Data Structures & Algorithms

2. Stacks & Queues

- stacks
- dynamic resizing
- queues
- generics
- applications

Slides are Reformatted From Lecture Note of Algorithms Course
by Robert Sedgewick, Princeton University, Fall, 2008.

Stacks and Queues

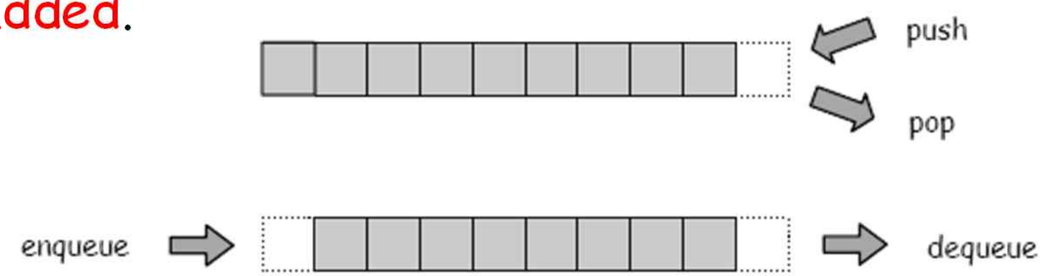
- Fundamental data types.
  - Values: sets of objects
  - Operations: insert, remove, test if empty.
  - Intent is clear when we insert.
  - Which item do we remove?

- Stack.
  - Remove the item most recently added.
  - Analogy: cafeteria trays, Web surfing.

LIFO="last in first out"

- Queue.
  - Remove the item least recently added.
  - Analogy: Registrar's line.

FIFO="first in first out"



Client, Implementation, Interface

- Separate interface and implementation so as to:
  - Build layers of abstraction.
  - Reuse software.
  - Ex: stack, queue, symbol table.

Interface: description of data type, basic operations.

Client: program using operations defined in interface.

Implementation: actual code implementing operations.

Client, Implementation, Interface

- Separate interface and implementation so as to:
  - Build layers of abstraction.
  - Reuse software.
  - Ex: stack, queue, symbol table.
  - Design: creates modular, re-usable libraries.
  - Performance: use optimized implementation where it matters.

Interface: description of data type, basic operations.

Client: program using operations defined in interface.

Implementation: actual code implementing operations.

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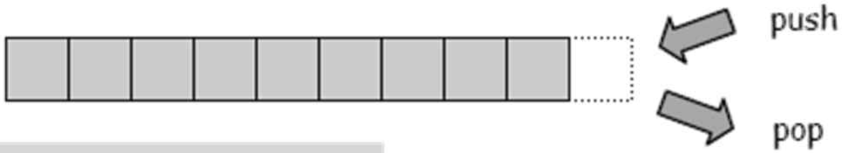
Stacks

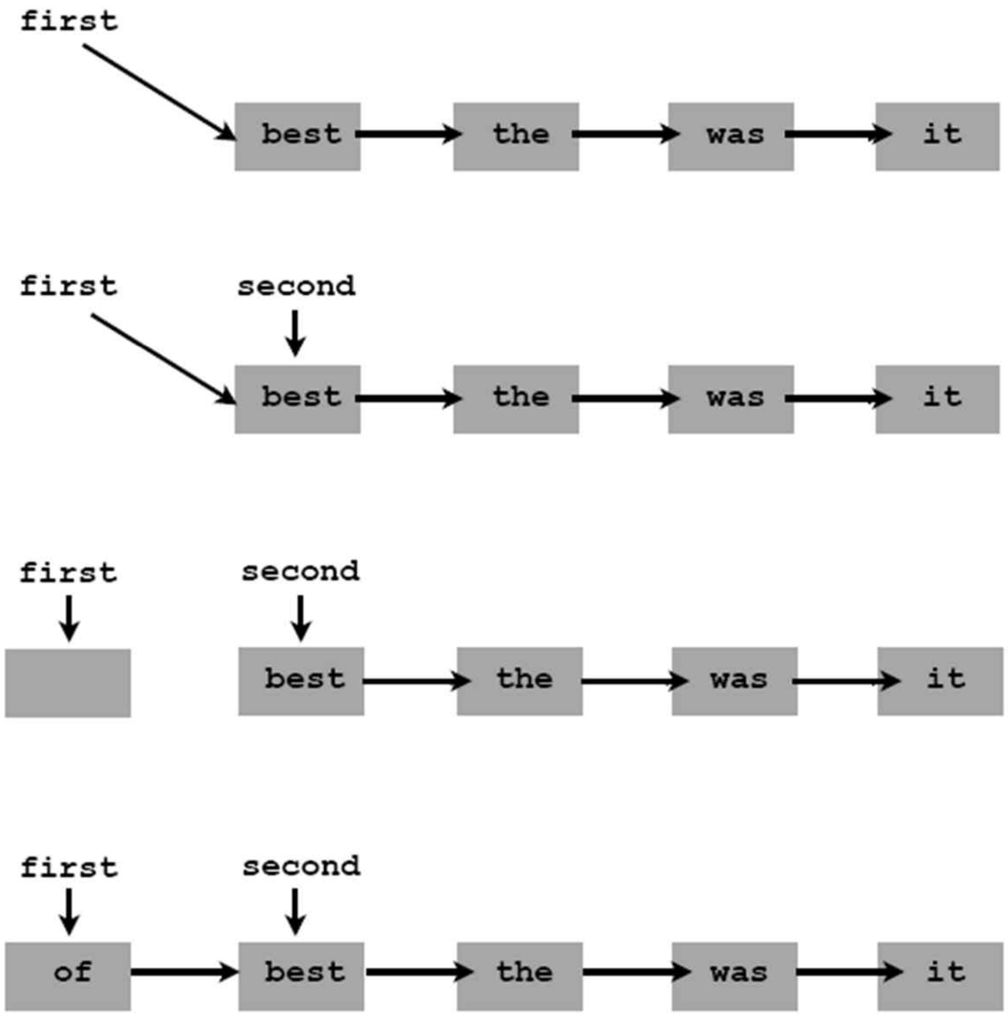
- Stack operations.
  - `push()` Insert a new item onto stack.
  - `pop()` Remove and return the item most recently added.
  - `isEmpty()` Is the stack empty?




```
public static void main(String[] args)
{
    StackOfStrings stack = new StackOfStrings();
    while(!StdIn.isEmpty())
    {
        String s = StdIn.readString();
        stack.push(s);
    }
    while(!stack.isEmpty())
    {
        String s = stack.pop();
        StdOut.println(s);
    }
}
```

a sample stack client

Stack push: Linked-list implementation

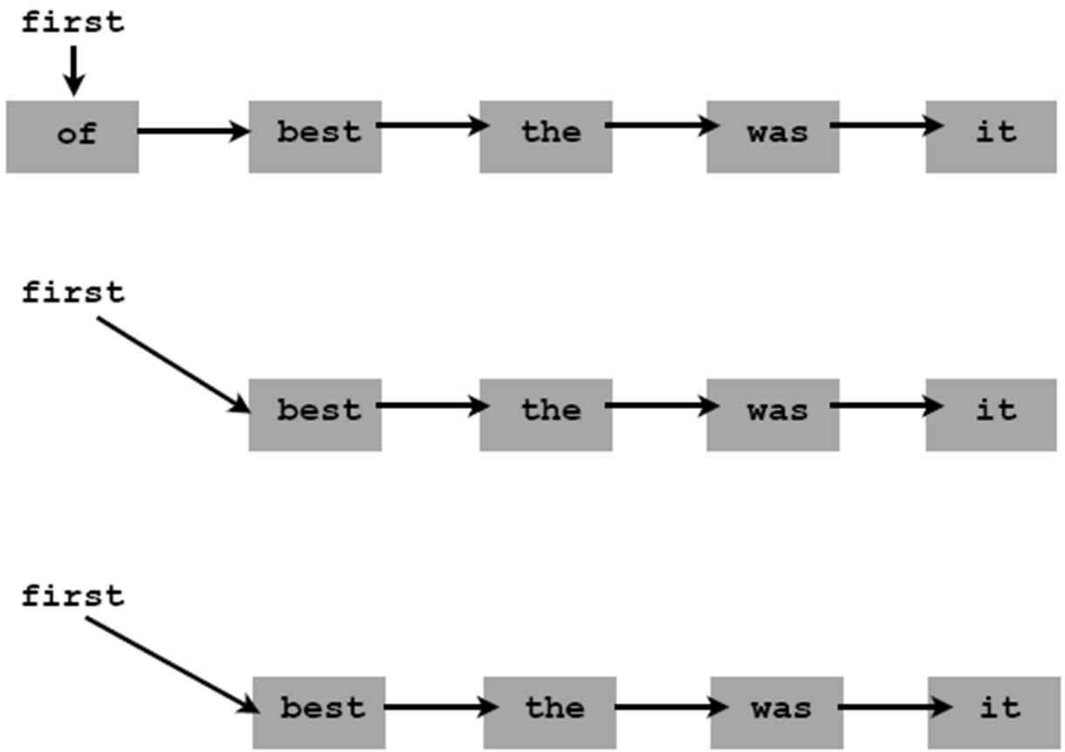



```
second = first;
```

```
first = new Node();
```

```
first.item = item;
first.next = second;
```

Stack pop: Linked-list implementation




```
item = first.item;
```

```
first = first.next;
```

```
return item;
```

Stack: Linked-list implementation

```
public class StackOfStrings
{
    private Node first = null;

    private class Node
    {
        String item;
        Node next;
    }                                  "inner class"

    public boolean isEmpty()
    {   return first == null;   }

    public void push(String item)
    {
        Node second = first;
        first = new Node();
        first.item = item;
        first.next = second;
    }

    public String pop()
    {
        String item = first.item;
        first = first.next;
        return item;
    }
}
```

- ❑ Error conditions?
- ❑ Example:
  - pop() an empty stack

Stack: Array implementation

- Array implementation of a stack.
  - Use array `s[]` to store `N` items on stack.
  - `push()` add new item at `s[N]`.
  - `pop()` remove item from `s[N-1]`.

`s[]`	it	was	the	best						
	0	1	2	3	4	5	6	7	8	9
					N					

Stack: Array implementation

```
public class StackOfStrings
{
   private String[] s;
   private int N = 0;

   public StringStack(int capacity)
   {  s = new String[capacity];   }

   public boolean isEmpty()
   { return N == 0; }

   public void push(String item)
   {  s[N++] = item;   }

   public String pop()
   {
      String item = s[N-1];
      s[N-1] = null;
      N--;
      return item;
   }

}
```

avoid loitering
(garbage collector only reclaims memory if no outstanding references)

2. Stacks & Queues

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Stack array implementation: Dynamic resizing

Q. How to grow array when capacity reached?

Q. How to shrink array (else it stays big even when stack is small)?

- First try:
  - `push()`: increase size of `s[]` by 1
  - `pop()` : decrease size of `s[]` by 1

- Too expensive
  - Need to copy all of the elements to a new array.
  - Inserting N elements: time proportional to $1 + 2 + \ldots + N \approx N^2/2$.

    infeasible for large N

  - Need to guarantee that array resizing happens infrequently

Stack array implementation: Dynamic resizing

Q. How to grow array?

A. Use repeated doubling:

- if array is full, create a new array of twice the size, and copy items

no-argument constructor

```
public StackOfStrings()
{ this(8); }

public void push(String item)
{
   if (N >= s.length) resize(2 * s.length);
   s[N++] = item;
}
```

create new array copy items to it

```
private void resize(int max)
{
   String[] dup = new String[max];
   for (int i = 0; i < N; i++)
      dup[i] = s[i];
   s = dup;
}
```

- Consequence. Inserting N items takes time proportional to N
  - (not N^2).

$8 + 16 + \ldots + N/4 + N/2 + N \approx 2N$

Stack array implementation: Dynamic resizing

Q. How (and when) to shrink array?

- How: create a new array of half the size, and copy items.
- When (first try): array is half full?
  - No, causes thrashing
  - push-pop-push-pop-... sequence: time proportional to N for each op
- When (solution): array is 1/4 full (then new array is half full).

```
public String pop(String item)
{
   String item = s[--N];
   s[N] = null;
   if (N == s.length/4)
      resize(s.length/2);
   return item;
}
```

- Consequences.
  - any sequence of N ops takes time proportional to N
  - array is always between 25% and 100% full

Stack Implementations: Array vs. Linked List

- Stack implementation tradeoffs.
  - Can implement with either array or linked list, and client can use interchangeably. Which is better?
- Array.
  - Most operations take constant time.
  - Expensive doubling operation every once in a while.
  - Any sequence of N operations (starting from empty stack) takes time proportional to N.

"amortized" bound

- Linked list.
  - Grows and shrinks gracefully.
  - Every operation takes constant time.
  - Every operation uses extra space and time to deal with references.
- Bottom line:
  - tossup for stacks but differences are significant when other operations are added

Stack implementations: Array vs. Linked list

- Which implementation is more convenient?

	array?	linked list?
return count of elements in stack		
remove the kth most recently added		
sample a random element		

2. Stacks & Queues

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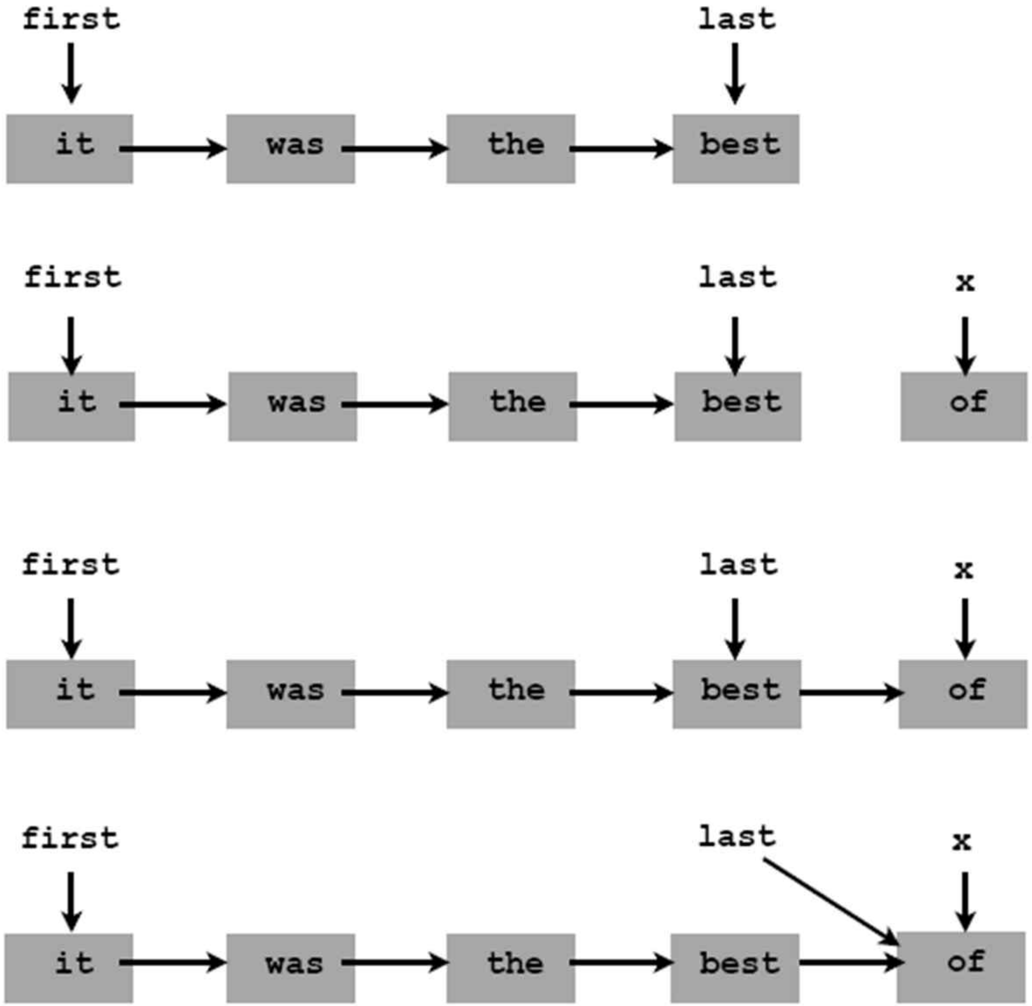
Queues

- Queue operations.
  - `enqueue()` Insert a new item onto queue.
  - `dequeue()` Delete and return the item least recently added.
  - `isEmpty()` Is the queue empty?

```
public static void main(String[] args)
{
   QueueOfStrings q = new QueueOfStrings();
   q.enqueue("Vertigo");
   q.enqueue("Just Lose It");
   q.enqueue("Pieces of Me");
   q.enqueue("Pieces of Me");
   System.out.println(q.dequeue());
   q.enqueue("Drop It Like It's Hot");

   while(!q.isEmpty()

      System.out.println(q.dequeue());
}
```

Enqueue: Linked List Implementation

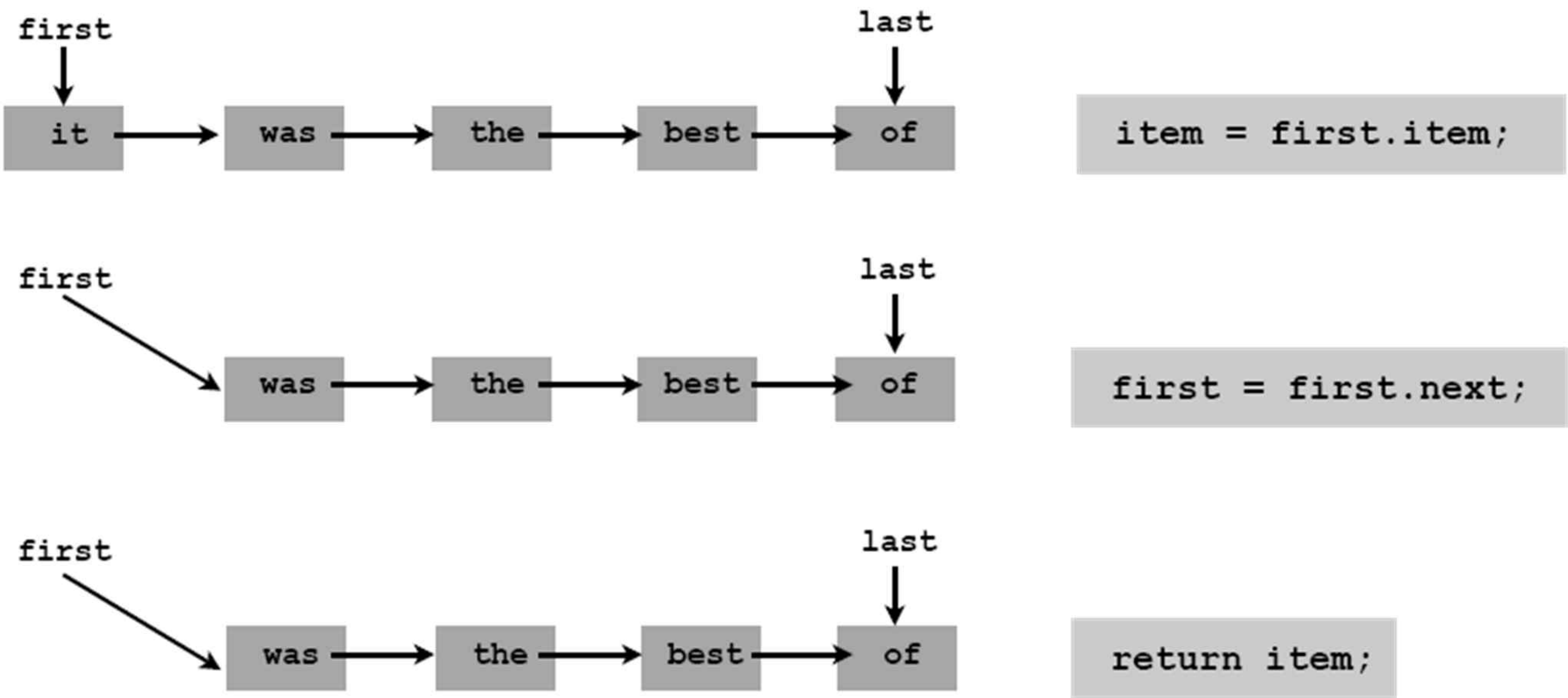



```
x = new Node();
x.item = item;
x.next = null;
```

```
last.next = x;
```

```
last = x;
```

Dequeue: Linked List Implementation



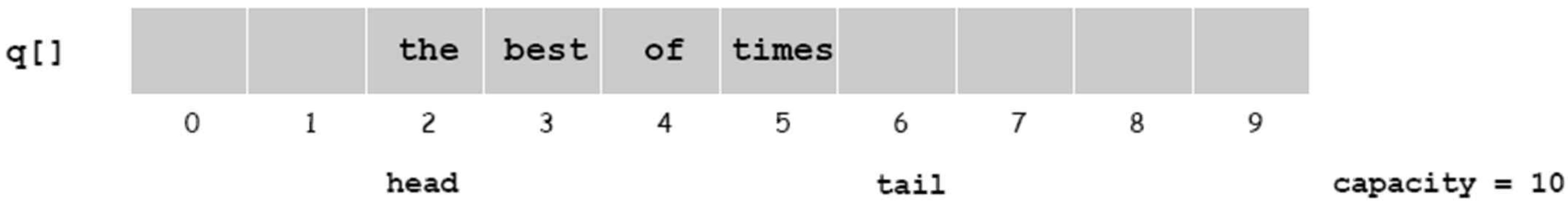
- Aside:
  - dequeue (pronounced “DQ”) means “remove from a queue”
  - deque (pronounced “deck”) is a data structure (see PA 1)

Queue: Linked List Implementation

```java
public class QueueOfStrings
{
   private Node first;
   private Node last;

   private class Node
   { String item; Node next; }

   public boolean isEmpty()
   { return first == null; }

   public void enqueue(String item)
   {
      Node x = new Node();
      x.item = item;
      x.next = null;
      if (isEmpty()) { first      = x; last = x; }
      else           { last.next = x; last = x; }
   }

   public String dequeue()
   {
      String item = first.item;
      first       = first.next;
      return item;
   }
}
```

Queue: Array implementation

- Array implementation of a queue.
  - Use array `q[]` to store items on queue.
  - `enqueue()` : add new object at `q[tail]`.
  - `dequeue()` : remove object from `q[head]`.
  - Update `head` and `tail` modulo the `capacity`.



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Generics (parameterized data types)

- We implemented: `StackOfStrings`, `QueueOfStrings`.
- We also want: `StackOfURLs`, `QueueOfCustomers`, etc?
- Attempt 1. Implement a separate stack class for each type.
  - Rewriting code is tedious and error-prone.
  - Maintaining cut-and-pasted code is tedious and error-prone.
- @#$*! most reasonable approach until Java 1.5

Stack of Objects

- We implemented: `StackOfStrings`, `QueueOfStrings`.
- We also want: `StackOfURLs`, `QueueOfCustomers`, etc?
- Attempt 2. Implement a stack with items of type `Object`.
  - Casting is required in client.
  - Casting is error-prone: run-time error if types mismatch.

```
Stack s = new Stack();
Apple a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = (Apple) (s.pop());
```

run-time error

Generics

- Generics. Parameterize stack by a single type.
  - Avoid casting in both client and implementation.
  - Discover type mismatch errors at compile-time instead of run-time.

parameter

```
Stack<Apple> s = new Stack<Apple>();
Apple a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);        compile-time error
a = s.pop();
```

no cast needed in client

- Guiding principles.
  - Welcome compile-time errors
  - Avoid run-time errors
- Why?

Generic Stack: Linked List Implementation

```java
public class StackOfStrings
{
    private Node first = null;

    private class Node
    {
        String item;
        Node next;
    }

    public boolean isEmpty()
    {   return first == null;   }

    public void push(String item)
    {
        Node second = first;
        first = new Node();
        first.item = item;
        first.next = second;
    }

    public String pop()
    {
        String item = first.item;
        first = first.next;
        return item;
    }
}
```

```java
public class Stack<Item>
{
    private Node first = null;

    private class Node
    {
        Item item;
        Node next ;
    }

    public boolean isEmpty()
    {   return first == null;   }

    public void push(Item item)
    {
        Node second = first;
        first = new Node();
        first.item = item;
        first.next  = second;
    }

    public Item pop()
    {
        Item item = first.item;
        first = first.next ;
        return item;
    }
}
```

Generic type name

Generic stack: array implementation

- The way it should be.

```
public class Stack<Item>
{
    private Item[] s;
    private int N = 0;

    public Stack(int cap)
    {   s = new Item[cap];   }

    public boolean isEmpty()
    { return N == 0; }

    public void push(Item item)
    {   s[N++] = item;   }

    public String pop()
    {
        Item item = s[N-1];
        s[N-1] = null;
        N--;
        return item;
    }

}
```

```
public class StackOfStrings
{
    private String[] s;
    private int N = 0;

    public StackOfStrings(int cap)
    {   s = new String[cap];   }

    public boolean isEmpty()
    { return N == 0; }

    public void push(String item)
    {   s[N++] = item;   }

    public String pop()
    {
        String item = s[N-1];
        s[N-1] = null;
        N--;
        return item;
    }

}
```

@#$*! generic array creation not allowed in Java

Generic stack: array implementation

- The way it is: an ugly cast in the implementation.

```
public class Stack<Item>
{
   private Item[] s;
   private int N = 0;

   public Stack(int cap)
   { s = (Item[]) new Object[cap]; }      the ugly cast

   public boolean isEmpty()
   { return N == 0; }

   public void push(Item item)
   {  s[N++] = item;   }

   public String pop()
   {
      Item item = s[N-1];
      s[N-1] = null;
      N--;
      return item;
   }

}
```

- Number of casts in good code: 0

Generic data types: autoboxing

- Generic stack implementation is object-based.
- What to do about primitive types?
- Wrapper type.
  - Each primitive type has a wrapper object type.
  - Ex: `Integer` is wrapper type for `int`.
- Autoboxing.
  - Automatic cast between a primitive type and its wrapper.
  - Syntactic sugar. Behind-the-scenes casting.

```
Stack<Integer> s = new Stack<Integer>();
s.push(17);          // s.push(new Integer(17));
int a = s.pop();     // int a = ((int) s.pop()).intValue();
```

- Bottom line: Client code can use generic stack for any type of data

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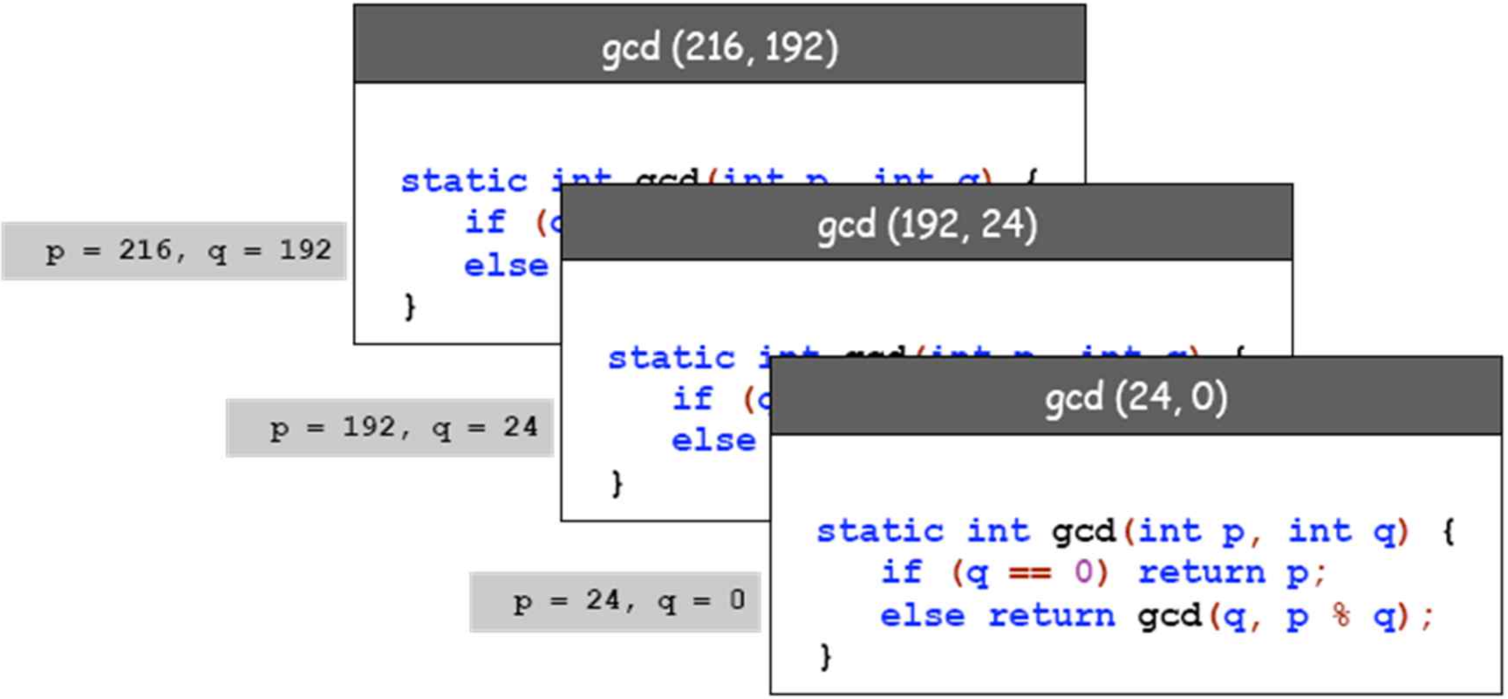
Stack Applications

- Real world applications.
  - Parsing in a compiler.
  - Java virtual machine.
  - Undo in a word processor.
  - Back button in a Web browser.
  - PostScript language for printers.
  - Implementing function calls in a compiler.

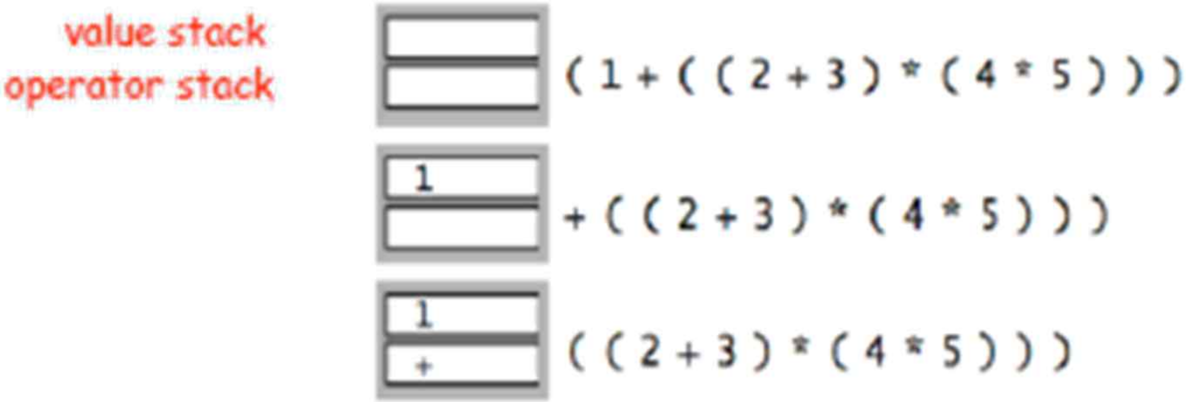
Function Calls

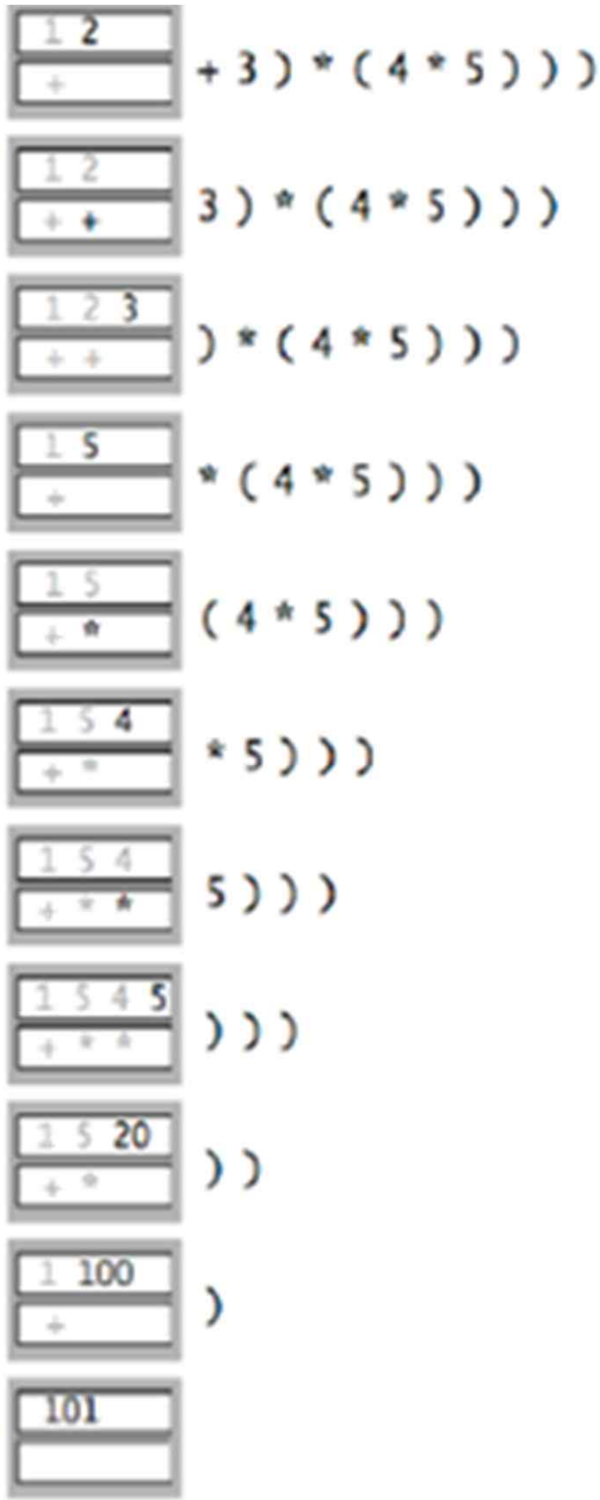
- How a compiler implements functions.
  - Function call: push local environment and return address.
  - Return: pop return address and local environment.
- Recursive function. Function that calls itself.
- Note. Can always use an explicit stack to remove recursion.



Arithmetic Expression Evaluation

- **Goal.** Evaluate infix expressions.
- **Two-stack algorithm.** [E. W. Dijkstra]
  - Value: push onto the value stack.
  - Operator: push onto the operator stack.
  - Left parens: ignore.
  - Right parens: pop operator and two values; push the result of applying that operator to those values onto the operand stack.
- **Context.** An interpreter!





Arithmetic Expression Evaluation

```
public class Evaluate {
   public static void main(String[] args) {
      Stack<String> ops  = new Stack<String>();
      Stack<Double> vals = new Stack<Double>();
      while (!StdIn.isEmpty()) {
         String s = StdIn.readString();
         if      (s.equals("("))               ;
         else if (s.equals("+"))    ops.push(s);
         else if (s.equals("*"))    ops.push(s);
         else if (s.equals(")")) {
            String op = ops.pop();
            if      (op.equals("+")) vals.push(vals.pop() + vals.pop());
            else if (op.equals("*")) vals.push(vals.pop() * vals.pop());
         }
         else vals.push(Double.parseDouble(s));
      }
      StdOut.println(vals.pop());
   }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

- Note: Old books have two-pass algorithm because generics were not available!

Correctness

- Why correct?
  - When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.
    - ✓ `( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )`
  - as if the original input were:
    - ✓ `( 1 + ( 5 * ( 4 * 5 ) ) )`
  - Repeating the argument:
    - ✓ `( 1 + ( 5 * 20 ) )`
    - ✓ `( 1 + 100 )`
    - ✓ `101`
- Extensions. More ops, precedence order, associativity.
    - ✓ `1 + (2 - 3 - 4) * 5 * sqrt(6 + 7)`

Stack-based programming languages

- Observation 1.
  Remarkably, the 2-stack algorithm computes the same value if the operator occurs after the two values.

```
( 1 ( ( 2 3 + ) ( 4 5 * ) * ) + )
```

- Observation 2.
  All of the parentheses are redundant!

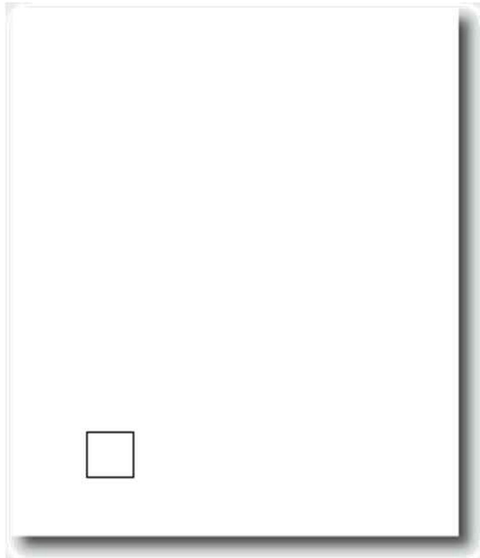
```
1 2 3 + 4 5 * * +
```

- Bottom line. Postfix or "reverse Polish" notation.

- Applications. Postscript, Forth, calculators, Java virtual machine, …

Stack-based programming languages: PostScript

- Page description language
  - explicit stack
  - full computational model
  - graphics engine
- Basics
  - %!: “I am a PostScript program”
  - literal: “push me on the stack”
  - function calls take args from stack
  - turtle graphics built in



a PostScript program

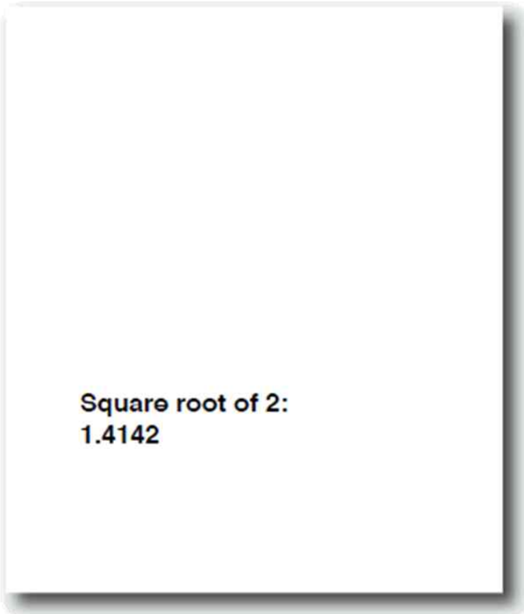
```
%!
72 72 moveto
0 72 rlineto
72 0 rlineto
0 -72 rlineto
-72 0 rlineto
2 setlinewidth
stroke
```

Stack-based programming languages: PostScript

- Data types
  - basic: integer, floating point, boolean, ...
  - graphics: font, path, ...
  - full set of built-in operators
- Text and strings
  - full font support
  - show (display a string, using current font)
  - cvs (convert anything to a string)

like System.out.print()

like toString()




```
%!
/Helvetica-Bold findfont 16 scalefont setfont
72 168 moveto
(Square root of 2:) show
72 144 moveto
2 sqrt 10 string cvs show
```

Stack-based programming languages: PostScript

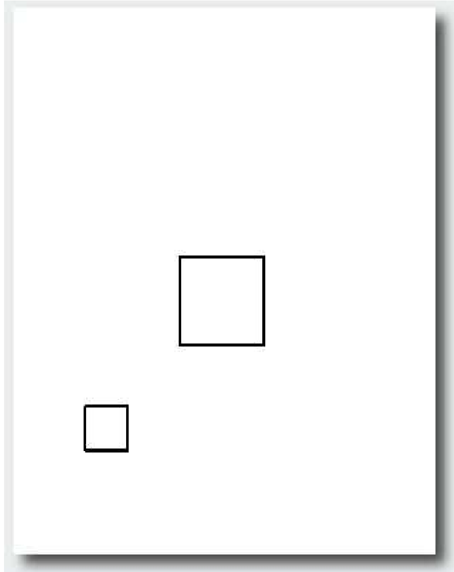
- Variables (and functions)
  - identifiers start with /
  - `def` operator associates id with value
  - Braces
  - args on stack

function definition

function calls

```
%!
/box
{
     /sz exch def
     0 sz rlineto
     sz 0 rlineto
     0 sz neg rlineto
     sz neg 0 rlineto
} def

72 144 moveto
72 box
288 288 moveto
144 box
2 setlinewidth
stroke
```

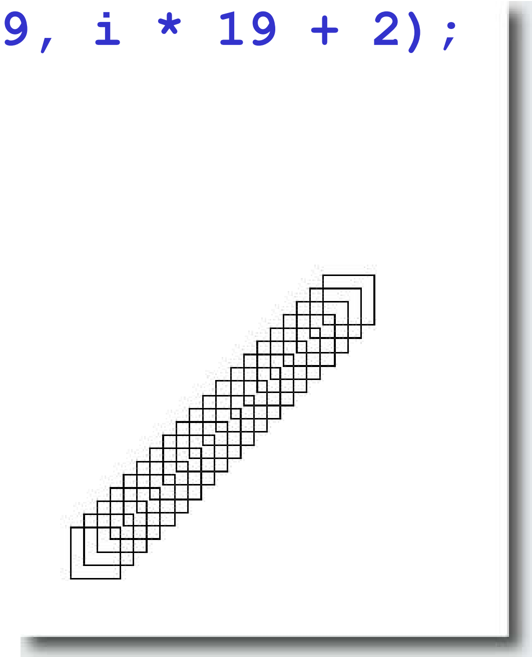


Stack-based programming languages: PostScript

- for loop
  - "from, increment, to" on stack
  - loop body in braces
  - for operator

```
1 1 20
{ 19 mul dup 2 add moveto 72 box }
for
```

```
for(i = 0; i <= 20; i++) {
    moveto(i * 19, i * 19 + 2);
    box(72);
}
```



- if-else
  - boolean on stack
  - alternatives in braces
  - if operator

... (hundreds of operators)

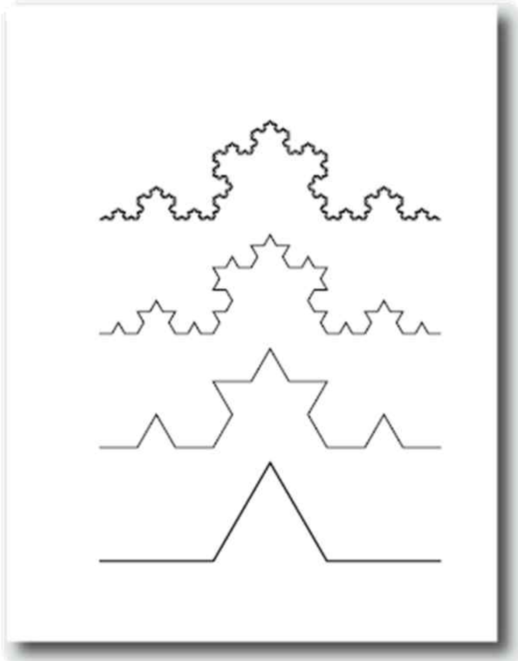
Stack-based programming languages: PostScript

An application: all figures in Algorithms in Java

```
%!
72 72 translate

/kochR
  {
    2 copy ge { dup 0 rlineto }
      {
        3 div
        2 copy kochR 60 rotate
        2 copy kochR -120 rotate
        2 copy kochR 60 rotate
        2 copy kochR
      } ifelse
    pop pop
  } def

0   0 moveto   81 243 kochR
0  81 moveto   27 243 kochR
0 162 moveto    9 243 kochR
0 243 moveto    1 243 kochR
stroke
```



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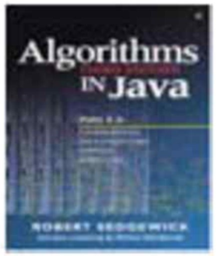


Queue applications

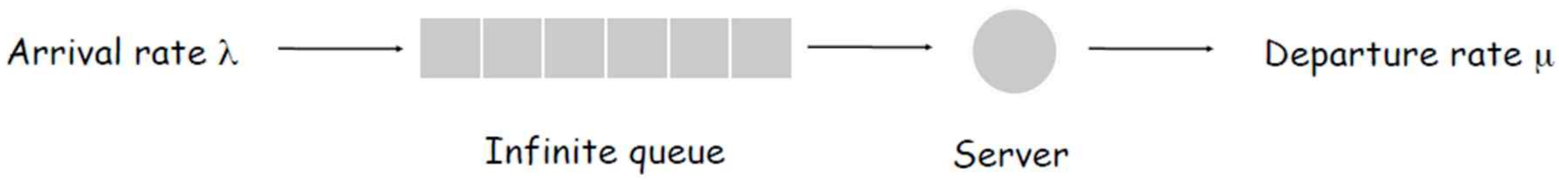
- Familiar applications.
  - iTunes playlist.
  - Data buffers (iPod, TiVo).
  - Asynchronous data transfer (file IO, pipes, sockets).
  - Dispensing requests on a shared resource (printer, processor).
- Simulations of the real world.
  - Traffic analysis.
  - Waiting times of customers at call center.
  - Determining number of cashiers to have at a supermarket.

M/D/1 queuing model

- M/D/1 queue.
  - Customers are serviced at fixed rate of μ per minute.
  - Customers arrive according to Poisson process at rate of λ per minute.

inter-arrival time has exponential distribution

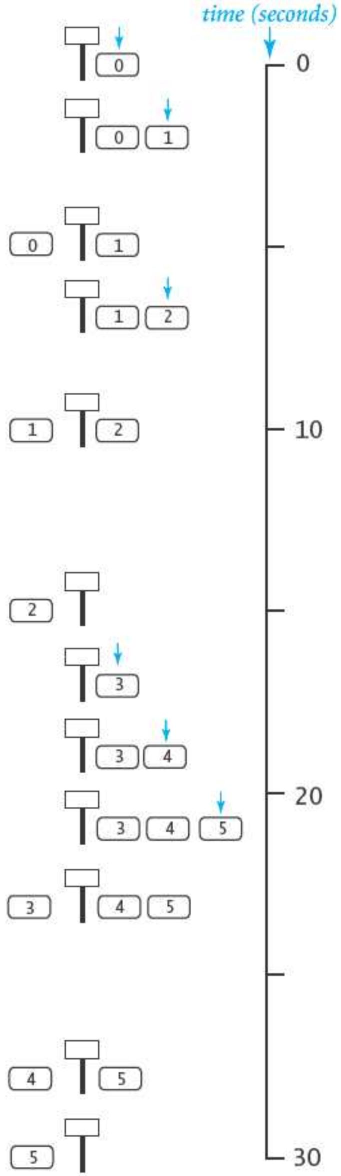
$$\Pr[X \le x] = 1 - e^{-\lambda x}$$



Q. What is average wait time W of a customer?

Q. What is average number of customers L in system?

M/D/1 queuing model: example

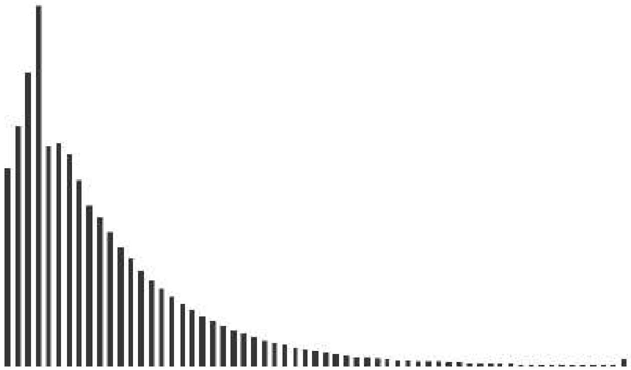


	arrival	departure	wait
0	0	5	5
1	2	10	8
2	7	15	8
3	17	23	6
4	19	28	9
5	21	30	9

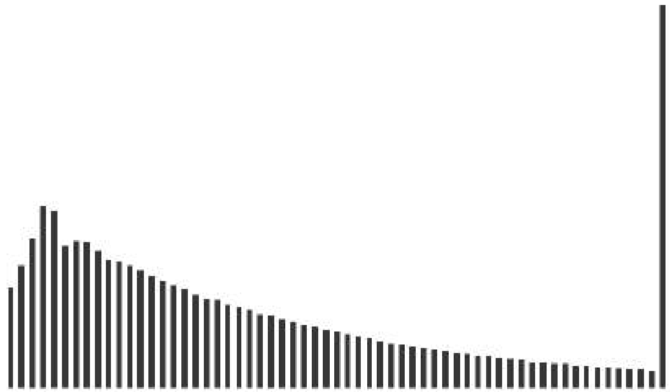
M/D/1 queuing model: experiments and analysis

- Observation.
  - As service rate μ approaches arrival rate λ , service goes to h***.

```
% java MD1Queue .167 .25
```



```
% java MD1Queue .167 .22
```



Little's Law

- Queueing theory (see ORFE 309).

$$W = \frac{\lambda}{2\mu(\mu-\lambda)} + \frac{1}{\mu}, \quad L = \lambda W$$

wait time W and queue length L approach infinity as service rate approaches arrival rate

M/D/1 queuing model: event-based simulation

```
public class MD1Queue
{
   public static void main(String[] args)
   {
      double lambda = Double.parseDouble(args[0]);   // arrival rate
      double mu     = Double.parseDouble(args[1]);   // service rate
      Histogram hist = new Histogram(60);
      Queue<Double> q = new Queue<Double>();
      double nextArrival = StdRandom.exp(lambda);
      double nextService = 1/mu;
      while (true)
      {
         while (nextArrival < nextService)
         {
            q.enqueue(nextArrival);
            nextArrival += StdRandom.exp(lambda);
         }
         double wait = nextService - q.dequeue();
         hist.addDataPoint(Math.min(60,  (int)  (wait)));
         if (!q.isEmpty())
            nextService = nextArrival + 1/mu;
         else
            nextService = nextService + 1/mu;
      }
   }
}
```