EE23BTECH11054 - Sai Krishna Shanigarapu*

Gate EE 2023

54. The circuit shown in the figure is initially in the steady state with the switch K in open condition and $\overline{K}$ in closed condition. The switch K is closed and $\overline{K}$ is opened simultaneously at the instant $t = t_1$, where $t_1 > 0$. The minimum value of t_1 in milliseconds such that there is no transient in the voltage across the 100 μF capacitor, is ____ (Round off to 2 decimal places).
(GATE EE 2023)

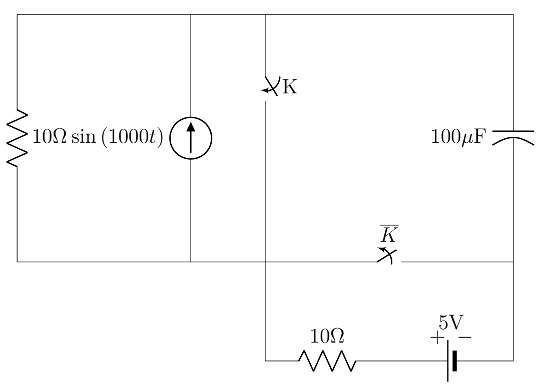

Fig. 1. Circuit 1

Solution:

1) Switch K is open and $\overline{K}$ is closed.

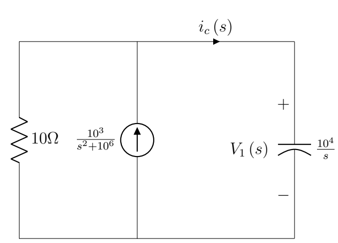

Fig. 2. Circuit 2

Using Current divider rule,

$$i_c(s) = \left(\frac{10^3}{s^2+10^6}\right)\left(\frac{10}{10+\frac{10^4}{s}}\right) \quad (1)$$

$$= \frac{10^3 s}{(s^2+10^6)(s+10^3)} \quad (2)$$

$$V_1(s) = \frac{10^7}{(s^2+10^6)(s+10^3)} \quad (3)$$

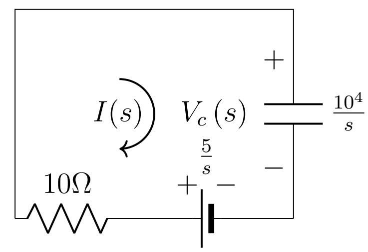

Fig. 3. Circuit 3

2) Switch K is closed and $\overline{K}$ is open.

$$I(s) = \frac{\frac{5}{s} - V_1(s)}{10+\frac{10^4}{s}} \quad (4)$$

$$V_c(s) = \frac{5}{s} - 10\left(\frac{5-V_1(s)}{1+10^{-3}s}\right) \quad (5)$$

For transient analysis,

$$\frac{5-V_1(s)}{1+10^{-3}s} = 0 \quad (6)$$

$$\implies V_1(s) = 5 \quad (7)$$

$$\frac{10^7}{(s^2+10^6)(s+10^3)} = \frac{5}{s} \quad (8)$$

$$\frac{5}{s+10^3} + \frac{10^3 - s}{s^2+10^6} = \frac{5}{s} \quad (9)$$

$$\frac{-s}{s^2+10^6} + \frac{10^3}{s^2+10^6} + \frac{1}{s+10^3} = \frac{1}{s} \quad (10)$$

$$-\cos(1000t_1) + \sin(1000t_1) + e^{-10^3 t_1} = 1 \quad (11)$$

$$\implies t_1 \approx 1.57\text{msec} \quad (12)$$

S Domain	Time Domain
$\frac{1}{s}$	$u(t)$
$\frac{-s}{a^2+s^2}$	$-\cos(at)$
$\frac{a}{a^2+s^2}$	$\sin(at)$
$\frac{1}{s+a}$	e^{-at}

TABLE I
Laplace transforms

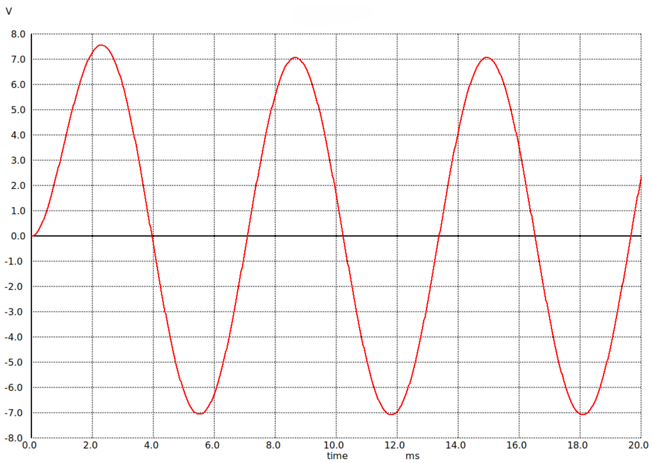

Fig. 4. plot of V_1

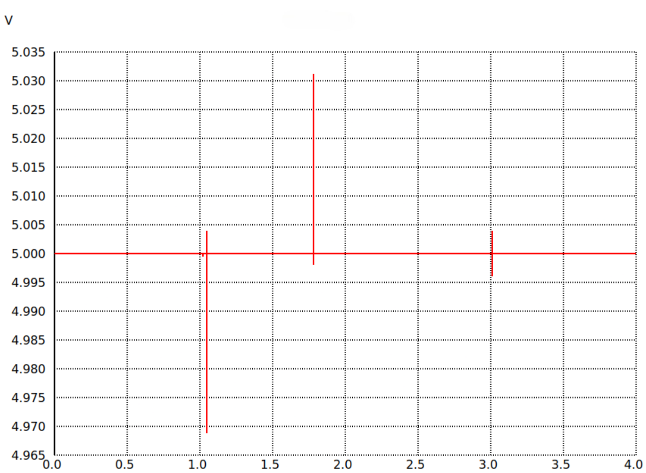

Fig. 5. plot of V_c