EE23BTECH11054 - Sai Krishna Shanigarapu*

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49. A sinusoidal message signal having root mean square value of 4V and frequency of 1 kHz fed to a phase modulator with phase deviation constant 2 rad/volt. If the carrier signal is $c(t) = 2\cos(2\pi 10^6 t)$, the maximum instantaneous frequency of the phase modulated signal (rounded off to one decimal place) is ______ Hz. (GATE 2021 EC)

Solution:

Parameter	Description	Value
f_m	Message signal frequency	1 kHz
$c(t)$	Carrier signal	$2\cos\left(2\pi 10^6 t\right)$
k_p	Phase sensitivity factor	2 rad V^{-1}
$m(t)$	message signal	$A_m \sin 2\pi f_m t$
f_c	Carrier signal frequency	1 kHz
A_c	Amplitude of carrier signal	2
A_m	Amplitude of message signal	

TABLE I
Input Parameters

Parameter	Description	Formula
$m(t)_{rms}$	rms value of $m(t)$	$\frac{A_m}{\sqrt{2}}$
$s(t)$	Phase modulation	$A_c \sin\left[2\pi f_c t + \theta_i(t)\right]$
$\theta_i(t)$	phase	$k_p\, m(t)$

TABLE II
Formulae

Phase Modulation Signal Proof:
let e_m and e_c be message and carrier signals, $2\pi f_m$ and $2\pi f_c$ be radial frequencies and A_m and A_c be their amplitudes respectively. Then,

$$e_m = A_m \cos(2\pi f_m t) \tag{1}$$
$$e_c = A_c \sin(2\pi f_c t) \tag{2}$$

On rewriting the equation 2

$$e = E_c \sin(\theta) \tag{3}$$
$$\theta = 2\pi f_c t + k_p e_m \tag{4}$$
$$= 2\pi f_c t + k_p E_m \cos(2\pi f_m t) \tag{5}$$
$$m_p = k_p E_m \tag{6}$$
$$\theta = 2\pi f_c t + m_p \cos(2\pi f_m t) \tag{7}$$
$$\implies s(t) = E_c \sin\left(2\pi f_c t + \underbrace{m_p \cos(2\pi f_m t)}_{\theta_i(t)}\right) \tag{8}$$

$$m(t)_{rms} = 4V \tag{9}$$
$$A_m = 4\sqrt{2} \tag{10}$$

From Table I, eq (9) and eq (10)

$$m(t) = 4\sqrt{2}\sin\left(2\pi 10^3 t\right) \tag{11}$$
$$\tag{12}$$

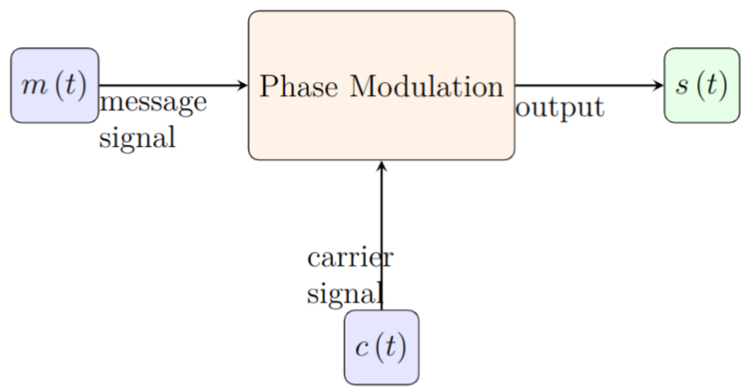

Fig. 1. Block diagram of phase modulation

From Table I, II and using eq (8) instantaneous frequency is given as,

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d}{dt}\theta_i(t) \tag{13}$$
$$= f_c + \frac{1}{2\pi}\frac{d}{dt}\left[k_p m(t)\right] \tag{14}$$
$$= f_c + \frac{1}{2\pi}\frac{d}{dt}\left(4\sqrt{2}\sin\left(2\pi 10^3 t\right)\right) \tag{15}$$
$$= f_c + \frac{2}{2\pi} 4\sqrt{2}\left(2\pi 10^3\right)\left(\cos\left(2\pi 10^3 t\right)\right) \tag{16}$$
$$= 1000 + 8\sqrt{2} \text{ x } 10^3 \cos\left(2\pi 10^3 t\right) \tag{17}$$

Thus,

$$\implies f_{i_{max}} = 1011313.7\, Hz \tag{18}$$

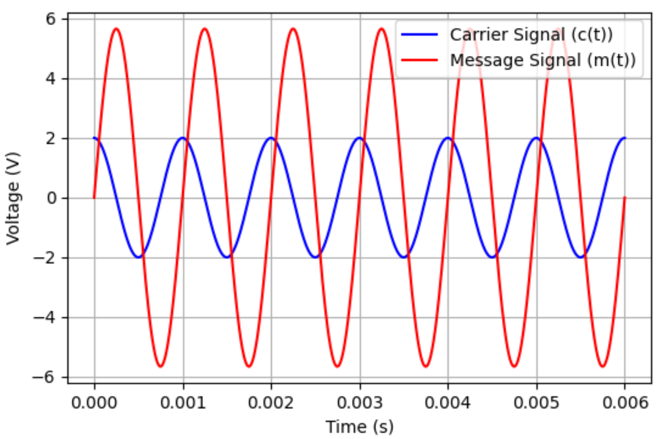

Fig. 2. plot of $m\left(t\right)$ and $c\left(t\right)$