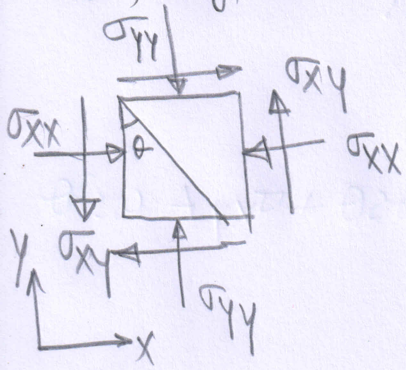
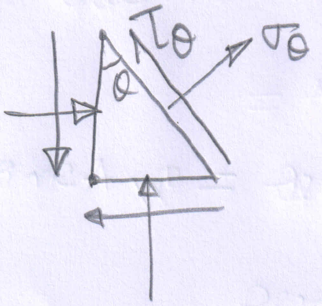
Mohr circle sign convention: (for pole method)

ⓐ If you take tension positive, take clockwise shear as positive.

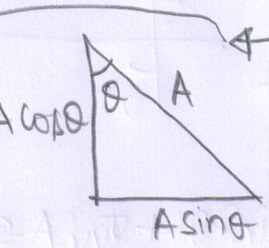
ⓑ If you take compression positive, take anticlockwise shear as positive.

Now, why do we have to follow this sign convention? ***



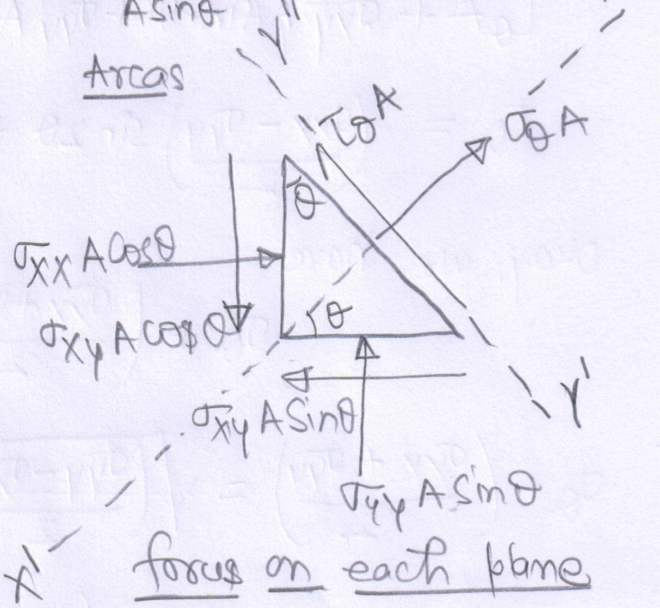


Stresses on a plane making an angle θ with vertical



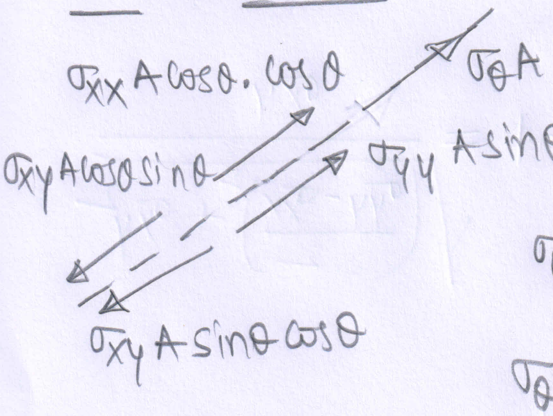
Areas

We have taken tension positive and anticlockwise shear positive.



forces on each plane

Resolving the forces along x'x'' & y'y''



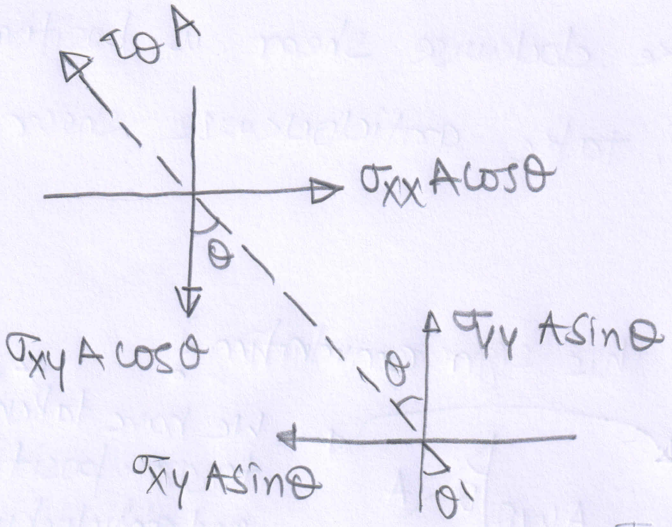
$$\sigma_\theta A + \sigma_{xx} A\cos^2\theta + \sigma_{yy} A\sin^2\theta = 2\sigma_{xy} A\sin\theta\cos\theta$$

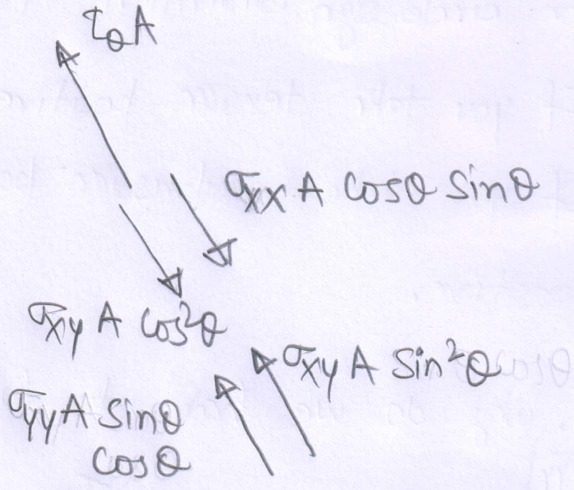
$$\sigma_\theta = -\sigma_{xx}\cos^2\theta - \sigma_{yy}\sin^2\theta + \sigma_{xy}\sin 2\theta$$

Put $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$;

$$\sigma_\theta = -\left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) + \left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)\cos 2\theta + \sigma_{xy}\sin 2\theta$$

$$\sigma_\theta + \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) = \left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)\cos 2\theta + \sigma_{xy}\sin 2\theta$$





$$\tau_\theta A + \sigma_{xy} A\sin^2\theta + \sigma_{yy} A\sin\theta\cos\theta = \sigma_{xx} A\sin\theta\cos\theta + \sigma_{xy} A\cos^2\theta$$

$$\tau_\theta = \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\sin 2\theta + \sigma_{xy}\cos 2\theta$$

Okay, we have

$$\sigma_\theta + \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) = \left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)\cos 2\theta + \sigma_{xy}\sin 2\theta$$

$$\sigma_\theta + \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) = \sqrt{\left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)^2 + \sigma_{xy}^2}\left[\cos\phi\cos 2\theta + \sin\phi\sin 2\theta\right]$$

Where $\cos\phi = \dfrac{(\sigma_{yy}-\sigma_{xx})/2}{\sqrt{\left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)^2 + \sigma_{xy}^2}}$; $\sin\phi = \dfrac{\sigma_{xy}}{\sqrt{\left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)^2 + \sigma_{xy}^2}}$

assuming $R = \sqrt{\left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)^2 + \sigma_{xy}^2}$

$$\sigma_\theta + \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) = R\cos(2\theta - \phi)$$

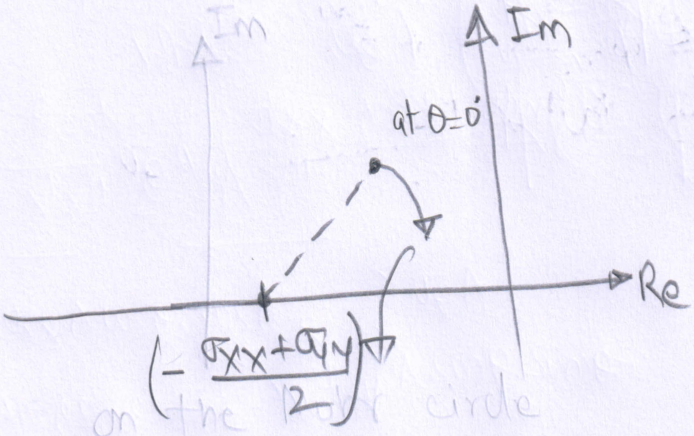
$$\tau_\theta = \sqrt{\left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)^2 + \sigma_{xy}^2}\left[-\left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)\frac{\sin(2\theta)}{R} + \frac{\sigma_{xy}}{R}\cos 2\theta\right]$$

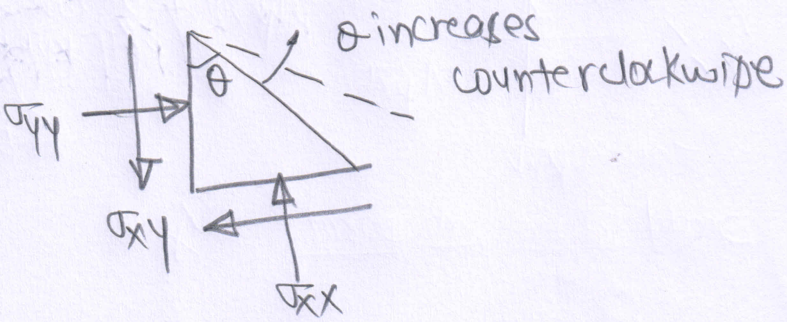
$$= R\left[-\cos\phi\,\sin 2\theta + \sin\phi\,\cos 2\theta\right] = -R\sin(2\theta-\phi)$$

$$\sigma_\theta + \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) = R\cos(2\theta-\phi)$$
$$= R\cos(\phi-2\theta)$$

$$\tau_\theta = -R\sin(2\theta-\phi) = R\sin(\phi-2\theta)$$

$$\left[\sigma_\theta + \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right)\right] + i\tau_\theta = R\cos(\phi-2\theta) + iR\sin(\phi-2\theta)$$
$$= Re^{i(\phi-2\theta)} = \left(Re^{i\phi}\right)e^{-i(2\theta)}$$



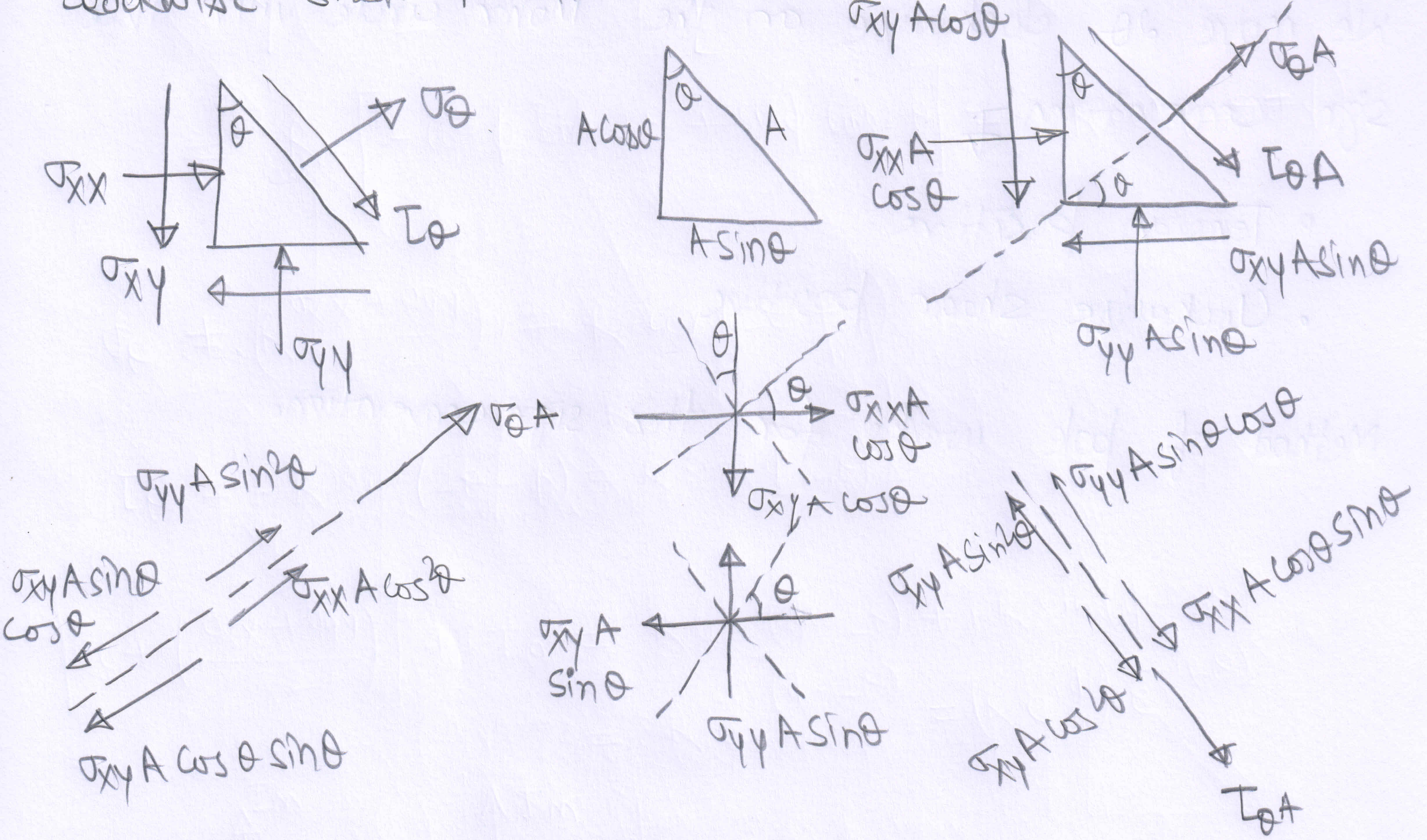


for every change θ counterclockwise rotation in the plane we move 2θ clockwise on the Mohr circle with the sign convention ↓

- Tension positive
- Anticlockwise shear positive

Method of pole does not work for this convention.

Let us see, what happens if we take tension positive and clockwise shear positive.

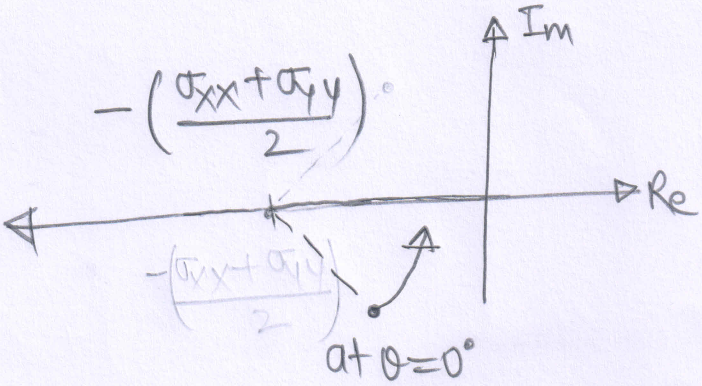


$$\sigma_\theta = \sigma_{xy}\sin 2\theta - \sigma_{xx}\cos^2\theta - \sigma_{yy}\sin^2\theta; \quad \tau_\theta = -\sigma_{xy}\cos 2\theta + \left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)\sin 2\theta$$

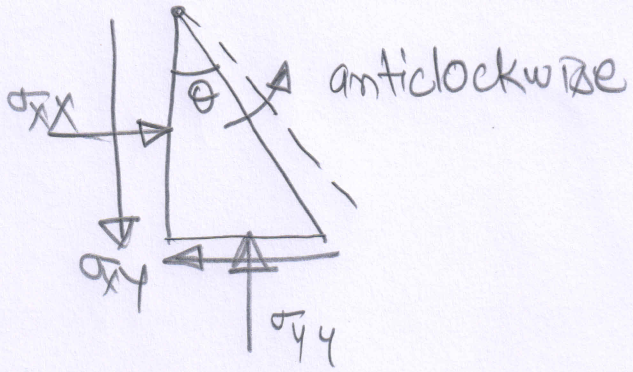
$$\sigma_\theta + \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) = \sigma_{xy}\sin 2\theta + \left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)\cos 2\theta;$$

$$\sigma_\theta + \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right) = R\cos(2\theta-\phi); \quad \tau_\theta = R\sin(2\theta-\phi)$$

$$\left[\sigma_\theta + \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right)\right] + i\tau_\theta = R e^{i(2\theta-\phi)}$$



Anticlockwise



for every θ increment counterclockwise of θ-plane, we move 2θ clockwise on the Mohr circle with the sign convention $\mathcal{F}$

- Tension positive
- Clockwise shear positive

Method of pole works for this sign convention.

In summary

If you take either of these sign convention

(a) Compression positive and counterclockwise (anticlockwise) shear positive

(b) Tension positive and clockwise shear positive

Your analysis would be correct.

With these conventions you can use

→ pole method

→ 2θ rotation method