EE23BTECH11054 - Sai Krishna Shanigarapu*

Gate EE 2023

54. The circuit shown in the figure is initially in the steady state with the switch K in open condition and $\overline{K}$ in closed condition. The switch K is closed and $\overline{K}$ is opened simultaneously at the instant $t = t_1$, where $t_1 > 0$. The minimum value of t_1 in milliseconds such that there is no transient in the voltage across the 100 μF capacitor, is _____ (Round off to 2 decimal places).
(GATE EE 2023)

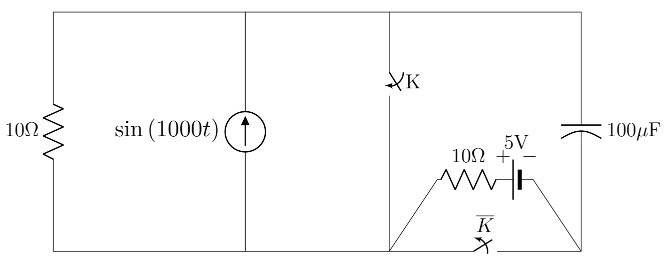

Fig. 1. Circuit 1

Solution:

1) Switch K is open and $\overline{K}$ is closed.

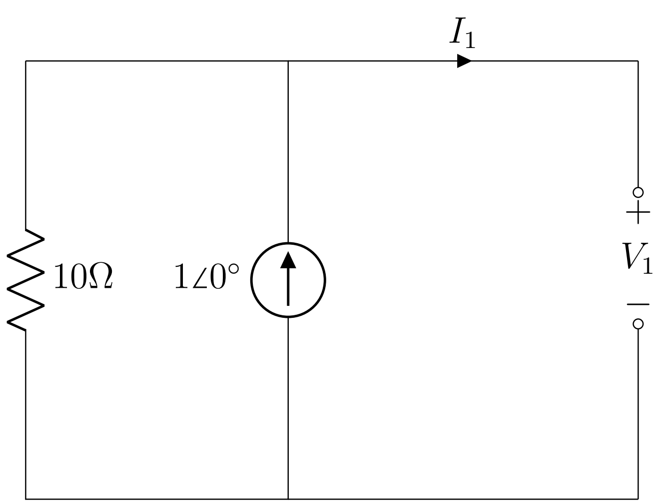

Fig. 2. K is open and $\overline{K}$ is closed

Using Current divider rule,

$$I_1\left(j\omega\right) = \frac{10}{10 + \frac{1}{j\omega C}} \tag{1}$$

$$V_1\left(j\omega\right) = \frac{10}{1 + 10j\omega C} \tag{2}$$

$$\left|V_1\left(j\omega\right)\right| = 5\sqrt{2} \tag{3}$$

From Table I

$$V_1\left(t\right) = 5\sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right) \tag{4}$$

2) Switch K is closed and $\overline{K}$ is open.

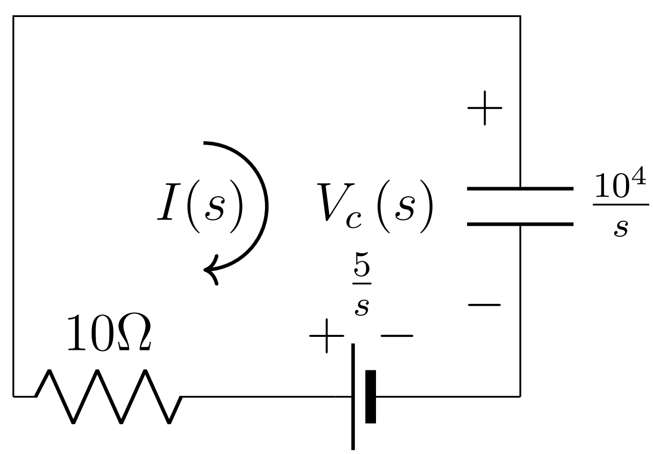

Fig. 3. K is closed and $\overline{K}$ is open

The capacitor is charged. Thus, acts as a voltage source.
From eq(4) and Table II

$$V_1\left(s\right) = \frac{5000 - 5s}{s^2 + 10^6} \tag{5}$$

$$I\left(s\right) = \frac{\frac{5}{s} - V_1\left(s\right)}{10 + \frac{10^4}{s}} \tag{6}$$

$$V_c\left(s\right) = \frac{5}{s} - 10\left(\frac{5 - V_1\left(s\right)}{1 + 10^{-3}s}\right) \tag{7}$$

For transient analysis,

$$\frac{5 - V_1(s)}{1 + 10^{-3}s} = 0 \tag{8}$$

$$\implies V_1(s) = 5 \tag{9}$$

$$\frac{10^7}{(s^2 + 10^6)(s + 10^3)} = \frac{5}{s} \tag{10}$$

$$\frac{5}{s + 10^3} + \frac{10^3 - s}{s^2 + 10^6} = \frac{5}{s} \tag{11}$$

$$\frac{-s}{s^2 + 10^6} + \frac{10^3}{s^2 + 10^6} + \frac{1}{s + 10^3} = \frac{1}{s} \tag{12}$$

From Table II

$$-\cos(1000t_1) + \sin(1000t_1) + e^{-10^3 t_1} = 1 \tag{13}$$

$$\implies t_1 \approx 1.57\text{msec} \tag{14}$$

Parameter	Description	Remarks
ω	frequency of sine-wave	1000 rad s^{-1}
$V_1(t)$	Voltage across capacitor	$\lvert V_1(j\omega)\rvert \sin(\omega t - \angle V_1(j\omega))$
$\angle V_1(j\omega)$	phase of $V_1(j\omega)$	$\frac{-\pi}{4}$
C	Capacitance	100μF

TABLE I
PARAMETERS

S Domain	Time Domain
$\frac{1}{s}$	$u(t)$
$\frac{-s}{a^2+s^2}$	$-\cos(at)$
$\frac{a}{a^2+s^2}$	$\sin(at)$
$\frac{1}{s+a}$	e^{-at}

TABLE II
LAPLACE TRANSFORMS

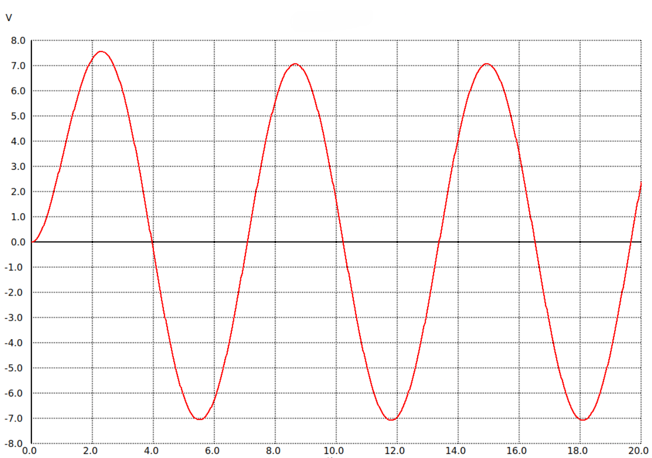

Fig. 4. plot of V_1 vs time

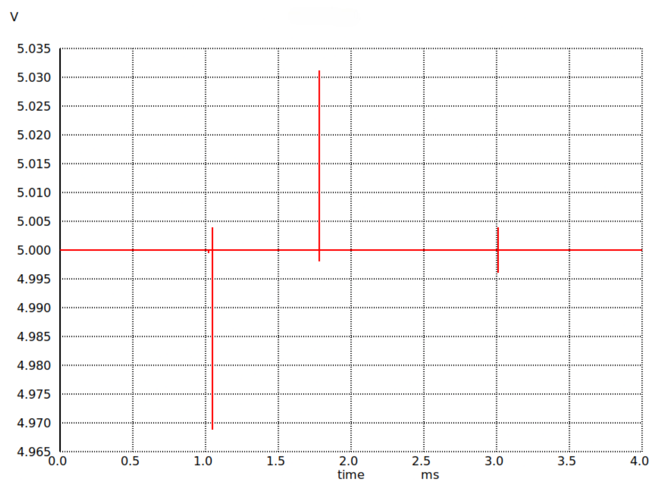

Fig. 5. plot of V_c vs time