FYP Assignment 01
Transient Modelling of Induction Machine

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Abbreviations

ψ_d	$d - axis\ flux\ linkage$
ψ_q	$q - axis\ flux\ linkage$
vds	d-axis stator voltage
vqs	q-axis stator voltage
ids	d-axis stator current
iqs	q-axis stator current
ψ_{ds}	$d - axis\ flux\ linkage$
ψ_{qs}	$q - axis\ flux\ linkage$
Rs	Stator resistance
Rr	Rotor resistance
L_{ls}	$leakage\ inductance\ of\ stator$
L_{lr}	$leakage\ inductance\ of\ rotor$
L_m	$main\ magnetizing\ inductance$
L_{ms}	$magnetizing\ inductance\ of\ stator$
L_{msr}	$mutual\ inductance\ statorand\ rotor$
θ_r	$rotor\ angle$
θ_s	$stator\ angle$
V_{as}	$stator\ voltage\ phase$
V_{bs}	$stator\ voltage\ phase$
V_{as}	$stator\ voltage\ phase$
V_{bs}	$stator\ voltage\ phase$
V_{as}	$stator\ voltage\ phase$

Chapter 1

DQ transformation Calculations

1.1 Problem Statement

2. With the above transformations, by utilising the supplied references "Induction machine SRF modelling part I and II" and any other resources you can find, carry on developing the transient model of such induction machine in the synchronous reference frame. Notice that the supplied references use DQ to represent the static two-axis and XY to represent the general rotating reference frame, which means the DQ and XY in these references correspond to α β and DQ in most modern references and the main lecture notes. Also, in the supplied references, rotor side variables are given as referred values by default and without using the prime notation.

Solution

Modelling in general frame of reference. Voltage equation of stator and rotor in natural frame of reference.

$$V_{abcs=}R_s i_{abcs+}\frac{d}{dt}\psi_{abcs}(1.1)$$

$$V_{abcr=}R_r i_{abcr+}\frac{d}{dt}\psi_{abcr}(1.2)$$

$$V_{abcs=}\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} \tag{1.3}$$

$$V_{abcr=} \begin{bmatrix} V_{ar} \\ V_{br} \\ V_{cr} \end{bmatrix} \tag{1.4}$$

$$I_{abcs=} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \tag{1.5}$$

$$I_{abcr=} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} \tag{1.6}$$

Now below are the equations for flux linkages

$$\psi_{abcs=} L_{abcs} i_{abcs+} L_{abcsr} i_{abcr} (1.7)$$

$$\psi_{abcr=} L_{abcrs}\ i_{abcs+} L_{abcr} i_{abcr} (1.8)$$

Now presenting self inductances in matrices.

$$L_{abcr=} \begin{bmatrix} L_{rr} & \frac{-1}{2} L_{mr} & \frac{-1}{2} L_{mr} \\ \frac{-1}{2} L_{mr} & \text{Ł}_{rr} & \frac{-1}{2} L_{mr} \\ \frac{-1}{2} L_{mr} & \frac{-1}{2} L_{mr} & \text{Ł}_{rr} \end{bmatrix} \tag{1.9}$$

$$L_{abcs=} \begin{bmatrix} L_{ss} & \frac{-1}{2} L_{ms} & \frac{-1}{2} L_{ms} \\ \frac{-1}{2} L_{ms} & \text{Ł}_{ss} & \frac{-1}{2} L_{ms} \\ \frac{-1}{2} L_{ms} & \frac{-1}{2} L_{mr} & \text{Ł}_{ss} \end{bmatrix} \tag{1.10}$$

L_{abcsr} and L_{abcrs}represent the coupling between stator and rotor variable.

$$L_{abcrs} \ = L_{abcsr} \tag{1.11}$$

$$L_{abcsr=}\begin{bmatrix} L_{sara} & L_{sarb} & L_{sarc} \\ L_{sbra} & L_{sbrb} & L_{sbrc} \\ L_{scra} & L_{scrb} & L_{scrc} \end{bmatrix} \tag{1.12}$$

$$L_{abcrs} = \begin{bmatrix} L_{rasa} & L_{rasb} & L_{rasc} \\ L_{rbsa} & L_{rbsb} & L_{rbsc} \\ L_{rcsa} & L_{rcsb} & L_{rcsc} \end{bmatrix} \tag{1.13}$$

$$L_{sara} = Lrasa = L_{srm} cos\Theta_r \tag{1.14}$$

$$L_{sarb} = L_{rbas} = L_{srm} cos(\Theta_r + \frac{2\pi}{3}) \tag{1.15}$$

$$L_{sarc} = L_{rcsa} = L_{srm} cos(\Theta_r - \frac{2\pi}{3}) \tag{1.16}$$

$$L_{sbra} = L_{rbsb} = L_{srm} cos(\Theta_r - \frac{2\pi}{3}) \tag{1.17}$$

$$L_{sbrb} = L_{rbsb} = L_{srm} cos(\Theta_r) \tag{1.18}$$

$$L_{sbrc} = L_{rcsb} = L_{srm} cos(\Theta_r + \frac{2\pi}{3}) \tag{1.19}$$

$$L_{scra} = L_{rasc} = L_{srm} cos(\Theta_r + \frac{2\pi}{3}) \tag{1.20}$$

$$L_{scrb} = L_{rbsc} = L_{srm} cos(\Theta_r - \frac{2\pi}{3}) \tag{1.21}$$

$$L_{scrc} = L_{rcsc} = L_{srm} cos(\Theta_r) \tag{1.22}$$

$$L_{abcsr=}L_{abcrs} \begin{bmatrix} cos(\Theta_r) & cos(\Theta_r + \frac{2\pi}{3}) & cos(\Theta_r - \frac{2\pi}{3}) \\ cos(\Theta_r - \frac{2\pi}{3}) & cos(\Theta_r) & cos(\Theta_r + \frac{2\pi}{3}) \\ cos(\Theta_r + \frac{2\pi}{3}) & cos(\Theta_r - \frac{2\pi}{3}) & cos(\Theta_r) \end{bmatrix} L_{srm} \tag{1.23}$$

$$L_{abcsr}^T = \begin{bmatrix} cos(\Theta_r) & cos(\Theta_r - \frac{2\pi}{3}) & cos(\Theta_r + \frac{2\pi}{3}) \\ cos(\Theta_r + \frac{2\pi}{3}) & cos(\Theta_r) & cos(\Theta_r - \frac{2\pi}{3}) \\ cos(\Theta_r - \frac{2\pi}{3}) & cos(\Theta_r + \frac{2\pi}{3}) & cos(\Theta_r) \end{bmatrix} L_{srm} \tag{1.24}$$

$$L_{srm} = \frac{N_s N_r}{R_m}$$

(1.25)

$$\int_0^t \omega_r \, dt \tag{1.26}$$

Now referring all the rotor variables to stator.

$$i'_{abcr} = (\frac{Nr}{Ns}) i_{abcr} \tag{1.27}$$

$$r'_{abcr} = (\frac{Ns}{Nr})^2 r_{abcr}$$

(1.28)

$$L'_{abcr} = (\frac{Ns}{Nr})^2 L_{abcr} \tag{1.29}$$

$$L'_{abcr=} \begin{bmatrix} L_{rs} + L_{mr} & \frac{-1}{2} L_{mr} & \frac{-1}{2} L_{mr} \\ \frac{-1}{2} L_{mr} & Ł_{rr} + L_{mr} & \frac{-1}{2} L_{mr} \\ \frac{-1}{2} L_{mr} & \frac{-1}{2} L_{mr} & Ł_{rr} + L_{mr} \end{bmatrix} (\frac{N_s}{N_r})^2 \tag{1.30}$$

$$L_{ms} = (\frac{Ns}{Nr}) L_{srm} \tag{1.31}$$

$$L'_{rr} = (\frac{Ns}{Nr})^2 L_{rr} \tag{1.32}$$

$$L_{ms} = (\frac{Ns}{Nr})^2 L_{mr} \tag{1.33}$$

$$L'_{abcr=} \begin{bmatrix} L'_{lr} + L_{ms} & \frac{-1}{2} L_{ms} & \frac{-1}{2} L_{ms} \\ \frac{-1}{2} L_{ms} & Ł'_{lr} + L_{ms} & \frac{-1}{2} L_{ms} \\ \frac{-1}{2} L_{ms} & \frac{-1}{2} L_{ms} & Ł'_{lr} + L_{ms} \end{bmatrix} \tag{1.34}$$

$$L_{abcs=} \begin{bmatrix} L_{ls} + L_{ms} & \frac{-1}{2} L_{ms} & \frac{-1}{2} L_{ms} \\ \frac{-1}{2} L_{ms} & Ł_{ls} + L_{ms} & \frac{-1}{2} L_{ms} \\ \frac{-1}{2} L_{ms} & \frac{-1}{2} L_{ms} & Ł_{ls} + L_{ms} \end{bmatrix} \tag{1.35}$$

Now eq 1.1 and 1.2 will become as follows;

$$L'_{abcsr} = (\frac{Ns}{Nr})L_{abcsr} = \begin{bmatrix} cos(\Theta_r) & cos(\Theta_r + \frac{2\pi}{3}) & cos(\Theta_r - \frac{2\pi}{3}) \\ cos(\Theta_r - \frac{2\pi}{3}) & cos(\Theta_r) & cos(\Theta_r + \frac{2\pi}{3}) \\ cos(\Theta_r + \frac{2\pi}{3}) & cos(\Theta_r - \frac{2\pi}{3}) & cos(\Theta_r) \end{bmatrix} L_{sm} \quad (1.36)$$

$$L'_{abcrs} = (\frac{Ns}{Nr})L_{abcrs} = \begin{bmatrix} cos(\Theta_r) & cos(\Theta_r - \frac{2\pi}{3}) & cos(\Theta_r + \frac{2\pi}{3}) \\ cos(\Theta_r + \frac{2\pi}{3}) & cos(\Theta_r) & cos(\Theta_r - \frac{2\pi}{3}) \\ cos(\Theta_r - \frac{2\pi}{3}) & cos(\Theta_r + \frac{2\pi}{3}) & cos(\Theta_r) \end{bmatrix} L_{sm} \quad (1.37)$$

$$V_{abcs=}R_s i_{abcs+}\frac{d}{dt}\psi_{abcs}(1.38)$$

$$V'_{abcr=}R_r i'_{abcr+}\frac{d}{dt}\psi'_{abcr}(1.39)$$

$$\psi_{abcs=}L_{abcs}i_{abcs+}L'_{abcsr}i'_{abcr}(1.40)$$

$$\psi'_{abcr=}L'_{abcrs}i_{abcs+}L'_{abcr}i'_{abcr}(1.41)$$

Putting equation 1.40 and 1.41 in equations 1.38 and 1.39 respectively

$$V_{abcs=}R_s i_{abcs+}\frac{d}{dt}(L_{abcs}i_{abcs+}L'_{abcsr}i'_{abcr)} \quad (1.42)$$

$$V'_{abcr=}R_r i'_{abcr+}\frac{d}{dt}\psi'(L'_{abcrs}i_{abcs+}L'_{abcr}i'_{abcr)} \quad (1.43)$$

Now transforming from abc to dq frame of reference i.e. Park Transformation

$$f_{dqs} = k_s f_{abcs} \quad (1.44)$$

$$k_s = \frac{2}{3}\begin{bmatrix} cos(\Theta_g) & cos(\Theta_g - \frac{2\pi}{3}) & cos(\Theta_g + \frac{2\pi}{3}) \\ -sin(\Theta_g) & -sin(\Theta_g - \frac{2\pi}{3}) & -sin(\Theta_g + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (1.45)$$

Now reverse Park Transformation will be;

$$f_{abcs} = k_s^{-1} f_{dqs} \tag{1.46}$$

$$k_s^{-1} = \begin{bmatrix} cos(\Theta_g) & -sin(\Theta_g) & 1 \\ cos(\Theta_g - \frac{2\pi}{3}) & -sin(\Theta_g - \frac{2\pi}{3}) & 1 \\ cos(\Theta_g + \frac{2\pi}{3}) & -sin(\Theta_g + \frac{2\pi}{3}) & 1 \end{bmatrix} \tag{1.47}$$

Similarly rotor variables;

$$f_{dqr} = k_r f_{abcr} \tag{1.48}$$

$$k_r = \frac{2}{3} \begin{bmatrix} cos(\Theta_g - \Theta_r) & cos(\Theta_g - \Theta_r - \frac{2\pi}{3}) & cos(\Theta_g - \Theta_r + \frac{2\pi}{3}) \\ -sin(\Theta_g - \Theta_r) & -sin(\Theta_g - \Theta_r - \frac{2\pi}{3}) & -sin(\Theta_g - \Theta_r + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{1.49}$$

Now reverse Park Transformation will be;

$$f_{abcr} = k_r^{-1} f_{dqr} \tag{1.50}$$

$$k_r^{-1} = \begin{bmatrix} cos(\Theta_g - \Theta_r) & -sin(\Theta_g - \Theta_r) & 1 \\ cos(\Theta_g - -\Theta_r \frac{2\pi}{3}) & -sin(\Theta_g - \Theta_r - \frac{2\pi}{3}) & 1 \\ cos(\Theta_g - \Theta_r + \frac{2\pi}{3}) & -sin(\Theta_g - \Theta_r + \frac{2\pi}{3}) & 1 \end{bmatrix} \tag{1.51}$$

Now transforming 1.37 and 1.38 equation

$$k_s^{-1} V_{dqs} = R_s k_s^{-1} i_{dqs} + \frac{d}{dt} k_s^{-1} \psi_{dqs} \tag{1.52}$$

Multiply both sides with k_s

$$V_{dqs} = R_s i_{dqs} + k_s(\frac{d}{dt} k_s^{-1} \psi_{dqs} + k_s^{-1} \frac{d}{dt} \psi_{dqs}) \tag{1.53}$$

$$V_{dqs} = R_s i_{dqs} + k_s \frac{d}{dt} k_s^{-1} \psi_{dqs} + k_s k_s^{-1} \frac{d}{dt} \psi_{dqs}) \tag{1.54}$$

$$V_{dqs} = R_s i_{dqs} + k_s \frac{d}{dt} k_s^{-1} \psi_{dqs} + \frac{d}{dt} \psi_{dqs}) \tag{1.55}$$

Now similarly for equation 1.38 or 1.44

$$k_r^{-1}V'_{dqr} = R_r k_r^{-1} i'_{dqr} + \frac{d}{dt} k_r^{-1} \psi'_{dqr} \tag{1.56}$$

Multiply by Kr on both sides;

$$k_r k_r^{-1} V'_{dqr} = k_r R_r k_r^{-1} i'_{dqr} + k_r (\frac{d}{dt} k_r^{-1} \psi'_{dqr} + k_r^{-1} \frac{d}{dt} \psi'_{dqr}) \tag{1.57}$$

$$V'_{dqr} = R_r i'_{dqr} + \frac{d}{dt} \psi'_{dqr} + \frac{d}{dt} k_r^{-1} \psi'_{dqr} \tag{1.58}$$

$$k_s \frac{d}{dt}[k_s^{-1}] = K_s \frac{d}{dt}[k_s^{-1}] = \omega_g \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \omega_g W_g \tag{1.59}$$

Now equation 1.56 will become

$$V_{dqs} = R_s i_{dqs} + \omega_g W_g \frac{d}{dt} \psi_{dqs} + \frac{d}{dt} \psi_{dqs}) \tag{1.60}$$

$$k_r \frac{d}{dt}[k_r^{-1}] = (\omega_g - \omega_r) \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = (\omega_g - \omega_r) W g \tag{1.61}$$

Now equation 1.59 will become

$$V'_{dqr} = R_r i'_{dqr} + (\omega_g - \omega_r) W g \psi'_{dqr} + \frac{d}{dt} \psi'_{dqr} \tag{1.62}$$

Now transforming equation 1.7 and 1.8

$$k_s^{-1} \psi_{dqs} = L_{abcs} k_s^{-1} i_{dqs} + L'_{abcsr} k_r^{-1} i_{dqr} \tag{1.63}$$

Multiply both sides with ks

$$\psi_{dqs} = k_s L_{abcs} k_s^{-1} i_{dqs} + k_s L'_{abcsr} k_r^{-1} i_{dqr} \tag{1.64}$$

Similarly for rotor;

$$k_r^{-1} \psi_{dqr} = L'_{abcrs} k_s^{-1} \ i_{dqs} + L'_{abcr} k_r^{-1} i'_{dqr} \tag{1.65}$$

Multiply both sides with kr

$$\psi_{dqr} = k_r L'_{abcrs} k_s^{-1}\ i_{dqs} + k_r L'_{abcr} k_r^{-1} i'_{dqr} \tag{1.66}$$

$$k_s L_{abcs} k_s^{-1} = L_s = \begin{bmatrix} L_{ls} + \frac{3}{2}L_{ms} & 0 & 0 \\ 0 & L_{ls} + \frac{3}{2}L_{ms} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \tag{1.67}$$

$$k_r L_{abcr} k_r^{-1} = L'_r = \begin{bmatrix} L_{lr} + \frac{3}{2}L_{ms} & 0 & 0 \\ 0 & L_{lr} + \frac{3}{2}L_{ms} & 0 \\ 0 & 0 & L_{lr} \end{bmatrix} \tag{1.68}$$

$$L_s L'_{abcsr} k_r^{-1} = \frac{3}{2} \begin{bmatrix} L_{ms} & 0 & 0 \\ 0 & L_{ms} & 0 \\ 0 & 0 & 0 \end{bmatrix} = M = \begin{bmatrix} L_m & 0 & 0 \\ 0 & L_m & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1.69}$$

$$k_r L'_{abcrs} k_s^{-1} = \frac{3}{2} \begin{bmatrix} L_{ms} & 0 & 0 \\ 0 & L_{ms} & 0 \\ 0 & 0 & 0 \end{bmatrix} = M = \begin{bmatrix} L_m & 0 & 0 \\ 0 & L_m & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1.70}$$

Now equation 1.65 and 1.67 will be

$$\psi_{dqs} = L_s i_{dqs} + M i'_{dqr} \tag{1.71}$$

$$\psi_{dqr} = M i_{dqs} + L'_r i'_{dqr} \tag{1.72}$$

Substituting above equations in 1.59 and 1.63

$$V_{dqs} = R_s i_{dqs} + \omega_g W_g L_s i_{dqs} + \omega_g W_g M i'_{dqr} + \frac{d}{dt} L_s i_{dqs} + \frac{d}{dt} M i'_{dqr} \tag{1.73}$$

$$V'_{dqr} = R'_r i'_{dqr} + (\omega_g - \omega_r) W_g M i_{dqs} + (\omega_g - \omega_r) W_g L'_r i'_{dqr} + \frac{d}{dt} M i_{dqs} + \frac{d}{dt} L'_r i'_{dqr} \tag{1.74}$$

expanding above equations

$$V_{ds} = (R_s + \frac{d}{dt} L_s) i_{ds} - \omega_g L_s i_{qs} + \frac{d}{dt} L_m i'_{dr} - \omega_g L_m i'_{dqr} \tag{1.75}$$

$$V_{qs} = \omega_g L_s i_{ds} + (R_s + \frac{d}{dt} L_s) i_{qs} + \omega_g L_m i'_{dr} + \frac{d}{dt} L_m i'_{qr} \tag{1.76}$$

$$V_{os} = (R_s + \frac{d}{dt} L_s) i_{os} \tag{1.77}$$

$$V'_{dr} = \frac{d}{dt} L_m i_{ds} - (\omega_g - \omega_r) L_m i_{qs} + (R'_r + \frac{d}{dt} L_r) i'_{dqr} - (\omega_g - \omega_r) L'_r i'_{qr} \tag{1.78}$$

$$V'_{qr} = \frac{d}{dt} L_m i_{qs} + (\omega_g - \omega_r) L_m i_{ds} + (R'_r + \frac{d}{dt} L_r) i'_{dqr} + (\omega_g - \omega_r) L'_r i'_{dr} \tag{1.79}$$

$$V'_{or} = (R'_r + \frac{d}{dt} L'_r) i'_{or} \tag{1.80}$$

Zero sequence is neglected for balanced system

$$\begin{bmatrix} V_{ds} \\ V_{qs} \\ V_{dr} \\ V_{qr} \end{bmatrix} = \begin{bmatrix} r_s + \frac{d}{dt} L_s & -\omega_g L_s & \frac{d}{dt} L_m & -\omega_g L_m \\ \omega_g L_s & r_s + \frac{d}{dt} L_s & \omega_g L_m & \frac{d}{dt} L_m \\ \frac{d}{dt} L_m & -(\omega_g - \omega_r) L_m & r_s + \frac{d}{dt} L_r & -(\omega_g - \omega_r) L_r \\ (\omega_g - \omega_r) L_m & \frac{d}{dt} L_m & (\omega_g - \omega_r) L_r & r_s + \frac{d}{dt} L_r \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} \tag{1.81}$$

where

$$L_s = L_l s + \frac{3}{2} L_{ms} \tag{1.82}$$

$$L_r = L_l r + \frac{3}{2} L_{ms} \tag{1.83}$$

$$L_m = \frac{3}{2} L_{ms} \tag{1.84}$$

$$L_{ss} = L_{ls} + L_m \ and \ L_{rr} = L_l r + L_m \tag{1.85}$$

$$V_{ds} = r_s i_{ds} + (L_{ls} + L_m) \frac{d}{dt} [i_{ds}] - \omega_g (L_{ls} + L_m) i_{qs} + L_m \frac{d}{dt} [i'_{dr}] - \omega_g L_m [i'_{qr}] \tag{1.86}$$

$$V_{qs} = \omega_g (L_{ls} + L_m) i_{ds} + (L_{ls} + L_m) \frac{d}{dt} [i_{qs}] + r_s i_{qs} + L_m \frac{d}{dt} [i'_{qr}] + \omega_g L_m [i'_{dr}] \tag{1.87}$$

$$V'_{dr} = L_m \frac{d}{dt} i_{ds} - (\omega_g - \omega_r) L_m i_{qs} + r'_r [i'_{dr}] + (L'_{lr} + L_{lm}) \frac{d}{dt} i'_{dr} - (\omega_g - \omega_r)(L'_{lr} + L_m)[i'_{qr}] \tag{1.88}$$

$$V'_{qr} = L_m \frac{d}{dt} i_{qs} + (\omega_g - \omega_r) L_m i_{ds} + r'_r [i'_{qr}] + (L'_{lr} + L_{lm}) \frac{d}{dt} i'_{qr} + (\omega_g - \omega_r)(L'_{lr} + L_m)[i'_{dr}] \tag{1.89}$$

Chapter 2

Transient Torque

2.1 Problem Statement

2. The transient expression for the machine torque is the derivative of the instantaneous total mutual inductance energy over the angular displacement $\frac{2}{P}\theta_r$ (assuming general case with pole-pair number of $\frac{P}{2}$). Then, we have:

$$T_e = [i_{sa} \quad i_{sb} \quad i_{sc}] \frac{d}{d(\frac{2}{P})} \begin{bmatrix} L_{ms}\cos\theta_r & L_{ms}\cos\theta_r + \frac{2\pi}{3} & L_{ms}\cos\theta_r - \frac{2\pi}{3} \\ L_{ms}\cos\theta_r - \frac{2\pi}{3} & L_{ms}\cos\theta_r & L_{ms}\cos\theta_r + \frac{2\pi}{3} \\ L_{ms}\cos\theta_r + \frac{2\pi}{3} & L_{ms}\cos\theta_r - \frac{2\pi}{3} & L_{ms}\cos\theta_r \end{bmatrix} \begin{bmatrix} i'_{ra} \\ i'_{rb} \\ i'_{rc} \end{bmatrix} \tag{2.1}$$

Find the transient torque expression as a function of the DQ variables in synchronous rotating reference frame.

Solution

$$[i_{abcs}]^T \frac{d}{d(\frac{2\theta_r}{P})}[L_{abcsr}]i'_{abcr} \tag{2.2}$$

where

$$i_{abcs} = \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \tag{2.3}$$

$$i'_{abcr} = \begin{bmatrix} i'_{ar} \\ i'_{br} \\ i'_{cr} \end{bmatrix} \tag{2.4}$$

$$L_{abcsr} = \begin{bmatrix} L_{ms}\cos(\theta_r) & L_{ms}\cos(\theta_r + \frac{2\pi}{3}) & L_{ms}\cos(\theta_r - \frac{2\pi}{3}) \\ L_{ms}\cos\theta_r - \frac{2\pi}{3} & L_{ms}\cos\theta_r & L_{ms}\cos(\theta_r + \frac{2\pi}{3}) \\ L_{ms}\cos(\theta_r + \frac{2\pi}{3}) & L_{ms}\cos(\theta_r - \frac{2\pi}{3}) & L_{ms}\cos(\theta_r) \end{bmatrix} \tag{2.5}$$

Now transforming to DQ frame of reference assume P=2

$$T_e = (k_s^{-1} i_{dqs})^T \frac{d}{d\theta_r}[L_{abcsr}][k_r^{-1} i'_{dqr}] \tag{2.6}$$

$$k_s^{-1} i_{dqs} = \begin{bmatrix} \cos(\theta_g) & -\sin(\theta_g) & 1 \\ \cos(\theta_g - \frac{2\pi}{3}) & -\sin(\theta_g - \frac{2\pi}{3}) & 1 \\ \cos(\theta_g - \frac{2\pi}{3}) & -\sin(\theta_g - \frac{2\pi}{3}) & 1 \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{0s} \end{bmatrix} \tag{2.7}$$

$$k_s^{-1} i_{dqs} = \begin{bmatrix} \cos(\theta_g) i_{ds} - \sin(\theta_g) i_{qs} + i_{0s} \\ \cos(\theta_g - \frac{2\pi}{3} i_{ds}) - \sin(\theta_g - \frac{2\pi}{3} i_{qs}) + i_{0s} \\ \cos(\theta_g - \frac{2\pi}{3}) i_{ds} - \sin(\theta_g - \frac{2\pi}{3}) i_{qs} + i_{0s} \end{bmatrix} \tag{2.8}$$

$$\frac{d}{d\theta_r} L_{abcsr} = \frac{d}{d\theta_r} \begin{bmatrix} L_{ms}\cos(\theta_r) & L_{ms}\cos(\theta_r + \frac{2\pi}{3}) & L_{ms}\cos(\theta_r - \frac{2\pi}{3}) \\ L_{ms}\cos(\theta_r - \frac{2\pi}{3}) & L_{ms}\cos(\theta_r) & L_{ms}\cos(\theta_r + \frac{2\pi}{3}) \\ L_{ms}\cos(\theta_r + \frac{2\pi}{3}) & L_{ms}\cos(\theta_r - \frac{2\pi}{3}) & L_{ms}\cos\theta_r \end{bmatrix} \tag{2.9}$$

$$\frac{d}{d\theta_r} L_{abcsr} = L_{ms} \begin{bmatrix} -\sin(\theta_r & -\sin\theta_r + \frac{2\pi}{3}) & -\sin(\theta_r - \frac{2\pi}{3}) \\ -\sin(\theta_r - \frac{2\pi}{3}) & -\sin(\theta_r) & -\sin(\theta_r + \frac{2\pi}{3}) \\ -\sin(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) & -\sin(\theta_r) \end{bmatrix} \tag{2.10}$$

$$[k_r'^{-1} i'_{dqr}] = \begin{bmatrix} \cos(\theta_g - \theta_r) & -\sin(\theta_g - \theta_r) & 1 \\ \cos(\theta_g - \theta_r - \frac{2\pi}{3}) & -\sin(\theta_g - \theta_r - \frac{2\pi}{3}) & 1 \\ \cos(\theta_g - \theta_r - \frac{2\pi}{3}) & -\sin(\theta_g - \theta_r - \frac{2\pi}{3}) & 1 \end{bmatrix} \begin{bmatrix} i'_{dr} \\ i'_{qr} \\ i'_{0r} \end{bmatrix} \tag{2.11}$$

$$[k_r'^{-1} i'_{dqr}] = \begin{bmatrix} \cos(\theta_g - \theta_r) i'_{dr} - \sin(\theta_g - \theta_r) i'_{qr} + i'_{0r} \\ \cos(\theta_g - \theta_r - \tfrac{2\pi}{3}) i'_{dr} - \sin(\theta_g - \theta_r - \tfrac{2\pi}{3}) i'_{qr} + i'_{0r} \\ \cos(\theta_g - \theta_r - \tfrac{2\pi}{3}) i'_{dr} - \sin(\theta_g - \theta_r - \tfrac{2\pi}{3}) i'_{qr} + i'_{0r} \end{bmatrix} \tag{2.12}$$

$$\mathrm{T}_e = \begin{bmatrix} \cos(\theta_g) i_{ds} - \sin(\theta_g) i_{qs} \\ \cos(\theta_g - \tfrac{2\pi}{3} i_{ds}) - \sin(\theta_g - \tfrac{2\pi}{3} i_{qs}) \\ \cos(\theta_g - \tfrac{2\pi}{3}) i_{ds} - \sin(\theta_g - \tfrac{2\pi}{3}) i_{qs} \end{bmatrix} \cdot \begin{bmatrix} -\sin(\theta_r & -\sin\theta_r + \tfrac{2\pi}{3}) & -\sin(\theta_r - \tfrac{2\pi}{3}) \\ -\sin(\theta_r - \tfrac{2\pi}{3}) & -\sin(\theta_r) & -\sin(\theta_r + \tfrac{2\pi}{3}) \\ -\sin(\theta_r + \tfrac{2\pi}{3}) & \cos(\theta_r - \tfrac{2\pi}{3}) & -\sin(\theta_r) \end{bmatrix} \cdot$$

$$\begin{bmatrix} \cos(\theta_g - \theta_r) i'_{dr} - \sin(\theta_g - \theta_r) i'_{qr} \\ \cos(\theta_g - \theta_r - \tfrac{2\pi}{3}) i'_{dr} - \sin(\theta_g - \theta_r - \tfrac{2\pi}{3}) i'_{qr} \\ \cos(\theta_g - \theta_r - \tfrac{2\pi}{3}) i'_{dr} - \sin(\theta_g - \theta_r - \tfrac{2\pi}{3}) i'_{qr} \end{bmatrix} \tag{2.13}$$

After this matrix multiplication, this will be the given results for 2 pole machine.

$$\frac{3}{2} L_m (i_{qs} i_{dr} - i_{ds} i_{qr}) \tag{2.14}$$

for any pole machine,

$$\frac{3}{2} \frac{p}{2} L_m (i_{qs} i_{dr} - i_{ds} i_{qr}) \tag{2.15}$$

Chapter 3

MATLAB SIMULINK MODEL

3.1 Simulink Model

3. An induction machine is given by the following parameters:

• Stator operating frequency 50Hz,

• Stator voltage 400V,

• Number of poles 2,

• Stator resistance 0.12,

• Stator leakage inductance 0.0001H

• Main magnetising inductance 0.01H,

• Referred rotor resistance 0.02,

• Referred rotor leakage inductance 0.0001H.

Derive MATLAB/SIMULINK model for this induction machine using the above transient model in the synchronous rotating reference frame.

4. Use SIMULINK model to simulate machine dynamics for a step change in the machine slip from 2% to 3% at 1s.

6. Determine and simulate the transient machine torque for the case of 2% and 3% slip.

Solution

We have made the entire model on the base of all the calculations previously done. Figure 3.1 is the complete transient model of induction motor. The input parameters are 3 phase voltages and output parameters Volatges and Currents of stator and rotor in dq reference frame. Rotor voltage are zero because its squirell cage induction motor and its bars are very short so voltage will be zero.

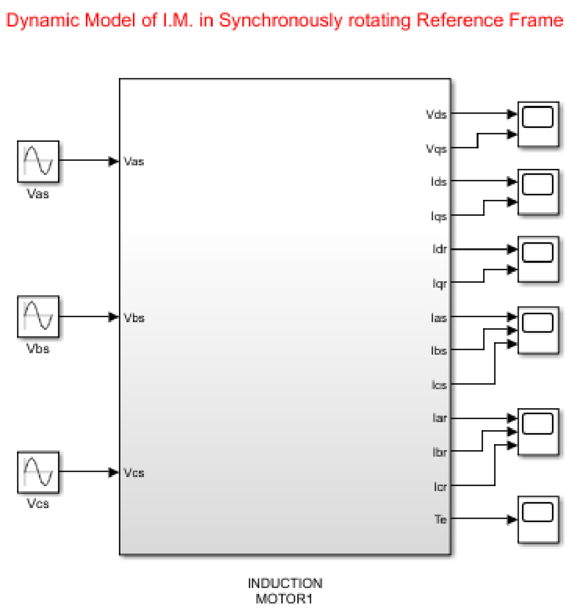

FIGURE 3.1: Complete Model

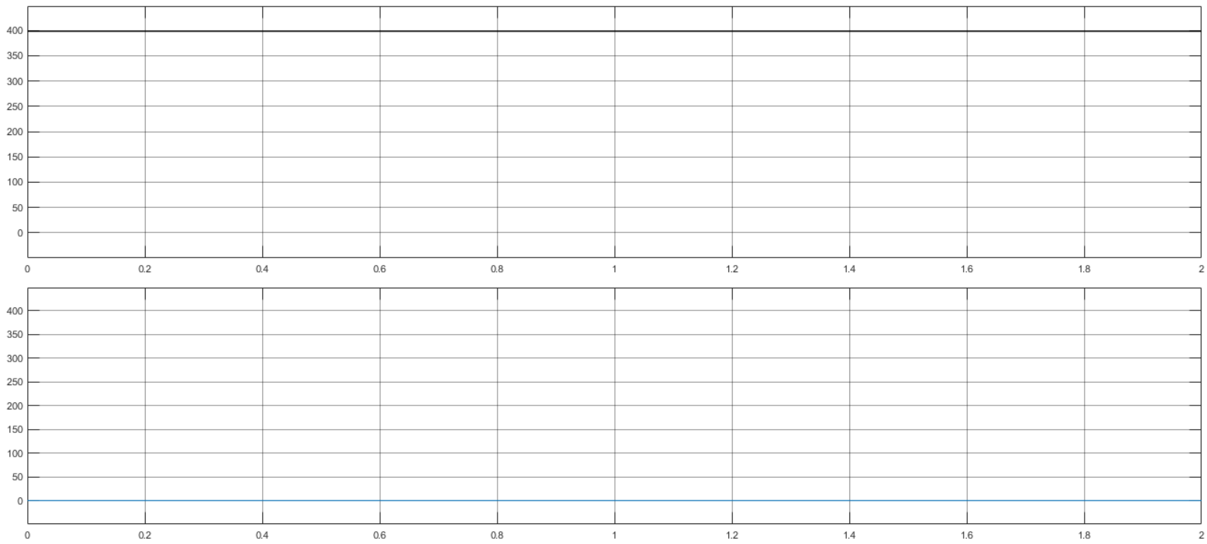

FIGURE 3.2: Vd and Vq of Stator Waveform

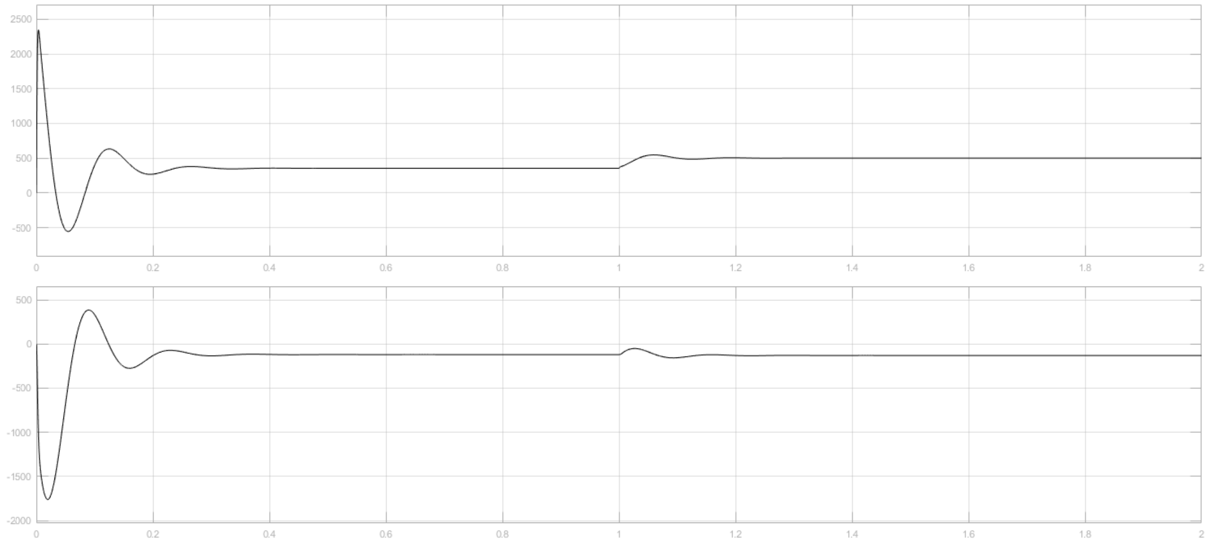

FIGURE 3.3: Id and Iq of Stator Waveform

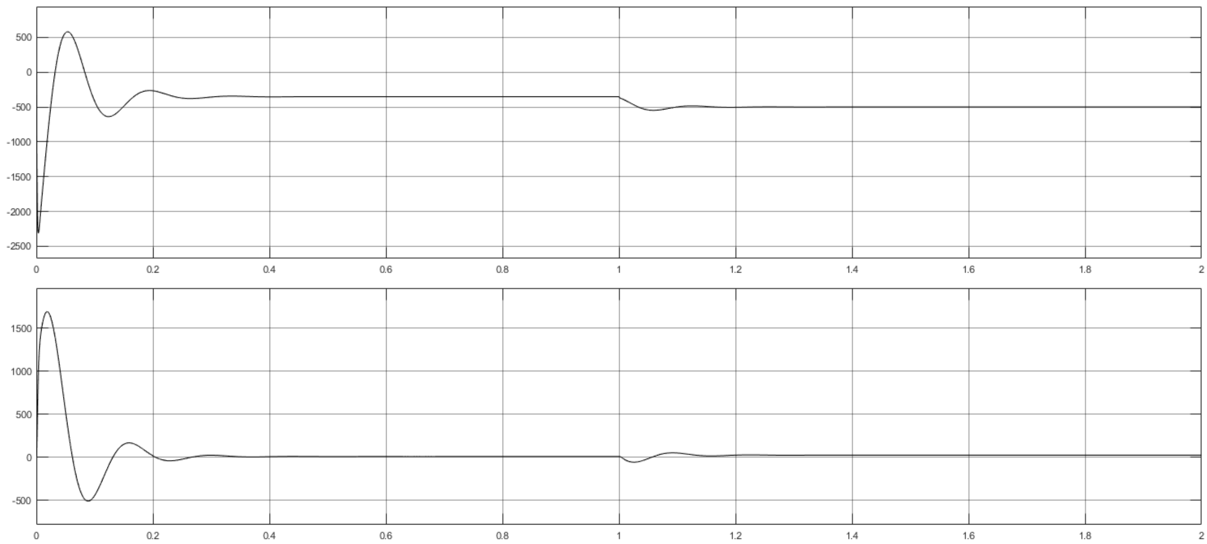

FIGURE 3.4: Id and Iq of Rotor Waveform

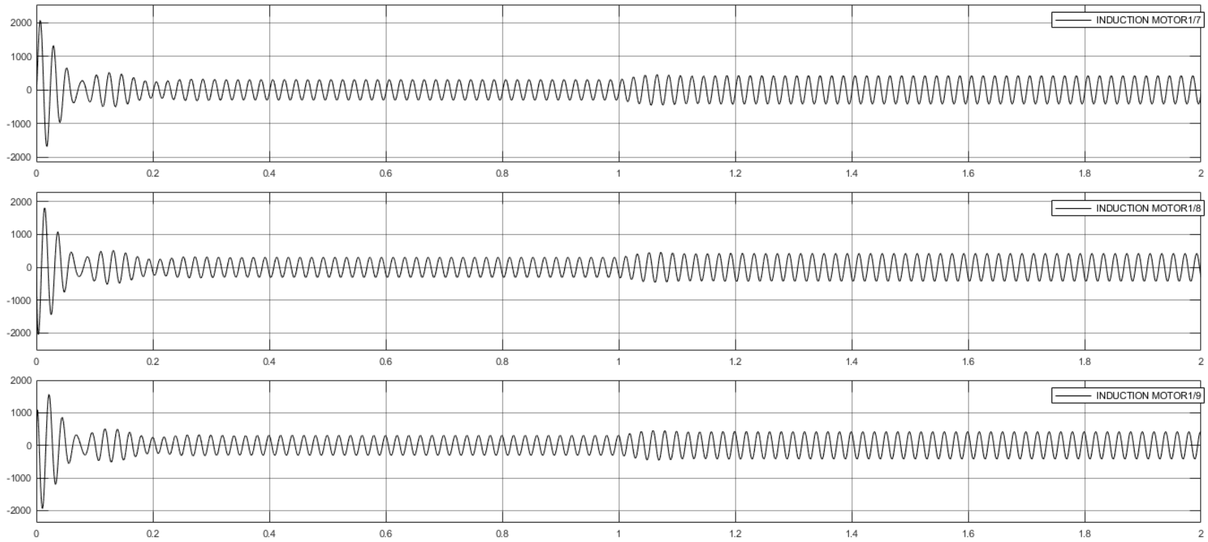

FIGURE 3.5: Iabc of Stator Waveform

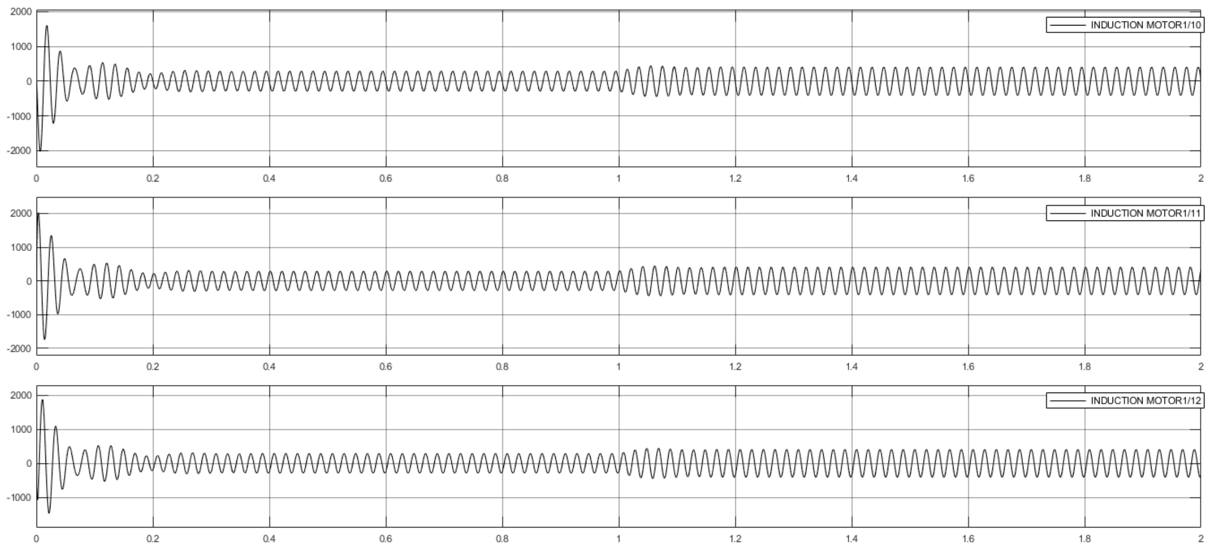

FIGURE 3.6: Iabc of Rotor Waveform

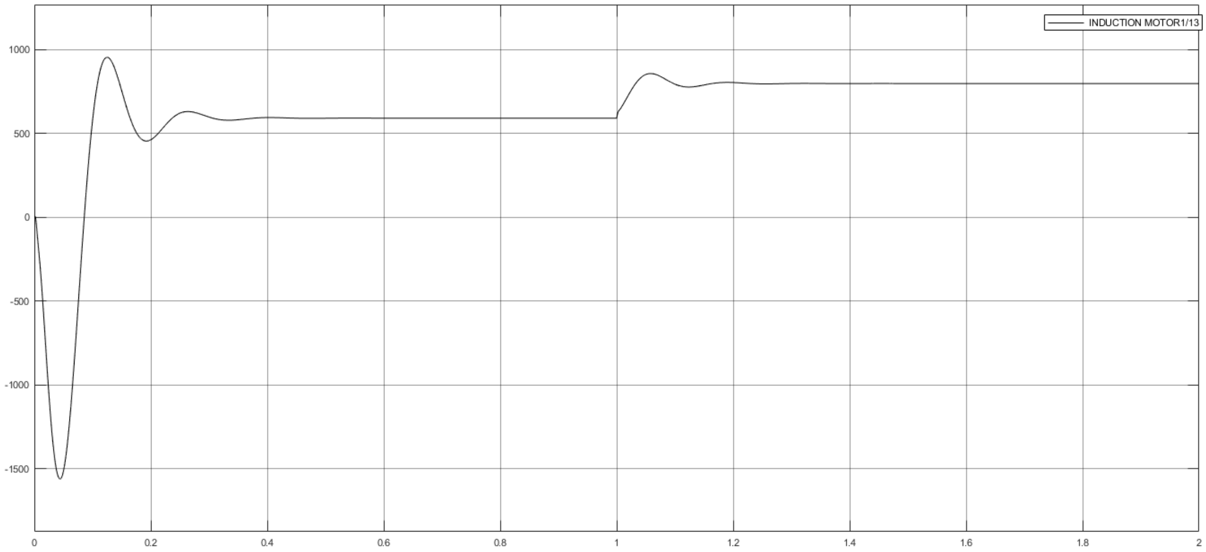

FIGURE 3.7: Torque Waveform from slip 2% to 3% at 1 sec

Now, This is the internal structure of mathematical model induction motor.3.8 First, 3 phase voltages are converted in dq frame of reference i.e. from natural abc frame to synchronous frame of reference frame because transient modeling of Induction motor is in synchronous ref frame so input will be in dq. Then motor' dq stator and rotor currnets are again converted to abc reference frame in next 2 blocks.

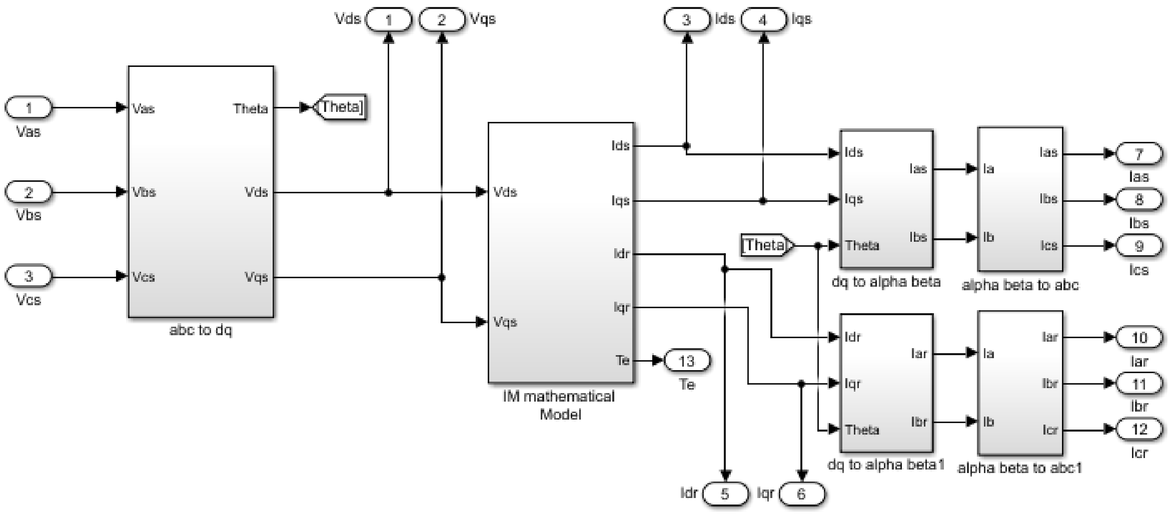

FIGURE 3.8: Internal Structure of Model

This is the internal structure of abc to dq transformation block.3.9 Now this is Park transformation block include conversion from abc to $\alpha\beta$ reference frame using the conversion matrix.

$$\begin{bmatrix} V_\alpha \\ V_\beta \\ V_\gamma \end{bmatrix} = \begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (3.1)$$

Using this conversion equation abc to $\alpha\beta$. Then we find angle which is rotating with synchronous frequency.

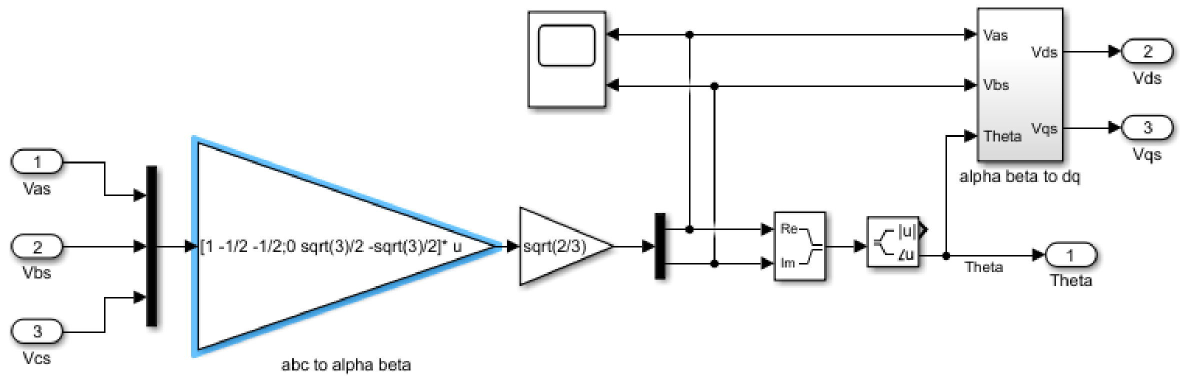

FIGURE 3.9: ABC TO DQ Transformation Block

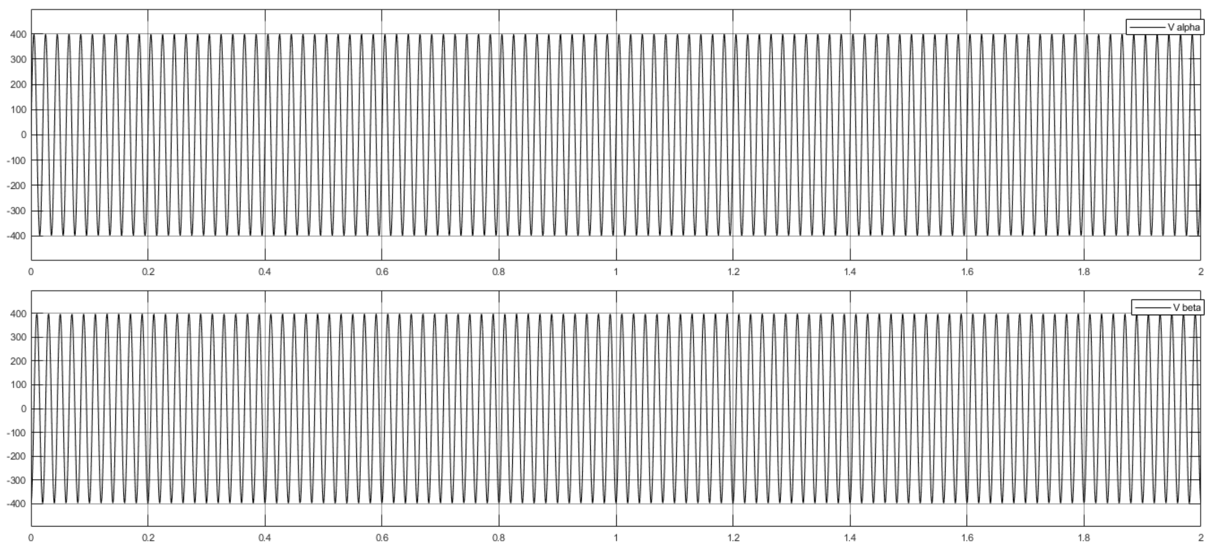

FIGURE 3.10: Va abd Vb of Stator Waveform

This is the $\alpha\beta$ to dq transformation block' internal structure.3.11 This is the conversion used to find V_d and V_q.

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos & \sin \\ -\sin & \cos \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \tag{3.2}$$

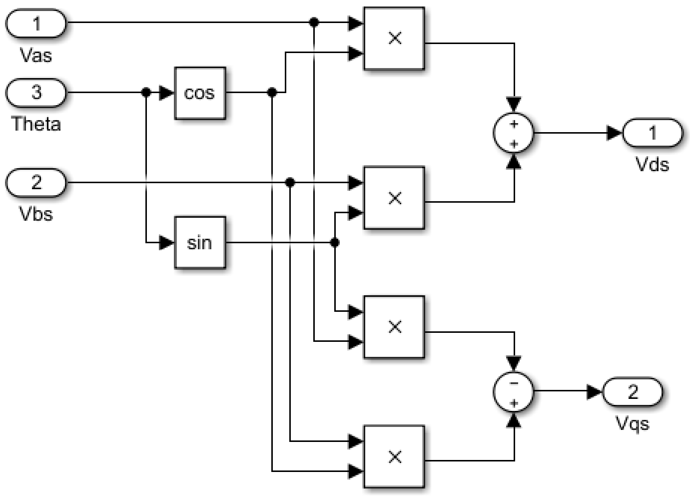

FIGURE 3.11: $\alpha\beta$ to dq transformation Block

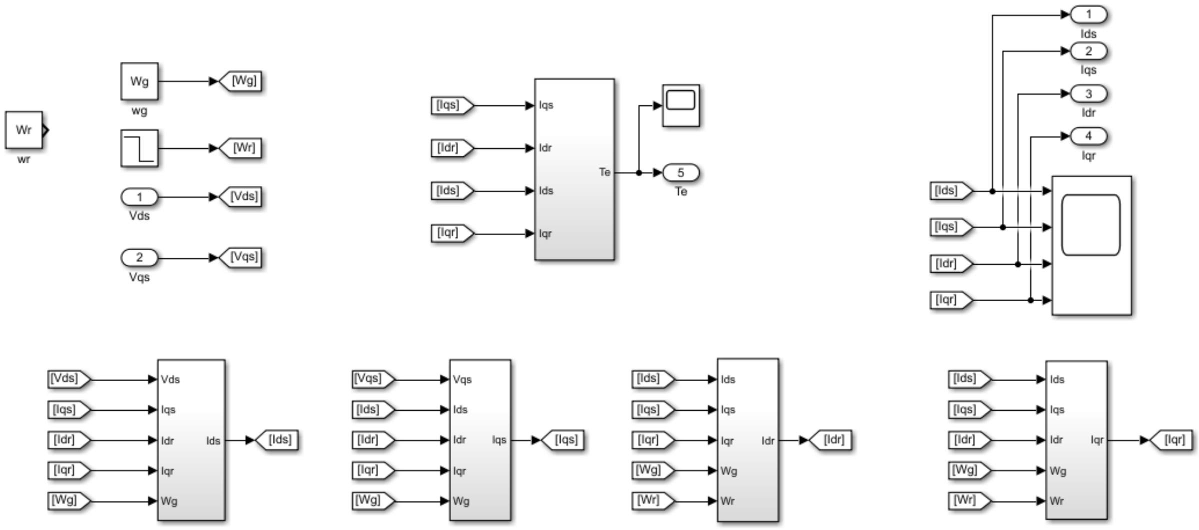

FIGURE 3.12: IM Mathematical Model Block Internal Structure

This is the internal structure of mathematical model of Induction Motor. 3.12

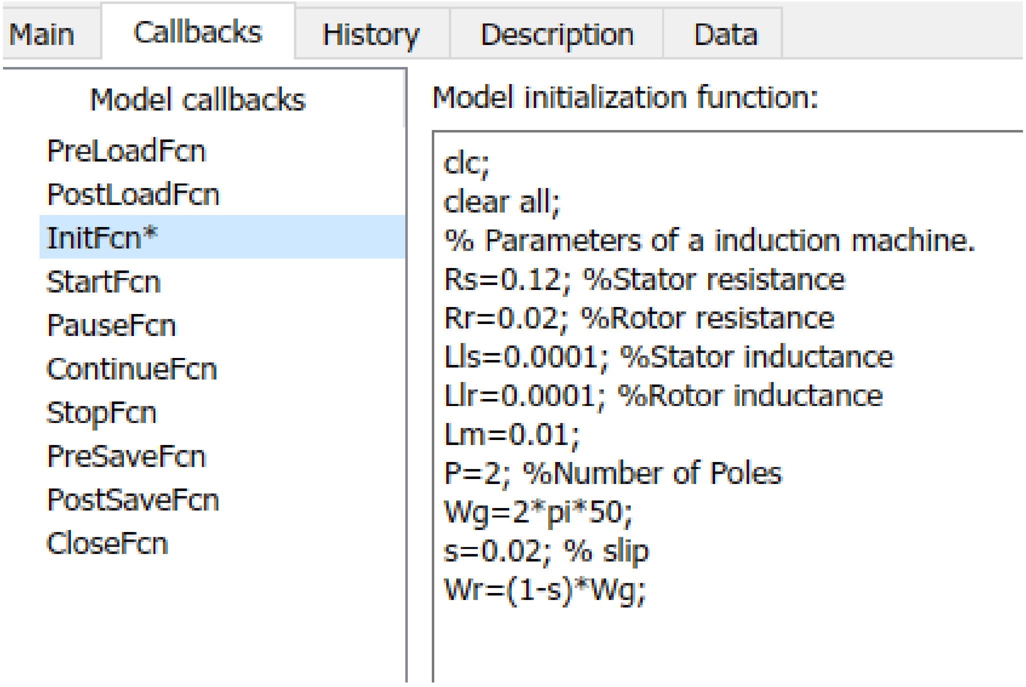

FIGURE 3.13: Model Parameters for Block

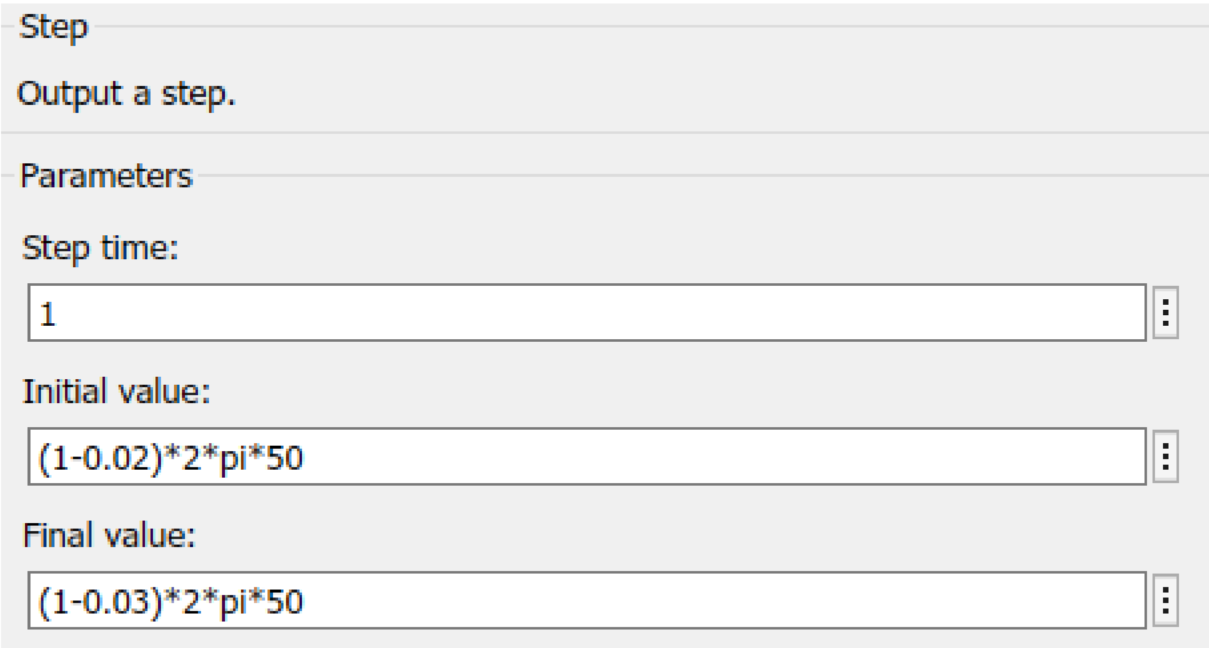

FIGURE 3.14: Step Given for 3% slip at 1second

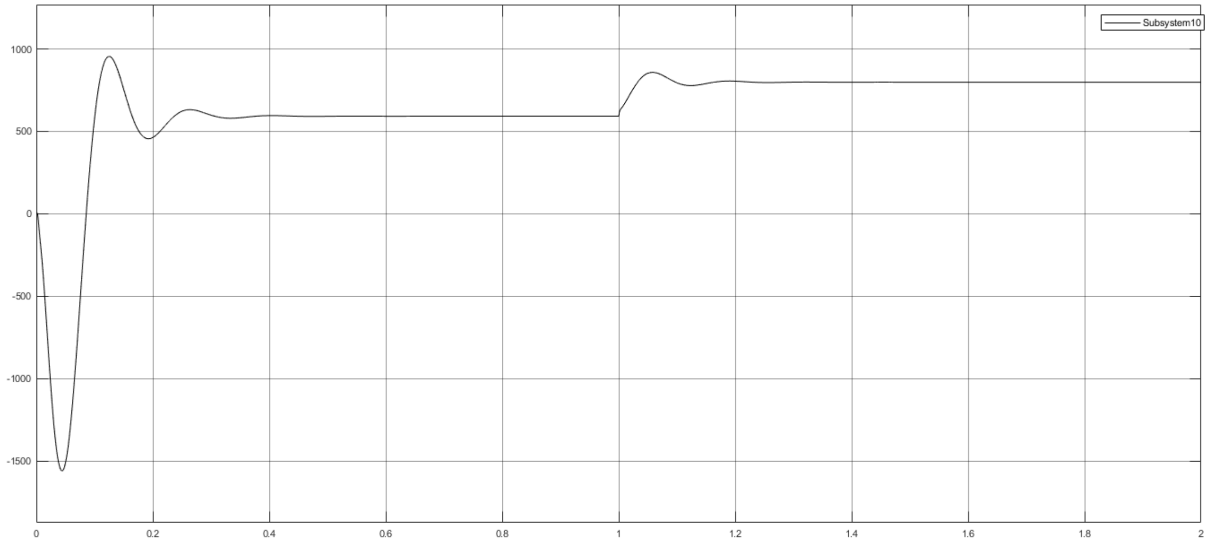

FIGURE 3.15: Torque Waeform

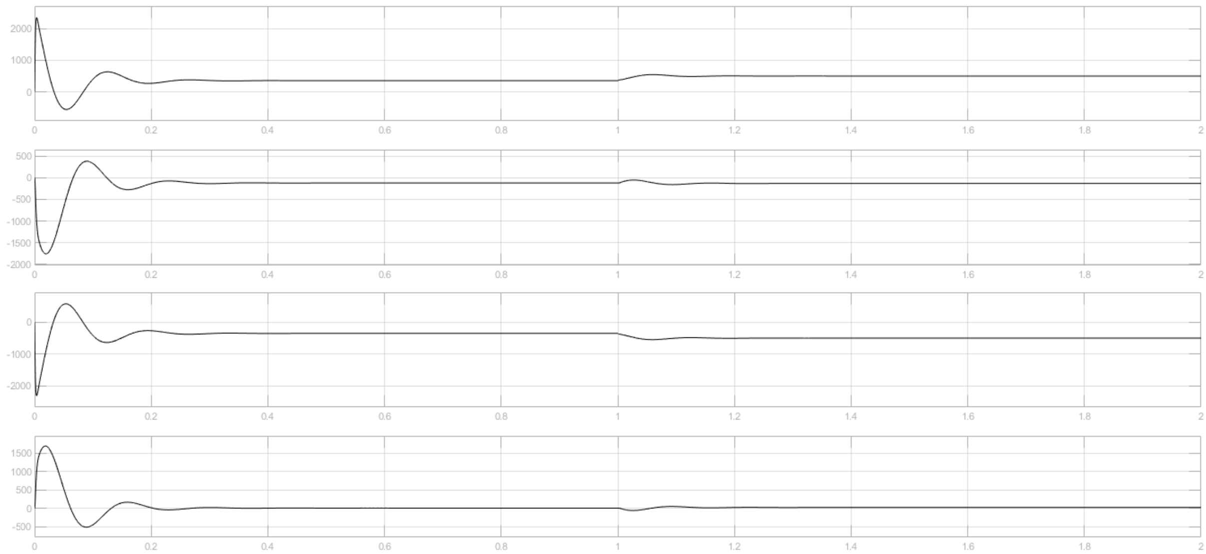

FIGURE 3.16: Ids Iqs Idr and Iqr Waveform

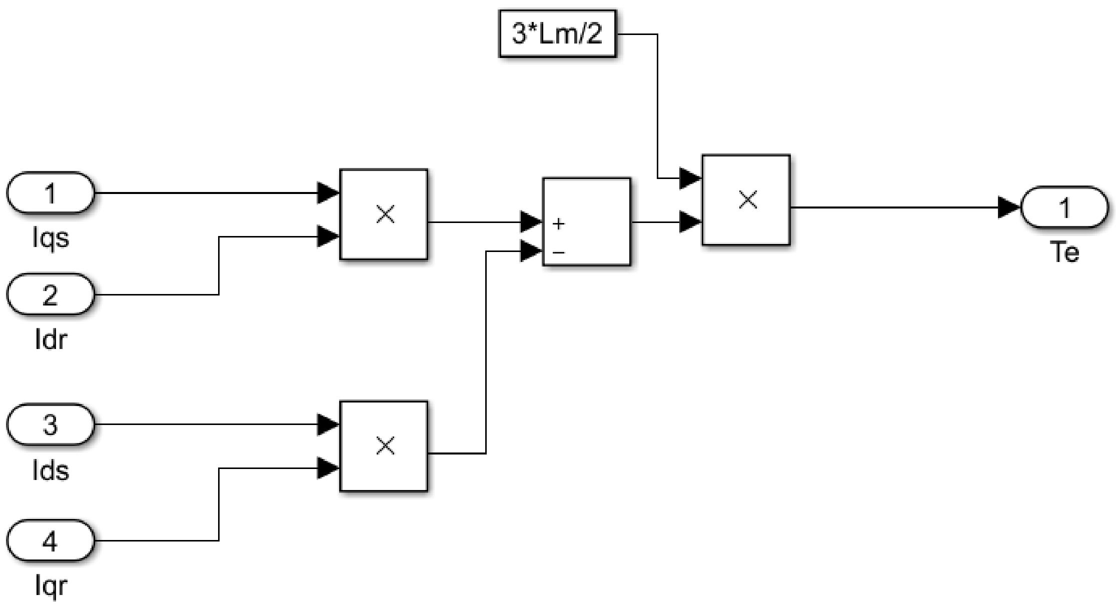

FIGURE 3.17: Torque Internal Structure

The equation used to find torque is given as;

$$T = \frac{3}{2}\frac{1}{2}\frac{d}{dt}L_m(i_{qs}i_{qr} - i_{ds}i_{dr}) \tag{3.3}$$

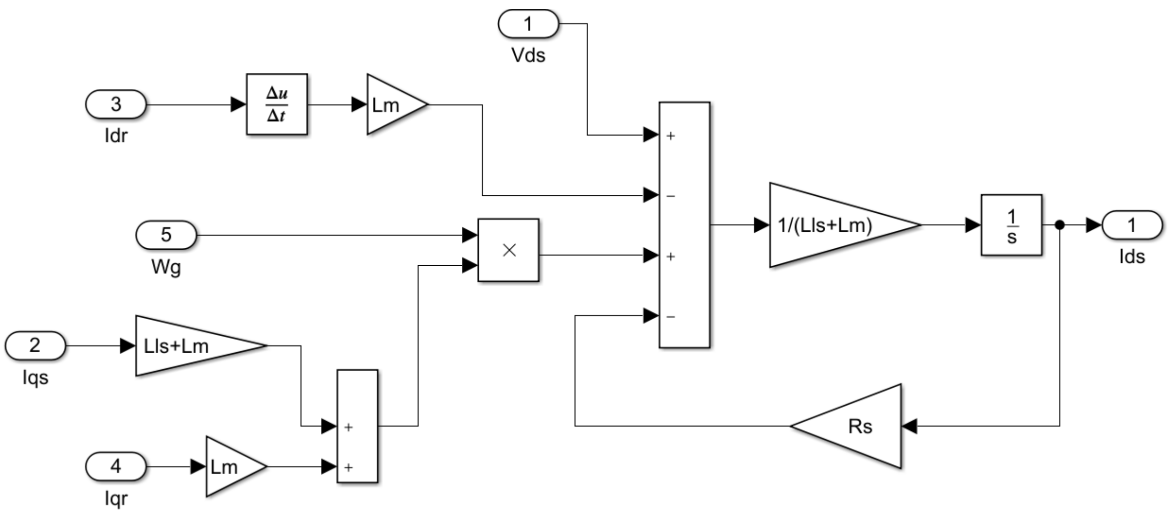

FIGURE 3.18: Ids Internal Structure

Figure 3.18 is implemented by this equation.

$$\frac{d}{dt}i_{ds} = [\frac{1}{L_{ls} + L_m}][V_{ds} - r_s i_{ds} + \omega_g[(L_{ls} + L_m)i_{qs} + L_m i'_{qr}] - L_m\frac{d}{dt}[i'_{dr}]] \tag{3.4}$$

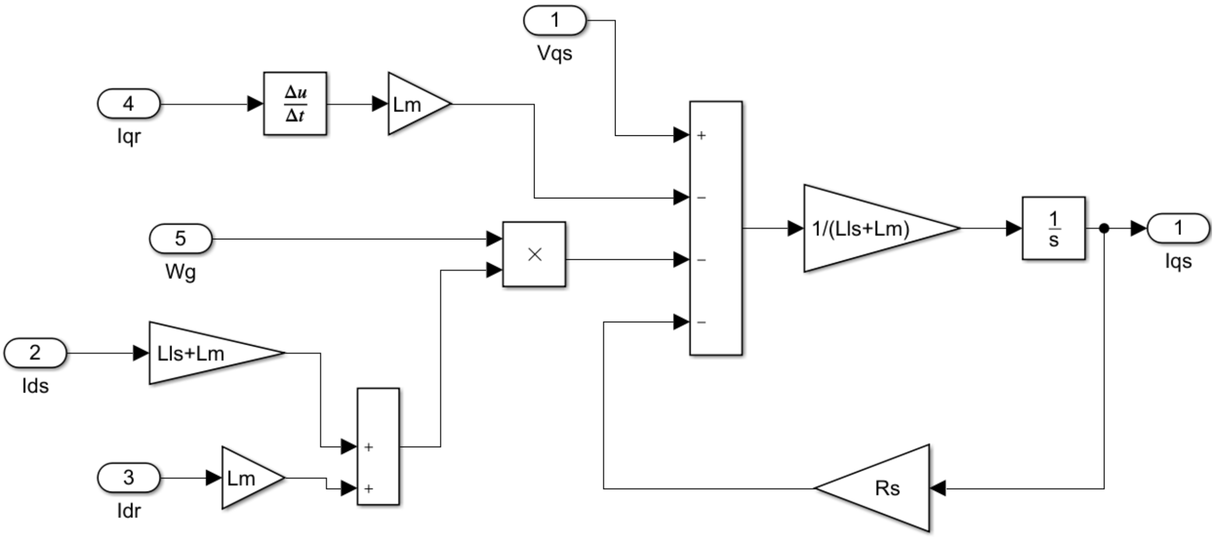

FIGURE 3.19: Iqs Internal Structure

Figure 3.19 is implemented by this equation.

$$\frac{d}{dt}i_{qs} = [\frac{1}{L_{ls} + L_m}][V_{qs} - r_s i_{qs} + \omega_g[(L_{ls} + L_m)i_{ds} + L_m i'_{dr}] - L_m \frac{d}{dt}[i'_{qr}]] \quad (3.5)$$

Figure 3.20 is implemented by this equation.

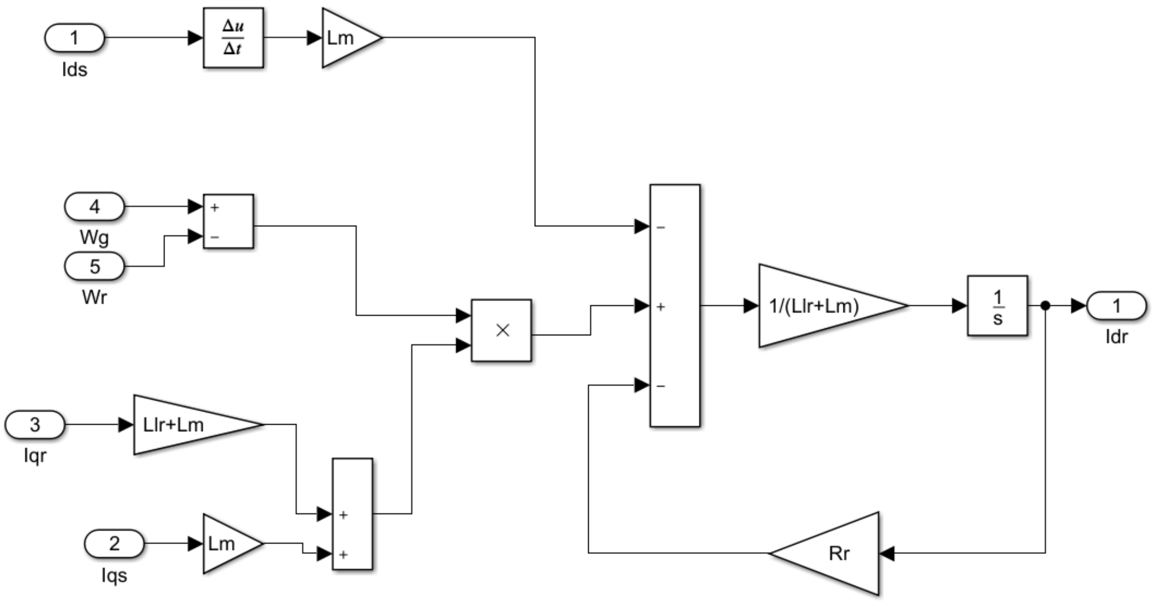

FIGURE 3.20: Iqr Internal Structure

$$\frac{d}{dt}i'_{ds} = [\frac{1}{L'_{lr} + L_m}][V'_{dr} - r'_r i'_{dr} + (\omega_g - \omega_r)[(L'_{lr} + L_m)i'_{qr} + L_m i'_{qs}] - L_m \frac{d}{dt}[i_{ds}]] \quad (3.6)$$

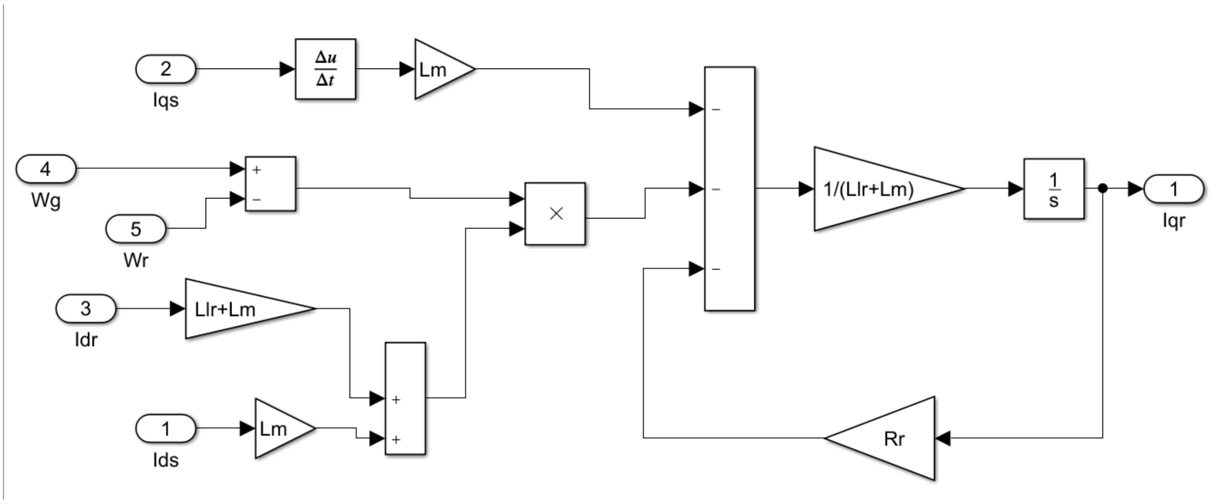

FIGURE 3.21: Idr Internal Structure

Figure 3.21 is implemented by this equation.

$$\frac{d}{dt}i'_{dr} = [\frac{1}{L'_{lr} + L_m}][V'_{qr} - r'_r i'_{qr} + (\omega_g - \omega_r)[(L'_{lr} + L_m)i'_{dr} + L_m i'_{ds}] - L_m \frac{d}{dt}[i_{qs}]] \quad (3.7)$$

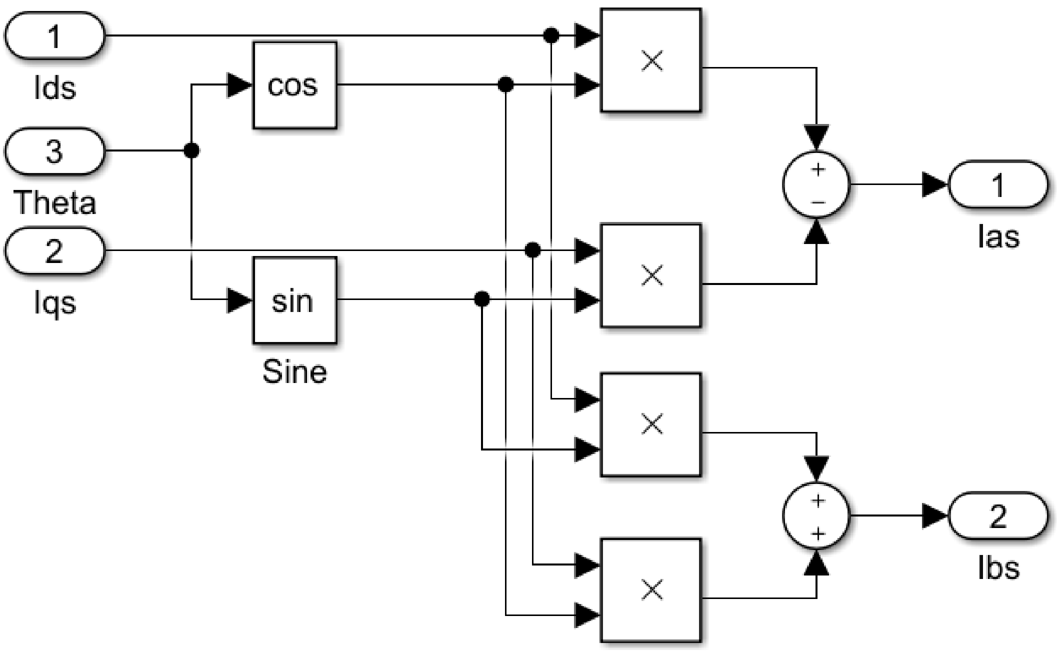

FIGURE 3.22: DQ to $\alpha\beta$ of Stator Transformation

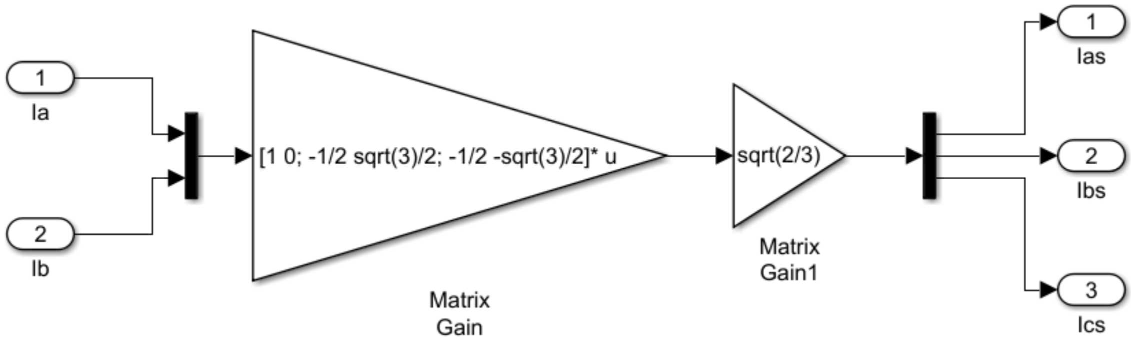

FIGURE 3.23: $\alpha\beta$ to DQ of Stator Transformation

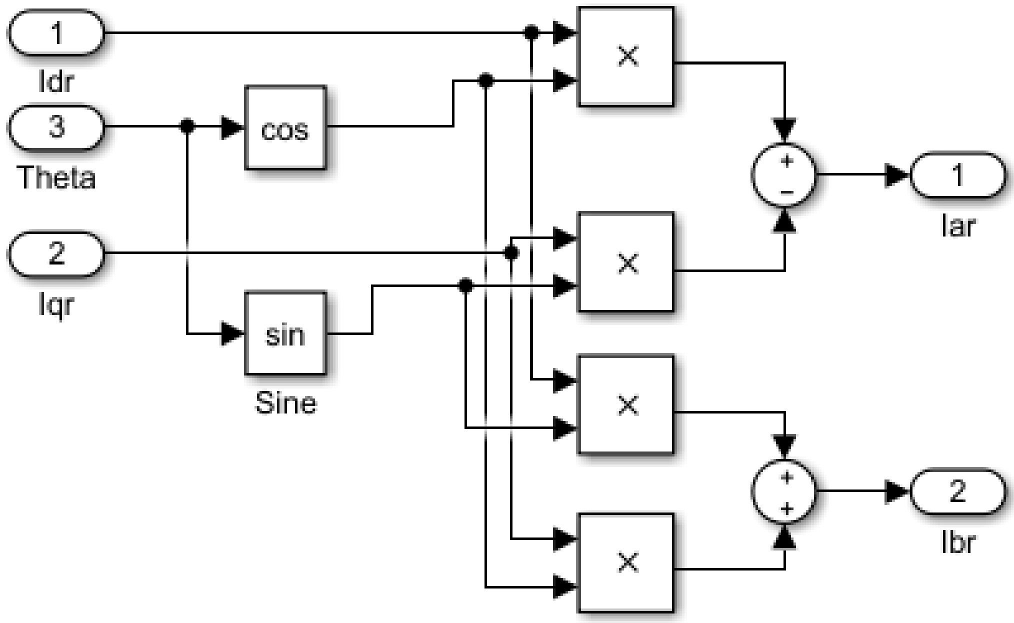

FIGURE 3.24: DQ to $\alpha\beta$ of Rotor Transformation

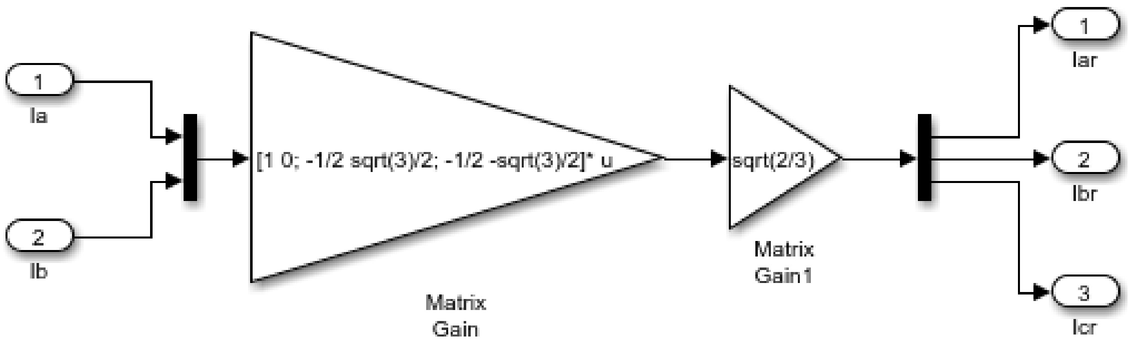

FIGURE 3.25: $\alpha\beta$ to DQ of Rotor Transformation

Chapter 4

Steady State Vs Transient State

4.1 Difference between Steady State and Transient State

7. Discuss difference between a transient model and a steady-state model. Show how to derive the steady-state model of such induction machine from the transient model equations.

Induction motor starts with an increasing slope of voltages from rest up to the desired speed, during which the motor will be running at an unbalanced condition, and reach to the balanced condition at the desirable speed. We have implemented the model of induction motor in which when we changes the slip of motor for a certain period of time it shows transient response i.e. time to settle down for machine to its desirable speed and when it achieve that speed it is at steady state. Any change in load and other parameters cause hindrance of operation of motor so it shows transients for some certain time which is very low usually in milliseconds. In this Figure 4.1 we can clearly see the transients up to .3s for 2% slip but when we changed the slip at 1s suddenly from 2% to 3% there are also transients for 0.1s, on the other hand steady state is achieved after 0.4 and 0.2 seconds respectively.

$$Ns = \frac{120f}{P} \tag{4.1}$$

$$Ns = \frac{120x50}{2} = 3000rpm \tag{4.2}$$

For Transients there is increasing current so this changes lead to differential calculations and we have to consider the inductive nature as well while in steady state

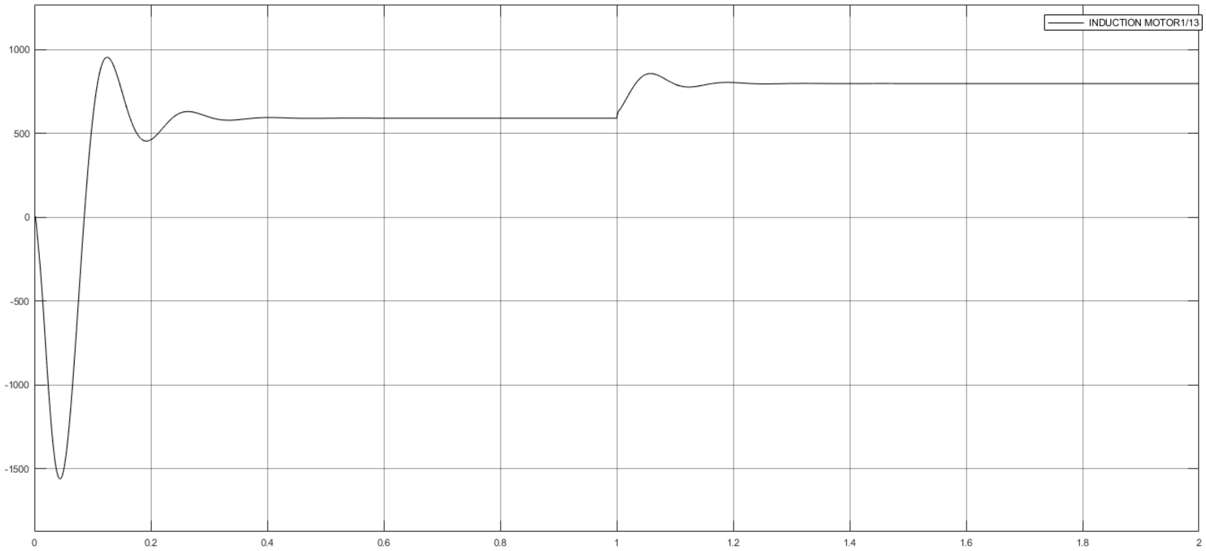

FIGURE 4.1: Torque at 2% and 3% slip

all the currents are constant so change in current is zero so in the calculations for steady state model we will neglect all the derivative terms making them zero.

4.2 Speed Choice

7. Discuss the speed choice of the rotating reference frame. Explain what responses we could expect for a different ω_g.

$$\omega_g = 2\pi f \tag{4.3}$$

$$\omega_g = 100\pi \tag{4.4}$$

When we increase ω_g in SIMULINK model then oscillations in torque increases. Induction motor operates on a slightly lower speed than Synschronous speed so by increasing ω_g motor speed increases.