EE23BTECH11054 - Sai Krishna Shanigarapu*

GATE EE 2023

54. In a circuit, there is a series connection of an ideal resistor and an ideal capacitor. The conduction current (in Amperes) through the resistor is $2\sin\left(t+\frac{\pi}{2}\right)$. The displacement current (in Amperes) through the capacitor is ____.

(A) $2\sin(t)$
(B) $2\sin(t+\pi)$
(C) $2\sin\left(t+\frac{\pi}{2}\right)$
(D) 0

(GATE EC 2022)

Solution:

Parameter	Description	Remarks
I_c	Conduction Current	$2\sin\left(t+\frac{\pi}{2}\right)$
I_d	Displacement current	?
A	Cross-sectional area	

TABLE I
PARAMETERS

Parameter	Description	Formula
Q	Charge	$\int I_c\,dt$
D	Electric Displacement	$\frac{Q}{A}$
J_D	Displacement current density	$\frac{\partial D}{\partial t}$

TABLE II
FORMULAE

S Domain	Time Domain
$\frac{1}{s}$	$u(t)$
$\frac{-s}{a^2+s^2}$	$-\cos(at)$
$\frac{a}{a^2+s^2}$	$\sin(at)$
$\frac{1}{s+a}$	e^{-at}

TABLE III
LAPLACE TRANSFORMS

$$\mathcal{L}\left[\int f(t)\,dt\right] = \int_0^\infty \left[\int f(t)\,dt\right] e^{-st}\,dt \tag{1}$$

$$= \int_0^\infty u\,dv \quad \text{where} \begin{cases} u = \int f(t)\,dt \\ dv = e^{-st}dt \end{cases} \tag{2}$$

$$= uv - v\int du \tag{3}$$

$$= \frac{1}{s}\int f(t)\,dt|_0 + \frac{1}{s}\int_0^\infty f(t)\,e^{-st}dt \tag{4}$$

$$\implies \frac{1}{s}\int f(t)\,dt|_0 + \frac{1}{s}F(s) \tag{5}$$

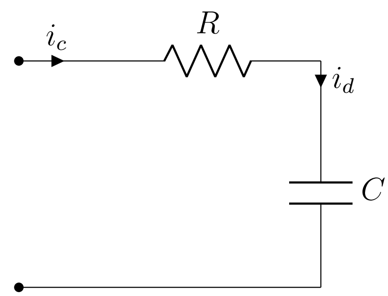

Fig. 1. Circuit 1

From Table II and Table III and eq 5

$$I_c(s) = \frac{2s}{s^2+1} \tag{6}$$

$$Q_c(s) = \frac{2}{s^2+1} \tag{7}$$

$$D(s) = \frac{1}{A}\left(\frac{2}{s^2+1}\right) \tag{8}$$

$$J_D(s) = \frac{2}{A}\left(\frac{s}{s^2+1}\right) \tag{9}$$

$$I_D(s) = \frac{2s}{s^2+1} \tag{10}$$

$$\implies I_D = 2\sin t \tag{11}$$

From figure 2, phase of I_d is $\frac{\pi}{2}$

$$\therefore I_d = 2\sin\left(t+\frac{\pi}{2}\right) \tag{12}$$

$\therefore$ (C) is correct.

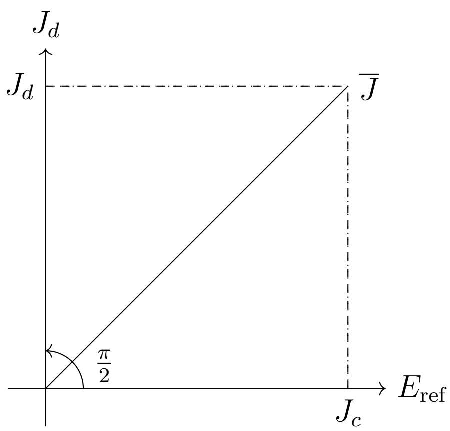

Fig. 2. Phasor plot

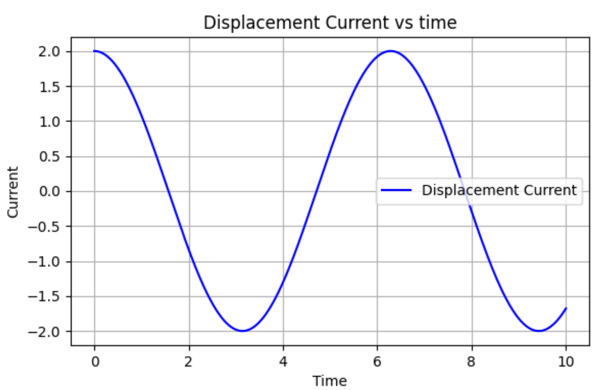

Fig. 3. plot of I_d vs time