

EE23BTECH11054 - Sai Krishna Shanigarapu*

GATE EC 2022

54. In a circuit, there is a series connection of an ideal resistor and an ideal capacitor. The conduction current (in Amperes) through the resistor is $2 \sin \left(t + \frac{\pi}{2} \right)$. The displacement current (in Amperes) through the capacitor is _____.

- (A) $2 \sin (t)$
 (B) $2 \sin \left(t + \pi \right)$
 (C) $2 \sin \left(t + \frac{\pi}{2} \right)$
 (D) 0

(GATE EC 2022)

Solution:

Parameter	Description	Value
I_c	Conduction Current	$2 \sin \left(t + \frac{\pi}{2} \right)$
A	Cross-sectional area	

TABLE I
PARAMETERS

Parameter	Description	Formula
Q	Charge	$\int I_c dt$
D	Electric Displacement	$\frac{Q}{A}$
J_D	Displacement current density	$\frac{\partial D}{\partial t}$
I_D	Displacement current	$J_D \times A$

TABLE II
FORMULAE

S Domain	Time Domain
$\frac{1}{s}$	$u(t)$
$\frac{-s}{a^2 + s^2}$	$-\cos(at)$
$\frac{a}{a^2 + s^2}$	$\sin(at)$
$\frac{1}{s+a}$	e^{-at}

TABLE III
LAPLACE TRANSFORMS

$$\mathcal{L} \left[\int f(t) dt \right] = \int_0^\infty \left[\int f(t) dt \right] e^{-st} dt \quad (1)$$

$$= \int_0^\infty u dv \quad \text{where} \begin{cases} u = \int f(t) dt \\ dv = e^{-st} dt \end{cases} \quad (2)$$

$$= uv - v \int du \quad (3)$$

$$= \frac{1}{s} \int f(t) dt \Big|_0^\infty + \frac{1}{s} \int_0^\infty f(t) e^{-st} dt \quad (4)$$

$$\Rightarrow \frac{1}{s} \int f(t) dt \Big|_0^\infty + \frac{1}{s} F(s) \quad (5)$$

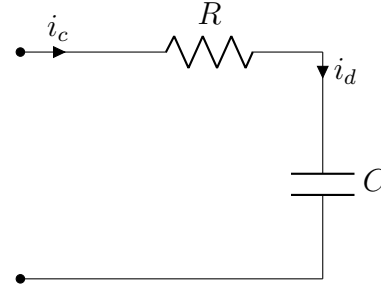


Fig. 1. Circuit 1

From Table II, Table III and eq (5)

$$I_c(s) = \frac{2s}{s^2 + 1} \quad (6)$$

$$Q_c(s) = \frac{2}{s(s^2 + 1)} \quad (7)$$

$$D(s) = \frac{1}{A} \left(\frac{2}{s(s^2 + 1)} \right) \quad (8)$$

$$J_D(s) = \frac{2}{A} \left(\frac{1}{s^2 + 1} \right) \quad (9)$$

$$I_D(s) = \frac{2}{s^2 + 1} \quad (10)$$

$$\Rightarrow I_D = 2 \sin t \quad (11)$$

From figure 2, phase of I_d is $\frac{\pi}{2}$

$$\therefore I_d = 2 \sin \left(t + \frac{\pi}{2} \right) \quad (12)$$

\therefore (C) is correct.

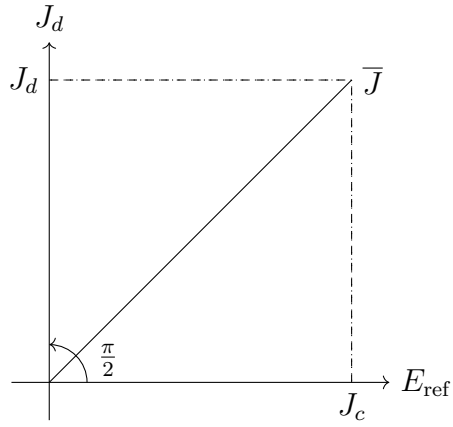


Fig. 2. Phasor plot

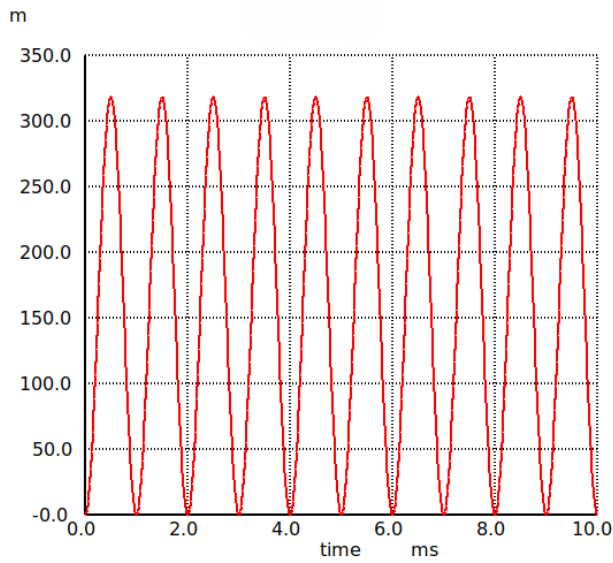


Fig. 3. Spice simulation of displacement current

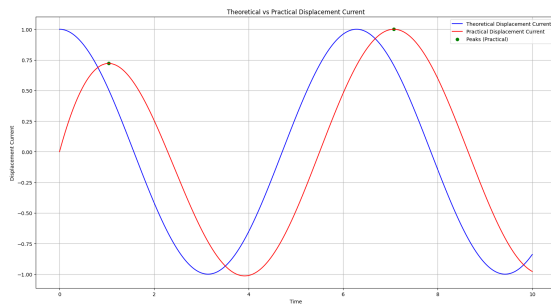


Fig. 4. Thoritical vs Practical simulation