

## EE23BTECH11054 - Sai Krishna Shanigarapu\*

GATE EC 2022

54. In a circuit, there is a series connection of an ideal resistor and an ideal capacitor. The conduction current (in Amperes) through the resistor is  $2 \sin \left( t + \frac{\pi}{2} \right)$ . The displacement current (in Amperes) through the capacitor is \_\_\_\_\_.

- (A)  $2 \sin (t)$   
 (B)  $2 \sin \left( t + \pi \right)$   
 (C)  $2 \sin \left( t + \frac{\pi}{2} \right)$   
 (D) 0

(GATE EC 2022)

**Solution:**

Parameter	Description	Value
$I_c$	Conduction Current	$2 \sin \left( t + \frac{\pi}{2} \right)$
$A$	Cross-sectional area	

TABLE I  
PARAMETERS

Parameter	Description	Formula
$Q$	Charge	$\int I_c dt$
$D$	Electric Displacement	$\frac{Q}{A}$
$J_D$	Displacement current density	$\frac{\partial D}{\partial t}$
$I_D$	Displacement current	$J_D \times A$

TABLE II  
FORMULAE

S Domain	Time Domain
$\frac{1}{s}$	$u(t)$
$\frac{-s}{a^2 + s^2}$	$-\cos(at)$
$\frac{a}{a^2 + s^2}$	$\sin(at)$
$\frac{1}{s+a}$	$e^{-at}$

TABLE III  
LAPLACE TRANSFORMS

$$\mathcal{L} \left[ \int f(t) dt \right] = \int_0^\infty \left[ \int f(t) dt \right] e^{-st} dt \quad (1)$$

$$= \int_0^\infty u dv \quad \text{where} \begin{cases} u = \int f(t) dt \\ dv = e^{-st} dt \end{cases} \quad (2)$$

$$= uv - v \int du \quad (3)$$

$$= \frac{1}{s} \int f(t) dt \Big|_0^\infty + \frac{1}{s} \int_0^\infty f(t) e^{-st} dt \quad (4)$$

$$\Rightarrow \frac{1}{s} \int f(t) dt \Big|_0^\infty + \frac{1}{s} F(s) \quad (5)$$

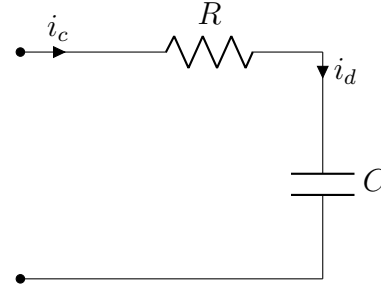


Fig. 1. Circuit 1

From Table II, Table III and eq (5)

$$I_c(s) = \frac{2s}{s^2 + 1} \quad (6)$$

$$Q_c(s) = \frac{2}{s(s^2 + 1)} \quad (7)$$

$$D(s) = \frac{1}{A} \left( \frac{2}{s(s^2 + 1)} \right) \quad (8)$$

$$J_D(s) = \frac{2}{A} \left( \frac{1}{s^2 + 1} \right) \quad (9)$$

$$I_D(s) = \frac{2}{s^2 + 1} \quad (10)$$

$$\Rightarrow I_D = 2 \sin t \quad (11)$$

From figure 2, phase of  $I_d$  is  $\frac{\pi}{2}$

$$\therefore I_d = 2 \sin \left( t + \frac{\pi}{2} \right) \quad (12)$$

$\therefore$  (C) is correct.

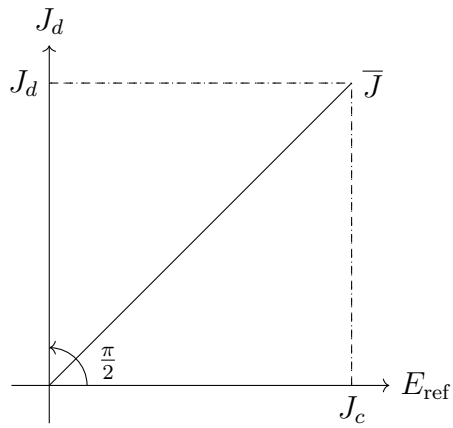


Fig. 2. Phasor plot

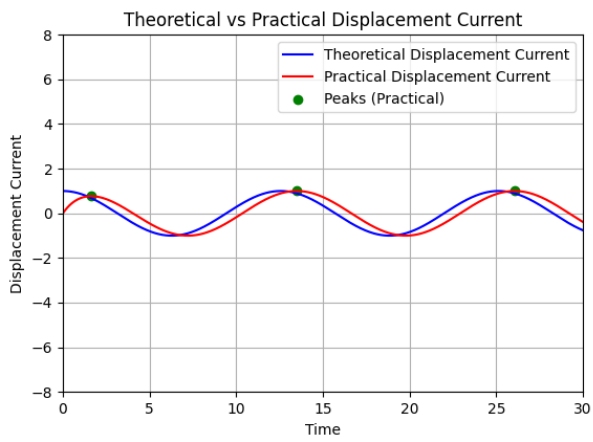


Fig. 3. Thoritical vs Practical simulation