1) Let skip weights vector = w_s
Input vector = x
Output vector = y
Output without using skip weights
= y'

$$y = f(y' + w_s x)$$

f → activation fn at output layer

Loss = $g(y, \cancel{label}\ l)$ (Loss function)

where l = labels / actual output / correct output

$$\frac{\partial g}{\partial w_s} = \left(\frac{\partial g}{\partial y}\right) \times \frac{\partial y}{\partial w_s}$$

$$= \left(\frac{\partial g}{\partial y}\right) \times \left(f'(y' + w_s x)\, x\right)$$

Ans 2 - H has w^2 elements.
By symmetry only lower triangular elements are independent.

number of ~~inde~~ independent terms in Hessian $= \frac{(w^2 - w)}{2} + w$

$$= \frac{w(w-1)}{2} + w \times \frac{2}{2}$$

$$= \frac{w(w-1+2)}{2}$$

$$= \frac{w(w+1)}{2}$$

Number of independent terms in $\nabla E|_{w=\bar{w}}$
$= w$

Error surface is determined by Hessian and $\nabla E|_{w=\bar{w}}$

Total terms $= w + \frac{(w+1)}{2} = \frac{w(w+3)}{2}$

Ans 3 - $z_i = (z_1, z_2 \dots z_n)$

$x_i = (x_1, x_2 \dots x_n)$

x = Convex hull of $x_i = \sum \alpha_i x_i$, where $\sum \alpha_i = 1$

z = Convex hull of $z_i = \sum \beta_i z_i$, where $\sum \beta_i = 1$

(i) $f(x) = w^T x + w_0$

$= w^T \sum \alpha_i x_i + \sum \alpha_i w_0$

$= \sum \alpha_i (w^T x_i + w_0)$

$f(z) = w^T z + w_0$

$= w^T \sum \beta_i z_i + w_0 \sum \beta_i$

$= \sum \beta_i (w^T z_i + w_0)$

if their convex hulls intersect and they are linearly separable

Then,

$f(x) > 0$ & $f(z) < 0$

& $x = z$

since $x = z$

$f(x) > 0$ & $f(x) < 0$

which is a contradiction

$\Rightarrow$ The points are not linearly separable.

(ii) As points are linearly separable

$w^T x + w_0 > 0$ & $w^T z + w_0 < 0$

$\Rightarrow$ $\Sigma \alpha_i (w^T x_i + w_0) > 0$

& $\Sigma \beta_i (w^T z_i + w_0) < 0$ — ①

Also, if we assume convex hulls intersect

then, $\Sigma \alpha_i x_i = \Sigma \beta_i z_i$ — ②

Conditions ① and ② can not be simultaneously satisfied as $f(\Sigma \alpha_i x_i) > 0$ & $f(\Sigma \beta_i z_i) < 0$

Thus we arrive at a contradiction

$\Rightarrow$ Convex hulls don't intersect.