Emission tomography

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PhD Winter School 2023
Advanced methods for mathematical image analysis

Bologna, Italy
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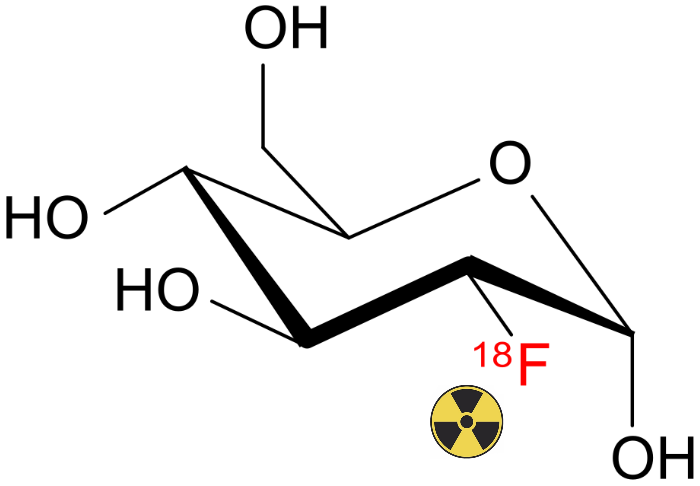
Outline

Medical positron emission tomography (PET)

Nuclear power 101

Passive Gamma Emission Tomography (PGET)

Fluorodeoxyglucose molecule



Fluorine-18 will decay (β^+) into stable oxygen-18 with a half-life of about two hours.

Recall the structure of (positive and negative) radioactive β-decay

In β^- decay, a neutron in an atomic nucleus transforms into a triplet of a proton, an electron and a neutrino:

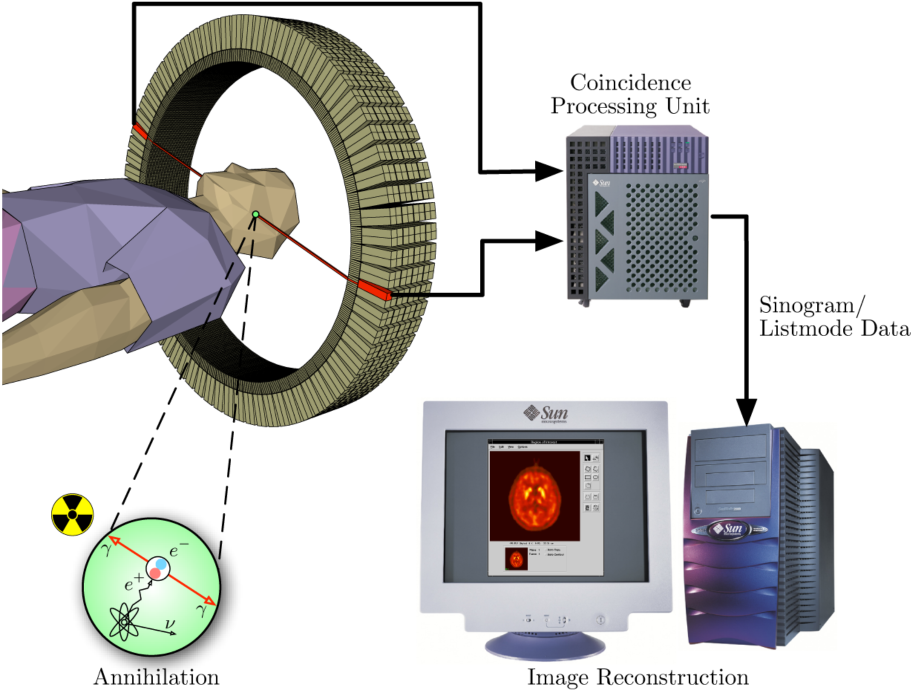
$$n \rightarrow \begin{cases} p^+ \\ e^- \\ \overline{\nu}_e \end{cases}$$

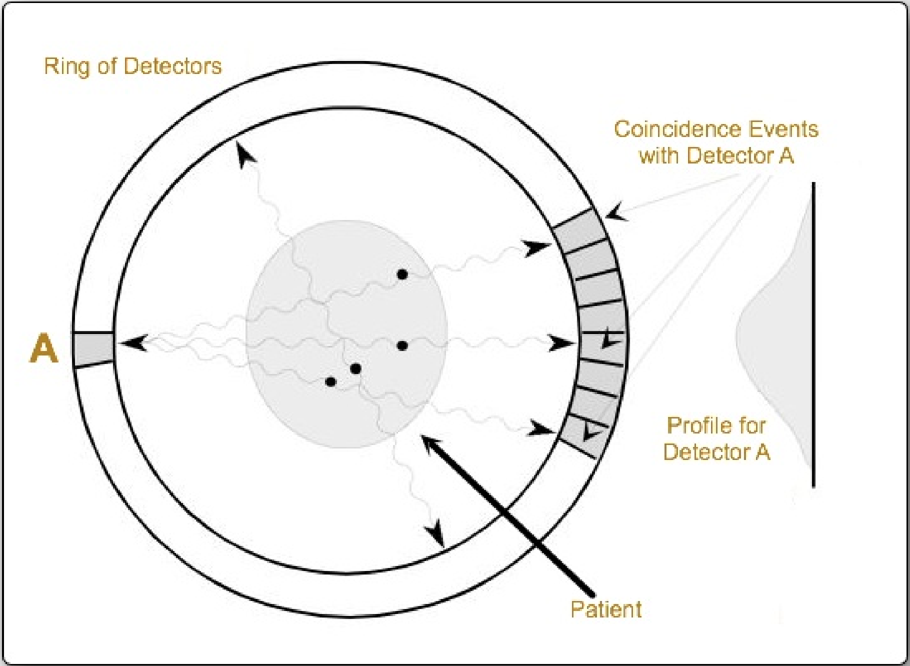
After the event, the nucleus has one more proton than before and so becomes another chemical element. It moves one position up in the periodic table of elements.

In β^+ decay, a proton in an atomic nucleus transforms into a triplet of a neutron, a positron (=anti-electron) and a antineutrino:

$$p^+ \rightarrow \begin{cases} n \\ e^+ \\ \nu_e \end{cases}$$

After the event, the nucleus has one less proton than before and so it moves one position down in the periodic table of elements.

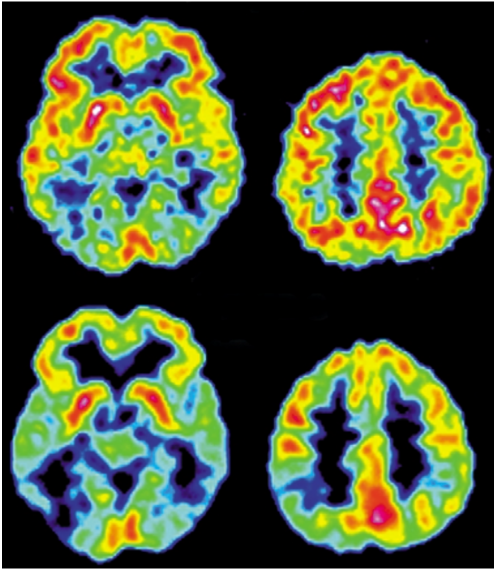

Coincidence
Processing Unit
Sinogram/
Listmode Data
γ
e^-
e^+
γ
ν
Annihilation
Image Reconstruction


Ring of Detectors
Coincidence Events
with Detector A
A
Profile for
Detector A
Patient

PET images show where nutrients are absorbed in the tissue, for example in the brain

Early Alzheimer's

Late Alzheimer's



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1

One uranium nucleus splits =
how many coal atoms burn to CO_2?

A	5 coal atoms burn
B	50 coal atoms burn
C	5000 coal atoms burn
D	50000000 coal atoms burn

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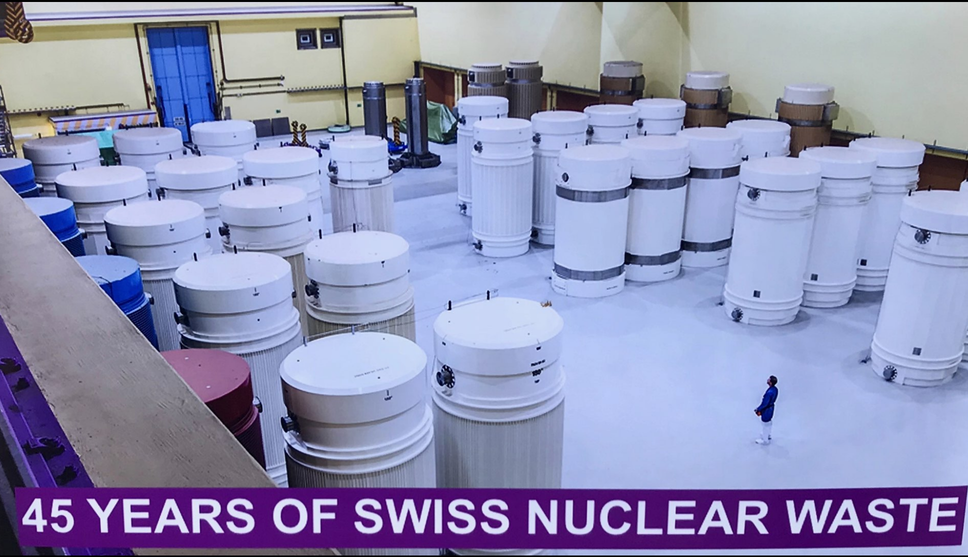
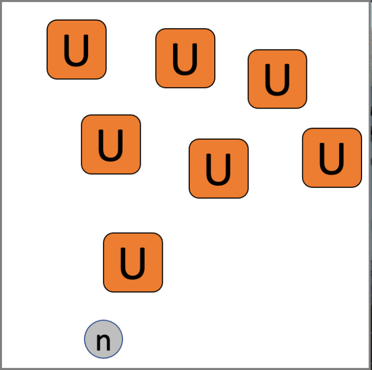

45 YEARS OF SWISS NUCLEAR WASTE

2 Who invented fission chain reaction?



A	Leonardo da Vinci
B	Leo Szilard
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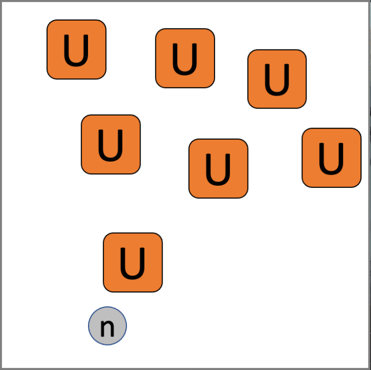

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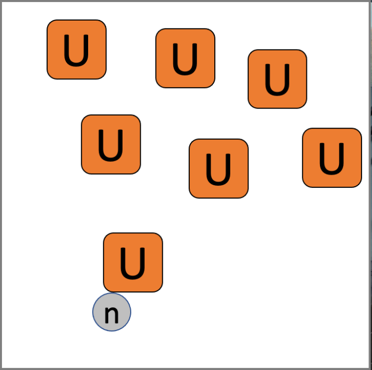

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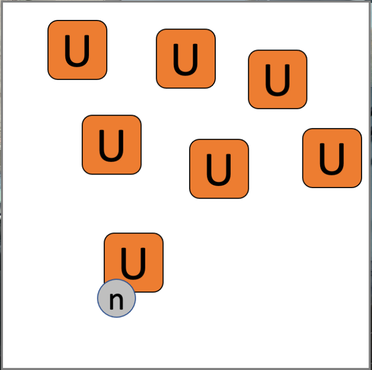

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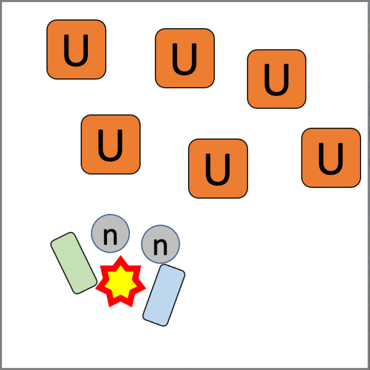

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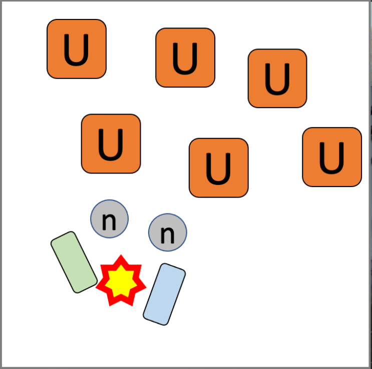

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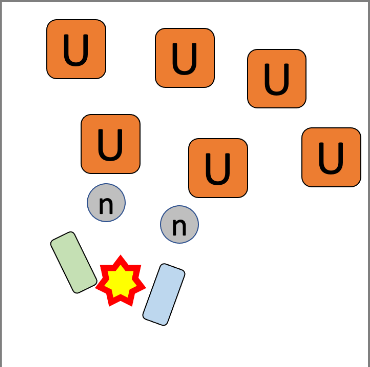

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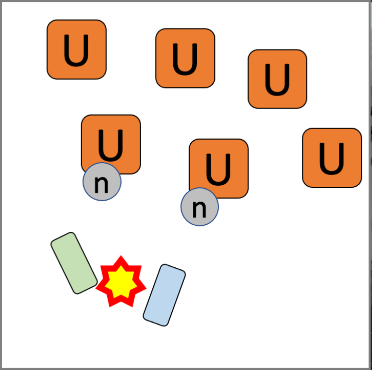

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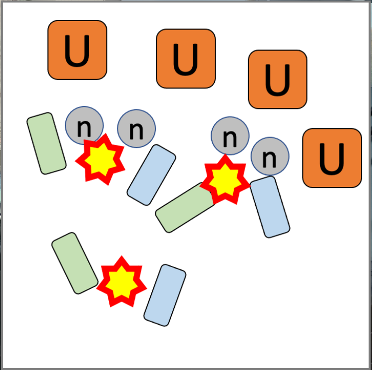

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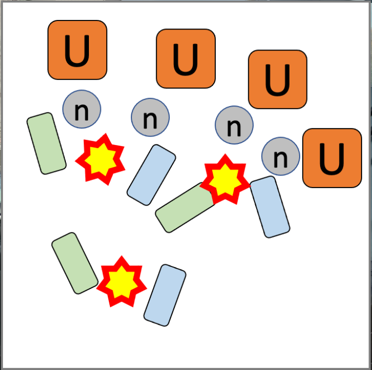

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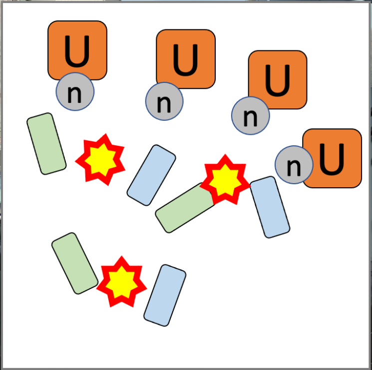

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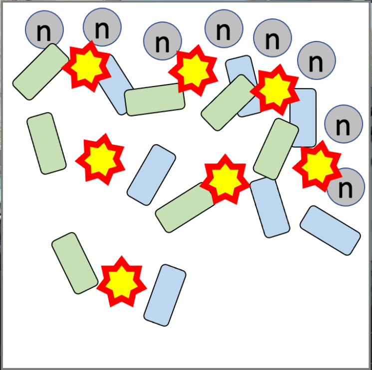

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Wait for it...

2 Who invented fission chain reaction?

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B Leo Szilard 1933

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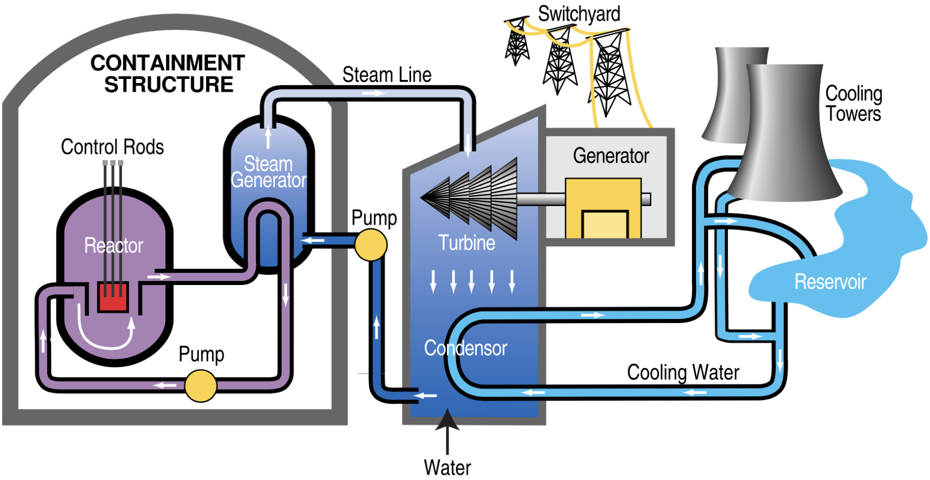
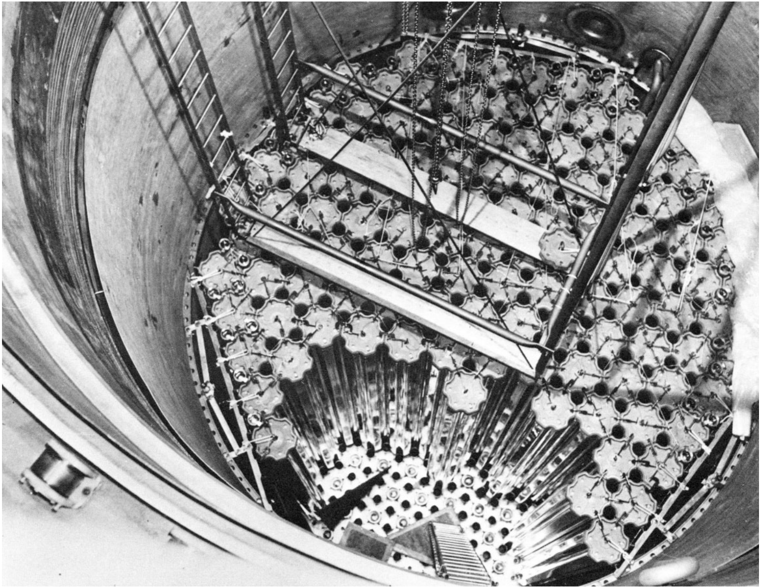
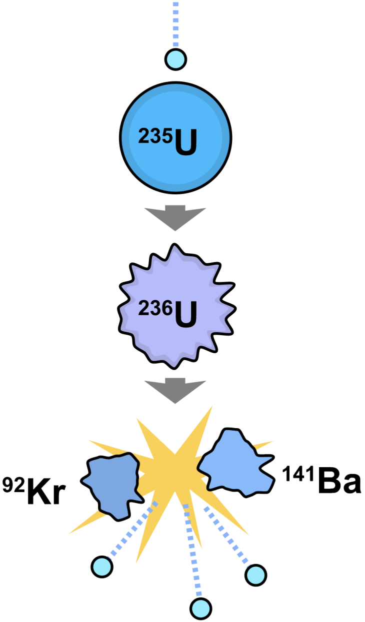
A nuclear power plant is a giant hot water kettle



Fuel is placed inside reactor core



Energy comes from splitting uranium-235 nuclei

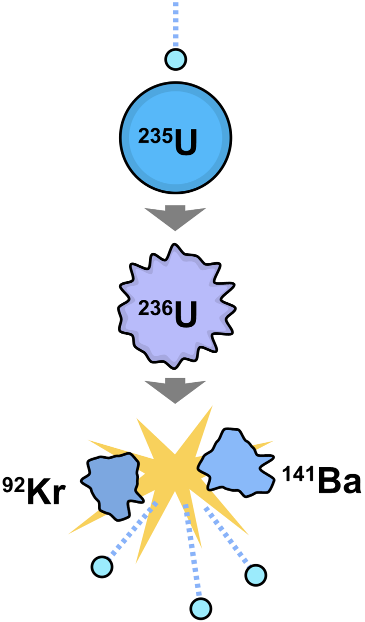


Krypton-92 will decay (β^-) into rubidium-92 with a half-life of 1.84 seconds.

Rubidium-92 will decay (β^-) into strontium-92 with half-life of 4.5 seconds, which in turn becomes yttrium 92 with half-life of about 3 hours.

The next β^- decay has half-life of three and a half hours, producing stable zirkonium-92.

Energy comes from splitting uranium-235 nuclei



Barium-141 will decay (β^-) into lanthanum-141 with a half-life of 18 minutes.

Lanthanum-141 will decay (β^-) into cerium-141 with half-life of about 4 hours. Cerium-141 becomes promethium-141 via β^- decay with a half-life of a month.

Promethium-141 undergoes β^+ decay into stable neudymium-142 with a half-life of 21 minutes.

Carbon emissions of energy production (median)

Method	CO_2-gram/kWh
Coal	820
Gas	490
Biomass	230
Solar	41
Geothermal	38
Hydropower	24
Nuclear	12
Wind	11

Source: IPCC; see page 7 in the document
https://www.ipcc.ch/site/assets/uploads/2018/02/ipcc_wg3_ar5_annex-iii.pdf

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Nuclear power 101

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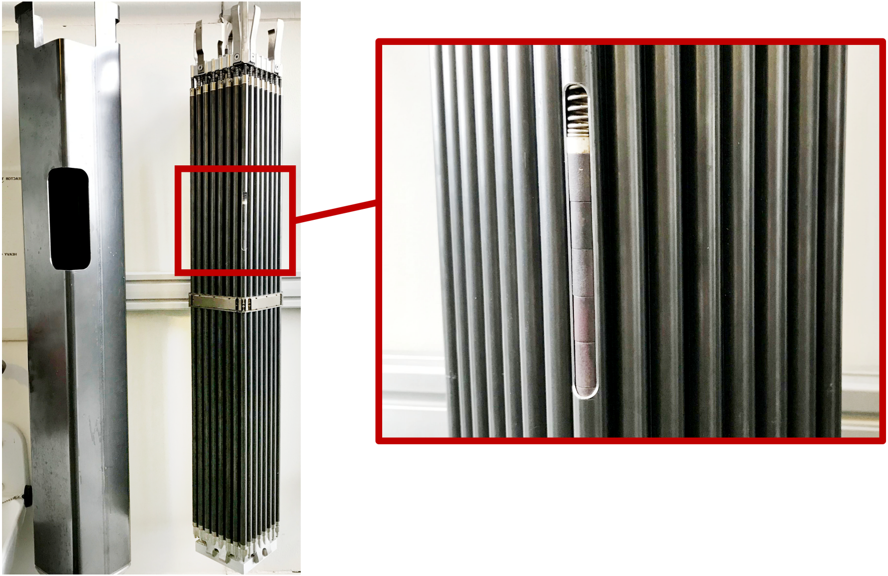
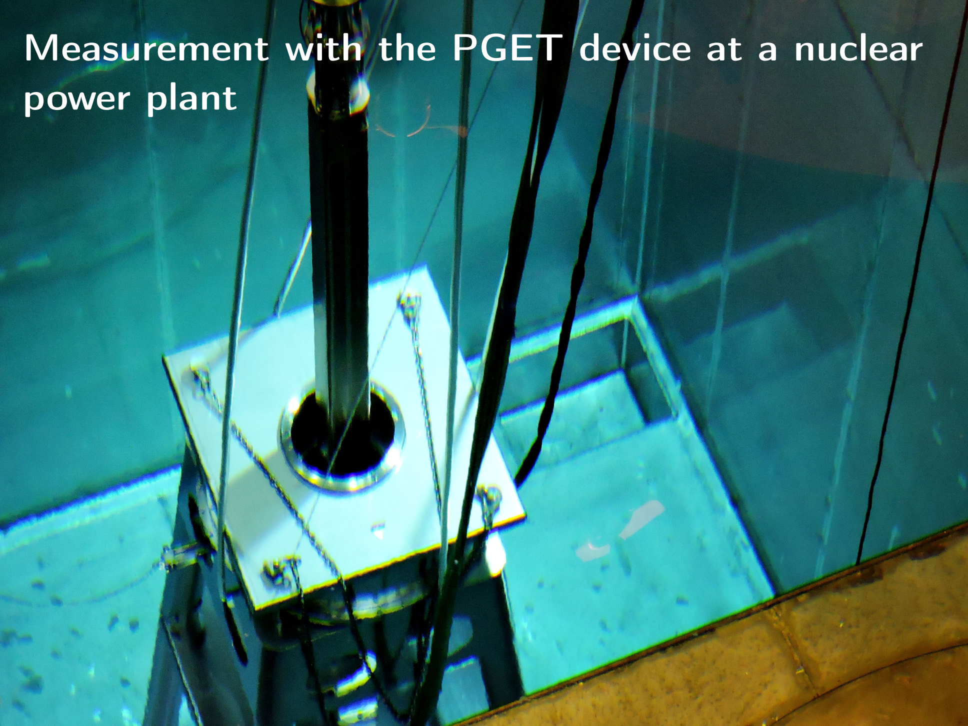
Let's start the PGET story with a video

VIDEO

A nuclear fuel assembly consists of rods filled with pellets containing uranium-235 and uranium-238



Measurement with the PGET device at a nuclear power plant

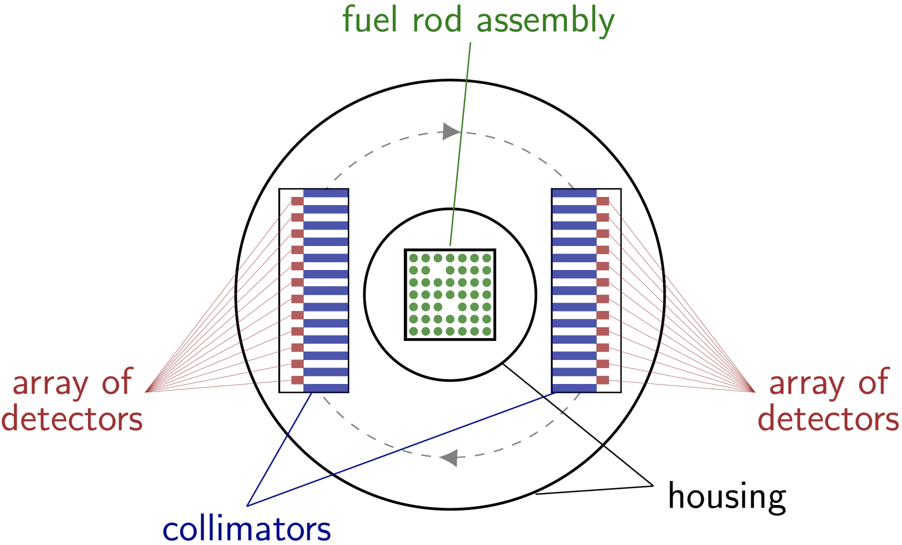


PGET measurement device

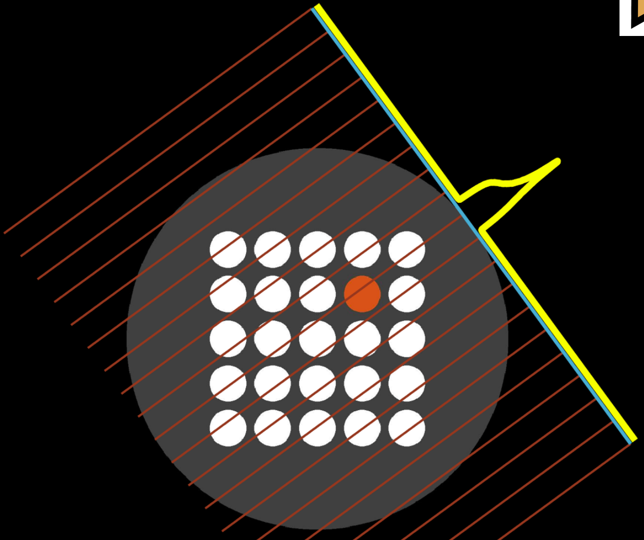
- **P**assive **G**amma **E**mission **T**omography
- Similar idea as in medical SPECT
- PGET strength: ability to image activity of single fuel pins
- IAEA started development in the 80's, approved for inspections in 2017
- Only one device exists at the moment, two more are being built

https://ideas.unite.un.org/iaea-tomography/Page/Home

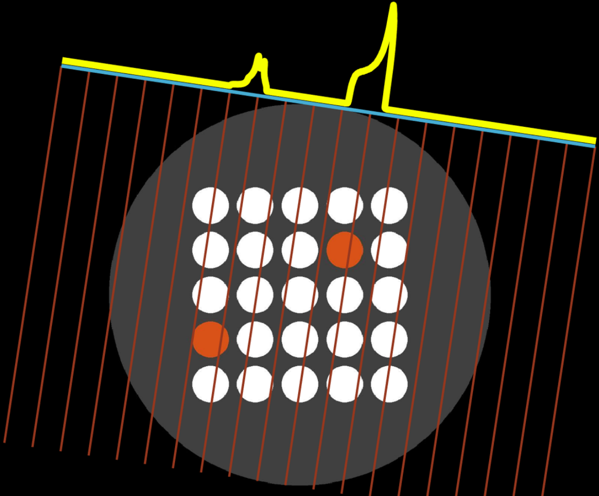
Measurement geometry of the PGET device



Data is collected by rotating the system around the fuel assembly



Data is collected by rotating the system around the fuel assembly



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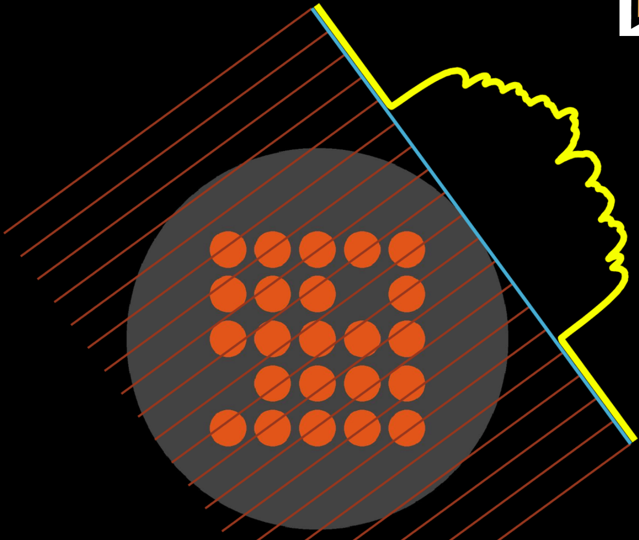
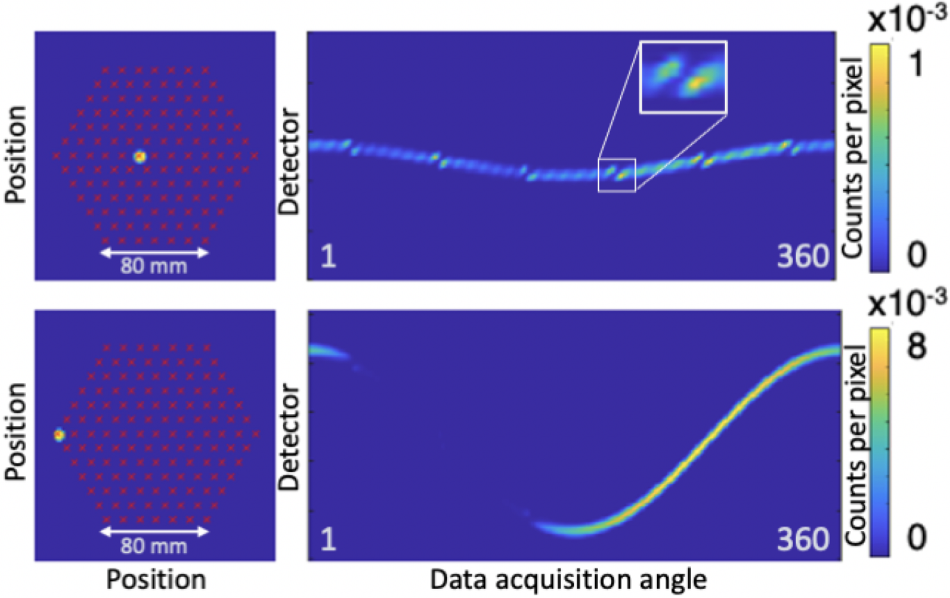
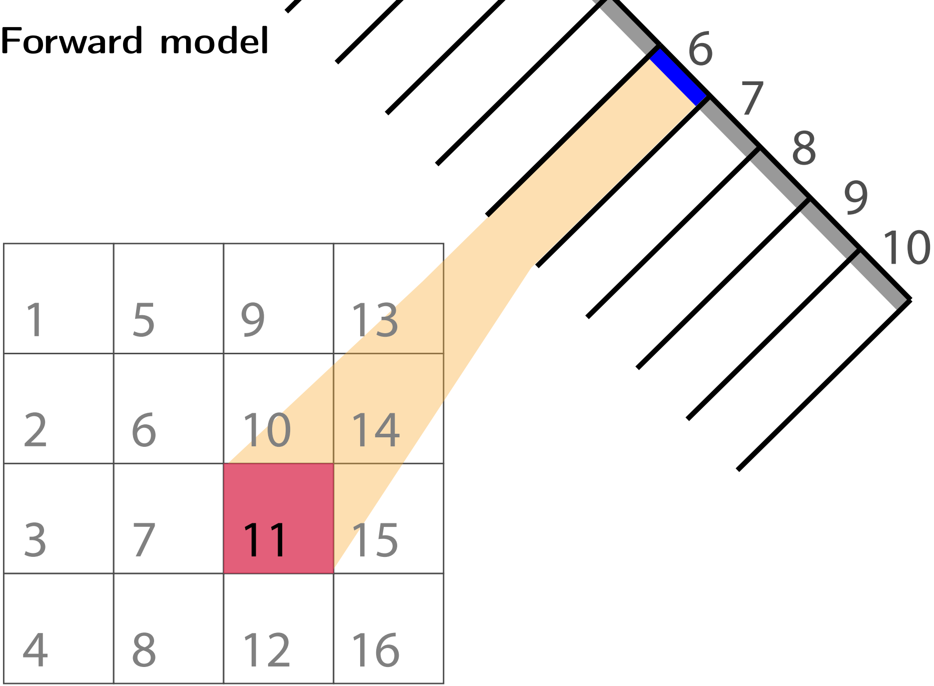


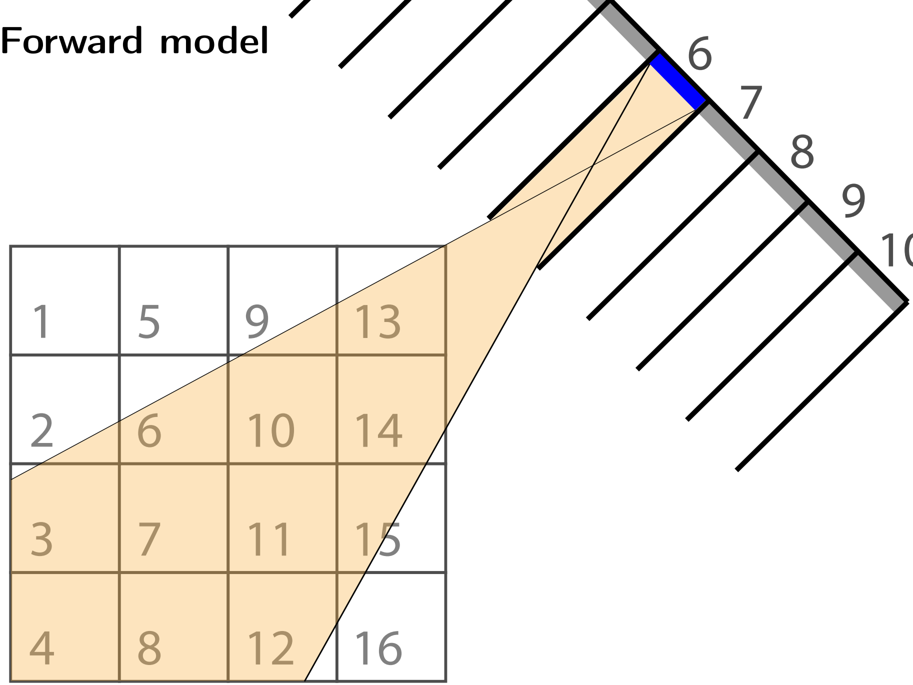
PGET sinograms



Forward model



Forward model



Reconstruction as a minimization problem

Reconstruction images are obtained by solving

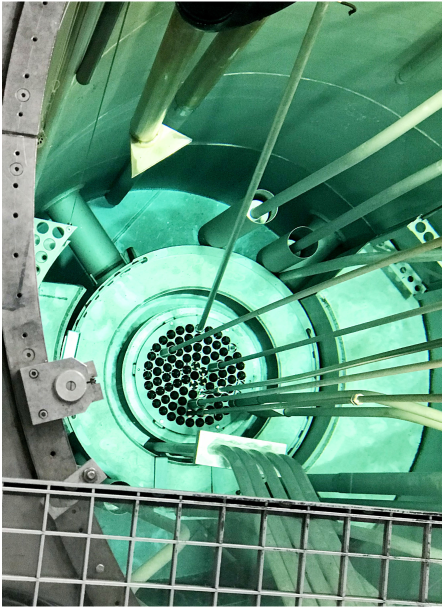
$$\min_{(\lambda,\mu)} \left\{ \| F(\lambda,\mu) - m \|_2^2 + \sum_i \alpha_i P_i(\lambda,\mu) \right\}$$

- λ is the emission image, μ is the attenuation image.
- Data fit term $\| F(\lambda,\mu) - m \|_2^2$ measures how well the forward projection $F(\lambda,\mu)$ matches the measurement m.
- The regularization terms $\sum_i \alpha_i P_i(\lambda,\mu)$ incorporate prior knowledge into the reconstruction process.

How We Won Silver in IAEA PGET Challenge

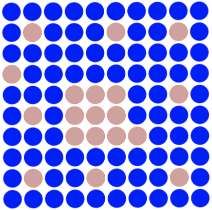
These are the mock-up fuel assemblies

The cobolt “fuel rods” were activated in a reactor

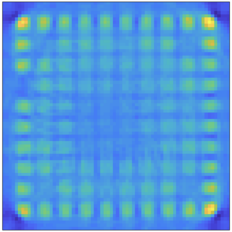


Reconstruction by Filtered Back-Projection

Ground truth,
present / missing



Activity
reconstruction

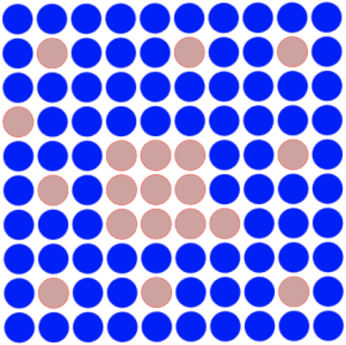


Attenuation
reconstruction

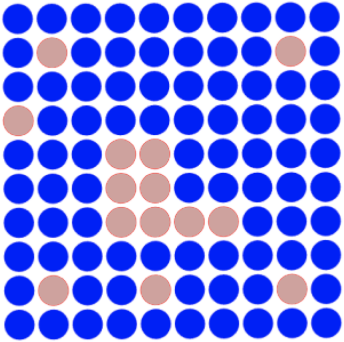
Not applicable

Classification by Filtered Back-Projection

Ground truth



Classification



Reconstruction as a minimization problem 1/2

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Reconstruction as a minimization problem 2/2

We write the Tikhonov regularization task as nonlinear least squares problem in stacked form

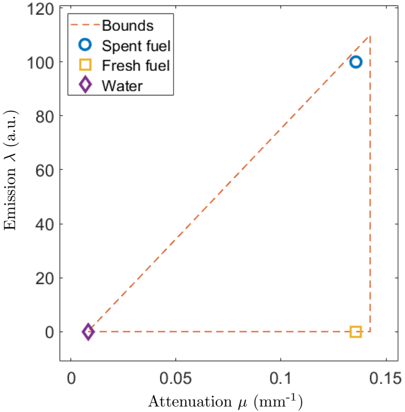
$$\arg\min_{\lambda,\mu} \left\| \begin{array}{c} F(\lambda,\mu) - m \\ \sqrt{\alpha_\lambda} M_\lambda \lambda \\ \sqrt{\alpha_\mu} M_\mu \mu \end{array} \right\|_2^2,$$

where the matrices M_λ and M_μ incorporate the *a priori* information into the reconstruction process. We solve the problem using the Levenberg-Marquardt method.

One piece of *a priori* information we put into the reconstruction is the physicality of materials

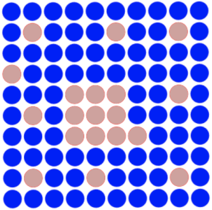
Need to set bounds for the emission and attenuation values in the minimization problem to produce reasonable images.

- Excludes the possibility of a material with high emission but low attenuation value.
- Some way of estimating these bounds before the minimization is needed.

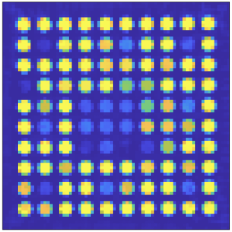


Reconstruction by geometry-aware prior

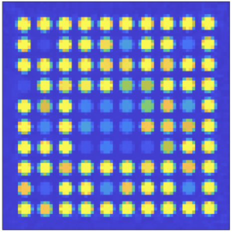
Ground truth,
present / missing



Activity
reconstruction

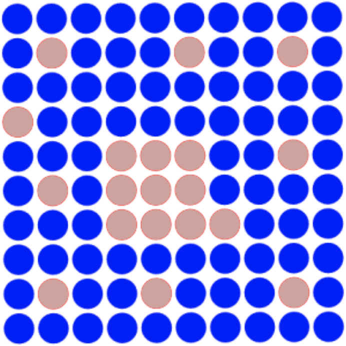


Attenuation
reconstruction



Classification by geometry-aware prior

Ground truth



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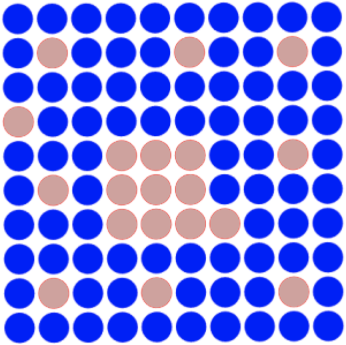
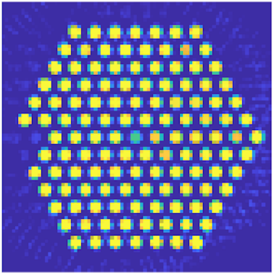


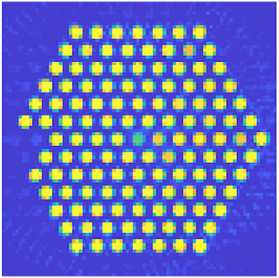
[Backholm, Bubba, Bélanger-Champagne, Helin, Dendooven & S 2020]

Many fuel rods are currently imaged in Finnish nuclear power plants. Here VVER-440 assembly

Emission



Attenuation



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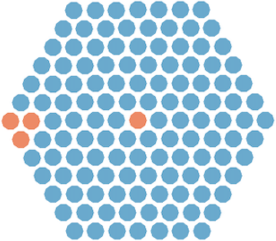

Calculations and images:
DI Riina Virta


Thank you for your attention!
⟵Slime mold called Lycogala conicum


Thank you for your attention!
Eiran sairaala 0443
LAURA