

# EE23BTECH11054 - Sai Krishna Shanigarapu\*

## GATE EE 2023

54. The circuit shown in the figure is initially in the steady state with the switch K in open condition and  $\overline{K}$  in closed condition. The switch K is closed and  $\overline{K}$  is opened simultaneously at the instant  $t = t_1$ , where  $t_1 > 0$ . The minimum value of  $t_1$  in milliseconds such that there is no transient in the voltage across the  $100 \mu F$  capacitor, is \_\_\_\_ (Round off to 2 decimal places). (GATE EE 2023)

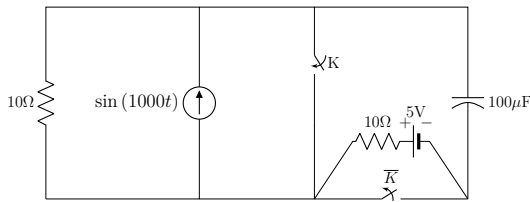


Fig. 1. Circuit 1

### Solution:

1) Switch K is open and  $\overline{K}$  is closed.

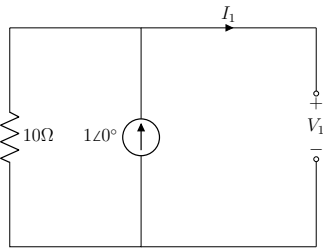


Fig. 2. K is open and  $\overline{K}$  is closed

Using Current divider rule,

$$I_1(j\omega) = \frac{10}{10 + \frac{1}{j\omega C}} \quad (1)$$

$$V_1(j\omega) = \frac{10}{1 + 10j\omega C} \quad (2)$$

$$|V_1(j\omega)| = 5\sqrt{2} \quad (3)$$

From Table I

$$V_1(t) = 5\sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right) \quad (4)$$

2) Switch K is closed and  $\overline{K}$  is open.

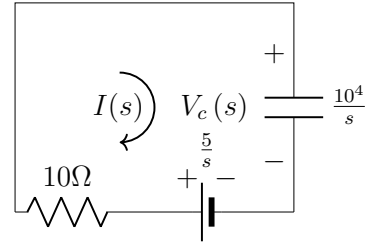


Fig. 3. K is closed and  $\overline{K}$  is open

The capacitor is charged. Thus, acts as a voltage source.

From eq(4) and Table II

$$V_1(s) = \frac{5000 - 5s}{s^2 + 10^6} \quad (5)$$

$$I(s) = \frac{\frac{5}{s} - V_1(s)}{10 + \frac{10^4}{s}} \quad (6)$$

$$V_c(s) = \frac{5}{s} - 10 \left( \frac{5 - V_1(s)}{1 + 10^{-3}s} \right) \quad (7)$$

For transient analysis,

$$\frac{5 - V_1(s)}{1 + 10^{-3}s} = 0 \quad (8)$$

$$\Rightarrow V_1(s) = 5 \quad (9)$$

$$\frac{10^7}{(s^2 + 10^6)(s + 10^3)} = \frac{5}{s} \quad (10)$$

$$\frac{5}{s + 10^3} + \frac{10^3 - s}{s^2 + 10^6} = \frac{5}{s} \quad (11)$$

$$\frac{-s}{s^2 + 10^6} + \frac{10^3}{s^2 + 10^6} + \frac{1}{s + 10^3} = \frac{1}{s} \quad (12)$$

From Table II

$$-\cos(1000t_1) + \sin(1000t_1) + e^{-10^3 t_1} = 1 \quad (13)$$

$$\Rightarrow t_1 \approx 1.57 \text{ msec} \quad (14)$$

Parameter	Description	Remarks
$\omega$	frequency of sine-wave	$1000 \text{ rad s}^{-1}$
$V_1(t)$	Voltage across capacitor	$ V_1(j\omega)  \sin(\omega t - \angle V_1(j\omega))$
$\angle V_1(j\omega)$	phase of $V_1(j\omega)$	$-\frac{\pi}{4}$
$C$	Capacitance	$100 \mu F$

TABLE I  
PARAMETERS

S Domain	Time Domain
$\frac{1}{s}$	$u(t)$
$\frac{-s}{a^2+s^2}$	$-\cos(at)$
$\frac{a}{a^2+s^2}$	$\sin(at)$
$\frac{1}{s+a}$	$e^{-at}$

TABLE II  
LAPLACE TRANSFORMS

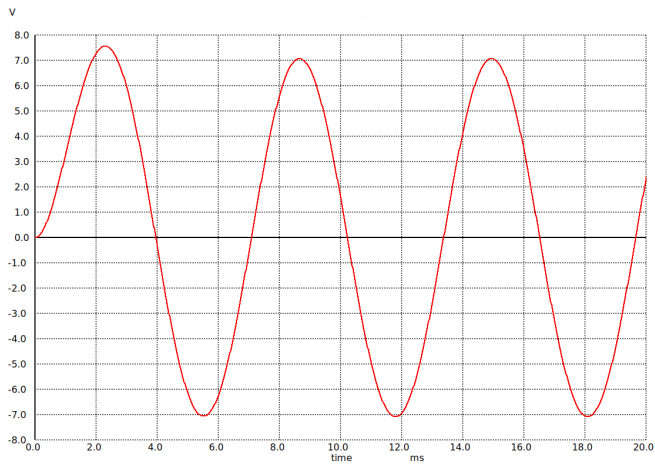


Fig. 4. plot of  $V_1$  vs time

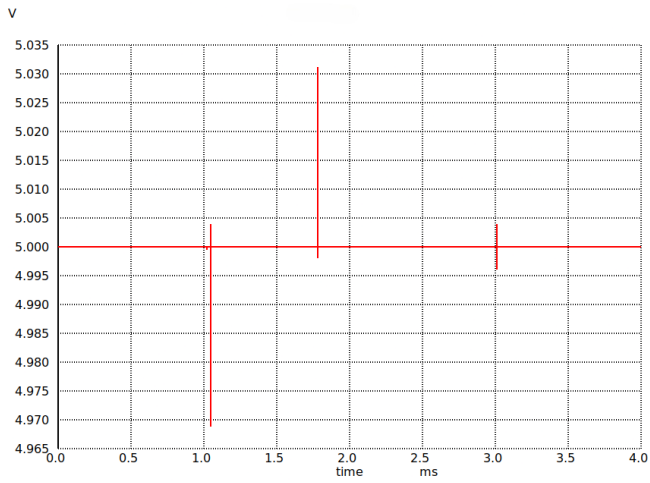


Fig. 5. plot of  $V_c$  vs time