Modeling Volatility Clustering in Financial Markets Using a Hybrid Agent-Based Model

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Abstract

We describe the development and calibration of a hybrid Agent-Based Model that can explain the presence of key stylized facts (i.e., volatility clustering and fat tails) found in financial markets compared to models utilizing Geometric Brownian Motion. The model presented in this paper consists of essentially three types of trader agents (fundamentalists, optimists, and pessimists, with the latter two considered noise traders), as well as an ordinary differential equation to model the stock price as a function of the states of the trader agents. The trader agents dynamically switch types over time based on simple (stochastic) rules. We show that the model can reproduce empirical market behaviors while providing a behavioral interpretation of how the stock market itself can cause periods of high volatility, even if the underlying economics of an asset grow at a deterministic and steady pace.

1. Introduction

The ability to accurately capture the stylized facts (fat tails, volatility clustering, etc.) of the returns of assets in a given financial market allows for better institutional decision-making when managing financial risks. While the Black–Scholes–Merton ("BSM") model (Black & Scholes, 1973) is a common way to model the price dynamics of financial derivatives, it is not without limitations. For example, a core assumption of the BSM model is that the price of the underlying asset of a given financial derivative follows Geometric Brownian Motion ("GBM"), which assumes a lognormal distribution. Given that the stylized facts from financial markets possess fat tails and volatility clustering, the BSM model's assumptions do not fit the actual data for financial markets (Alfarano & Lux, 2007).

More recently, applying agent-based models ("ABM") to financial markets has allowed for explaining the stylized facts that are commonly seen in these markets. (Alfarano & Lux, 2007; LeBaron, 2001)

Given the shortcomings of the assumptions underlying GBM and the strengths of agent-based models, we sought to build an agent-based model to replicate the historical behavior of stock prices while giving plausible explanations for the stylized facts that are ubiquitous in financial markets. Using this framework, we sought to understand if we could reproduce the volatility clustering of a given market by simulating the behavior of two different types of agents that are found in virtually every financial market, fundamentalists and noise traders, as well as an opinion dynamic (pessimistic vs. optimistic) for the latter. In any given instance, a market contains a mixture of noise traders and fundamentalists. Generally speaking, a fundamentalist buys and sells based on perceived mispricings in the market (e.g., whether the asset is over or undervalued). In contrast, a noise trader can be defined as a trader driven by herd instincts. It is well known that the composition of noise traders in the market can have adverse impacts, especially when it comes to flash crashes.

This paper is structured as follows: In Section 2, we introduce the model that was developed to understand how the behaviors of noise traders and fundamentalists can affect a given market. In Section 3, we describe our approach for calibrating our ABM to actual data. In Section 4, we describe the simulations that we conducted using our ABM. Finally, in Section 5, we present a discussion, conclusion, and limitations of our work. The working code for our ABM can be found on GitHub.[1]

2. Explanation of Model and Associated System

The system modeled in this paper is the stock market for a single Exchange Traded Fund (ETF), which we take to be the SPDR S&P 500 ETF Trust[2] (symbol, SPY). Prices for this ETF fluctuate over time based on buy and sell orders placed by traders for various reasons. However, the time series of the returns generated by stock prices universally show volatility clustering and fat tail (kurtosis) behavior. We seek to capture these stylized facts by modeling the market

[1]https://github.com/ssie-projects/ssie523-abm-markets/tree/develop/research/MarketABM.ipynb

[2]https://www.google.com/finance/quote/SPY:NYSEARCA

as an agent-based model.

2.1. Basic Model

This basic model used in this paper was originally proposed in (Alfarano & Lux, 2007). The model consists of two types of agents (traders): fundamentalists and noise traders. Fundamentalists buy and sell shares of stock based on perceived mispricing, measured as the difference between the market price and the fundamental price. We can think of the fundamental price as the true worth of the stock. Noise traders, on the other hand, trade based on their opinion about the future. For simplicity, the opinion of each noise trader agent is designated as either optimistic or pessimistic.

Given the number of each type of trader, an ordinary differential equation ("ODE") is used to model the behavior of a broker agent, which executes orders from all traders at a given timestep and settles on an equilibrium price [3]. The market price of the stock is updated based on the following ODE:

$$\frac{dp}{dt} = \beta\left[N_F T_F (p_f - p) + N_C T_C x\right] p, \tag{1}$$

where

$$x = \frac{N_o - N_p}{N_C}, \tag{2}$$

p is the market price of the stock, p_f is the fundamental value of the stock, and β is a parameter that controls the speed of price adjustment per unit time. T_F and T_C are the numbers of shares traded by each fundamentalist and noise trader, respectively, and they are fixed throughout the simulation. N_F is the number of fundamentalist traders, and N_C is the number of noise traders, both of which are also fixed parameters in the basic model. N_o is the number of noise traders who are optimists and N_p is the number of noise traders who are pessimists, where $N_o + N_p = N_C$. Note that N_o and N_p are dynamic and change throughout the simulation (described below). To avoid absorbing states, boundary conditions are also required to ensure that there is at least one of each type of trader at all times (i.e., $N_o > 0$ and $N_p > 0$). The simulation is implemented using Euler Forward Method, with a simulation timestep Δt. The simulation initially starts by randomly designating each noise trader an optimist or pessimist. For each timestep, each optimist has a probability p_{op} of switching to a pessimist and each pessimist has a probability p_{po} of switching to an optimist, where the switches are Bernoulli and are assigned the following probabilities:

$$p_{op} = \nu_1 \Delta t \frac{N_p}{N}, \qquad p_{po} = \nu_1 \Delta t \frac{N_o}{N} \tag{3}$$

We see from these definitions that when the majority of noise traders are pessimists (optimists), the few remaining optimists (pessimists) will have a high probability of switching to pessimists (optimists), while the pessimists (optimists) will have a low probability of switching to optimists (pessimists). In other words, the system is attracted to the states where most traders are either pessimists or optimists, and there is a low probability of switching out of these states. However, since the model is stochastic, the mood of the market does occasionally switch between the two, in what we refer to as a regime shift.

Some intuition about Equation (1) can be gained by rewriting it as follows:

$$\frac{dp}{dt} = p\left[\beta N_F T_F\right](p_f - p) + p\left[\beta N_C T_C\right] x \tag{4}$$

Note that all values in the brackets are constants for the basic model. The first expression on the right is just an exponential with a mean-reverting term added to it. In other words, the model pulls the price back to its fundamental value when it diverges too far. The second expression on the right is also essentially an exponential form but has noise x added to it. Note that, given the definition of x in Equation (2), this noise is always in the interval $x \in (-1, 1)$ and has (stochastic) attractors at -1 and 1, so it can pull the value of the stock above or below the intrinsic value for long periods during the simulation. This emergent herding behavior, along with the recurrent regime switches between optimists and pessimists, is the primary driver of volatility clustering in the model.

2.2. Enhanced Model

The basic model has two major limitations, both of which we addressed in our project. First, the value of p_f is constant over time in the original model. However, given that stock prices tend to grow exponentially and that the intrinsic value drives the market price of a stock over time, we chose to model p_f as an exponential function.

$$\frac{dp_f}{dt} = \mu p_f \tag{5}$$

Second, the number of fundamentalists and noise traders is fixed in the original model. Because of this, the dynamics of the stock price showed unnaturally smooth behavior when

[3] An alternative model for the broker agent is a limit order book, which takes buy and sell orders from each trader and mimics the mechanics of an actual stock exchange (see (Paddrik et al., 2012)). In our preliminary work for this project, we found the stock price dynamics to be much less stable when using a limit order book (data not shown).

the value of x remained close to 1 or -1 for long periods. In other words, since $x = 1$ and $x = -1$ are stochastic attractors, the stock price would very closely follow the trajectory of p_f without much deviation, except during regime shifts between majority optimists and majority pessimists. This assumption is not very realistic, so we incorporated the ability of trader agents to switch between fundamentalist and noise traders based on the following transition probabilities

$$p_{fc} = \nu_2 \Delta t e^{-\alpha\rho}, \qquad p_{cf} = \nu_2 \Delta t \left(1 - e^{-\alpha\rho}\right) \tag{6}$$

where p_{fc} is the probability that a fundamentalist trader switches to a noise trader, and p_{cf} is the probability that a noise trader switches to a fundamentalist trader, α is a parameter, and

$$\rho = \frac{|p_f - p|}{p_f} \tag{7}$$

is the absolute percent deviation from the intrinsic value. Note that the total number of traders, $N_F + N_C = N$, is fixed throughout the simulation.

Intuitively, p_{cf} grows as the price diverges from the fundamental value, so noise traders increasingly switch to fundamental traders to take advantage of the perceived mispricing. On the other hand, p_{fc} is greatest when there is no deviation from the intrinsic value, in which case there is no perceived advantage to being a fundamental trader, so traders increasingly switch to noise traders. On average, we'd like the switching probabilities to be approximately equal so that the asymptotic behavior of the model doesn't tend to all fundamentalists or all noise traders. We can use this reasoning along with Equation (6) to choose a value for α as follows:

$$\begin{aligned} e^{-\alpha\rho} &\approx 1 - e^{-\alpha\rho} \\ 2e^{-\alpha\rho} &\approx 1 \\ -\alpha\rho &\approx \ln\frac{1}{2} \\ \alpha &\approx -\frac{1}{\rho}\ln\frac{1}{2}. \end{aligned} \tag{8}$$

In this case, if we allow for an approximate 10% percent deviation from the fundamental value over time, we would set $\alpha = 6.9$ which is the value we chose for our simulations.

$$\alpha \approx \frac{1}{0.1}\ln\frac{1}{2} \approx 6.9 \tag{9}$$

2.3. Visualization of Model

To illustrate the model's overall dynamics, each model component is plotted over time in Figure 1. The top plot shows the percentage of noise traders through time who are optimists and pessimists. The middle plot shows the number of fundamentalists and noise traders through time. The bottom plot shows both the stock's intrinsic value and the stock price generated by the ABM.

The stochastic attractor behavior of x is evident in the top plot, as at any given time, the vast majority of noise traders are either optimists or pessimists. Note that while these dynamics are independent of the stock price, the stock price relative to the fundamental value is highly dependent on them. This can be seen by comparing the top and bottom plots and noting that the stock price is generally above the fundamental value when the market is optimistic and below the fundamental value when the market is pessimistic.

We also see that, due to Equation (6), and in particular its dependence on Equation (7), the number of fundamental traders tends to grow during prolonged periods of either optimism or pessimism. This can be seen, for instance, in the period to the left of the first vertical line in Figure 1, where the market is overwhelmingly optimistic. We also see similar behavior between the second and third vertical lines. Intuitively, since the stock price is above the fundamental value, more traders will switch to fundamentalists, which tends to pull the price back down towards the fundamental value with more force due to the mean-reverting term in Equation (1).

We also see that the number of fundamentalists drops dramatically when there is a regime shift in market sentiment, as can be seen between the first and second vertical lines, where x breaks away from its attractor and the stock price reverts to the fundamental value. Similar behavior can also be seen between the third and fourth vertical lines. This makes intuitive sense, as there's less incentive for fundamentalists to take advantage of mispricing in the market when the herding behavior of noise traders isn't pushing the stock price away from its fundamental value.

3. Model Calibration

While an ABM is a preferred method to model complex phenomena, the absence of robust methods to calibrate and validate these models is frequently reported in the literature. (Lamperti et al., 2018) In this section, we describe our approach for calibrating and validating our ABM. As noted previously, we sought to recreate the daily log returns for SPY ETF from Jan 1, 2006, to April 23, 2021. Using these data, we computed summary statistics for the SPY data and used these values in our calibration procedure.

We elected to fix certain parameters in our ABM and calibrate others. Specifically, we set $N = 200$, with initial values $N_C = 100$ and $N_F = 100$. We also set $\beta = 0.1$ and $T_F = 10$. Furthermore, we set μ equal to the annualized

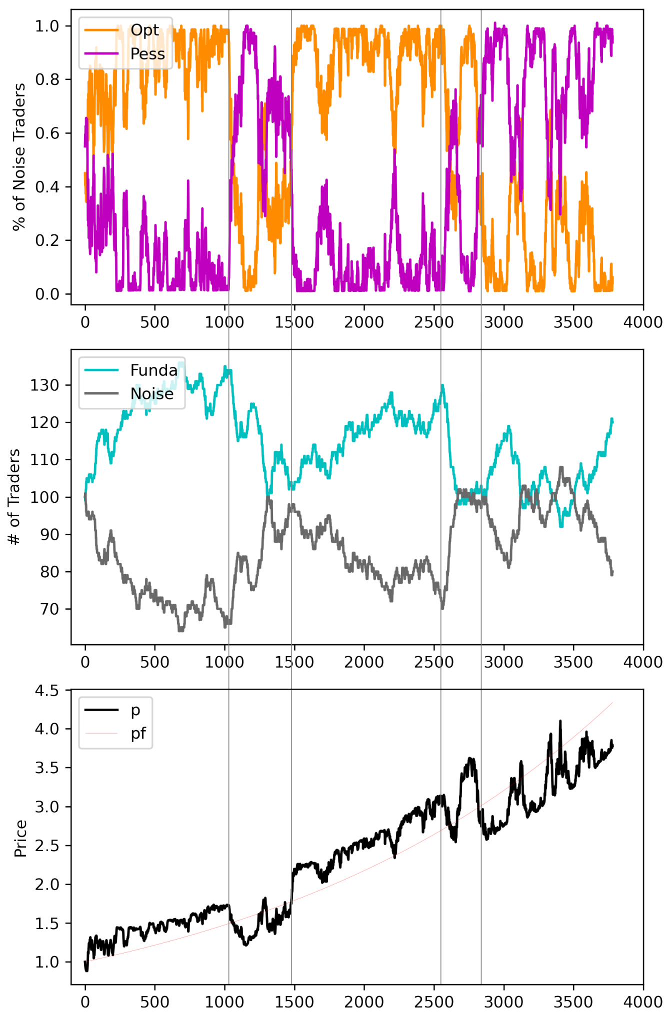

Figure 1. Daily time series dynamics of the Hybrid ABM. Top: optimists vs pessimists, Middle: fundamentalists vs noise traders, Bottom: simulated stock price vs fundamental value.

historical mean of the SPY log returns, since p moves symmetrically around p_f, and therefore p_f drives the long-run return of the stock. Finally, since we were primarily interested in returns, we set the initial values of p and p_f to 1. The vector of simulated prices can easily be converted to any starting price by simply multiplying it by the desired starting price. This choice of parameters put the model in the general ballpark of returns we sought to replicate.

Given the fixed parameter values above, three hyperparameters remained to be calibrated: ν_1, ν_2, and T_C. We created a parameter set of these three hyperparameters and performed a grid search between 729 ordered triplets to find the set of hyperparameters that achieved the lowest weighted mean-square error of the variance, skew, and kurtosis between the data from our model and the ground-truth dataset. Since variance is reported in a smaller unit than skew or kurtosis, we decided to weigh the variance by a factor of 10 so that kurtosis did not dominate the calibration. Upon completion of the grid search, we found that an optimum weighted mean-square error of 28.5% was achieved when $T_C = 5$ (`opt_Tc = 5`), $\nu_1 \Delta t = 0.7$ (`opt_nu1 = 0.7`), and $\nu_2 \Delta t = 0.007$ (`opt_nu2 = 0.007`). These parameters suggest that volatility clustering can be achieved under three conditions. First, when the fundamental traders initiate trades that are twice the size as noise traders (that is, when `opt_Tc = 5` and `opt_Tf = 10`), second, when the probability that an optimist trader switches to pessimist (and vice-versa) is scaled by a factor of `opt_nu1 = 0.7`, and third when the probability that a fundamentalist switches to a noise trader is scaled by a factor by `opt_nu2 = 0.007`.

4. Modeling Simulations

We seeded both our ABM and GBM with the annualized average return from the ground-truth dataset (GBM was also seeded with annualized variance) for the modeling simulations. We also chose $\Delta t = 1/252$, since there are 252 trading days in a year (daily timesteps). Once these values were set, we simulated the daily returns from the ABM and GBM and compared these results to the ground-truth values. The time series of daily returns and histograms from our simulations with the optimized parameter set are shown below in Figure 2. In Figure 2, the charts containing the red data refer to the ground-truth (or historical data), the charts containing green data refer to the GBM model, and the charts containing the blue data refer to the ABM.

Upon qualitative review of Figure 2, the daily returns from our ABM do exhibit a noticeable level of volatility clustering compared to the GBM. That is if you compare the Blue and Green traces in 2, the volatility in the returns clusters together in the Blue trace, but not in the Green trace.

Similarly, the distribution of returns from our ABM tends to be similar to that of the ground-truth data instead of the GBM. Taken together, these observations suggest that our approach for capturing the behaviors of the fundamentalists and noise traders via an ABM produces results that are more consistent with historical returns compared to the GBM baseline.

5. Discussion

The chief contribution of this work allows for an economic explanation of the behavioral origins of the universal characteristics seen in financial markets using a rather simplistic agent based model with very few underlying assumptions.

As shown in Figure 2, our hybrid ABM can recreate the

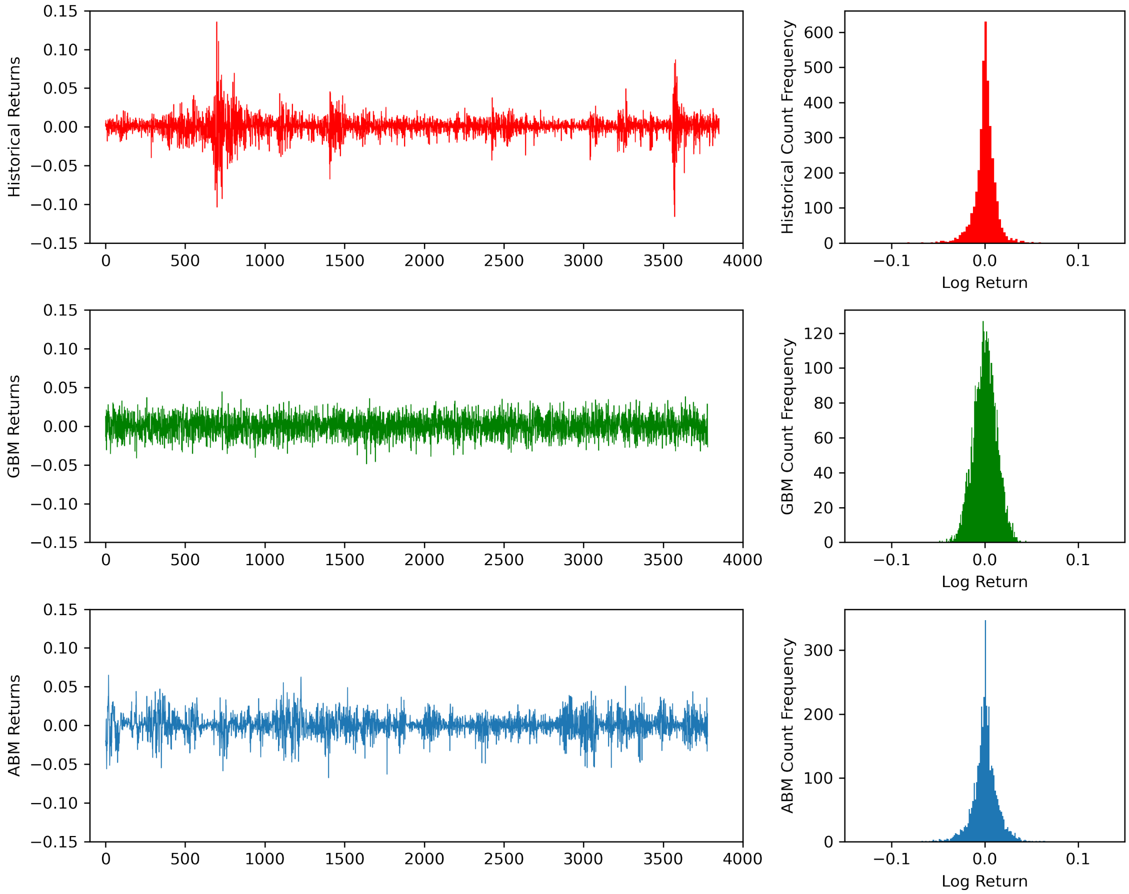

Figure 2. Returns of the historical returns (Red trace), the GBM model (Green trace), and our hybrid ABM (Blue trace).

salient set of stylized facts of financial markets. For example, when comparing the output from our GBM model to that of our ABM (the green and blue traces seen in Figure 2) volatility clustering is seen in the output of the ABM, and the historical returns, *but not* in the returns from the GBM. In addition, when comparing the histogram in Figure 2, the ABM (panel in blue) can produce a more pronounced leptokurtic distribution compared to that of the GBM model (panel in green). Taken together, the output from the ABM can more accurately produce the stylized facts that are seen in financial markets when compared to the GBM.

Finally, there are two areas in which our work can be improved upon. First, work can be undertaken to develop better calibration methodologies for the ABM. Second, it is desirable to conduct a statistical analysis of the results from our model. In particular, it is possible to perform a Bera-Jarque test to assess the normality of the simulated data or even the Box-Ljung test to understand the amount of auto-correlations in the simulated data.

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