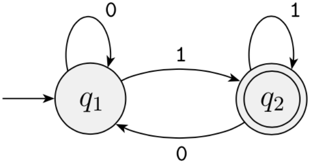
Lec 02. More on DFA & Nondeterministic Finite Automata

Eunjung Kim

Formal definition of computation

- Let $w = w_1 w_2 \cdots w_n \in \Sigma^*$, where $w_i \in \Sigma$.
- The extended transition function δ^* is a mapping from $Q \times \Sigma^*$ to Q defined as: $\delta^*(q, w) = q'$ if there is a sequence of states $r_0, \ldots, r_n$ in Q such that
  - $r_0 = q$,
  - $r_i = \delta(r_{i-1}, w_i)$ for every $1 \leq i \leq n$,
  - $r_n = q'$
- Equivalently, there is a walk in the transition diagram of M from q to q' labelled by w.

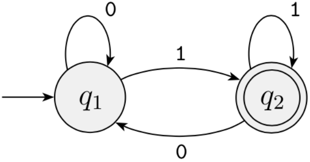


Computation history

- Configuration of a finite automata $M = (Q, \Sigma, \delta, q_0, F)$ is a pair $(q, w) \in Q \times \Sigma^*$.
- We interpret a configuration (q, w) as...
- $(q, w) \rightsquigarrow_M (q', w')$ if...
- $(q, w) \rightsquigarrow_M^* (q', w')$ if there is...
- A sequence of configuration is a computation history if the first configuration is in the form (q_0, w) for some $w \in \Sigma^*$.
- A sequence of configurations is an accepting computation history if the last configuration is in the form ??????.

DFA M ACCEPTS A STRING

- Let $w_1 w_2 \cdots w_n$ be a string in Σ^*.
- $M = (Q, \Sigma, \delta, q_0, F)$ accepts w if
  - $\delta^*(q_0, w) \in F$, or equivalently
  - In the transition diagram of M, there is an walk from q_0 to an accept state labelled by w.



Language recognized by DFA

Definition: language recognized by DFA

- Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automata.
- A string $w \in \Sigma^*$ is <u>accepted</u> by M if
  - $\delta^*(q_0, w) \in F$, or equivalently
  - in the transition diagram of M, there is an walk from q_0 to an accept state labelled by w.
- Let $L(M)$ be the set of all strings which are accepted by M.
- A language A is said to be <u>recognized</u> by M if $A = L(M)$.

Regular Language

Regular language = recognized by some DFA

- A language L over a finite alphabet is said to be regular if there is a finite-state automaton M which recognizes L.

From Languages to DFA: examples

Show that the following language is regular.

- $L = \{$all 0,1-strings containing 01$\}$
- $L = \{$ all 0,1-strings containing exactly even numbers of 0's and 1's respectively $\}$.
- $L = \{$ all strings containing at least two a's$\} \subseteq \{a, b\}^*$.
- $L = \{awa : w \in \{a, b\}^*\}$.

From Languages to DFA: examples

Suppose $L \subseteq \Sigma^*$ is regular. Is the complement of L, i.e. $\Sigma^* - L$, is regular?

From Languages to DFA: examples

$L = \{awa : w \in \{a,b\}^*\}$, $L^2 = \{aw_1aaw_2a : w_i \in \{a,b\}^*\}$

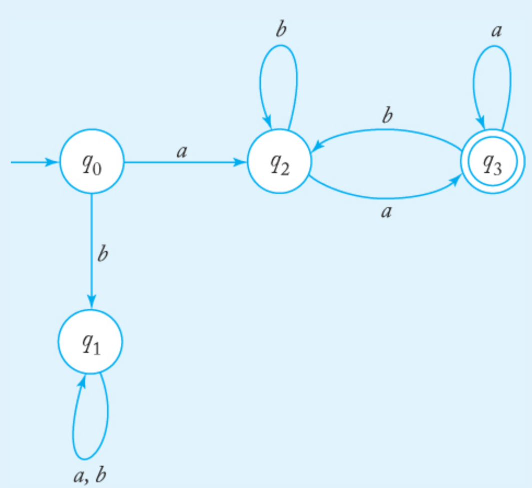


Figure 2.6 from Linz 2017.

Nondeterminism

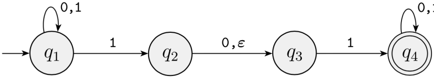


Figure 1.27, Sipser 2012.

	Deterministic FA	Nondeterministic FA
each state & symbol	one leaving arc	multiple arcs or none
labels	Σ	$\Sigma \cup \{\epsilon\}$
computation history	single path	multiple paths (tree)

Nondeterminism: computation tree and ϵ

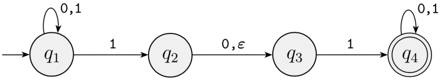
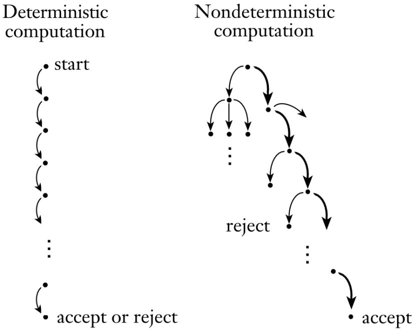


Figure 1.27, Sipser 2012.



Examples of NFA

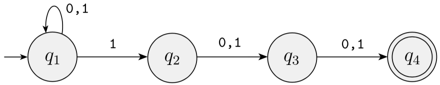


Figure 1.31, Sipser 2012.

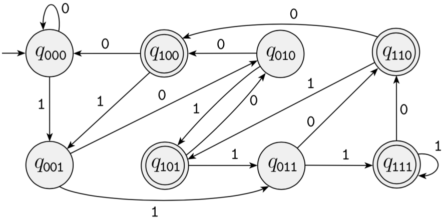


Figure 1.32, Sipser 2012.

Examples of NFA

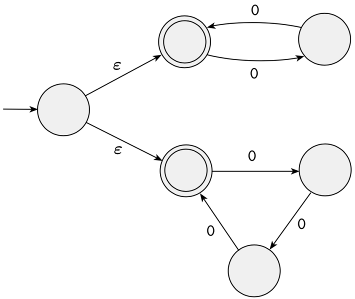


Figure 1.33, Sipser 2012.

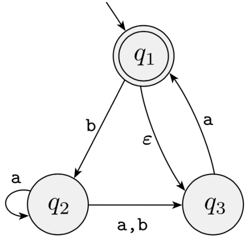
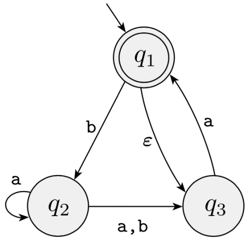


Figure 1.36, Sipser 2012.

Formal definition of NFA

Nondeterministic FA is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$

- Q a finite set called the states,
- Σ a finite set called the alphabet,
- δ a function from $Q \times \Sigma_\epsilon$ to 2^Q called the transition function,
- $q_0 \in Q$ the start state,
- $F \subseteq Q$ the set of accept states.



Write a formal description of this NFA