
- Delivered power: $P_L = 100W - 4W = 96W$
- Efficiency: $\eta = 96\%$
- Return loss: $RL = -20\log(0.2) = 14$ dB

Case 3: Low Impedance Load (50 - 50 - 25)
Antenna impedance lower than system impedance:
Reflection coefficient:

$$\Gamma = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = -0.333 \tag{3.20}$$

Standing wave ratio:

$$SWR = \frac{1 + 0.333}{1 - 0.333} = \frac{1.333}{0.667} = 2.0 \tag{3.21}$$

Power analysis:

- Reflected power: $P_r = (0.333)^2 \times 100W = 11.1W$
- Delivered power: $P_L = 100W - 11.1W = 88.9W$
- Efficiency: $\eta = 88.9\%$
- Return loss: $RL = -20\log(0.333) = 9.5$ dB

Note: Negative Γ indicates phase inversion of reflected signal.
Case 4: Severe Mismatch (50 - 50 - 200)
Significant impedance mismatch:
Reflection coefficient:

$$\Gamma = \frac{200 - 50}{200 + 50} = \frac{150}{250} = 0.6 \tag{3.22}$$

Standing wave ratio:

$$SWR = \frac{1 + 0.6}{1 - 0.6} = \frac{1.6}{0.4} = 4.0 \tag{3.23}$$

Power analysis:

- Reflected power: $P_r = (0.6)^2 \times 100W = 36W$
- Delivered power: $P_L = 100W - 36W = 64W$
- Efficiency: $\eta = 64\%$
- Return loss: $RL = -20\log(0.6) = 4.4$ dB

Dangerous Reflections:

- 36W reflected power returns to transmitter
- Can cause overheating of output transistors
- May trigger protection circuits
- Reduces transmitted power significantly
- Creates hot spots on transmission line

Simulation Analysis: Four Termination Cases

Circuit Setup:

- Source: Square wave generator, 50 impedance
- Transmission line: 50 coaxial cable
- Four different termination resistors
- Measurement: Power dissipated in each load

Case 1: Proper Termination (50)

- Load resistance: 50
- Power dissipated: 83 mW
- Reflection coefficient: $\Gamma = 0$
- Result: All energy absorbed, no reflections
- Efficiency: 100% (maximum possible with matched system)

Case 2: High Termination Resistance

- Load resistance: Much higher than 50
- Power dissipated: 2 mW (much lower)
- Positive reflection coefficient
- Result: Most energy reflected back
- High voltage, low current at load

Case 3: Low Termination Resistance

- Load resistance: Much lower than 50

- Power dissipated: 34 mW
- Negative reflection coefficient
- Result: Significant energy reflection despite low resistance
- Low voltage, high current at load

Paradox Explanation: Why does the low-resistance load (Case 3) dissipate less power than the matched load (Case 1) even though it has lower resistance?

Answer: Impedance mismatch causes reflections. The reflected energy doesn't reach the load, so despite the lower resistance that should draw more current, the actual delivered power is less due to the impedance mismatch.

Case 4: Double Mismatch

- Both source and load mismatched
- Power dissipated: 7 mW (very poor)
- Multiple reflections back and forth
- Result: Energy bounces between mismatches
- Very low efficiency

Practical Matching Techniques

L-Section Matching Network:

To match R_L to Z_0 using inductor and capacitor:

For $R_L > Z_0$:

$$X_L = Z_0\sqrt{\frac{R_L}{Z_0} - 1} \tag{3.24}$$

$$X_C = \frac{R_L Z_0}{X_L} \tag{3.25}$$

For $R_L < Z_0$:

$$X_C = Z_0\sqrt{\frac{Z_0}{R_L} - 1} \tag{3.26}$$

$$X_L = \frac{R_L Z_0}{X_C} \tag{3.27}$$

Quarter-Wave Transformer:

For matching two real impedances:

$$Z_{transformer} = \sqrt{Z_1 \times Z_2} \tag{3.28}$$

Length: $\lambda/4$ at operating frequency

Example: Match 75 to 50 system:

$$Z_{transformer} = \sqrt{75 \times 50} = \sqrt{3750} = 61.2 \tag{3.29}$$

Stub Matching:

- Open or shorted transmission line sections
- Act as reactive components
- Length determines reactance value
- Tunable by adjusting stub length

High-Speed Digital Applications

Why Matching Matters in Digital Circuits:

- Fast rise times create high-frequency content
- PCB traces act as transmission lines
- Reflections cause signal integrity problems
- Overshoot, undershoot, ringing

Critical Length:
For 10% error tolerance:

$$l_{critical} = \frac{t_r}{6} \times v_{prop} \tag{3.30}$$

Where:

- t_r = signal rise time
- v_{prop} = propagation velocity on PCB

Example: 1ns rise time, PCB velocity = 150 mm/ns:

$$l_{critical} = \frac{1ns}{6} \times 150mm/ns = 25mm \tag{3.31}$$

Any trace longer than 25mm needs impedance control!
PCB Impedance Control:

- Microstrip: trace over ground plane
- Stripline: trace between ground planes
- Differential pairs: controlled spacing
- Via impedance: drill size and pad size

Frequency Effects

Frequency-Dependent Matching:
Real components have parasitic elements:

- Resistors: parasitic inductance at high frequency
- Capacitors: parasitic inductance (ESL)
- Inductors: parasitic capacitance

- Transmission lines: frequency-dependent losses

Broadband Matching Challenges:

- Perfect match only possible at one frequency
- Broadband systems require compromise
- Multiple matching networks for wide bandwidth
- Transformer coupling for moderate bandwidth

Skin Effect in Transmission Lines:
At high frequencies, current concentrates near conductor surface:

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \tag{3.32}$$

Where:

- δ = skin depth
- ω = angular frequency
- μ = permeability
- σ = conductivity

Result: Resistance increases with $\sqrt{f}$, affecting matching.

Important Notes & Caveats

- **Impedance Matching vs. Voltage Transfer:**
  - Impedance matching is for POWER transfer (50% efficiency)
  - Voltage transfer wants mismatched impedances (¿90% efficiency)
  - Don't confuse these two different objectives!
  - Most circuits want voltage transfer, not impedance matching
  - Impedance matching mainly for: RF, antennas, transmission lines
- **When to Match, When NOT to Match:**
  - MATCH: RF systems, antennas, high-speed digital, cable driving
  - DON'T MATCH: Audio amplifiers, op-amps, most analog circuits
  - MATCH: When transmission line effects dominate
  - DON'T MATCH: When circuit dimensions ¡¡ wavelength
- **Transmission Line Length Matters:**
  - Short lines (¡$\lambda/10$): behave like lumped components
  - Long lines (¿$\lambda/4$): transmission line effects dominate
  - Critical length depends on rise time and propagation velocity
  - PCB traces can be transmission lines even at moderate frequencies
- **Reflection Dangers:**
  - Reflected power can damage source (especially RF transmitters)
  - Standing waves create voltage/current hot spots on lines
  - Poor signal integrity in digital circuits
  - EMI problems from multiple reflections
  - Protection circuits may trigger and reduce power
- **Frequency Limitations:**
  - Perfect matching only possible at discrete frequencies
  - Broadband matching requires multiple sections or compromise
  - Component parasitic elements affect high-frequency matching
  - Transmission line losses increase with frequency
- **Real-World Complications:**
  - Load impedance may vary with signal level (nonlinear loads)
  - Temperature affects component values and impedance
  - Manufacturing tolerances create impedance variations
  - Aging components can change matching over time
  - Connector impedance discontinuities
- **Measurement Challenges:**
  - Impedance is frequency-dependent (use vector network analyzer)
  - Loading effects of measurement equipment
  - Calibration is critical for accurate measurements
  - Time domain vs. frequency domain measurements

Summary - Quick Revision

Impedance Matching Essentials:

Core Concepts:

- Match for maximum power transfer
- $Z_{source} = Z_{line} = Z_{load}$
- Eliminates reflections
- 50% efficiency at perfect match
- SWR = 1 for perfect match

Applications:

- RF transmitters & antennas
- High-speed digital circuits
- Transmission line systems
- Cable TV distribution
- Test equipment (50 standard)

Reflection Effects:

- Power loss back to source
- Standing waves on lines
- Signal integrity problems
- Potential source damage
- EMI issues

Key Formulas:
Reflection Coefficient:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Standing Wave Ratio:

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Delivered Power:

$$P_L = P_f(1 - |\Gamma|^2)$$

Return Loss:

$$RL = -20\log_{10}(|\Gamma|) \text{ dB}$$

Quarter-Wave Match:

$$Z_t = \sqrt{Z_1 \times Z_2}$$

Design Example - RF Power Amplifier:
Specification: 1 GHz transmitter driving 50 antenna through coax cable

- Transmitter power: 100W
- Cable: 50 coax, 10 meters
- Antenna: 50 (nominal), but measured 45
- Requirement: ¡10% power loss due to reflections

Analysis:
Reflection coefficient with 45 load:

$$\Gamma = \frac{45 - 50}{45 + 50} = \frac{-5}{95} = -0.0526$$

Standing wave ratio:

$$SWR = \frac{1 + 0.0526}{1 - 0.0526} = \frac{1.0526}{0.9474} = 1.11$$

Power loss:

$$P_{reflected} = (0.0526)^2 \times 100W = 0.277W$$

Efficiency = 99.7% Meets ¡10% loss requirement!
Return loss:

$$RL = -20\log(0.0526) = 25.6 \text{ dB}$$

Verdict: Excellent match, minimal reflections, no matching network needed.

Impedance Matching vs. Voltage Transfer Summary:

Objective	Impedance Relation	Efficiency	Applications
Voltage Transfer	$Z_{[illegible]} << Z_{[illegible]}$	¡90%	Amplifiers, Logic

Chapter 4

Measuring Input Impedance

TL;DR - Quick Summary

Input impedance measurement uses voltage divider principle with a variable resistor in series with signal generator. Method: Set variable resistor to zero, note output voltage. Increase resistor until output voltage drops to exactly half. At this point, resistor value equals input impedance of circuit. Cannot use ohmmeter (measures only DC resistance). Need AC signal generator, oscilloscope, and variable resistor for proper AC impedance measurement.

Key Concepts

Why Special Measurement Technique is Needed

Impedance vs. Resistance:

- **Impedance:** AC property, frequency-dependent
- **Resistance:** DC property, frequency-independent
- Ohmmeter only measures DC resistance
- Input impedance includes reactive components (L, C)
- Must use AC signal for impedance measurement

Impedance Components:

$$Z_{in} = R + jX = R + j(\omega L - \frac{1}{\omega C}) \tag{4.1}$$

Where:

- R = resistive component (DC resistance)
- X = reactive component (inductive or capacitive)
- $\omega = 2\pi f$ = angular frequency

Magnitude:

$$|Z_{in}| = \sqrt{R^2 + X^2} \tag{4.2}$$

Phase angle:

$$\phi = \arctan\left(\frac{X}{R}\right) \tag{4.3}$$

Voltage Divider Method (Basic Technique)

Circuit Setup:
Signal Generator → Variable Resistor (R_v) → Circuit Under Test (Z_{in}) → Output
Voltage Divider Equation:

$$V_{in,circuit} = V_{gen} \times \frac{Z_{in}}{R_v + Z_{in}} \tag{4.4}$$

Key Principle: When $R_v = |Z_{in}|$, the input voltage to the circuit becomes half the generator voltage.
Measurement Condition: When $R_v = Z_{in}$:

$$V_{in,circuit} = V_{gen} \times \frac{Z_{in}}{Z_{in} + Z_{in}} = \frac{V_{gen}}{2} \tag{4.5}$$

Method Summary:

1. Set $R_v = 0$, measure output voltage $V_{out,max}$
2. Increase R_v until output voltage $= \frac{V_{out,max}}{2}$
3. At this point: $R_v = |Z_{in}|$
4. Remove and measure R_v with ohmmeter

Equipment Requirements

Essential Equipment:

- **Signal Generator:** Provides AC test signal (usually sine wave)
- **Variable Resistor:** Precision decade box or potentiometer
- **Oscilloscope:** Monitors output voltage changes
- **Ohmmeter:** Measures final resistor value

Signal Generator Requirements:

- Frequency range: Cover frequency of interest
- Low output impedance: ¡10 (to avoid loading effects)
- Amplitude control: Adjust for proper signal levels
- Waveform options: Sine wave preferred for impedance measurement

Variable Resistor Requirements:

- Range: 1 to 100k or higher
- Resolution: Fine adjustment capability

- Accuracy: ±1
- Power rating: Handle test signal power

Oscilloscope Requirements:

- Bandwidth: ¿10× test frequency
- Input impedance: High (¿1M) to avoid loading
- Sensitivity: Measure small voltage changes
- AC coupling: Remove DC offsets

Measurement Procedure - Step by Step

Step 1: Initial Setup

- Connect signal generator to variable resistor
- Connect variable resistor to circuit input
- Connect oscilloscope to circuit output
- Set variable resistor to 0 (short circuit)

Step 2: Signal Configuration

- Set signal generator to 1 kHz sine wave
- Adjust amplitude for clean, distortion-free signal
- Ensure output signal is large and easily readable
- Avoid clipping or saturation

Step 3: Reference Measurement

- With $R_v = 0$, measure output voltage $V_{out,ref}$
- Record amplitude and waveform characteristics
- This is the maximum output voltage condition
- Ensure signal is stable and noise-free

Step 4: Impedance Measurement

- Slowly increase variable resistor value
- Monitor output voltage on oscilloscope
- Stop when output voltage = $\frac{V_{out,ref}}{2}$
- Fine-tune for exact half-amplitude condition

Step 5: Result Recording

- Turn off all equipment
- Remove variable resistor from circuit
- Measure resistor value with precision ohmmeter
- This value equals the magnitude of input impedance

Frequency Dependency

Single-Frequency Measurement:

- Basic method gives impedance at one frequency only
- Input impedance typically varies with frequency
- Multiple measurements needed for frequency response

Frequency Sweep Procedure:

1. Measure impedance at low frequency (100 Hz)
2. Repeat at 1 kHz (standard reference)
3. Measure at higher frequencies (10 kHz, 100 kHz)
4. Plot impedance vs. frequency curve
5. Identify resonant frequencies and trends

Typical Frequency Dependencies:

- **Resistive inputs:** Impedance constant with frequency
- **Capacitive inputs:** Impedance decreases with frequency
- **Inductive inputs:** Impedance increases with frequency
- **Resonant circuits:** Impedance varies dramatically near resonance

Accuracy Considerations

Sources of Error:

- Generator output impedance effects
- Variable resistor tolerance and parasitic elements
- Oscilloscope loading effects
- Frequency-dependent behavior of test resistor
- Cable impedances and stray capacitances

Improving Accuracy:

- Use low-impedance signal generator (¡1)
- Use high-impedance oscilloscope (¿1M)
- Minimize cable lengths
- Use precision decade resistance box
- Calibrate equipment before measurement

Measurement Resolution:
Best achievable resolution:

$$\Delta Z = \frac{\Delta V_{out}}{V_{out}} \times Z_{in} \tag{4.6}$$

For 1% voltage measurement accuracy: $\Delta Z = 0.01 \times Z_{in}$

Detailed Explanation

Complete Audio Amplifier Measurement Example

Circuit Under Test:

- Audio amplifier input stage
- Operating frequency: 1 kHz
- Expected input impedance: 10-50 k range
- Load: 8 loudspeaker

Equipment Setup:

- Signal generator: Function generator, 0-20V output
- Variable resistor: Decade box, 1 to 100k
- Oscilloscope: 100 MHz, 1M input impedance
- Ohmmeter: Digital multimeter, 0.1% accuracy

Detailed Procedure:
Step 1: Initial Connection

- Generator output → Decade box input
- Decade box output → Amplifier input
- Oscilloscope probe → Speaker terminals (amplifier output)
- Decade box set to 0 (all switches off)

Step 2: Signal Setup

- Generator frequency: 1000 Hz
- Generator amplitude: Start with 1V peak-to-peak
- Waveform: Pure sine wave
- Check for clean, undistorted output signal

Step 3: Reference Measurement
With decade box = 0:

- Oscilloscope reading: 4.8V peak-to-peak (example)
- This represents maximum signal transfer
- Record this as $V_{out,max} = 4.8V_{pp}$
- Verify signal is stable and noise-free

Step 4: Impedance Measurement Process
Target voltage: $V_{target} = \frac{4.8V}{2} = 2.4V_{pp}$
Measurement sequence:

- Set decade box to 1k: $V_{out} = 4.5V_{pp}$ (too high)
- Set decade box to 10k: $V_{out} = 3.2V_{pp}$ (still too high)
- Set decade box to 20k: $V_{out} = 2.8V_{pp}$ (getting close)
- Set decade box to 25k: $V_{out} = 2.5V_{pp}$ (very close)
- Set decade box to 27k: $V_{out} = 2.4V_{pp}$ (exact match!)

Step 5: Final Measurement

- Turn off all equipment
- Remove decade box from circuit
- Verify setting: 27k
- Double-check with ohmmeter: 27.1k ±0.1k

Result: Input impedance = 27.1k at 1 kHz

Mathematical Analysis of the Measurement

Voltage Divider Analysis:
With variable resistor R_v and input impedance Z_{in}:

$$V_{in} = V_{gen} \times \frac{Z_{in}}{R_v + Z_{in}} \tag{4.7}$$

Amplifier gain relationship: If amplifier has voltage gain A_v:

$$V_{out} = A_v \times V_{in} = A_v \times V_{gen} \times \frac{Z_{in}}{R_v + Z_{in}} \tag{4.8}$$

At measurement condition ($R_v = Z_{in}$):

$$V_{out} = A_v \times V_{gen} \times \frac{Z_{in}}{2Z_{in}} = \frac{A_v \times V_{gen}}{2} \tag{4.9}$$

This is exactly half the output voltage when $R_v = 0$:

$$V_{out,max} = A_v \times V_{gen} \quad \text{(when } R_v = 0\text{)} \tag{4.10}$$

Verification:

$$\frac{V_{out}}{V_{out,max}} = \frac{\frac{A_v \times V_{gen}}{2}}{A_v \times V_{gen}} = \frac{1}{2} = 0.5 \tag{4.11}$$

This confirms the half-voltage method is mathematically correct.

Frequency Response Measurement

Multi-Frequency Testing:
For the same amplifier, measure at different frequencies:
100 Hz measurement:

- Reference voltage: $4.7V_{pp}$

- Half voltage target: 2.35V$_{pp}$
- Variable resistor at half voltage: 31k
- Result: $Z_{in}(100Hz) = 31k$

1 kHz measurement: (from previous example)

- Result: $Z_{in}(1kHz) = 27k$

10 kHz measurement:

- Reference voltage: 4.2V$_{pp}$
- Half voltage target: 2.1V$_{pp}$
- Variable resistor at half voltage: 18k
- Result: $Z_{in}(10kHz) = 18k$

Analysis of Results:
Input impedance decreases with frequency: 31k → 27k → 18k
This suggests capacitive input (input capacitance in parallel with resistance):

$$Z_{in}(f) = \frac{R_{in}}{1 + j\omega R_{in} C_{in}} = \frac{R_{in}}{\sqrt{1 + (\omega R_{in} C_{in})^2}} \tag{4.12}$$

Estimating Input Capacitance:
From 1 kHz measurement: $R_{in} 27k$
At 10 kHz, impedance magnitude is 18k:

$$18k = \frac{27k}{\sqrt{1 + (2\pi \times 10^4 \times 27 \times 10^3 \times C_{in})^2}} \tag{4.13}$$

Solving for C_{in}:

$$\frac{18k}{27k} = \frac{2}{3} = \frac{1}{\sqrt{1 + (2\pi \times 10^4 \times 27 \times 10^3 \times C_{in})^2}} \tag{4.14}$$

$$\sqrt{1 + (1.7 \times 10^9 \times C_{in})^2} = 1.5 \tag{4.15}$$

$$1 + (1.7 \times 10^9 \times C_{in})^2 = 2.25 \tag{4.16}$$

$$C_{in} = \frac{\sqrt{1.25}}{1.7 \times 10^9} = 656pF \tag{4.17}$$

Verification at 100 Hz:

$$Z_{in}(100Hz) = \frac{27k}{\sqrt{1 + (2\pi \times 100 \times 27 \times 10^3 \times 656 \times 10^{-12})^2}} \tag{4.18}$$

$$Z_{in}(100Hz) = \frac{27k}{\sqrt{1 + (0.111)^2}} = \frac{27k}{1.006} 26.8k \tag{4.19}$$

Close to measured 31k (within measurement accuracy).

Advanced Measurement Techniques

Impedance Bridge Method:
More accurate for precise measurements:

- AC bridge circuit with null detection
- Balances unknown impedance against known standards
- Provides both magnitude and phase information
- Higher accuracy than voltage divider method

Network Analyzer Method:
Professional measurement approach:

- Vector network analyzer (VNA)
- Measures S-parameters directly
- Provides magnitude and phase vs. frequency
- Very high accuracy and wide frequency range
- Automatic calibration and error correction

Current-Voltage Method:
Alternative approach using current measurement:

$$Z_{in} = \frac{V_{applied}}{I_{measured}} \tag{4.20}$$

- Apply known AC voltage across input
- Measure resulting AC current
- Calculate impedance using Ohm's law
- Requires precision AC current meter

Special Considerations for Different Circuit Types

Op-Amp Input Impedance:

- Very high impedance (¿1M typically)
- May require larger variable resistor range
- Bias current effects at very high impedances
- Input capacitance dominates at high frequencies

Transistor Input Impedance:

- Varies with bias point and signal level
- May be nonlinear (use small test signals)

- Temperature dependent

- Frequency dependent due to junction capacitances

Digital Circuit Input Impedance:

- Often specified at logic threshold voltages

- May have different values for high and low states

- Loading due to input protection diodes

- Fast switching may affect measurements

RF Circuit Input Impedance:

- Transmission line effects become important

- May need 50 measurement system

- Standing wave effects on cables

- Calibration plane considerations

Important Notes & Caveats

- **AC vs. DC Measurement Critical Distinction:**
  - Ohmmeter gives DC resistance only (can be misleading)
  - Input impedance is AC property, frequency-dependent
  - Reactive components (L, C) don't affect DC measurements
  - Always use AC signal for impedance measurements
  - DC resistance may be completely different from AC impedance
- **Loading Effects Must Be Minimized:**
  - Signal generator output impedance should be ¡¡ test impedance
  - Oscilloscope input impedance should be ¿¿ test impedance
  - Cable capacitance can affect high-impedance measurements
  - Use shortest practical cable lengths
  - High-impedance probes for ¿10k measurements
- **Signal Level Considerations:**
  - Use small signal levels to avoid nonlinearity
  - Avoid overdriving the circuit under test
  - Ensure output signal is distortion-free
  - Signal should be large enough for accurate measurement
  - Different signal levels may give different impedance values
- **Frequency Dependency is Normal:**
  - Input impedance typically varies with frequency
  - Single-frequency measurement gives limited information
  - Capacitive coupling and parasitic elements affect response
  - RF circuits may have resonant behavior
  - Always specify frequency when reporting impedance
- **Variable Resistor Limitations:**
  - Parasitic inductance and capacitance at high frequencies
  - Temperature coefficient affects accuracy
  - Contact resistance in mechanical switches
  - Self-heating at higher power levels
  - Limited resolution in continuous potentiometers
- **Measurement Accuracy Factors:**
  - Half-voltage determination accuracy is critical
  - Variable resistor tolerance directly affects result
  - Oscilloscope voltage measurement accuracy
  - Frequency stability of signal generator
  - Environmental factors (temperature, humidity)
- **Circuit State Considerations:**
  - Powered vs. unpowered circuit may have different impedance

Summary - Quick Revision

Input Impedance Measurement Essentials:

Basic Principle:

- Voltage divider method
- Variable resistor in series
- Half-voltage condition indicates match
- $R_{variable} = |Z_{input}|$ when $V_{out} = V_{max}/2$
- AC measurement essential

Equipment Needed:

- AC signal generator
- Variable resistor (decade box)
- Oscilloscope or AC voltmeter
- Ohmmeter for final measurement
- Connecting cables

Measurement Steps:

- Connect equipment
- Set R = 0, measure V_{max}
- Increase R until $V = V_{max}/2$
- Remove and measure R
- R value = input impedance magnitude

Key Formulas:
Voltage Divider:

$$V_{in} = V_{gen} \times \frac{Z_{in}}{R_v + Z_{in}}$$

Half-Voltage Condition:

$$R_v = |Z_{in}| \text{ when } V_{out} = \frac{V_{max}}{2}$$

Impedance Magnitude:

$$|Z_{in}| = \sqrt{R^2 + X^2}$$

Capacitive Input:

$$|Z_{in}(f)| = \frac{R}{\sqrt{1 + (\omega RC)^2}}$$

Measurement Error:

$$\Delta Z = \frac{\Delta V}{V} \times Z_{in}$$

Measurement Example - Op-Amp Input Stage:
Circuit: Non-inverting op-amp amplifier with feedback

- Test frequency: 1 kHz
- Signal generator: 1V$_{pp}$ sine wave
- Expected impedance: ¿1M

Procedure:

- $R_v = 0$: Output = 8.2V$_{pp}$ (amplifier gain 8)
- Target: $V_{target} = 4.1V_{pp}$
- Adjust variable resistor until output = 4.1V$_{pp}$
- Final resistor reading: 1.2M

Result: Input impedance = 1.2M at 1 kHz
Verification: Typical for op-amp input stage

Frequency Response Example:
For audio amplifier with capacitive input coupling:

Frequency	Measured $\|Z_{in}\|$	Analysis
100 Hz	31 k	Low frequency limit
1 kHz	27 k	Mid-band response
10 kHz	18 k	Capacitive loading

Chapter 5

Measuring Output Impedance

TL;DR - Quick Summary

Output impedance measurement uses load variation method: Replace normal load with variable resistor. Start with no load (infinite resistance), measure output voltage. Decrease load resistance until output voltage drops to exactly half. At this point, load resistance equals output impedance. Critical: Use adequate power rating on variable resistor and don't run amplifier at full power during test to avoid damage.

Key Concepts

Load Variation Method Principle

Circuit Model:
Amplifier can be modeled as Thévenin equivalent circuit:

- V_{th} = Thévenin voltage (open-circuit voltage)
- Z_{out} = Output impedance (what we want to measure)
- Connected to variable load resistance R_L

Voltage Divider Analysis:

$$V_{load} = V_{th} \times \frac{R_L}{Z_{out} + R_L} \tag{5.1}$$

Key Measurement Points:
No load condition ($R_L = \infty$):

$$V_{no-load} = V_{th} \times \frac{\infty}{Z_{out} + \infty} = V_{th} \tag{5.2}$$

Half-voltage condition ($R_L = Z_{out}$):

$$V_{half} = V_{th} \times \frac{Z_{out}}{Z_{out} + Z_{out}} = \frac{V_{th}}{2} \tag{5.3}$$

Measurement Principle: When load resistance equals output impedance, the output voltage drops to exactly half the no-load voltage.

Equipment and Setup Requirements

Essential Equipment:

- **Variable Load Resistor:** High power rating, wide resistance range
- **AC Voltmeter/Oscilloscope:** Monitor output voltage changes
- **Ohmmeter:** Measure final resistor value
- **Power Rating Calculator:** Ensure safe operation

Variable Load Resistor Requirements:

- **Resistance range:** 0.1 to 100k (depending on expected Z_{out})
- **Power rating:** Must handle $P = \frac{V_{out}^2}{4R_L}$ safely
- **Accuracy:** ±1
- **Heat dissipation:** Adequate thermal management

Power Considerations:
Maximum power in load occurs at impedance match:

$$P_{max} = \frac{V_{th}^2}{4Z_{out}} \tag{5.4}$$

For safety, variable resistor should be rated for at least:

$$P_{rating} 2 \times P_{max} = \frac{V_{th}^2}{2Z_{out}} \tag{5.5}$$

Detailed Measurement Procedure

Step 1: Initial Setup

- Disconnect normal load from amplifier output
- Set variable resistor to maximum resistance (effectively no load)
- Connect oscilloscope to monitor output voltage
- Ensure amplifier is operating at reduced power level

Step 2: No-Load Measurement

- Turn on amplifier with variable resistor at maximum
- Measure and record no-load output voltage $V_{no-load}$
- This represents the Thévenin voltage V_{th}
- Verify signal is clean and stable

Step 3: Load Variation Process

- Calculate target voltage: $V_{target} = \frac{V_{no-load}}{2}$

- Gradually decrease variable resistor value
- Monitor output voltage continuously
- Stop when output voltage equals V_{target}
- Fine-tune for exact half-voltage condition

Step 4: Result Recording

- Turn off amplifier
- Remove variable resistor from circuit
- Measure resistance value with precision ohmmeter
- This value equals the output impedance magnitude

Safety Considerations

Power Dissipation Safety:

- Variable resistor must handle expected power levels
- Use reduced amplifier drive levels during testing
- Monitor resistor temperature during measurement
- Allow cooling time between measurements

Equipment Protection:

- Don't exceed amplifier current ratings
- Avoid short-circuit conditions (minimum load resistance)
- Use current-limiting if available
- Monitor for thermal protection activation

Power Rating Calculation Example:

For amplifier with 10V no-load output and expected 1 output impedance:

$$P_{max} = \frac{(10V)^2}{4 \times 1} = 25W \tag{5.6}$$

Recommended variable resistor rating: ¿50W

Comparison with Input Impedance Measurement

Key Differences:

Aspect	**Input Impedance**	**Output Impedance**
Variable component	Series resistor	Load resistor
Measurement point	Circuit input	Circuit output
Power concerns	Minimal	Significant
Safety issues	Low voltage	High current/power
Reference voltage	Generator voltage	No-load voltage

Circuit Configurations:

Input impedance: Generator → $R_{variable}$ → Circuit → Load **Output impedance:** Generator → Circuit → $R_{variable}$ (replaces load)

Frequency Response Measurements

Multi-Frequency Testing:

- Output impedance typically varies with frequency
- Repeat measurement at different frequencies
- Use appropriate signal generator frequency
- Account for reactive components at higher frequencies

Typical Frequency Dependencies:

- **Audio amplifiers:** Constant impedance in mid-band
- **Op-amp outputs:** May increase at high frequencies
- **RF circuits:** Complex impedance with reactive components
- **Switch-mode supplies:** Frequency-dependent due to filters

Detailed Explanation

Audio Power Amplifier Measurement Example

Circuit Under Test:

- Audio power amplifier, 50W rated output
- Normal load: 8 loudspeaker
- Expected output impedance: ¡1 (good amplifier)
- Test frequency: 1 kHz

Equipment Setup:

- Variable load: High-power rheostat, 0.1 to 100, 100W rating
- Voltmeter: True RMS digital multimeter
- Signal source: Audio generator at reduced level
- Power calculation: Monitor power dissipation

Safety Calculations:
For testing, limit amplifier output to 10W instead of full 50W:

- Test voltage: $V_{test} = \sqrt{10W \times 8} = 8.94V_{RMS}$
- Expected maximum load power: $P_{max} = \frac{(8.94V)^2}{4 \times Z_{out}}$
- If $Z_{out} = 0.5$: $P_{max} = \frac{80}{2} = 40W$
- Variable resistor rating (100W) is adequate

Detailed Measurement Process:
Step 1: No-Load Measurement

- Variable resistor set to maximum (100)
- Amplifier input adjusted for reasonable output level
- No-load voltage measured: $V_{no-load} = 8.94V_{RMS}$
- This represents the Thévenin equivalent voltage

Step 2: Target Calculation

$$V_{target} = \frac{V_{no-load}}{2} = \frac{8.94V}{2} = 4.47V_{RMS} \tag{5.7}$$

Step 3: Load Variation
Measurement sequence:

- Set load to 10: $V_{out} = 8.1V_{RMS}$ (too high)
- Set load to 5: $V_{out} = 7.2V_{RMS}$ (still too high)
- Set load to 2: $V_{out} = 6.1V_{RMS}$ (getting closer)
- Set load to 1: $V_{out} = 5.2V_{RMS}$ (close)
- Set load to 0.8: $V_{out} = 4.9V_{RMS}$ (very close)
- Set load to 0.7: $V_{out} = 4.7V_{RMS}$ (close)
- Set load to 0.6: $V_{out} = 4.47V_{RMS}$ (exact match!)

Step 4: Verification and Results

- Turn off amplifier
- Remove and measure variable resistor: 0.6
- Verify with ohmmeter: 0.61 ± 0.02

Result: Output impedance = 0.61 at 1 kHz
Power Verification: At measurement point (impedance match):

$$P_{dissipated} = \frac{V_{out}^2}{R_{load}} = \frac{(4.47V)^2}{0.6} = 33.3W \tag{5.8}$$

Within 100W rating of variable resistor

Mathematical Analysis and Verification

Thévenin Model Verification:
From measurements:

- $V_{th} = 8.94V$ (no-load voltage)
- $Z_{out} = 0.6$ (from load variation)

Verification at Different Load Values:
At 1 load:

$$V_{predicted} = 8.94V \times \frac{1}{0.6 + 1} = 8.94V \times \frac{1}{1.6} = 5.59V \tag{5.9}$$

Measured: 5.2V (within measurement tolerance)
At 2 load:

$$V_{predicted} = 8.94V \times \frac{2}{0.6 + 2} = 8.94V \times \frac{2}{2.6} = 6.88V \tag{5.10}$$

Measured: 6.1V (reasonable agreement)
Load Regulation Analysis:
Load regulation from no-load to full-load (8):

$$V_8 = 8.94V \times \frac{8}{0.6 + 8} = 8.94V \times \frac{8}{8.6} = 8.32V \tag{5.11}$$

$$\text{Regulation} = \frac{8.94V - 8.32V}{8.32V} \times 100\% = 7.5\% \tag{5.12}$$

Good regulation (¡10

Op-Amp Output Impedance Measurement

Special Considerations for Op-Amps:

- Very low output impedance (typically ¡1)
- Limited output current capability
- Feedback affects output impedance
- May require different measurement approach

Modified Procedure for Low Impedance:
For expected output impedance ¡0.1:

1. Use lower voltage levels (1-2V) to avoid current limiting
2. Use precision low-value resistors (0.01 resolution)
3. Monitor for current limiting or thermal protection
4. May need current measurement method instead

Current Measurement Alternative:
For very low output impedances:

$$Z_{out} = \frac{\Delta V_{out}}{\Delta I_{out}} \tag{5.13}$$

1. Measure no-load voltage and current (0A)
2. Apply known load, measure voltage and current
3. Calculate impedance from voltage change per current change
4. More accurate for very low impedances

Frequency Response Analysis

Multi-Frequency Measurement Example:
For the same audio amplifier, testing at different frequencies:
100 Hz measurement:

- No-load voltage: 8.9V$_{RMS}$
- Half-voltage at load: 0.55
- Result: $Z_{out}(100Hz) = 0.55$

1 kHz measurement: (from previous example)

- Result: $Z_{out}(1kHz) = 0.61$

10 kHz measurement:

- No-load voltage: 8.8V$_{RMS}$
- Half-voltage at load: 0.85
- Result: $Z_{out}(10kHz) = 0.85$

Analysis of Results:
Output impedance increases with frequency: 0.55 $\rightarrow$ 0.61 $\rightarrow$ 0.85
This suggests inductive output impedance components:

$$Z_{out}(f) = R_{out} + j\omega L_{out} \tag{5.14}$$

Estimating Output Inductance:
At 10 kHz: $Z_{out} = 0.85$ At 1 kHz: $Z_{out} = 0.61$
Assuming $R_{out} = 0.55$ (100 Hz value):
At 10 kHz:

$$0.85 = \sqrt{(0.55)^2 + (2\pi \times 10^4 \times L_{out})^2} \tag{5.15}$$

Solving for inductance:

$$(0.85)^2 = (0.55)^2 + (6.28 \times 10^4 \times L_{out})^2 \tag{5.16}$$

$$0.72 = 0.30 + (6.28 \times 10^4 \times L_{out})^2 \tag{5.17}$$

$$L_{out} = \frac{\sqrt{0.42}}{6.28 \times 10^4} = 10.3 \text{ H} \tag{5.18}$$

This inductance could be from output transformer or circuit layout.

Practical Applications and Implications

Audio Amplifier Design Verification:

- Good amplifiers: $Z_{out} < 0.1$ for 8 speakers
- Adequate amplifiers: $Z_{out} < 1$

- Poor amplifiers: $Z_{out} > 2$ (audible performance degradation)

Power Supply Load Regulation:

- Low output impedance indicates good regulation
- High output impedance causes voltage droop under load
- Target: $Z_{out} < 0.01$ for precision supplies

Buffer Circuit Effectiveness:

- Input buffer: High input impedance, low output impedance
- Measure both input and output to verify performance
- Impedance conversion ratio indicates buffering effectiveness

Transmission Line Interface:

- For 50 systems: Want output impedance 50
- Measured impedance should match characteristic impedance
- Deviations indicate mismatch and potential reflections

Important Notes & Caveats

- **Power Safety is Critical:**
  - Variable load resistor must handle full power at impedance match
  - Maximum power occurs when load equals output impedance
  - Use reduced amplifier levels during testing
  - Monitor resistor temperature to prevent damage
  - Calculate power dissipation before starting measurement
- **Current Limiting Can Affect Results:**
  - Many amplifiers have current limiting or thermal protection
  - Limiting circuits can make output impedance appear higher
  - Use signal levels well below protection thresholds
  - Monitor for distortion or protection activation
  - May need multiple measurements at different power levels
- **Load Resistance Range Requirements:**
  - Variable resistor must cover expected output impedance range
  - For audio amps: Need 0.1 to 10 range
  - For op-amps: May need 0.01 to 1 range
  - For RF circuits: 1 to 1k range typical
  - Consider decade resistance boxes for flexibility
- **Measurement Accuracy Factors:**
  - Half-voltage determination accuracy is critical
  - Variable resistor tolerance directly affects result
  - Contact resistance in switched resistors
  - Temperature coefficient of test resistor
  - Voltmeter accuracy and resolution
- **Frequency Dependency Considerations:**
  - Output impedance typically varies with frequency
  - Parasitic inductances increase impedance at high frequencies
  - Output capacitance may decrease impedance at high frequencies
  - Feedback loop characteristics affect frequency response
  - Always specify frequency when reporting output impedance
- **Circuit Operating State Effects:**
  - Output impedance may vary with signal level
  - DC bias conditions can affect measurements
  - Temperature changes during measurement affect results
  - Supply voltage variations affect active circuit impedance
  - Aging components can change impedance over time
- **Alternative Methods for Special Cases:**
  - Very low impedance (¡0.1): Use current variation method

Summary - Quick Revision

Output Impedance Measurement Essentials:

Basic Principle:

- Load variation method
- Replace normal load with variable resistor
- Half no-load voltage indicates impedance match
- $R_{load} = Z_{out}$ when $V_{out} = V_{no-load}/2$
- Power safety is critical

Equipment Needed:

- High-power variable resistor
- AC voltmeter/oscilloscope
- Ohmmeter for final reading
- Power calculation tools
- Thermal monitoring

Measurement Steps:

- Set variable load to maximum
- Measure no-load voltage
- Decrease load until voltage halves
- Remove and measure load value
- Load value = output impedance

Key Formulas:
Thévenin Voltage Divider:

$$V_{load} = V_{th} \times \frac{R_L}{Z_{out} + R_L}$$

Half-Voltage Condition:

$$R_L = Z_{out} \text{ when } V_{load} = \frac{V_{th}}{2}$$

Maximum Load Power:

$$P_{max} = \frac{V_{th}^2}{4Z_{out}}$$

Load Regulation:

$$\text{Reg} = \frac{V_{no-load} - V_{load}}{V_{load}} \times 100\%$$

Current Method (Low Z):

$$Z_{out} = \frac{\Delta V}{\Delta I}$$

Power Rating Example - 50W Audio Amplifier:
Safety Calculation:

- Test at 10W level: $V_{test} = \sqrt{10W \times 8} = 8.94V$
- Expected $Z_{out} = 0.5$
- Maximum test power: $P_{max} = \frac{(8.94V)^2}{4 \times 0.5} = 40W$
- Required resistor rating: ¿50W

Measurement Results:

- No-load voltage: 8.94V
- Half-voltage target: 4.47V
- Load at half-voltage: 0.6
- Result: Output impedance = 0.6

Typical Output Impedance Values:

Circuit Type	Typical Z_{out}	Performance
Good audio amplifier	¡0.1	Excellent damping
Average audio amplifier	0.1-1	Good performance
Poor audio amplifier	¿2	Audible degradation
Op-amp (feedback)	¡1	Very good
DC power supply	¡0.01	Excellent regulation

Appendix A

Quick Reference Guide

A.1 Key Formulas Summary

A.1.1 Impedance Relationships

Complex Impedance:

$$Z = R + jX = |Z|e^{j\phi} \tag{A.1}$$

Impedance Magnitude:

$$|Z| = \sqrt{R^2 + X^2} \tag{A.2}$$

Phase Angle:

$$\phi = \arctan\left(\frac{X}{R}\right) \tag{A.3}$$

A.1.2 Voltage Division

General Voltage Divider:

$$V_{out} = V_{in} \times \frac{Z_2}{Z_1 + Z_2} \tag{A.4}$$

Input Impedance Loading:

$$V_{circuit} = V_{gen} \times \frac{Z_{in}}{Z_{source} + Z_{in}} \tag{A.5}$$

Output Impedance Loading:

$$V_{load} = V_{source} \times \frac{Z_{load}}{Z_{out} + Z_{load}} \tag{A.6}$$

A.1.3 Power Transfer

Maximum Power Transfer:

$$P_{max} = \frac{V_{th}^2}{4R_{th}} \quad \text{when } R_L = R_{th} \tag{A.7}$$

Power Efficiency at Match:

$$\eta_{max} = 50\% \quad \text{at impedance match} \tag{A.8}$$

Delivered Power:

$$P_L = \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2} \tag{A.9}$$

A.1.4 Transmission Lines

Characteristic Impedance (Coax):

$$Z_0 = \frac{138}{\sqrt{\epsilon_r}} \log\left(\frac{D}{d}\right) \tag{A.10}$$

Reflection Coefficient:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{A.11}$$

Standing Wave Ratio:

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \tag{A.12}$$

Return Loss:

$$RL = -20 \log_{10}(|\Gamma|) \text{ dB} \tag{A.13}$$

A.2 Standard Impedance Values

Application	Impedance	Notes
RF Systems	50	Most common, test equipment
Video/Cable TV	75	Broadcast television
Ethernet (Differential)	100	High-speed digital
Professional Audio	600	Balanced audio lines
Telephone Systems	600	Analog telephone
Antenna Feedlines	50, 75, 300	Depends on application

A.3 Design Guidelines

A.3.1 Voltage Transfer (Most Circuits)

Objective: Maximize voltage delivered to load **Rule:** $Z_{source} << Z_{load}$ (at least 10:1 ratio) **Applications:** Amplifiers, logic circuits, most analog circuits

A.3.2 Power Transfer (RF/Specialized)

Objective: Maximize power delivered to load **Rule:** $Z_{source} = Z_{load}$ (impedance matching) **Applications:** RF systems, antennas, transmission lines

A.3.3 Typical Design Values

Input Impedance (Should be HIGH):

- Op-amp inputs: ¿1M
- Audio line inputs: 10k - 50k
- Digital logic inputs: 1k - 10k
- RF inputs: 50 (matched systems)

Output Impedance (Should be LOW):

- Audio amplifiers: ¡0.1 (for 8 speakers)
- Op-amp outputs: ¡1
- Power supplies: ¡0.01
- Digital logic outputs: 10 - 100
- RF outputs: 50 (matched systems)

Appendix B

Measurement Equipment Guide

B.1 Signal Generators

Requirements for Impedance Measurements:

- Frequency range: DC to at least 10× highest test frequency
- Output impedance: ¡10 for voltage transfer measurements
- Amplitude stability: ¡1% over measurement time
- Low distortion: ¡1% THD for accurate measurements
- Calibrated output levels for quantitative measurements

Recommended Features:

- Multiple waveforms (sine, square, triangle)
- Swept frequency capability
- External modulation inputs
- 50 output option for RF measurements
- Digital display with high resolution

B.2 Variable Resistors

Types and Applications:

- **Decade boxes:** High accuracy, stepped values
- **Potentiometers:** Continuous adjustment, lower accuracy
- **Rheostats:** High power capability for output measurements
- **Precision resistors:** Fixed values for calibration

Specifications:

- Resistance range: 0.1 to 10M (depending on application)

- Accuracy: $\pm 0.1\%$ for precision measurements
- Power rating: Calculate based on expected test conditions
- Frequency response: Consider parasitic inductance/capacitance
- Temperature coefficient: ¡100 ppm/°C for stable measurements

B.3 Measurement Instruments

Oscilloscopes:

- Bandwidth: ¿10× highest test frequency
- Input impedance: ¿1M —— ¡20pF
- Sensitivity: Adequate for expected signal levels
- AC coupling capability to remove DC offsets
- Multiple channels for simultaneous measurements

Multimeters:

- True RMS capability for accurate AC measurements
- High input impedance for voltage measurements
- AC frequency response adequate for test frequencies
- 4-wire resistance capability for low impedance measurements
- Data logging for automated measurements

Network Analyzers:

- Vector capability for magnitude and phase measurements
- Wide frequency range for frequency response analysis
- Calibration standards for accurate impedance measurements
- S-parameter capability for RF circuit analysis
- Time domain capability for transmission line analysis

Appendix C

Troubleshooting Guide

C.1 Common Measurement Problems

C.1.1 Input Impedance Measurements

Problem: Cannot achieve half-voltage condition

- **Cause:** Variable resistor range inadequate
- **Solution:** Use decade box with wider range

Problem: Measurement varies significantly with signal level

- **Cause:** Nonlinear circuit behavior
- **Solution:** Reduce signal level, use small-signal analysis

Problem: Results inconsistent between frequencies

- **Cause:** Reactive components in input circuit
- **Solution:** Plot frequency response, analyze trends

C.1.2 Output Impedance Measurements

Problem: Variable resistor overheats during measurement

- **Cause:** Insufficient power rating
- **Solution:** Use higher power resistor, reduce test levels

Problem: Output voltage drops suddenly during measurement

- **Cause:** Current limiting or thermal protection
- **Solution:** Reduce amplifier drive level, allow cooling

Problem: Measured impedance much higher than expected

- **Cause:** Circuit damage or protection activation
- **Solution:** Check circuit operation, verify bias conditions

C.2 Accuracy Improvement Techniques

Minimize Loading Effects:

- Use high-impedance measurement equipment
- Keep connecting cables short
- Use active probes for high-impedance measurements
- Consider measurement equipment input capacitance

Reduce Measurement Errors:

- Calibrate equipment before measurements
- Use precision decade boxes instead of potentiometers
- Make multiple measurements and average results
- Account for temperature effects on components
- Use appropriate measurement bandwidth

Environmental Considerations:

- Maintain stable temperature during measurements
- Shield sensitive measurements from EMI
- Use proper grounding techniques
- Allow equipment warm-up time for stability
- Control humidity for high-impedance measurements

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