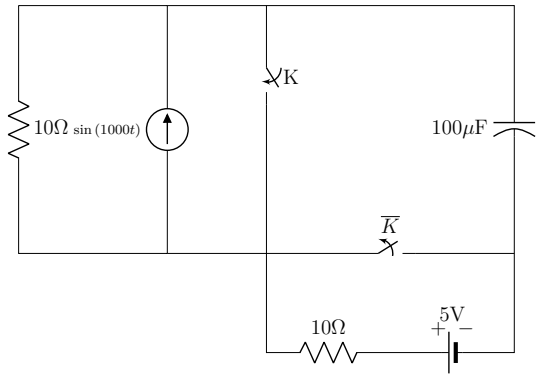
EE23BTECH11054 - Sai Krishna Shanigarapu*

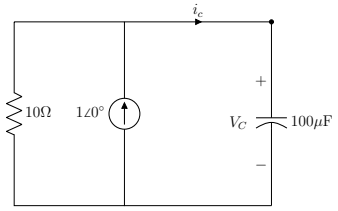
Gate EE 2023

54. The circuit shown in the figure is initially in the steady state with the switch K in open condition and $\overline{K}$ in closed condition. The switch K is closed and $\overline{K}$ is opened simultaneously at the instant $t = t_1$, where $t_1 > 0$. The minimum value of t_1 in milliseconds such that there is no transient in the voltage across the 100 μF capacitor, is ____ (Round off to 2 decimal places).



Solution:

Case(i) Switch K is open and $\overline{K}$ is closed.



$$X_c = -10j \tag{1}$$

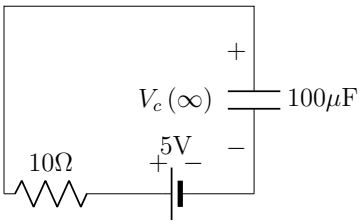
Using Current divider rule,

$$i_c = \frac{10}{10 + X_c} 1\angle 0^\circ \tag{2}$$

$$= \frac{1\angle 0^\circ}{1 - j} \tag{3}$$

$$\implies V_c = 7.07 \sin\left(1000t - 45^\circ\right) V \tag{4}$$

Case(ii) Switch K is closed and $\overline{K}$ is open.



Symbol	description	value
$V_C(\infty)$	Voltage across capacitor after long time	$5V$
τ	Time constant	1 msec
R	Resistance	10Ω
C	capacitance	100μF
f	frequency of the current source	$\frac{500}{\pi}$

TABLE I
Parameters

The voltage across capacitor at time t is given as,

$$V_c(t) = V_c(\infty) + \left(V_c(0) - V_c(\infty)\right) e^{-t/\tau} \tag{5}$$

$$\implies V_c(t) = 5 + \left(7.07 \sin\left(1000t - 45^\circ\right) - 5\right) e^{-(t-t_1)/\tau} \tag{6}$$

For transient free voltage,

$$7.07 \sin\left(1000t_1 - 45^\circ\right) = 5 \tag{7}$$

$$1000t_1 - \frac{\pi}{4} = \frac{5}{7.07} \tag{8}$$

$$\implies t_1 \approx 1.57\text{msec} \tag{9}$$

Symbol	Description	Formula
τ	Time constant	RC
X_c	Capacitive reactance	$\frac{1}{2\pi fC}$
V_c	Voltage across capacitor	$i_c X_c$

TABLE II
Formulae