1. **Sobolev Approximation and P-Cygni Line Profiles**

In this exercise, the Sobolev Approximation is used to calculate a P-Cygni within a Monte Carlo radiative transfer simulation. For this purpose, we consider a shell in homologous expansion which is illuminated by a radiative source at its lower boundary. Within the expanding material, photons may perform resonant line-interactions with one atomic transition only. The emergent radiation field is recorded and thus line profile, shaped by the interactions within the shell, calculated. Your task is to write a simple Monte Carlo program, which solves the propagation of the radiation field. Use the Sobolev approximation to treat the line interactions and thus determine the P-Cygni line profile in the emergent spectrum. For this problem you may adopt the following assumptions:

- the shell extends from $R_{\rm min} = 1 \times 10^{11}$ cm to $R_{\rm max} = 1 \times 10^{13}$ cm. In velocity space, it covers the linear range from $0.0001\,c$ to $0.01\,c$.
- the shell is entirely composed of hydrogen and the only allowed atomic transition is the Lyman-α line at the rest frame wavelength of 1215.6 Å
- the material within the shell has constant density and temperature. These material properties are such that the Sobolev optical depth of the relevant transition amounts to $\tau_{\rm sob} = 0.1$
- at the lower boundary, an isotropic radiative source is located, which emits uniformly within the wavelength range $\lambda_{\rm min} = 1100$ Å and $\lambda_{\rm max} = 1300$ Å

In developing the Monte Carlo routine you may adhere to the following strategy:

(a) initialise the Monte Carlo packets at the lower boundary. Assign an initial frequency and direction (assuming no limb darkening, i.e. $N(\mu)\mathrm{d}\mu \propto \mu \mathrm{d}\mu$)

(b) packets may only interact at the Sobolev point, i.e. at the location where their CMF frequency is equal to the rest-frame frequency of the line transition. Determine this point using the Doppler effect formula, $\nu_0 = \gamma\nu(1 - \beta\mu)$[1]

(c) at the Sobolev point, packets interact if $\tau > \tau_{\rm sob}$. The optical depth of the packet is determined in the usual way, i.e. by $\tau = -\ln z$

(d) if a line interaction occurs, change the packet properties by assuming isotropy of the resonant scatterings and by according for energy conservation in the CMF

(e) follow the propagation of each packet in the described fashion until it leaves through either of the two boundaries. Discard the packet if it intersects the lower boundary, but record its LF frequency if it emerges through the outer boundary

(f) construct a spectrum from the recorded frequencies

[1]For the problem at hand, where $v \ll c$, you may set $\gamma = 1$.