Remote controlled phantom for dynamic tomography

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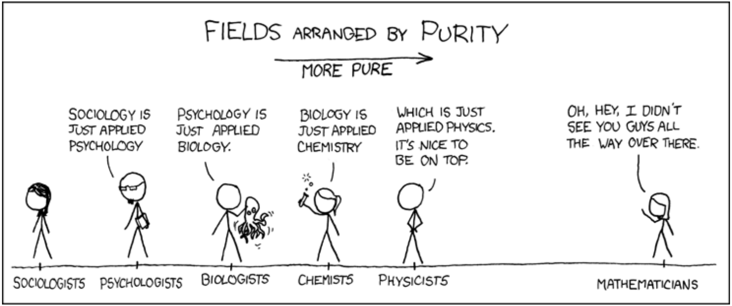
UNIVERSITY OF HELSINKI
DEPARTMENT OF MATHEMATICS
AND STATISTICS

Overview

Combining theory and practice



XKCD#435 by Randall Munroe

Combining theory and practice

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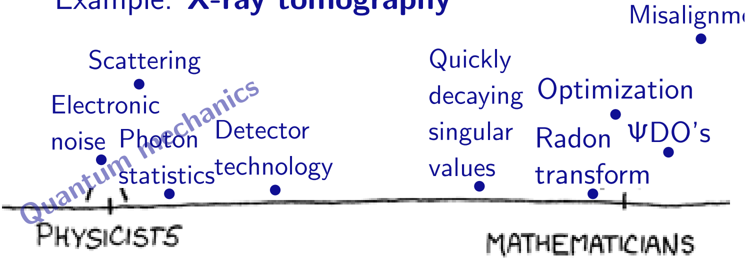
Example: **X-ray tomography**



Combining theory and practice

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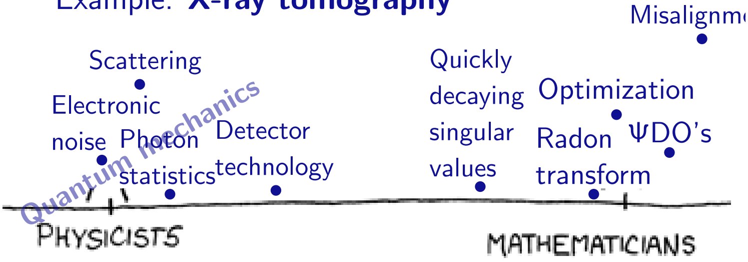

No plan of operations extends with any certainty beyond the first encounter with the main enemy forces.

– Helmuth von Moltke (1871), Prussian field marshall

Combining theory and practice

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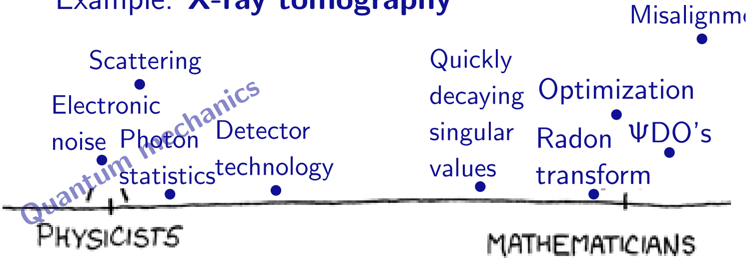


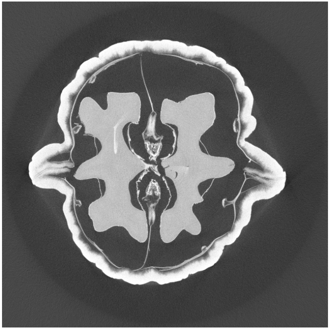
No plan of operations extends with any certainty beyond the first encounter with ~~the main enemy forces~~ realistic setting and data.

– Helmuth von Moltke (1871), Prussian ~~field marshall~~ inversionist*

[citation needed]

Tomographic data

- Finnish Inverse Problems Society: fips.fi/dataset.php



X-ray data of a walnut
K. Hämäläinen, L. Harhanen, A. Kallonen, A. Kujanpää, E. Niemi & S. Siltanen

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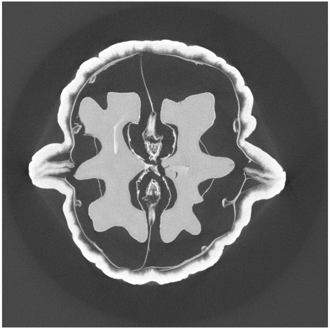

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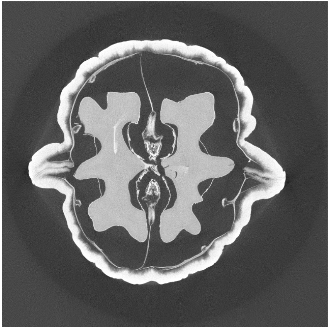

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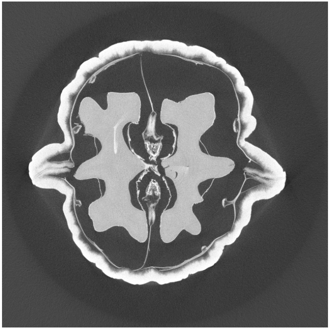
- Variety of formats, methods and sources
- New data is always welcome!



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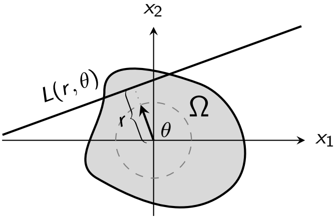
Movement = trouble



Radon transform

$$\mathcal{R} f(x) \longmapsto p(r,\theta) = \int_{\{x\cdot\theta=r\}} f(x)\,\mathrm{d}x$$

- Projections (information) of the target from multiple directions.

Movement = trouble

$t = 1$

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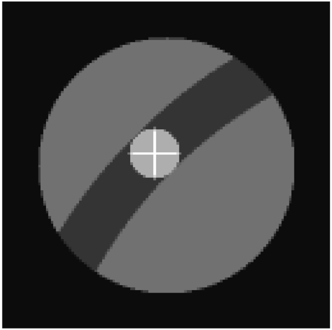

$t = 180$

Radon transform

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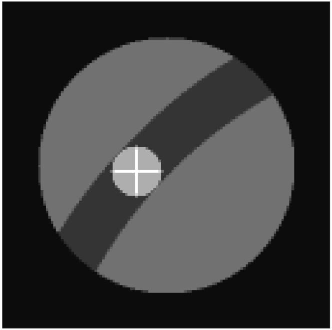
- Projections (information) of the target from multiple directions.
- Measurements are not instant!

Movement = trouble



$t = 360$

Radon transform

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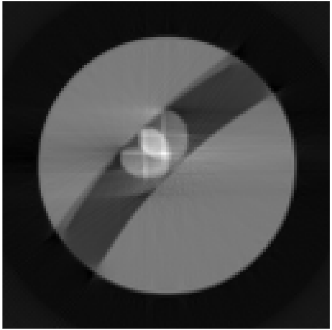

FBP reconstruction

Radon transform

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- Special methods are required

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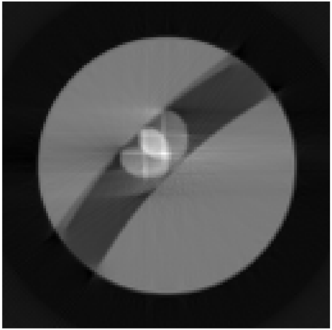

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- Measurements are not instant!
- Special methods are required
- ...and equally special data

Dynamic data examples

Emoji phantom
A. Meaney, Z. Purisha & S. Siltanen

- Some options are available

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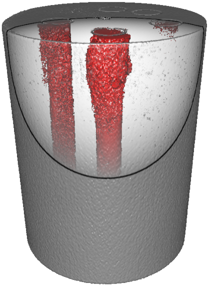

Gel phantom

TH, H. Help & A. Meaney

Dynamic data examples

- Some options are available
- Limited temporal resolution

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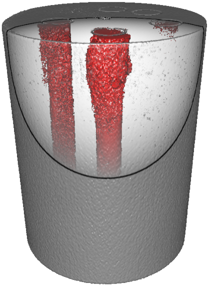

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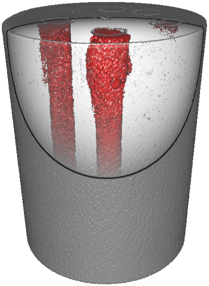

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Dynamic data examples

- Some options are available
- Limited temporal resolution
- Hard or impossible to repeat
- What if you need special measurement setup?

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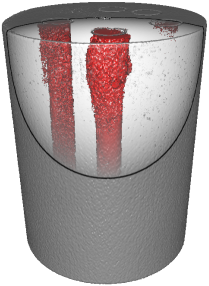

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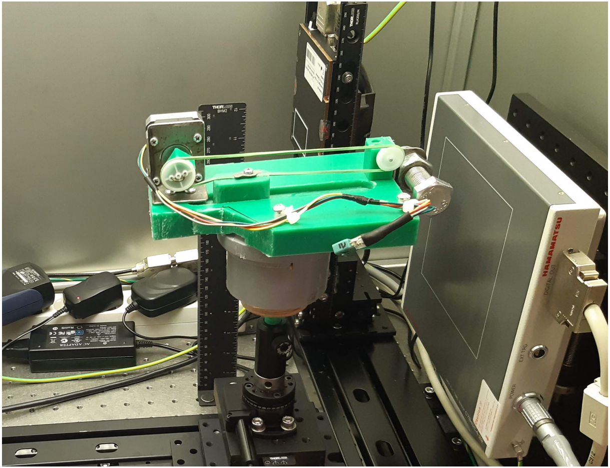
Spatio-TEmporal Motor POwered phantom

STEMPO



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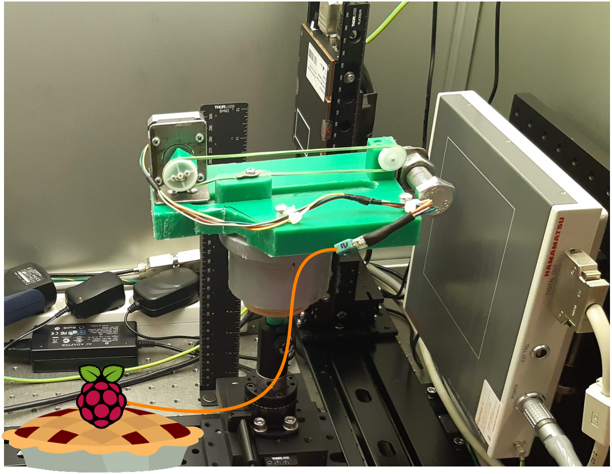

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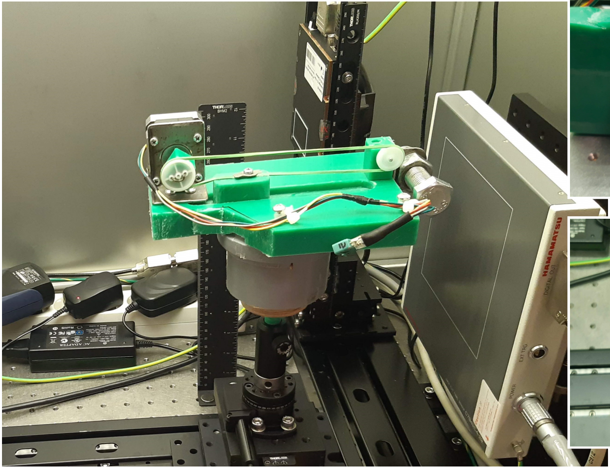

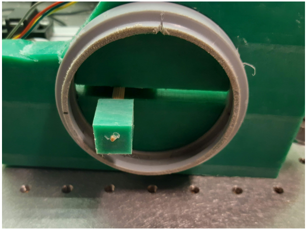

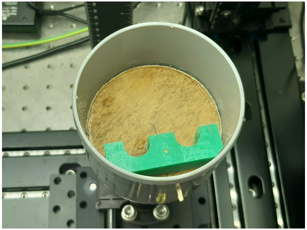

STEMPO data: 10.5281/zenodo.7081688

- Static measurement using 360 projections
  `stempo_static_*d_b*.mat`

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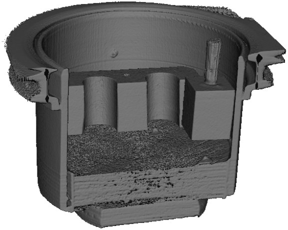

3D FDK recon.

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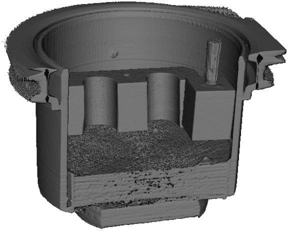

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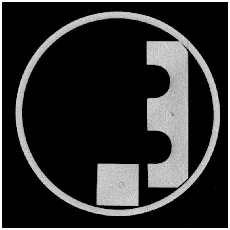

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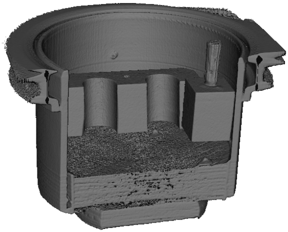

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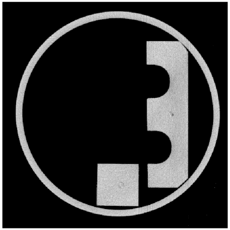

Recommended:

- ASTRA toolbox
- Spot operators
- HelTomo Toolbox
- CUDA GPU*

Example 1: 2D + time wavelet regularization

Minimize functional J_w:

$$J_w(f) = \sum_{t=1}^{T} \frac{1}{2} \| \mathcal{R} f_t - m_t^{\eta} \|_2^2 + \lambda \| Wf \|_1$$

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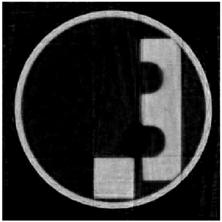

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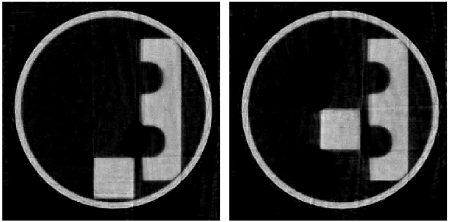

$t = 1$ $\qquad$ $t = 6$

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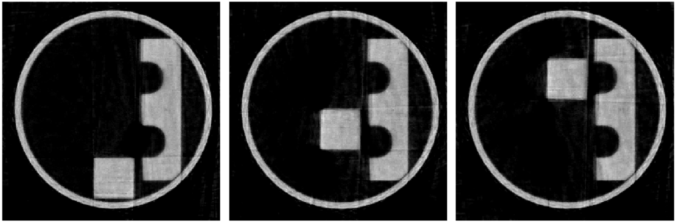

$t = 1$ $t = 6$ $t = 11$

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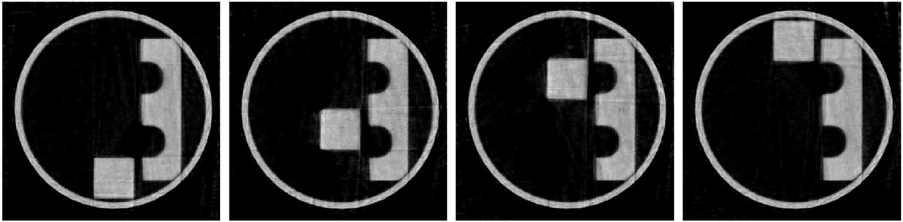

$t = 1$ $t = 6$ $t = 11$ $t = 16$

Bonus: 3D + time wavelet regularization

Minimize functional J_w:

$$J_w(f) = \sum_{t=1}^{T} \frac{1}{2} \| \mathcal{R} f_t - m_t^\eta \|_2^2 + \lambda \| \mathcal{C} f \|_1$$

- `stempo_seq8x45_3d_b8.mat` data, $T = 16$, each with 23 projections.
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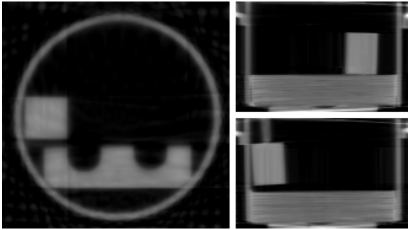

Example 2: Low-rank + sparse decomposition

Minimize functional J_{L+S}:

$$J_{L+S}(L,S) = \frac{1}{2}\|\mathcal{R}(L+S) - m^{\eta}\|_2^2 + \mu_L\|L\|_* + \mu_S\|\widetilde{W}S\|_1$$

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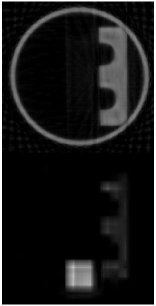

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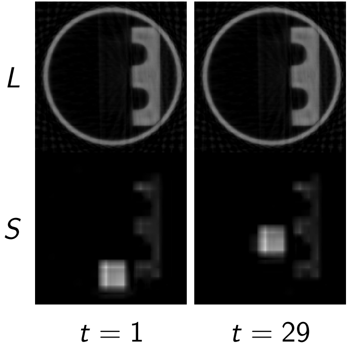

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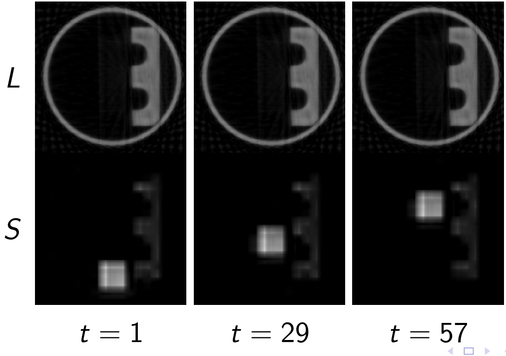

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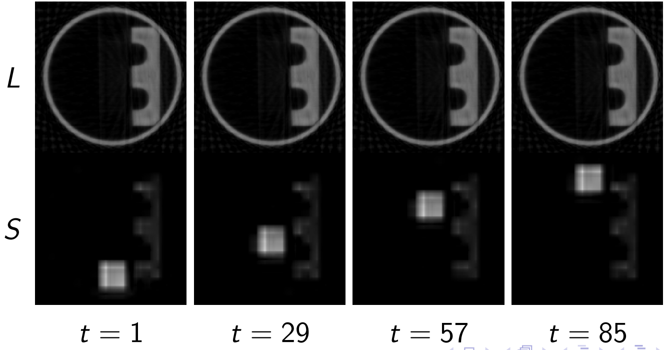

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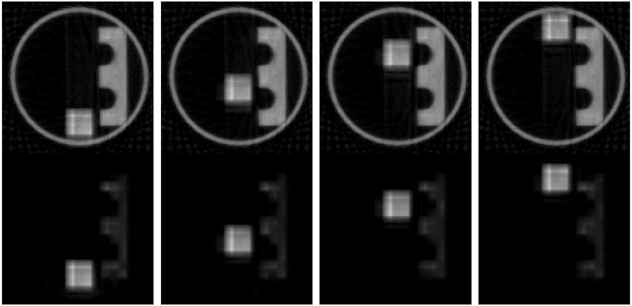
$L + S$

S



$t = 1$ $\quad$ $t = 29$ $\quad$ $t = 57$ $\quad$ $t = 85$

Future development

- **More data!**

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- Random angles, multiple source-detector pairs

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- More difficult moving and static objects

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- Random angles, multiple source-detector pairs
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- Far future: true 3D motion, spectral CT, deformations...

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- Dynamic tomography data is limited
- STEMPO is a computer controlled device which can produce repeatable measurements
- Goal: variety of easy to use data sets for many dynamic tomography problems

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- Real data is important for validation
- Dynamic tomography data is limited
- STEMPO is a computer controlled device which can produce repeatable measurements
- Goal: variety of easy to use data sets for many dynamic tomography problems