

EE23BTECH11054 - Sai Krishna Shanigarapu*

EXERCISE 9.2

13. If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m .

Solution:

$$Y(z) = \sum_{n=0}^{\infty} y(n) z^{-n} \quad (1)$$

$$= \frac{2(4 - z^{-1})}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (2)$$

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (3)$$

$$X(z) = \frac{Y(z)}{U(z)} \quad (4)$$

$$= 2 \left(\frac{1}{1 - z^{-1}} \right) + 6 \left(\frac{1}{1 - z^{-1}} \right)^2 \quad (5)$$

$$= \frac{8z^2 - 2z}{(z - 1)^2} \quad (6)$$

Using Contour Integration to find the inverse Z-transform,

$$x[n] = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \quad (7)$$

$$= \frac{1}{2\pi j} \oint_C \frac{(8z^{n+1} - 2z^n) dz}{(z - 1)^2} \quad (8)$$

We can observe that the pole is repeated 2 times and thus $m = 2$,

$$x[n] = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (9)$$

$$= \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{8z^{n+1} - 2z^n}{(z-1)^2} \right) \quad (10)$$

$$= \lim_{z \rightarrow 1} (8(n+1)z^n - 2nz^{n-1}) \quad (11)$$

$$= 6n + 8 \quad (12)$$

$$\Rightarrow x(n) = (6n + 8)(u(n)) \quad (13)$$

$$164 = (6m + 8)(u(m)) \quad (14)$$

$$\Rightarrow m = 26 \quad (15)$$

Symbol	Remarks
$y(n) = (3n^2 + 11n + 8)(u(n))$	Sum of n terms
$x(m-1)$	164
$y(n)$	$x(n) * u(n)$

TABLE I
PARAMETERS

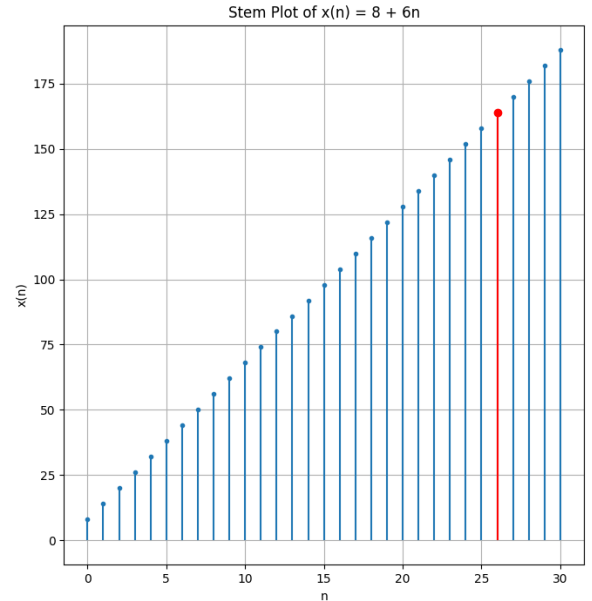


Fig. 1. Plot of $x(n)$ vs n