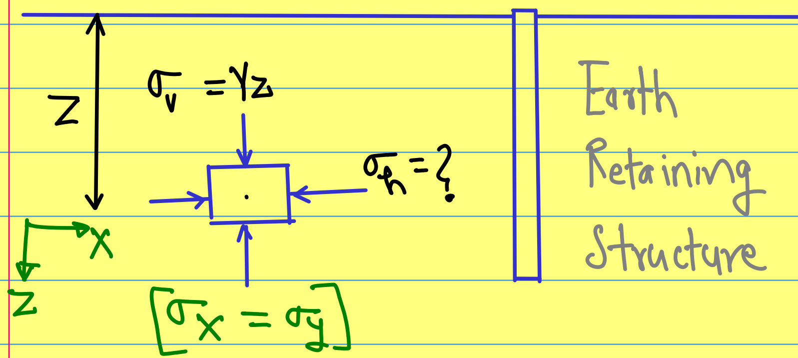
Lateral earth pressure

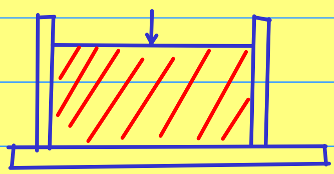
* Earth pressure at rest
* Active earth pressure
* Passive earth pressure



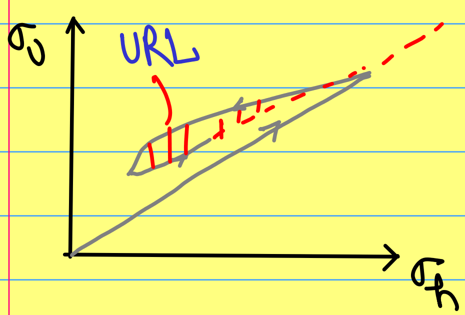
Earth pressure at rest

$[\varepsilon_x = 0]$

At rest condition is $[\varepsilon_{lateral} = 0] \rightarrow$ Simulated in oedometer or 1-D consolidation.



Due to lateral constraint, σ_h responds to σ_v.



In elastic limit,

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E} = 0$$

$$\sigma_x = \sigma_y = \sigma_h \quad ; \quad \sigma_z = \sigma_v$$

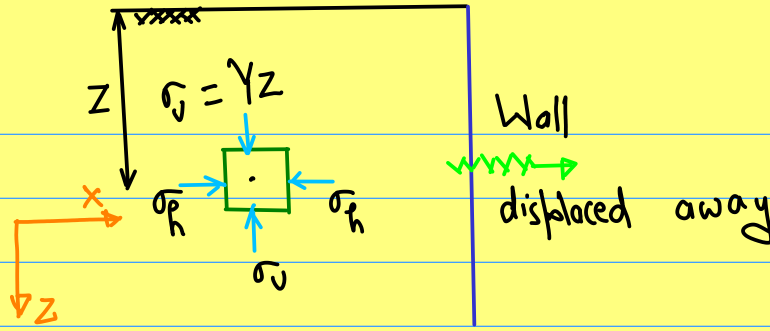
$$(1-\nu)\sigma_h = \nu \sigma_v \Rightarrow \sigma_h / \sigma_v = \nu/(1-\nu)$$

For normally consolidated soil $\left[K_{o(nc)} = 1 - \sin\phi\right]$

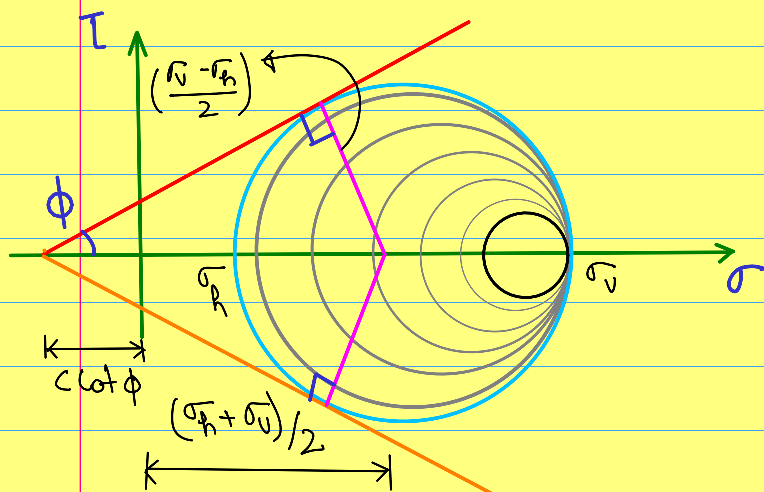
For over consolidated soil $\left[K_{o(oc)} = K_{o(nc)} f(OCR)\right]$

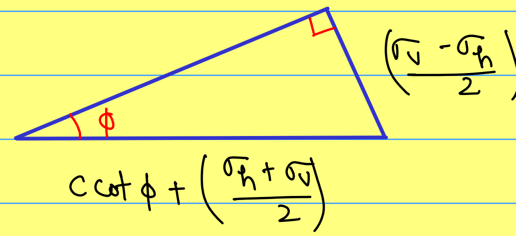
The $K_{o(oc)}$ equation also depends on the unload-reload path.

Active earth pressure



Active earth pressure condition

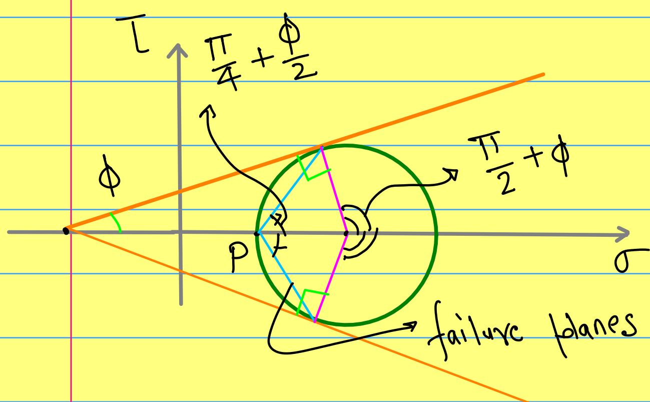


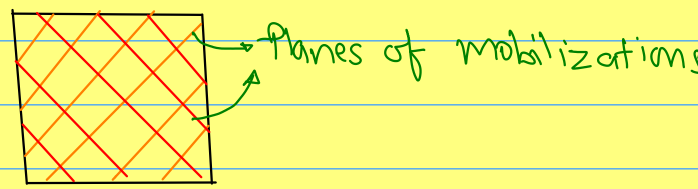


$$\sin\phi = \frac{\sigma_v - \sigma_h}{2c\cot\phi + (\sigma_v + \sigma_h)}$$

$$\sigma_v = \sigma_h N_\phi + 2c\sqrt{N_\phi}\,; \qquad N_\phi = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

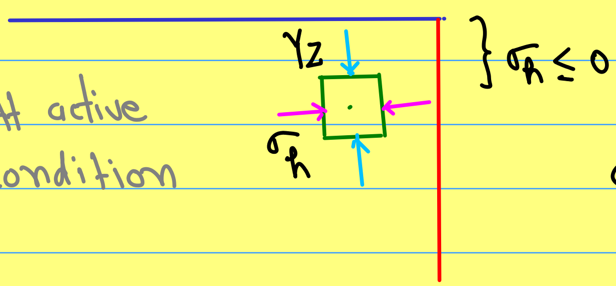
$$\boxed{\sigma_h = \frac{\sigma_v}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}}$$





$$\sigma_h = \frac{\sigma_v}{N_\phi} - \frac{2c}{\sqrt{N_\phi}} = \frac{\gamma z}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}$$

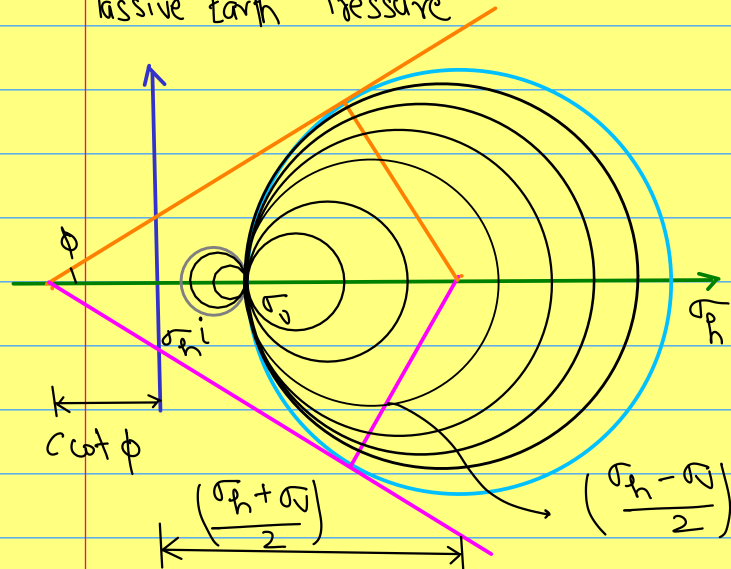
At active condition

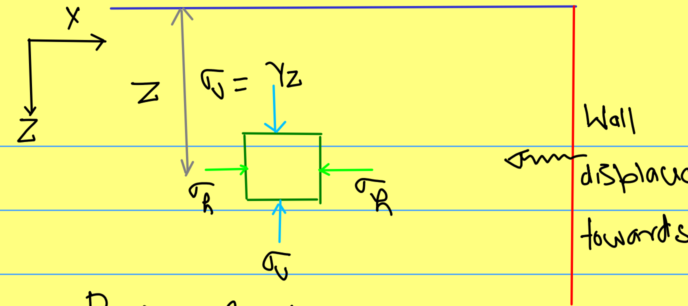


$$\sigma_h = 0 \Rightarrow Z_c = \frac{2c\sqrt{N_\phi}}{\gamma}$$

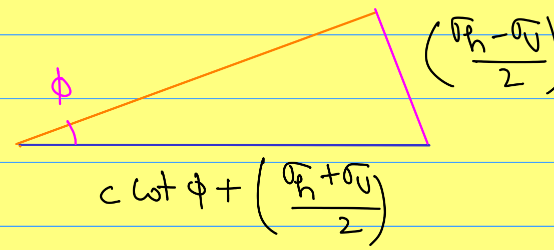
depth of tensile crack

Passive Earth Pressure



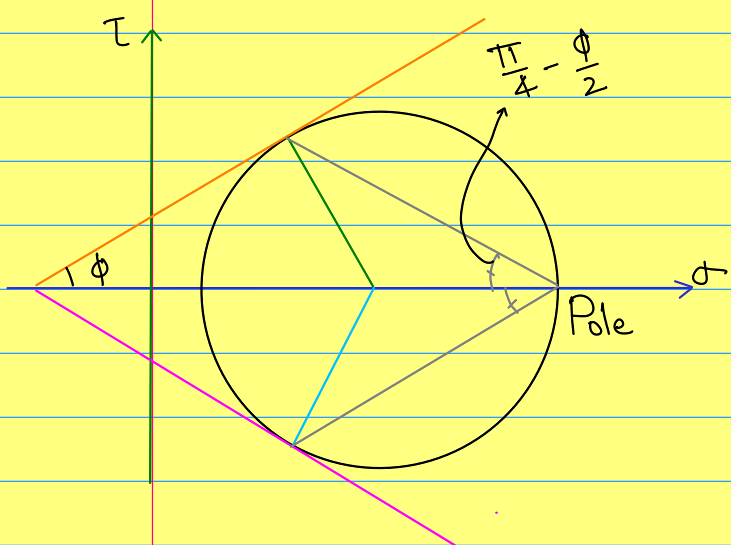


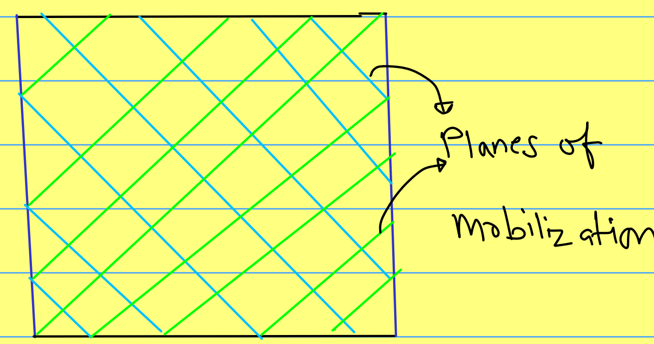
Passive Condition



$$\sin\phi = \frac{\sigma_h - \sigma_v}{2c\cot\phi + (\sigma_h + \sigma_v)} \Rightarrow \sigma_v(1+\sin\phi) + 2c\cos\phi = \sigma_h(1-\sin\phi)$$

$$\Rightarrow \sigma_h = \sigma_v \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) + 2c\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right) = \sigma_v N_\phi + 2c\sqrt{N_\phi}$$

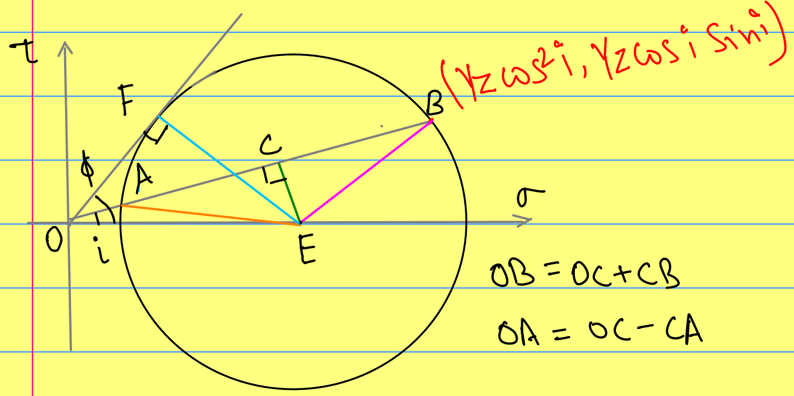




<u>Note</u> For above discussion

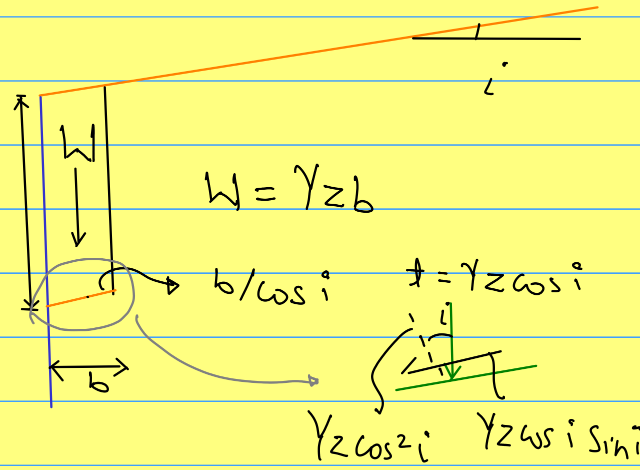
1. Earth retaining structure is vertical.
2. This structure is also frictionless
3. The soil backfill is horizontal.
4. Soil follows Mohr Coulomb criterion.

Inclined backfill



$OB = OC + CB$

$OA = OC - CA$



$EF = OE\sin\phi$ $\qquad EC = OE\sin i$

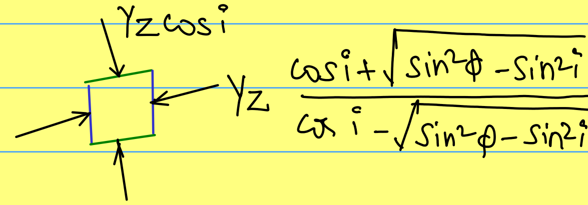
$CA = \sqrt{EA^2 - CE^2} = OE\sqrt{\sin^2\phi - \sin^2 i}$ $\qquad CB = OE\sqrt{\sin^2\phi - \sin^2 i}$

$OC = OE\cos i$

$$OB = OE\left\{\cos i + \sqrt{\sin^2\phi - \sin^2 i}\right\}$$

$$\frac{OA}{OB} = \frac{\cos i + \sqrt{\sin^2\phi - \sin^2 i}}{\cos i - \sqrt{\sin^2\phi - \sin^2 i}} \qquad OA = OE\left\{\cos i - \sqrt{\sin^2\phi - \sin^2 i}\right\}$$

$$\boxed{OA = \gamma z\,\frac{\cos i + \sqrt{\sin^2\phi - \sin^2 i}}{\cos i - \sqrt{\sin^2\phi - \sin^2 i}}}$$

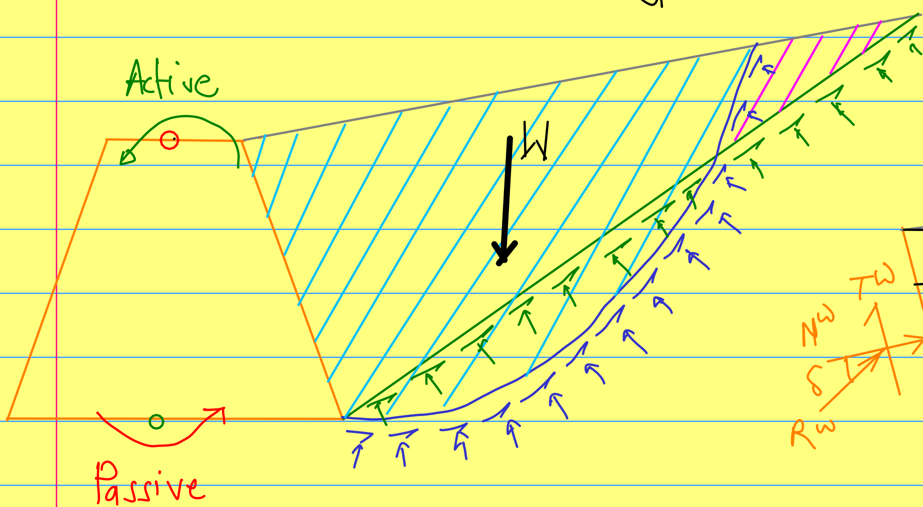


$$OE = \gamma z\,\frac{\cos i}{\cos i + \sqrt{\sin^2\phi - \sin^2 i}} \qquad EF = \gamma z\,\frac{\cos i\sin\phi}{\cos i + \sqrt{\sin^2\phi - \sin^2 i}}$$

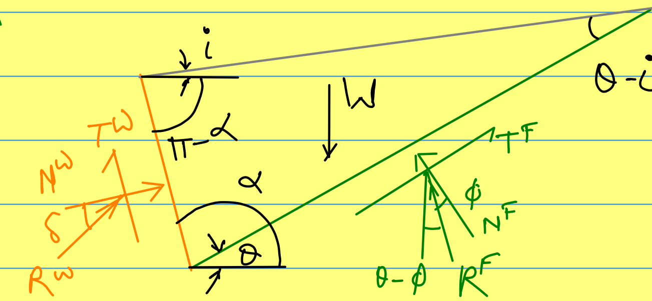
Similar procedure can be used to derive the traction on the wall in passive case.

In all the above cases, we have a frictionless wall and planar backfill with planar failure surface.

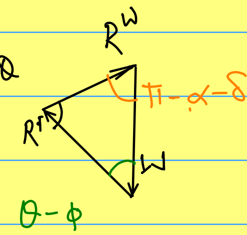
Coulomb's Earth Pressure Theory



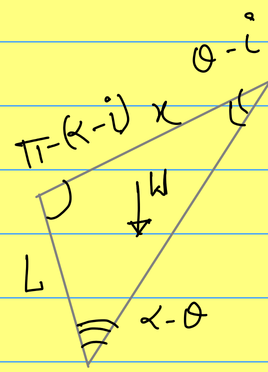
Active Case



$$\frac{W}{\sin(\alpha+\delta+\phi-\theta)} = \frac{R^F}{\sin(\alpha+\delta)} = \frac{R^W}{\sin(\theta-\phi)}$$



$$\boxed{R^W = \frac{W\sin(\theta-\phi)}{\sin(\alpha+\delta+\phi-\theta)}}$$



$$\frac{x}{\sin(\alpha-\theta)} = \frac{L}{\sin(\theta-i)}$$

$$x = L\,\frac{\sin(\alpha-\theta)}{\sin(\theta-i)}$$

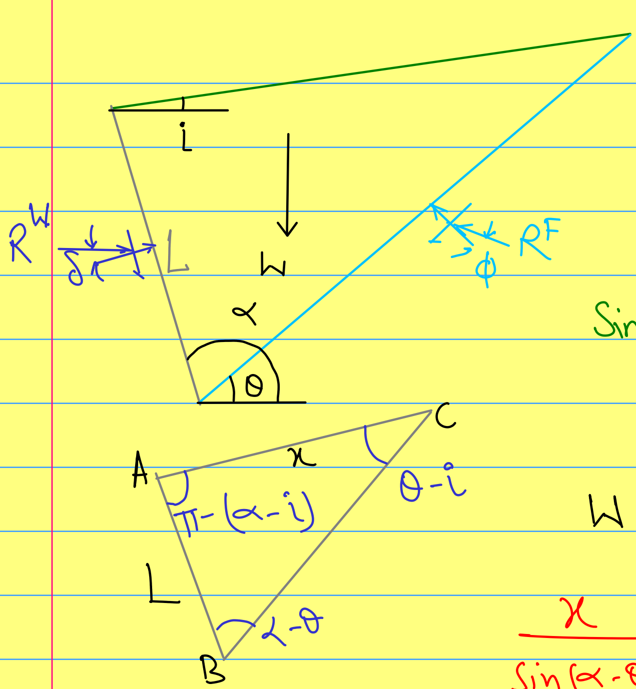
$$W = \gamma\left\{\frac{1}{2}L^2\,\frac{\sin(\alpha-\theta)\sin(\alpha-i)}{\sin(\theta-i)}\right\}$$

$$R^W = \frac{1}{2}\gamma L^2\,\frac{\sin(\alpha-\theta)\sin(\alpha-i)\sin(\theta-\phi)}{\sin(\alpha+\delta+\phi-\theta)\,\sin(\theta-i)}$$

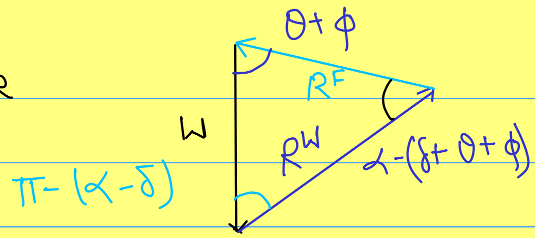
Active earth resistance is $F_a = \max_{\theta}\{R^W\}$

$$R^W = \frac{1}{2}\gamma L^2\sin(\alpha-i)\,\frac{\sin(\alpha-\theta)\,\sin(\phi-\theta)}{\sin(\theta-\{\alpha+\delta+\phi\})\,\sin(\theta-i)} = \frac{1}{2}\gamma L^2\sin(\alpha-i)\,\frac{\sin(\theta-\alpha)\sin(\theta-\phi)}{\sin(\theta-i)\,\sin(\theta-[\alpha+\delta+\phi])}$$

$$F_a = \frac{1}{2}\gamma L^2\sin(\alpha-i)\,\frac{\sin^2(\alpha-\phi)}{\left\{\sqrt{\sin(\alpha+\delta)\sin(\alpha-i)}+\sqrt{\sin(\phi+\delta)\sin(\phi-i)}\right\}^2}$$



Passive Case



$$\frac{W}{\sin(\alpha-\delta-\theta-\phi)} = \frac{R^W}{\sin(\theta+\phi)} = \frac{R^F}{\sin(\alpha-\delta)}$$

$$W = \gamma\left\{\frac{1}{2} L x \sin(\alpha-i)\right\}$$

$$\frac{x}{\sin(\alpha-\theta)} = \frac{L}{\sin(\theta-i)} \Rightarrow x = L\,\frac{\sin(\alpha-\theta)}{\sin(\theta-i)}$$

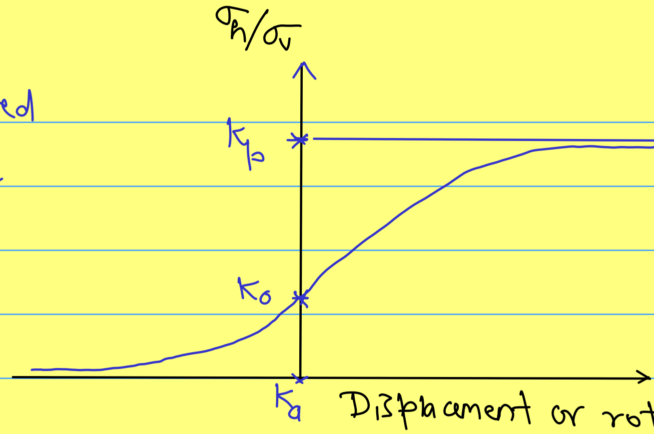
$$W = \frac{1}{2}\gamma L^2 \sin(\alpha-i)\,\frac{\sin(\alpha-\theta)}{\sin(\theta-i)}\,; \quad R^W = W\,\frac{\sin(\theta+\phi)}{\sin(\alpha-\delta-\phi-\theta)}\,;$$

$$R^W = \frac{1}{2}\gamma L^2 \sin(\alpha-i)\,\frac{\sin(\theta-\alpha)\,\sin(\theta+\phi)}{\sin(\theta-i)\,\sin(\theta-\{\alpha-\delta-\phi\})}$$

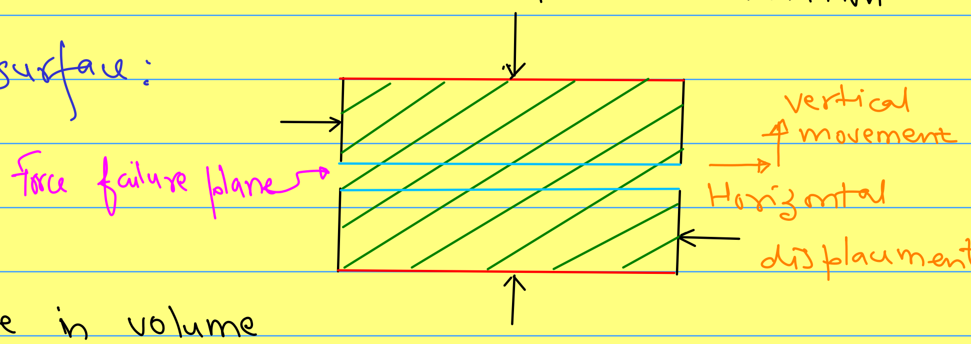
The passive earth resistance is $F_p = \max_{\theta}\{R^W(\theta)\}$;

$$F_p = \frac{1}{2}\gamma L^2 \sin(\alpha-i)\,\frac{\sin^2(\phi+\alpha)}{\left\{\sqrt{\sin(\alpha-\delta)\sin(\alpha-i)} + \sqrt{\sin(\delta+\phi)\sin(i+\phi)}\right\}^2}$$

Displacement or rotation required to reach active or passive failure conditions

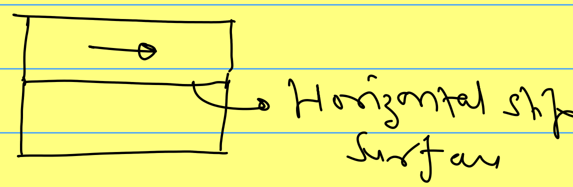


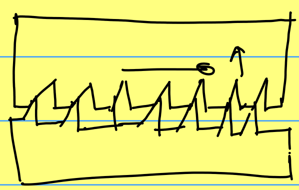
Log spiral slip surface:



Dilation is increase in volume when material is sheared.

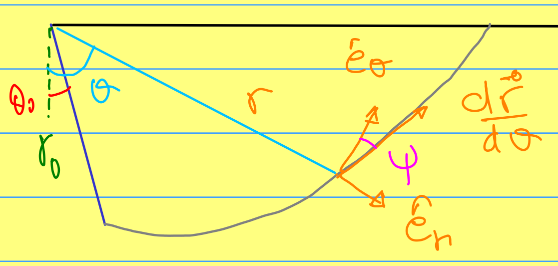
$$\boxed{\psi = \frac{d\varepsilon_v}{d\varepsilon_q}}$$





Saw tooth model

This implies, in a soil which dilates, the failure surface should not be linear.



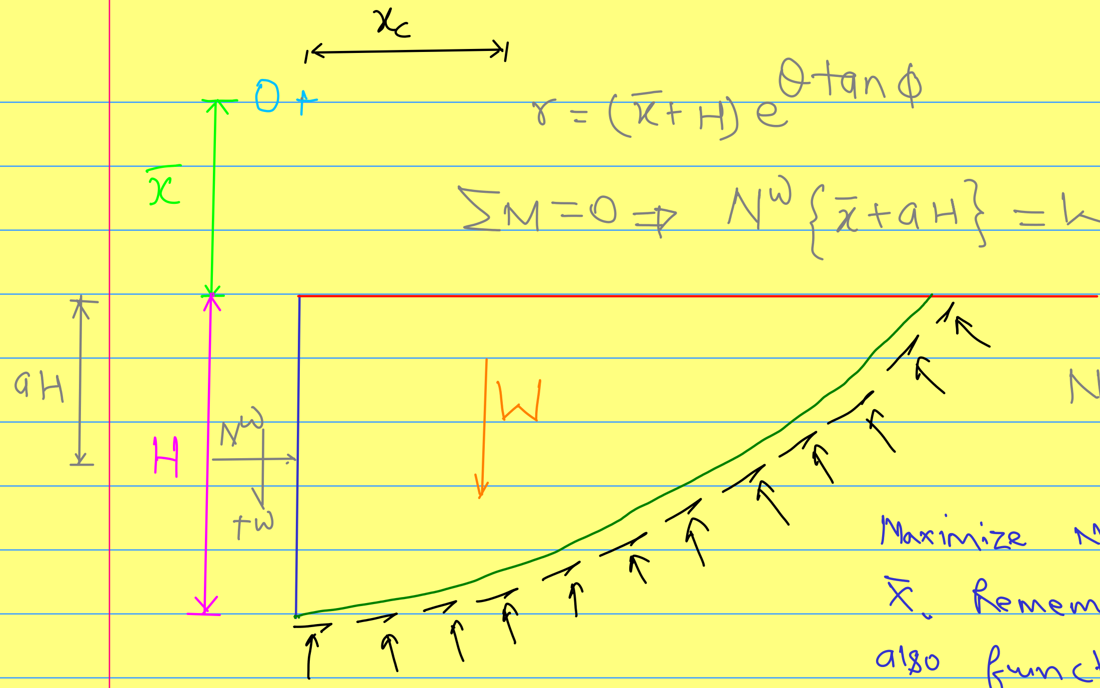
$$\Rightarrow \quad \frac{d\vec{r}}{d\theta} \cdot \hat{e}_\theta = \left|\frac{d\vec{r}}{d\theta}\right| \cos\psi$$

$$\frac{d\vec{r}}{d\theta} = \frac{dr}{d\theta}\hat{e}_r + r\hat{e}_\theta$$

$$\Rightarrow \quad r = \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2} \cos\psi \quad \Rightarrow \quad r^2 \sin^2\psi = \left(\frac{dr}{d\theta}\right)^2 \cos^2\psi$$

$$\Rightarrow \quad \frac{dr}{d\theta} = r\tan\psi \quad \Rightarrow \quad \frac{dr}{r} = (\tan\psi)\, d\theta$$

$$\Rightarrow \quad \Big[\ln r\Big]_{r_0}^{r} = \Big[\theta\tan\psi\Big]_{\theta_0}^{\theta} \quad \Rightarrow \quad \boxed{r = r_0\, e^{(\theta-\theta_0)\tan\psi}}$$



$$r = (\bar{x}+H)e^{\theta\tan\phi}$$

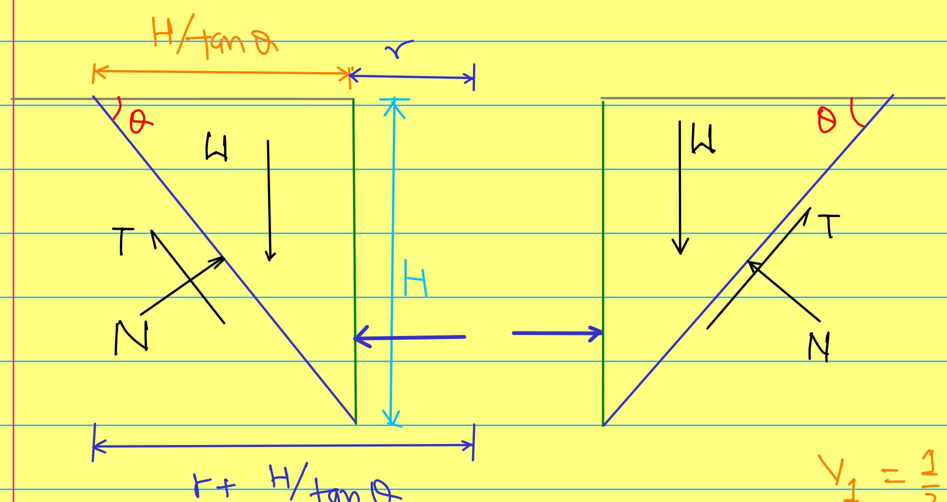
$$\sum M = 0 \Rightarrow N^w\{\bar{x}+aH\} = Wx_c\,;$$

$$N^w = \frac{Wx_c}{\bar{x}+aH}$$

Maximize N^w with respect to $\bar{x}$. Remember W and x_c are also function of $\bar{x}$.

We are seeking maximum because we want to work for worst case scenario.

Lateral earth pressure on shaft linings



V_0

$$= \frac{1}{3}\pi\left(r+\frac{H}{\tan\theta}\right)^2\left(r+\frac{H}{\tan\theta}\right)\tan\theta$$

$$= \frac{1}{3}\pi\tan\theta\left(r+\frac{H}{\tan\theta}\right)^3.$$

$$V_1 = \frac{1}{3}\pi r^3\tan\theta$$

$$V_2 = \pi r^2 H$$

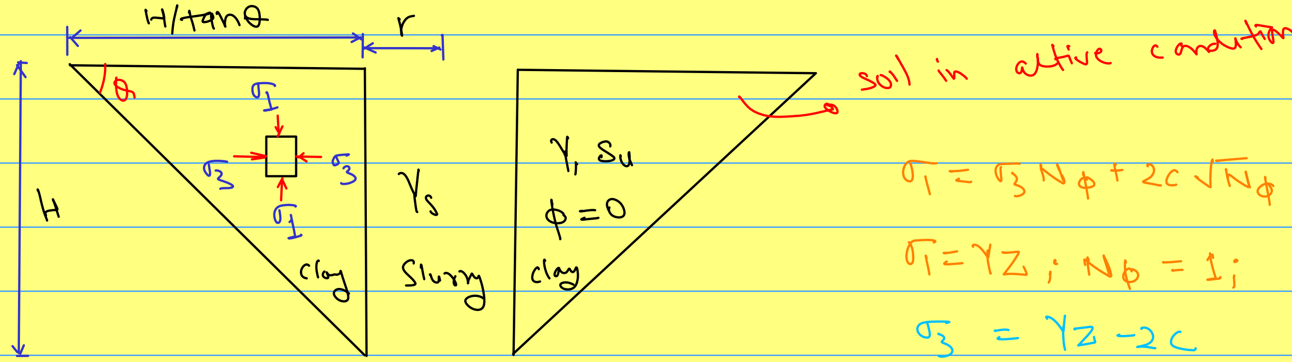
$$V = V_0 - V_1 - V_2$$

$$= \frac{1}{3}\pi\tan\theta\left(r+\frac{H}{\tan\theta}\right)^3 - \frac{1}{3}\pi r^3\tan\theta - \pi r^2 H$$

$$W = \frac{\pi}{3}\gamma\left[\left(r+\frac{H}{\tan\theta}\right)^3\tan\theta - r^3\tan\theta - 3r^2 H\right]$$

$$= \frac{\pi}{3}\gamma\left[\frac{H^3}{\tan^2\theta} + \frac{3rH^2}{\tan\theta}\right] = \frac{\pi\gamma H^2}{3\tan\theta}\left[\frac{H}{\tan\theta} + 3r\right]$$

Shaft lining in saturated clay trench



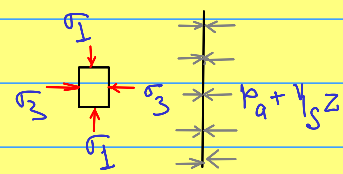
$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$

$\sigma_1 = \gamma z$; $N_\phi = 1$;

$\sigma_3 = \gamma z - 2c$

$\sigma_3 = \gamma z - 2S_u$

Equilibrium of soil pressure and slurry pressure



$$\Rightarrow p_a = \sigma_3 - \gamma_s z$$
$$= \gamma z - 2S_u - \gamma_s z$$
$$= \{\gamma - \gamma_s\} z - 2S_u$$

$$P_a = \int_0^H p_a \, dz = \int_0^H \left[(\gamma - \gamma_s)z - 2S_u\right] dz$$

$$= (\gamma - \gamma_s)\frac{H^2}{2} - 2S_u H$$