ANALOG 12.8 8

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Question 8:
Suppose that the electric field amplitude of an electromagnetic wave is E_0 = 120N/C and that its frequency is f = 50.0 MHz.
(a) Determine, B_0, ω, k and λ
(b) Find expressions for **E** and **B**

Solution (a):

Table 1

Input Parameters		
Symbol	Description	value
f	frequency of source	50.0 MHz
E_0	Electric field amplitude	120 N/C
c	speed of light	3 x 10^8 m/s
$\mathbf{e_2}$, $\mathbf{e_3}$	Standard basis unit vectors	$\lvert\mathbf{e_2}\rvert = \lvert\mathbf{e_3}\rvert = 1$

General representation of electric and magnetic field is:

$$\mathbf{E} = E_0 sin(kx - \omega t)\mathbf{e_2} \tag{1}$$

$$\mathbf{B} = B_0 sin(kx - \omega t)\mathbf{e_3} \tag{2}$$

Magnitude of magnetic field strength is given as:

$$B_0 = \frac{E_0}{c} = \frac{120}{3x10^8} = 400nT \tag{3}$$

Angular frequency of source is given as:

$$\omega = 2\pi f = 3.14x10^8 rad/s \tag{4}$$

Propagation constant is given as:

$$k = \frac{\omega}{c} = 1.05 rad/m \tag{5}$$

Wavelength of the wave is given as:

$$\lambda = \frac{c}{f} = \frac{3x10^8}{50x10^6} = 6.0m \tag{6}$$

Solution (b)
Suppose the wave is propagating in the positive x direction. Then, the electric field vector will be in the positive y direction and the magnetic field vector will be in the positive z direction. This is because all three vectors are mutually perpendicular.

The standard basis vectors in Cartesian coordinates are denoted as e_1 (unit vector along the x-axis), e_2 (unit vector along the y-axis) and e_3 (unit vector along the z axis. The component of vectors **E** and **B** in terms of these basis vectors can be expressed as follows.

Equation of the Electric field vector is given as:
$\mathbf{E} = E_0 \sin(kx - \omega t)\mathbf{e_2}$

$$\boxed{\mathbf{E} = 120 \sin[1.05x - 3.1 \times 10^8 t]\mathbf{e_2}} \tag{7}$$

Magnetic field vector is given as:

$\mathbf{B} = B_0 \sin(kx - \omega t)\mathbf{e_3}$

$$\boxed{\mathbf{B} = (4 \times 10^{-7}) \sin[1.05x - 3.14 \times 10^8 t]\mathbf{e_3}} \tag{8}$$

Table 2

Output parameters		
Symbol	Description	value
B_0	Magnetic field strength	400nT
ω	Angular frequency	3.14 x 10^8m/s
k	Propagation constant	1.05rad/s
λ	Wavelength	6.0m
E	Electric field vector	120 sin[1.05x - 3.1 x 10^8t]$\mathbf{e_2}$
B	Magnetic field vector	(4 x 10^{-7})sin[1.05x - 3.14 x 10^8t]$\mathbf{e_3}$

Graphs of E and B