Predicting and Classifying European Tesla Option Values with Machine Learning Algorithms

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Executive Summary

This study aims to explore the performance of various machine learning algorithms in predicting option pricing, comparing their results to the widely used Black-Scholes model. The analysis focused on both regression and classification tasks, employing a range of models including linear regression, ridge and lasso regression, support vector machines (SVM), k-nearest neighbors (KNN), decision trees, and random forests. The performance of regression models was evaluated using in-sample R-squared and mean squared error (MSE) metrics, while the classification models were assessed using the mean cross-validation score. The Random Forest algorithm emerged as the top-performing model in both tasks, achieving an R-squared of 99.68% in regression and a mean cross-validation score of 93.52% in classification. The decision tree model also demonstrated strong performance, with a mean cross-validated R-squared of 99.11% in regression. Despite the random forest model's superior performance, it is crucial to consider the business aspects and implications when selecting a model for real-world application. While the random forest algorithm excels in predictive accuracy, its complex nature may hinder interpretability, a key requirement in the financial industry. Stakeholders may prefer a more transparent and explainable model, such as decision trees. In high-stakes scenarios, the use of a more established model like the Black-Scholes equation may be preferred, as it has been thoroughly tested and validated over time. To further validate the superiority of the machine learning models, it is essential to gather more extensive and diverse data to develop models that are robust and can be generalized to the volatile financial market while meeting business requirements.

1. Introduction

The Black-Scholes formula, developed in 1973, is a widely used mathematical model for pricing European call options. The formula provides a theoretical estimate of fair price of an option based on certain key variables such as current price of underlying asset (S), strike price of option (K), risk-free interest rate (r), time to maturity in years (τ), volatility of underlying asset's returns (σ). The objective is to determine whether machine learning models can outperform traditional Black-Scholes, mitigate its limitations and provide more accurate predictions for informed investment decisions. The project predicts Tesla's European option values, classifying them as either overvalued or undervalued. Multiple algorithms including linear, ridge and lasso regression, KNN, decision tree, random forest, support vector machines (SVM), linear discriminant analysis (LDA), and logistic regression were explored. The models were evaluated and the best model finalized for final prediction.

2. Methodology

2.1 Data Preprocessing and Validation

The dataset underwent preprocessing, which involved addressing missing and null values, as well as scaling features when necessary. Initially, a simple train-test split was employed to validate a linear regression model; however, upon further consideration, it was determined that a 10-fold cross-validation approach applied to the training data would be more appropriate. This entails evaluating the models across 10 train-test splits, each utilizing different subsets of the training data, thereby ensuring that every training point is ultimately used for both training and evaluation purposes. Additionally, this approach reduces the variance of the performance metric that arises when using a single train-test split and mitigates the influence of a single random split, as the final metric is derived from the mean across all folds. For classification task, a similar 10-fold cross-validation process was employed, albeit with a stratified and shuffled approach to maintain consistent class distribution of the binary response label across each fold.

2.2 Evaluation Metrics

For regression models predicting Tesla option values, the performance was evaluated using both the mean cross-validated R-squared and mean squared error (MSE) (the additive inverse of the negative MSE) metrics. In cases where two models had marginally close mean cross-validated R-squared scores, the model with the lower mean cross-validated mean squared error was preferred. For classification models, the performance was assessed using the mean cross-validated accuracy score, which measures the trained model's ability to accurately label whether the Tesla option value was either overvalued or undervalued.

2.3 Model Selection and Hyperparameter Tuning

Models used for predicting Tesla's option price include linear regression, ridge and lasso regression, KNN, decision tree, random forest, and SVM. For classifying whether the option price was either overvalued or undervalued, logistic regression, LDA, KNN, decision tree, random forest, and SVM were utilized.

Linear regression was chosen as a baseline model for its simplicity and interpretability, while regularization techniques such as ridge and lasso were used to address overfitting. Logistic regression was employed for binary classification due to its straightforward and interpretable results. LDA was considered in the case of linearly separable labels. KNN was selected for its ability to capture non-linear relationships better than linear regression and classification.

Decision tree model was utilized for its ability to handle outliers despite its susceptibility to overfitting in both regression and classification, meanwhile, random forests were employed to mitigate this issue. SVM was considered for both regression and classification because of its ability to handle non-linear relationships in higher dimensions.

For some models, such as KNN, decision tree, random forest, and SVM, a grid search cross-validation was employed to find the optimal hyperparameters while simultaneously using the cross-validation approach. The use of dense neural networks for predicting Tesla's option value and classifying was considered but ultimately dismissed. It was concluded that the regression problem, predicting Tesla's option value, was better off with a simpler model because the dataset was not complex enough for a neural network technique. Additionally, using neural networks for classification tasks would have numerous hyperparameters involved in tuning, such as the number of neurons in each hidden layer, the number of hidden layers, and the number of epochs for forward and backpropagation, making this approach less feasible.

3. Results

3.1 Regression Outcome

The linear regression model, serving as a baseline, achieved a mean cross-validated R-squared of 92.44% and an MSE of 1,178. Lasso and Ridge regularization, with optimized alpha parameters, yielded no improvement over the linear regression model. The SVM model performed slightly worse, with an R-squared of 91.56% and an MSE of 1,316. The KNN algorithm, optimized using grid search and k-means clustering (K=3), demonstrated superior performance, achieving an R-squared of 98.43% and an MSE of 246. The decision tree model further improved the results, with an R-squared of 99.11% and an MSE of 139. The random forest algorithm emerged as the best-performing model, achieving an impressive R-squared of 99.68% and a low MSE of 50.75.

3.2 Classification Outcome

As a baseline model, the logistic regression model achieved a mean cross-validated accuracy score of 87.94%. The LDA model performed slightly worse at 86.92%. The SVM model performed better at 89.26%. Breaking the 90% threshold, the decision tree model achieved a 91.52% mean cross-validated accuracy score, while KNN at K = 3, achieved a mean cross-validated accuracy score of 91.58%. Finally, the best classification model, random forest algorithm, achieved a mean cross-validated accuracy score of 93.52%.

4. Conclusion

The random forest algorithm emerged as the superior model for both predicting Tesla's European option value and classifying whether the option was overvalued or undervalued. In the regression task, random forest achieved a 99.68% mean cross-validated R-squared score with a minimal mean cross-validated MSE, demonstrating its ability to accurately predict the option value with minimal error. Similarly, in the classification task, random forest outperformed the second-best model, the decision tree, by 2%, achieving an accuracy score of 93.52%. Consequently, the random forest model was selected to predict Tesla's option value and classify them as either overvalued (0) or undervalued (1). The model's mean cross-validated ROC-AUC score of 0.9736 further confirmed its excellent performance in distinguishing classes across multiple evaluations. Despite the random forest model's high mean cross-validated R-squared score, indicating strong predictive performance, its ability to mitigate overfitting through the bagging algorithm is a notable advantage. However, the absence of true observed values for the test data limits capacity to comprehensively assess the model's generalization performance, leaving some concerns regarding overfitting unaddressed.

Appendix

May 1, 2024

EXPLORATORY DATA ANALYSIS (EDA)

```
[ ]: import pandas as pd
     import numpy as np

     from google.colab import files
     uploaded = files.upload()

     #load training dataset
     df_train = pd.read_csv('option_train.csv')
     df_train.head()
```

```
<IPython.core.display.HTML object>

Saving option_train.csv to option_train.csv
```

```
[ ]:    Unnamed: 0    Value        S     K       tau       r     BS
     0           1  348.500  1394.46  1050  0.128767  0.0116  Under
     1           2  149.375  1432.25  1400  0.679452  0.0113  Under
     2           3  294.500  1478.90  1225  0.443836  0.0112  Under
     3           4    3.375  1369.89  1500  0.117808  0.0119   Over
     4           5   84.000  1366.42  1350  0.298630  0.0119  Under
```

```
[ ]: #dropping the first column
     df_train = df_train.drop(['Unnamed: 0'], axis = 1)

     #check for nulls
     df_train.isnull().sum()
```

```
[ ]: Value    0
     S        0
     K        0
     tau      0
     r        0
     BS       0
     dtype: int64
```

```
[ ]: #check for zero or negative values
     print(df_train.describe())
```

```
              Value            S            K          tau            r
count   5000.000000  5000.000000  5000.000000  5000.000000  5000.000000
mean     140.316869  1426.643916  1370.244000     0.327615     0.011468
std      125.155000    56.051523   172.679107     0.231184     0.000448
min        0.281250  1264.740000   750.000000     0.084932     0.010600
25%       45.750000  1387.670000  1275.000000     0.142466     0.011100
50%      105.125000  1434.320000  1400.000000     0.238356     0.011400
75%      200.406250  1469.440000  1475.000000     0.463014     0.011700
max      685.500000  1527.460000  1995.000000     0.989041     0.012900
```

[ ]:
```
#check for BS count
df_train.BS.value_counts()
```

[ ]:
```
BS
Under    3868
Over     1132
Name: count, dtype: int64
```

[ ]:
```
#encode under as 0 and over as 1 in the BS column
mapping = {'Under': 0, 'Over': 1}
df_train['BSE'] = df_train['BS'].map(mapping)
print(df_train)
```

```
        Value        S     K       tau       r     BS  BSE
0     348.500  1394.46  1050  0.128767  0.0116  Under    0
1     149.375  1432.25  1400  0.679452  0.0113  Under    0
2     294.500  1478.90  1225  0.443836  0.0112  Under    0
3       3.375  1369.89  1500  0.117808  0.0119   Over    1
4      84.000  1366.42  1350  0.298630  0.0119  Under    0
…         …        …   …         …       …     …  …
4995  325.250  1465.15  1175  0.424658  0.0111  Under    0
4996   36.000  1480.87  1480  0.101370  0.0111   Over    1
4997   90.000  1356.56  1500  0.673973  0.0120  Under    0
4998  175.875  1333.36  1200  0.309589  0.0122  Under    0
4999  106.375  1480.87  1475  0.504110  0.0111  Under    0

[5000 rows x 7 columns]
```

[ ]:
```
#normalize training features data
from sklearn.preprocessing import MinMaxScaler

scaler = MinMaxScaler()

numerical_features = ['S', 'K', 'tau', 'r']

df_train[numerical_features] = scaler.
 ↪fit_transform(df_train[numerical_features])
```

```
print(df_train)
```

```
        Value         S         K       tau         r     BS  BSE
0     348.500  0.493758  0.240964  0.048485  0.434783  Under    0
1     149.375  0.637599  0.522088  0.657576  0.304348  Under    0
2     294.500  0.815164  0.381526  0.396970  0.260870  Under    0
3       3.375  0.400236  0.602410  0.036364  0.565217   Over    1
4      84.000  0.387028  0.481928  0.236364  0.565217  Under    0
…         …         …         …         …         …    …   …
4995  325.250  0.762827  0.341365  0.375758  0.217391  Under    0
4996   36.000  0.822663  0.586345  0.018182  0.217391   Over    1
4997   90.000  0.349498  0.602410  0.651515  0.608696  Under    0
4998  175.875  0.261191  0.361446  0.248485  0.695652  Under    0
4999  106.375  0.822663  0.582329  0.463636  0.217391  Under    0

[5000 rows x 7 columns]
```

```
[ ]: #training and features for regression
X_train = df_train.drop(columns = ['Value', 'BS', 'BSE']) #dropping value, bs,␣
 ↪and bse columns
y_train = df_train['Value'] #all rows column 1
training = df_train.drop(columns = ['BSE', 'BS']) #dropping columns bs and bse
print("Features: ")
print(X_train)
print()
print("Target: ")
print(y_train)
```

```
Features:
             S         K       tau         r
0     0.493758  0.240964  0.048485  0.434783
1     0.637599  0.522088  0.657576  0.304348
2     0.815164  0.381526  0.396970  0.260870
3     0.400236  0.602410  0.036364  0.565217
4     0.387028  0.481928  0.236364  0.565217
…          …         …         …         …
4995  0.762827  0.341365  0.375758  0.217391
4996  0.822663  0.586345  0.018182  0.217391
4997  0.349498  0.602410  0.651515  0.608696
4998  0.261191  0.361446  0.248485  0.695652
4999  0.822663  0.582329  0.463636  0.217391

[5000 rows x 4 columns]

Target:
0       348.500
1       149.375
2       294.500
```

```
3         3.375
4        84.000
         …
4995    325.250
4996     36.000
4997     90.000
4998    175.875
4999    106.375
Name: Value, Length: 5000, dtype: float64
```

REGRESSION MODELS:

Linear Regression

```
[ ]: from sklearn.model_selection import KFold, cross_val_score
     from sklearn.linear_model import LinearRegression

     #setup ols
     ols_model = LinearRegression()

     #setup k-fold cross-validation with k = 10 and compute mse
     folds = 10
     k_fold = KFold(n_splits = folds, random_state = 42, shuffle = True)
     cv_score_ols_r2 = cross_val_score(ols_model, X_train, y_train, cv = k_fold,␣
      ↪scoring = 'r2')
     cv_score_ols_mse = cross_val_score(ols_model, X_train, y_train, cv = k_fold,␣
      ↪scoring = 'neg_mean_squared_error')

     #print the mean cv rmse score
     print("Mean CV MSE score for OLS:", round(-cv_score_ols_mse.mean(), 6))
     print()
     #print the mean cv r2 score
     print("Mean CV R-squared score for OLS:", round(cv_score_ols_r2.mean(), 6))
```

```
Mean CV MSE score for OLS: 1178.304931

Mean CV R-squared score for OLS: 0.924414
```

K Means Clustering

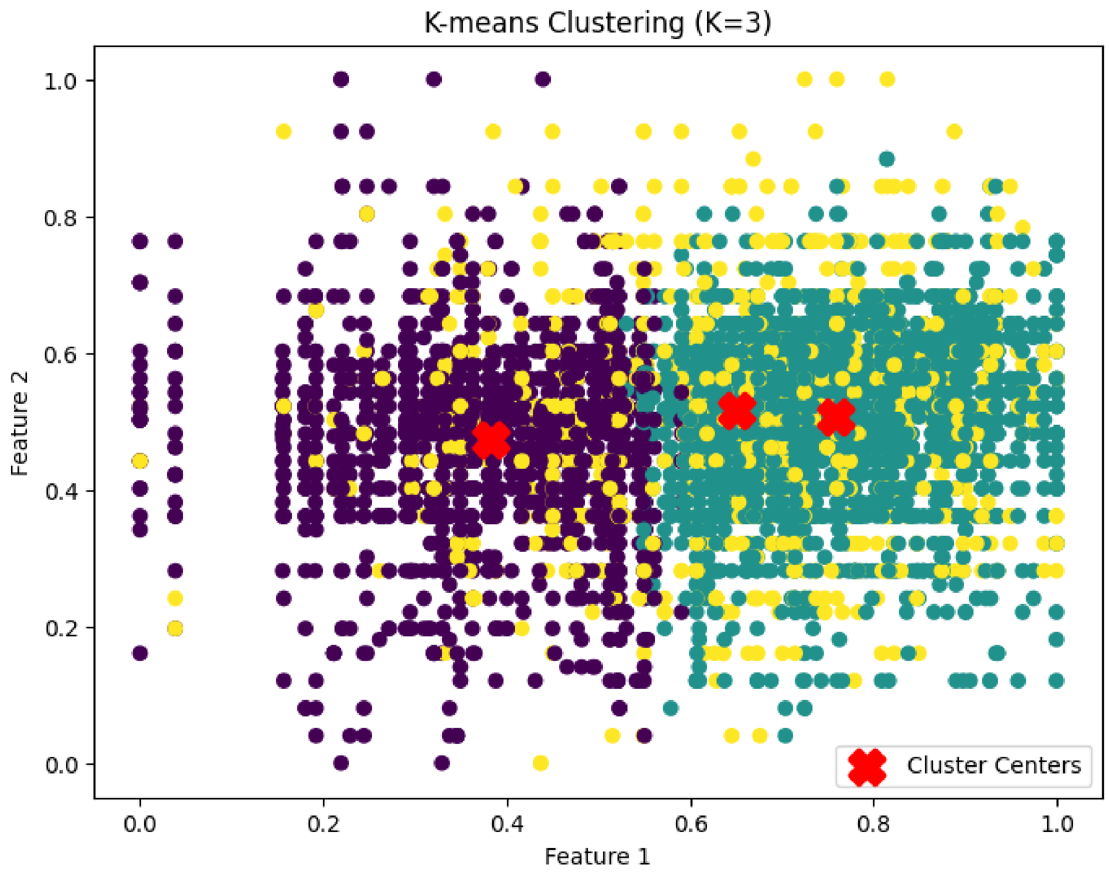
```
[ ]: from sklearn.cluster import KMeans
     from sklearn.metrics import silhouette_score
     import matplotlib.pyplot as plt
     import pandas as pd

     # Assuming X_train is your training feature DataFrame
     max_clusters = 5

     # Function to perform clustering and return silhouette score
```

```python
def perform_clustering(k):
    kmeans = KMeans(n_clusters=k, random_state=42)
    kmeans.fit(X_train)
    cluster_labels = kmeans.labels_
    silhouette_avg = silhouette_score(X_train, cluster_labels)
    return silhouette_avg

# Perform clustering for different values of K and store silhouette scores
silhouette_scores = pd.DataFrame({'K': range(2, max_clusters+1)})
silhouette_scores['Silhouette Score'] = silhouette_scores['K'].
 ↪apply(perform_clustering)

# Find the best number of clusters based on silhouette score
best_k = silhouette_scores.loc[silhouette_scores['Silhouette Score'].idxmax(),␣
 ↪'K']
print(f"Best number of clusters: {best_k}")

# Perform clustering with the best K value
kmeans = KMeans(n_clusters=best_k, random_state=42)
kmeans.fit(X_train)
cluster_labels = kmeans.labels_

# Visualize the clusters (assuming 2D data)
plt.figure(figsize=(8, 6))
plt.scatter(X_train.iloc[:, 0], X_train.iloc[:, 1], c=cluster_labels,␣
 ↪cmap='viridis')
plt.scatter(kmeans.cluster_centers_[:, 0], kmeans.cluster_centers_[:, 1],␣
 ↪marker='X', s=200, linewidths=3,
            color='red', label='Cluster Centers')
plt.xlabel('Feature 1')
plt.ylabel('Feature 2')
plt.title(f'K-means Clustering (K={best_k})')
plt.legend()
plt.show()
```

```
/usr/local/lib/python3.10/dist-packages/sklearn/cluster/_kmeans.py:870:
FutureWarning: The default value of `n_init` will change from 10 to 'auto' in
1.4. Set the value of `n_init` explicitly to suppress the warning
  warnings.warn(
/usr/local/lib/python3.10/dist-packages/sklearn/cluster/_kmeans.py:870:
FutureWarning: The default value of `n_init` will change from 10 to 'auto' in
1.4. Set the value of `n_init` explicitly to suppress the warning
  warnings.warn(
/usr/local/lib/python3.10/dist-packages/sklearn/cluster/_kmeans.py:870:
FutureWarning: The default value of `n_init` will change from 10 to 'auto' in
1.4. Set the value of `n_init` explicitly to suppress the warning
  warnings.warn(
/usr/local/lib/python3.10/dist-packages/sklearn/cluster/_kmeans.py:870:
FutureWarning: The default value of `n_init` will change from 10 to 'auto' in
1.4. Set the value of `n_init` explicitly to suppress the warning
  warnings.warn(
```

```
Best number of clusters: 3
```

```
/usr/local/lib/python3.10/dist-packages/sklearn/cluster/_kmeans.py:870:
FutureWarning: The default value of `n_init` will change from 10 to 'auto' in
1.4. Set the value of `n_init` explicitly to suppress the warning
  warnings.warn(
```



KNN - K Nearest Neighbor

```
[ ]: from sklearn.neighbors import KNeighborsRegressor

     #we need to find an optimal value of k for knn
     #define a range of k values to try
     k_values = [1, 3, 5, 7, 9]
```

```
#dictionary to store mean scores for each k value, k value = key, mean score =␣
 ↪value
knn_mean_scores = {}

#loop over each k value
for k in k_values:
    #create knn model to loop
    knn_reg = KNeighborsRegressor(n_neighbors=k)

    #perform cv score loop
    folds = 10
    k_fold = KFold(n_splits = folds, random_state = 42, shuffle = True)

    #take the mean of the mse score for that k and put taht into␣
 ↪knn_mean_scores dictionary
    knn_mean_scores[k] = -cross_val_score(knn_reg, X_train, y_train, cv =␣
 ↪k_fold, scoring = 'neg_mean_squared_error').mean()

#loop dictionary to print
for k, mse in knn_mean_scores.items():
    print("For k =", k, ", Mean Squared Error (MSE):", round(mse, 6))
```

```
For k = 1 , Mean Squared Error (MSE): 299.427719
For k = 3 , Mean Squared Error (MSE): 246.229558
For k = 5 , Mean Squared Error (MSE): 255.42395
For k = 7 , Mean Squared Error (MSE): 275.004414
For k = 9 , Mean Squared Error (MSE): 287.492181
```

From the loop, we can see that the minimal mse obtained from knn from k = 1, 3, 5, 7, 9, the optimal k value is k = 3

```
[ ]: #use the optimal k value to find the cv R-squared score associated with that␣
      ↪model
     optimal_k = 3
     knn_model = KNeighborsRegressor(n_neighbors = optimal_k).fit(X_train, y_train)
     cv_score_knn_r2 = cross_val_score(knn_model, X_train, y_train, cv = k_fold,␣
      ↪scoring = 'r2')
     print("Mean CV R-squared score for KNN:", round(cv_score_knn_r2.mean(), 6))
     print()
     print("Mean CV RMSE score for KNN (k = 3):", round(knn_mean_scores[3], 6))
```

```
Mean CV R-squared score for KNN: 0.984249

Mean CV RMSE score for KNN (k = 3): 246.229558
```

Decision Tree

```
[ ]: #setup decision tree
     from sklearn.tree import DecisionTreeRegressor
     from sklearn.model_selection import GridSearchCV

     #define the parameter grid with different values for max_depth
     param_dt = {'max_depth': [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]}

     #setup decision tree model
     dt_model = DecisionTreeRegressor()

     #perform grid search with cv to find the best max depth parameter using mse as␣
      ↪metric and n_jobs -1 for parallel processing
     gs_dt_mse = GridSearchCV(dt_model, param_dt, cv = k_fold, scoring =␣
      ↪'neg_mean_squared_error', n_jobs = -1)
     gs_dt_mse.fit(X_train, y_train)

     print("Best parameters:", gs_dt_mse.best_params_)
     print("Best mean squared error:", round(-gs_dt_mse.best_score_, 6))
     print()

     #perform grid search with cv to find the best max depth parameter using␣
      ↪r-squared as metric and n_jobs -1 for parallel processing
     gs_dt_r2 = GridSearchCV(dt_model, param_dt, cv = k_fold, scoring = 'r2', n_jobs␣
      ↪= -1)
     gs_dt_r2.fit(X_train, y_train)
     print("Best parameters:", gs_dt_r2.best_params_)
     print("Best R-squared:", round(gs_dt_r2.best_score_, 6))
```

```
Best parameters: {'max_depth': 10}
Best mean squared error: 139.223729

Best parameters: {'max_depth': 10}
Best R-squared: 0.991017
```

Random Forest

```
[ ]: #setup random forest for baseline MSE to compare if overfitting
     from sklearn.ensemble import RandomForestRegressor

     rf_model = RandomForestRegressor()

     rf_model.fit(X_train, y_train)

     cv_score_rf_mse = cross_val_score(rf_model, X_train, y_train, cv = k_fold,␣
      ↪scoring = 'neg_mean_squared_error')
     cv_score_rf_r2 = cross_val_score(rf_model, X_train, y_train, cv = k_fold,␣
      ↪scoring = 'r2')
     print("Mean CV MSE score for Random Forest:", round(-cv_score_rf_mse.mean(), 6))
     print()
     print("Mean CV R-squared score for Random Forest:", round(cv_score_rf_r2.
      ↪mean(), 6))
```

```
Mean CV MSE score for Random Forest: 50.942357

Mean CV R-squared score for Random Forest: 0.99669
```

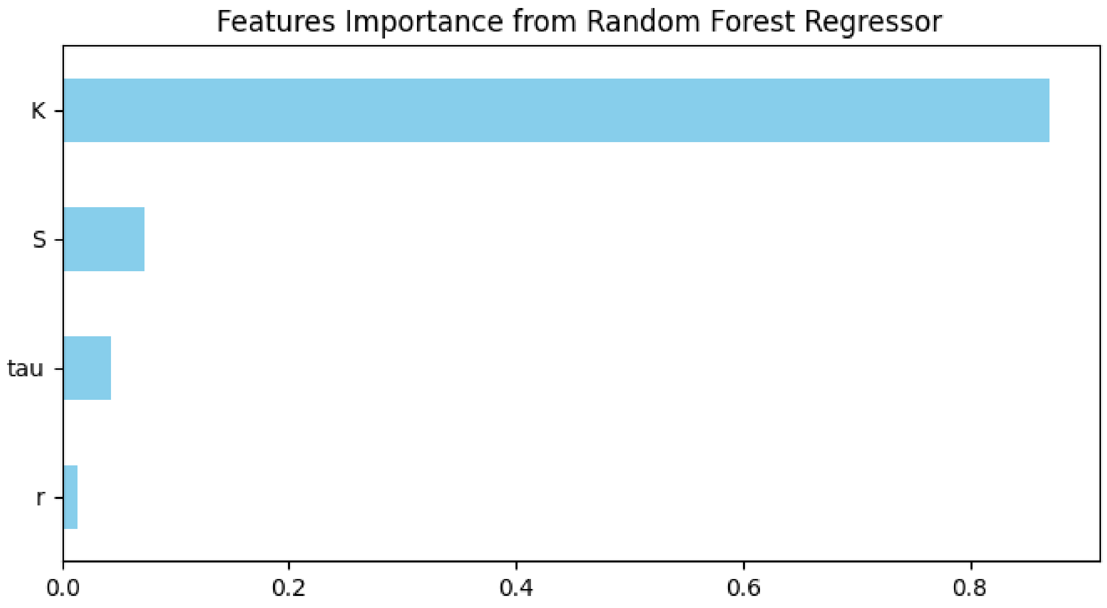
[ ]:
```
#perform a gridsearch to find the optimal mse with the best parameters
#this will take about 25 minutes to run
#define the parameter grid with different values for hyperparameters
param_rf = {
    'n_estimators': [100, 200, 300],  #number of trees in the forest
    'max_depth': [3, 5, 7, 9, 20],   #max depth of the trees aka the # of␣
 ↪splits each decision tree is allowed to make
    'min_samples_split': [2, 5, 10],   #min observations need to split a node
    'min_samples_leaf': [1, 2, 4]}      #min observations need to be at a leaf␣
 ↪node

#setup random forest model
rf_model = RandomForestRegressor()

#perform grid search with cv using mse as metric and njobs -1 to parallel␣
 ↪process
gs_rf_mse = GridSearchCV(rf_model, param_rf, cv = k_fold, scoring =␣
 ↪'neg_mean_squared_error', n_jobs = -1)
gs_rf_mse.fit(X_train, y_train)

#print the best parameters and best mean squared error
print("Best parameters:", gs_rf_mse.best_params_)
print("Best mean squared error:", round(-gs_rf_mse.best_score_, 6))
```

```
Best parameters: {'max_depth': 20, 'min_samples_leaf': 1, 'min_samples_split':
2, 'n_estimators': 300}
Best mean squared error: 50.505522
```

[ ]:
```
#using the parameters found in the best mean squared error
rf_model_best_param = RandomForestRegressor(n_estimators = 300, max_depth = 20,␣
 ↪min_samples_split = 2, min_samples_leaf = 1)

#perform cross-validation
cv_score_rf = cross_val_score(rf_model_best_param, X_train, y_train, cv =␣
 ↪k_fold, scoring = 'r2', n_jobs = -1)

#print the mean R-squared score
print("Mean CV R-squared score for Random Forest:", round(cv_score_rf.mean(),␣
 ↪6))
```

```
Mean CV R-squared score for Random Forest: 0.996751
```

[ ]:
```
import matplotlib.pyplot as plt

#fit best model on entire training data to obtain feature importances
rf_reg = rf_model_best_param.fit(X_train, y_train)
rf_feat_importances = rf_reg.feature_importances_

#create a pandas series to map feature importances to their corresponding␣
↪feature names
importances = pd.Series(rf_feat_importances, index = ['S', 'K', 'tau', 'r'])

#sort the features by importance
sorted_importances = importances.sort_values()

#plotting
plt.figure(figsize=(8,4))
sorted_importances.plot(kind='barh', color='skyblue')
plt.title('Features Importance from Random Forest Regressor')
plt.show()
```



Ridge and Lasso Regression

Ridge:

[ ]:
```
from sklearn.linear_model import Ridge, RidgeCV

#alpha scale
```

```
alphas = 10**np.linspace(10,-2,100)*0.5

ridgecv = RidgeCV(alphas = alphas, scoring = 'r2', cv = k_fold)
ridgecv.fit(X_train, y_train)
optimal_ridge_alpha = ridgecv.alpha_
print('Optimal alpha value for Ridge regression:', round(optimal_ridge_alpha,␣
  ↪4))
print()

#use optimal alpha in ridge model to find cv r-squared and mse
ridge_model = Ridge(alpha = ridgecv.alpha_)

cv_score_ridge_mse = cross_val_score(ridge_model, X_train, y_train, cv =␣
  ↪k_fold, scoring = 'neg_mean_squared_error')
cv_score_ridge_r2 = cross_val_score(ridge_model, X_train, y_train, cv = k_fold,␣
  ↪scoring = 'r2')

print("Mean CV MSE score for Ridge:", round(-cv_score_ridge_mse.mean(), 6))
print()
print("Mean CV R-squared score for Ridge:", round(cv_score_ridge_r2.mean(), 6))
```

```
Optimal alpha value for Ridge regression: 0.0202

Mean CV MSE score for Ridge: 1178.302928

Mean CV R-squared score for Ridge: 0.924414
```

Lasso:

```
[ ]: from sklearn.linear_model import Lasso, LassoCV
     from sklearn.metrics import make_scorer, r2_score #need for finding r2 with␣
       ↪lasso regression

     lassocv = LassoCV(alphas = alphas, cv = k_fold) #lassoCV only uses scoring =␣
       ↪neg mean squared error
     lassocv.fit(X_train, y_train)
     optimal_lasso_alpha = lassocv.alpha_
     print('Optimal alpha value for Lasso regression:', round(optimal_lasso_alpha,␣
       ↪4))
     print()

     #use optimal alpha in ridge model to find cv mse
     lasso_model = Lasso(alpha = lassocv.alpha_)

     cv_score_lasso_mse = cross_val_score(lasso_model, X_train, y_train, cv =␣
       ↪k_fold, scoring = 'neg_mean_squared_error')
```

```
#because lasso doesn't have r2 we have to make it ourselves so we define a␣
↪custom scorer for R2 score to put in cv scoring argumernt
r2_scorer = make_scorer(r2_score, greater_is_better = True)

#use r2_scorer as scoring argument in cv
cv_score_lasso_r2 = cross_val_score(lasso_model, X_train, y_train, cv = k_fold,␣
↪scoring = r2_scorer)

#print mean cv mse and r2 score
print("Mean CV MSE score for Lasso:", round(-cv_score_lasso_mse.mean(), 6))
print()
print("Mean CV R-squared score for Lasso:", round(cv_score_lasso_r2.mean(), 6))
```

```
Optimal alpha value for Lasso regression: 0.0116

Mean CV MSE score for Lasso: 1178.245307

Mean CV R-squared score for Lasso: 0.924418
```

SVM - Support Vector Machine

```
[ ]: from sklearn.svm import SVR

#takes 9 minutes to run
#set up SVM model
svm_model = SVR()

#set up parameters to check
param_svm = {
    'C': [0.1, 1, 10],          #budget for amount that margin can be violated
    'kernel': ['linear', 'poly', 'rbf'],  #kernel type
    'gamma': [0.1, 0.01, 0.001]}  #kernel coefficient for radial kernel

#grid search cv for mse
gs_svr_mse = GridSearchCV(svm_model, param_svm, cv = k_fold, scoring =␣
↪'neg_mean_squared_error', n_jobs = -1)
gs_svr_mse.fit(X_train, y_train)

#grid search cv for r-squared
gs_svr_r2 = GridSearchCV(svm_model, param_svm, cv = k_fold, scoring = 'r2',␣
↪n_jobs = -1)
gs_svr_r2.fit(X_train, y_train)

print("Best parameters:", gs_svr_mse.best_params_)
print("Best MSE:", round(-gs_svr_mse.best_score_, 6))
print()
print("Best parameters:", gs_svr_r2.best_params_)
print("Best R-squared:", round(gs_svr_r2.best_score_, 6))
```

```
Best parameters: {'C': 10, 'gamma': 0.1, 'kernel': 'linear'}
Best MSE: 1316.661485

Best parameters: {'C': 10, 'gamma': 0.1, 'kernel': 'linear'}
Best R-squared: 0.915584
```

EVALUATION METRIC SUMMARY FOR REGRESSION TECHNIQUES:

```
[ ]: print('OLS:')
print("Best mean CV MSE score for OLS:", round(-cv_score_ols_mse.mean(), 6))
print("Best mean CV R-squared score for OLS:", round(cv_score_ols_r2.mean(), 6))
print()

print('KNN:')
print("Best mean CV MSE score for KNN (k = 3):", round(knn_mean_scores[3], 6))
print("Best mean CV R-squared score for KNN (k = 3):", round(cv_score_knn_r2.
  ↪mean(), 6))
print()

print('Decision Tree:')
print("Best mean CV MSE score for Decision Tree", round(-gs_dt_mse.best_score_,␣
  ↪6))
print("Best mean CV R-squared score for Decision Tree:", round(gs_dt_r2.
  ↪best_score_, 6))
print()

print('Random Forest:')
print("Best mean CV MSE score for Random Forest:", round(-gs_rf_mse.
  ↪best_score_, 6))
print("Best mean CV R-squared score for Random Forest:", round(cv_score_rf.
  ↪mean(), 6))
print()

print('Ridge Regression:')
print('Best mean CV MSE score for Ridge at alpha = 0.0202:',␣
  ↪round(-cv_score_ridge_mse.mean(), 6))
print('Best mean CV R-squared score for Ridge at alpha = 0.0202:',␣
  ↪round(cv_score_ridge_r2.mean(), 6))
print()

print('Lasso')
print('Best mean CV MSE score for Lasso at alpha = 0.0116:',␣
  ↪round(-cv_score_lasso_mse.mean(), 6))
print('Best mean CV R-squared score for Lasso at alpha = 0.0116:',␣
  ↪round(cv_score_lasso_r2.mean(), 6))
print()
print('Support Vector Machine:')
print("Best mean CV MSE score for SVM:", round(-gs_svr_mse.best_score_, 6))
print("Best mean CV R-squared score for SVM:", round(gs_svr_r2.best_score_, 6))
```

```
OLS:
Best mean CV MSE score for OLS: 1178.304931
Best mean CV R-squared score for OLS: 0.924414

KNN:
Best mean CV MSE score for KNN (k = 3): 246.229558
Best mean CV R-squared score for KNN (k = 3): 0.984249

Decision Tree:
Best mean CV MSE score for Decision Tree 139.223729
Best mean CV R-squared score for Decision Tree: 0.991017

Random Forest:
Best mean CV MSE score for Random Forest: 50.505522
Best mean CV R-squared score for Random Forest: 0.996751

Ridge Regression:
Best mean CV MSE score for Ridge at alpha = 0.0202: 1178.302928
Best mean CV R-squared score for Ridge at alpha = 0.0202: 0.924414

Lasso
Best mean CV MSE score for Lasso at alpha = 0.0116: 1178.245307
Best mean CV R-squared score for Lasso at alpha = 0.0116: 0.924418

Support Vector Machine:
Best mean CV MSE score for SVM: 1316.661485
Best mean CV R-squared score for SVM: 0.915584
```

CLASSFICATION MODELS:

```
[ ]: #set up data to train classificaiton models

X_clf_train = df_train.drop(['BS', 'BSE'], axis = 1)
y_clf_train = df_train['BSE']

print('Classification Training Features:')
print()
print(X_clf_train)
print()
print('Classification Training Target:')
print()
print(y_clf_train)
```

```
Classification Training Features:
        Value         S         K       tau         r
0     348.500  0.493758  0.240964  0.048485  0.434783
1     149.375  0.637599  0.522088  0.657576  0.304348
2     294.500  0.815164  0.381526  0.396970  0.260870
3       3.375  0.400236  0.602410  0.036364  0.565217
4      84.000  0.387028  0.481928  0.236364  0.565217
…         …         …       …         …         …
4995  325.250  0.762827  0.341365  0.375758  0.217391
4996   36.000  0.822663  0.586345  0.018182  0.217391
4997   90.000  0.349498  0.602410  0.651515  0.608696
4998  175.875  0.261191  0.361446  0.248485  0.695652
4999  106.375  0.822663  0.582329  0.463636  0.217391

[5000 rows x 5 columns]

Classification Training Target:

0       0
1       0
2       0
3       1
4       0
       ..
4995    0
4996    1
4997    0
4998    0
4999    0
Name: BSE, Length: 5000, dtype: int64
```

Logistic Regression:

```
[ ]: from sklearn.linear_model import LogisticRegression
     from sklearn.model_selection import StratifiedKFold

     #set up stratified kfold
     folds_clf = 10
     skf = StratifiedKFold(n_splits = folds_clf, random_state = 42, shuffle = True)

     #set up logit model
     logit_model = LogisticRegression(max_iter = 1_000)

     cv_score_logit_acc = cross_val_score(logit_model, X_train, y_clf_train, cv =␣
      ↪skf, scoring = 'accuracy')

     print('Mean CV accuracy score for Logit:', round(cv_score_logit_acc.mean(), 4))
```

```
Mean CV accuracy score for Logit: 0.8794
```

LDA

```
[ ]: from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
     from sklearn.metrics import accuracy_score, confusion_matrix,␣
      ↪classification_report

     #initialize and train the Linear Discriminant Analysis Classifier
     lda_classifier = LinearDiscriminantAnalysis()
     lda_classifier.fit(X_train, y_clf_train)

     cv_score_lda_acc = cross_val_score(lda_classifier, X_train, y_clf_train, cv =␣
      ↪skf, scoring = 'accuracy')

     print('Mean CV accuracy score for LDA:', round(cv_score_lda_acc.mean(), 6))
     print()

     #predict on the validation set to get a confusion matrix
     y_pred_lda = lda_classifier.predict(X_train)

     #confusion matrix
     conf_matrix_lda = confusion_matrix(y_clf_train, y_pred_lda)
     print('Confusion Matrix:\n', conf_matrix_lda)
     print()

     report_lda = classification_report(y_clf_train, y_pred_lda)
     print('Classification Report:\n', report_lda)
```

```
Mean CV accuracy score for LDA: 0.8692

Confusion Matrix:
 [[3764  104]
 [ 551  581]]

Classification Report:
               precision    recall  f1-score   support

           0       0.87      0.97      0.92      3868
           1       0.85      0.51      0.64      1132

    accuracy                           0.87      5000
   macro avg       0.86      0.74      0.78      5000
weighted avg       0.87      0.87      0.86      5000
```

KNN - K Nearest Neighbor - Classification:

```
[ ]: from sklearn.neighbors import KNeighborsClassifier
```

```python
#define parameters
param_knn_clf = {'n_neighbors': [3, 5, 7, 9]}  #k neighbors

#setup KNN clf model
knn_clf_model = KNeighborsClassifier()

#grid search with cv
gs_knn_clf = GridSearchCV(knn_clf_model, param_knn_clf, cv = skf, scoring =␣
 ↪'accuracy', n_jobs = -1)
gs_knn_clf.fit(X_train, y_clf_train)

print('KNN best parameter: ', gs_knn_clf.best_params_)
print('KNN best mean accuracy: ', round(gs_knn_clf.best_score_, 4))
```

```
KNN best parameter:  {'n_neighbors': 3}
KNN best mean accuracy:  0.9158
```

Decision Tree - Classification:

```python
[ ]: from sklearn.tree import DecisionTreeClassifier

#define parameters
param_dt_clf = {
    'max_depth': [None, 10, 20, 30],  #max depth of the tree
    'min_samples_split': [2, 5, 10],  #min samples need to split a node
    'min_samples_leaf': [1, 2, 4]}  #min samples need at each leaf node

#set up decision tree model for clf
dt_clf_model = DecisionTreeClassifier()

#grid search with cv
gs_dt_clf = GridSearchCV(dt_clf_model, param_dt_clf, cv = skf, scoring =␣
 ↪'accuracy', n_jobs = -1)
gs_dt_clf.fit(X_train, y_clf_train)

print("Decision Tree best parameters:", gs_dt_clf.best_params_)
print("Decision Tree best mean accuracy:", round(gs_dt_clf.best_score_, 4))
```

```
Decision Tree best parameters: {'max_depth': 30, 'min_samples_leaf': 1,
'min_samples_split': 2}
Decision Tree best mean accuracy: 0.9152
```

Random Forest - Classification:

```python
[ ]: from sklearn.ensemble import RandomForestClassifier

#set up random foreset model for clf
rf_clf_model = RandomForestClassifier(n_estimators = 300, max_depth = 20,␣
 ↪min_samples_split = 2, min_samples_leaf = 1)
```

```
#cv using same optimal parameters as in regression for random forest
cv_rf_clf = cross_val_score(rf_clf_model, X_train, y_clf_train, cv = skf,␣
↪scoring = 'accuracy')

print("Random Forest best mean accuracy:", round(cv_rf_clf.mean(), 4))
```

```
Random Forest best mean accuracy: 0.9352
```

SVM - Support Vector Maching - Classification:

[ ]:
```
from sklearn.svm import SVC

#setup SVM model
svm_clf_model = SVC(C = 10, gamma = 0.1, kernel = 'linear')

#cv using same optimal parameters as svm regression
cv_svm_clf = cross_val_score(svm_clf_model, X_train, y_clf_train, cv = skf,␣
↪scoring = 'accuracy')

print("SVM best mean accuracy:", round(cv_svm_clf.mean(), 4))
```

```
SVM best mean accuracy: 0.8926
```

EVALUATION METRIC SUMMARY FOR CLASSIFICATION TECHNIQUES:

[ ]:
```
print('Best mean CV accuracy score for Logit:', round(cv_score_logit_acc.
↪mean(), 4))
print('Best mean CV accuracy score for LDA:', round(cv_score_lda_acc.mean(), 4))
print('Best mean CV accuracy score for KNN: ', round(gs_knn_clf.best_score_, 4))
print("Best mean CV accuracy score for Decision Tree:", round(gs_dt_clf.
↪best_score_, 4))
print("Best mean CV accuracy score for Random Forest:", round(cv_rf_clf.mean(),␣
↪4))
print("Best mean CV accuracy score for SVM:", round(cv_svm_clf.mean(), 4))
```

```
Best mean CV accuracy score for Logit: 0.8794
Best mean CV accuracy score for LDA: 0.8692
Best mean CV accuracy score for KNN:  0.9158
Best mean CV accuracy score for Decision Tree: 0.9152
Best mean CV accuracy score for Random Forest: 0.9352
Best mean CV accuracy score for SVM: 0.8926
```

[ ]:
```
#ROC-AUC for best model

cv_rf_roc_auc = cross_val_score(rf_clf_model, X_train, y_clf_train,
                                cv = skf, scoring = 'roc_auc')

print("Mean ROC AUC for Random Forest:", np.mean(cv_rf_roc_auc))
```

```
Mean ROC AUC for Random Forest: 0.973565356619061
```