EE23BTECH11054 - Sai Krishna Shanigarapu*

Gate EE 2022

28. The network shown below has a resonant frequency of 150 kHz and bandwidth of 600 Hz. The Q-factor of the network is _____
(rounded off to one decimal place).
(GATE 2022 EC)

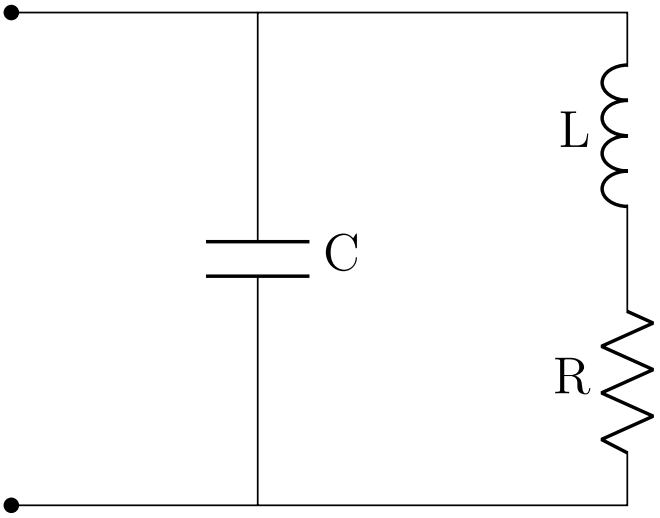

Fig. 1. Circuit 1

Solution:

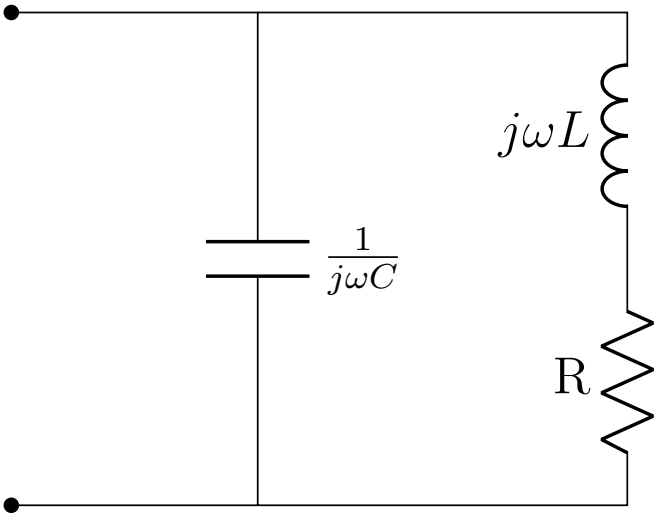

Fig. 2. Circuit 2

Parameter	Description	Value
f_0	Resonant frequency	150 kHz
B	Bandwidth	600 Hz

TABLE I
Parameters

Parameter	Description	Formula
Q	Quality factor	$\frac{X_L}{R}$
B	Bandwidth	$\frac{R}{2\pi L}$
ω_0	Radial resonant frequency	$2\pi f_0$
X_L	Inductive reactance	ωL
X_C	Capacitive reactance	$\frac{1}{\omega C}$

TABLE II
Formulae

At Resonance,

$$X_L = X_C \tag{1}$$

$$\omega_0 L = \frac{1}{\omega_0 C} \tag{2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{3}$$

$$2\pi f_0 = \frac{1}{\sqrt{LC}} \tag{4}$$

$$\implies f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{5}$$

Using Table II,

$$Q = \frac{X_L}{R} \tag{6}$$

$$= \frac{\omega_0 L}{R} \tag{7}$$

$$= \left(\frac{1}{\sqrt{LC}}\right)\frac{L}{R} \tag{8}$$

$$\implies Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{9}$$

From eq (5) and Table II

$$\frac{f_0}{B} = \left(\frac{1}{2\pi\sqrt{LC}}\right)\frac{2\pi L}{R} \tag{10}$$

$$= \left(\frac{1}{\sqrt{LC}}\right)\frac{L}{R} \tag{11}$$

$$\implies \frac{f_0}{B} = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{12}$$

From Table I, eq (9) and eq (12),

$$Q = \frac{f_0}{B} \tag{13}$$

$$= \frac{150 \text{ x } 10^3}{600} \tag{14}$$

$$= 250 \tag{15}$$

$\therefore$ Q-factor is 250

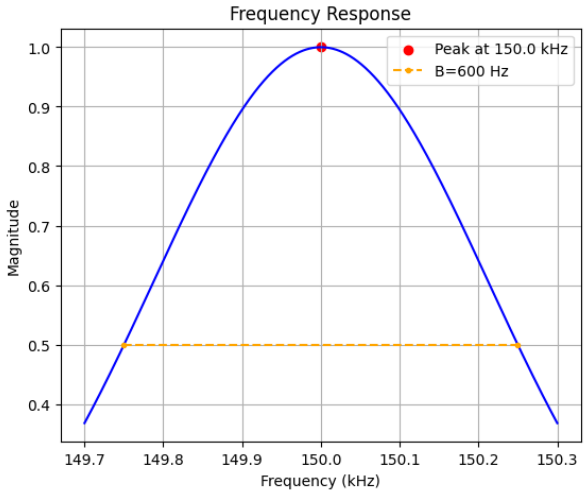

Fig. 3. Plot of Q-factor