

# EE23BTECH11054 - Sai Krishna Shanigarapu\*

## EXERCISE 9.2

13. If the sum of  $n$  terms of an A.P. is  $3n^2 + 5n$  and its  $m^{th}$  term is 164, find the value of  $m$ .

**Solution:**

$$Y(z) = \sum_{n=0}^{\infty} y(n) z^{-n} \quad (1)$$

$$= \frac{2(4 - z^{-1})}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (2)$$

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (3)$$

$$X(z) = \frac{Y(z)}{U(z)} \quad (4)$$

$$= 2 \left( \frac{1}{1 - z^{-1}} \right) + 6 \left( \frac{1}{(1 - z^{-1})^2} \right) \quad (5)$$

$$= \frac{8z^2 - 2z}{(z - 1)^2} \quad (6)$$

Using Contour Integration to find the inverse Z-transform,

$$= \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \quad (7)$$

$$= \frac{1}{2\pi j} \oint_C \frac{(8z^{n+1} - 2z^n) dz}{(z - 1)^2} \quad (8)$$

$$= \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \quad (9)$$

$$= \lim_{z \rightarrow 1} \frac{d}{dz} \left( (z - 1)^2 \frac{8z^{n+1} - 2z^n}{(z - 1)^2} \right) \quad (10)$$

$$= \lim_{z \rightarrow 1} (8(n+1)z^n - 2nz^{n-1}) \quad (11)$$

$$= 6n + 8 \quad (12)$$

$$\Rightarrow x(n) = (6n + 8)(u(n)) \quad (13)$$

$$164 = (6m + 8)(u(m)) \quad (14)$$

$$\Rightarrow m = 26 \quad (15)$$

Symbol	Remarks
$y(n) = (3n^2 + 11n + 8)(u(n))$	Sum of $n$ terms
$x(m-1)$	164
$y(n)$	$x(n) * u(n)$

TABLE I  
PARAMETERS

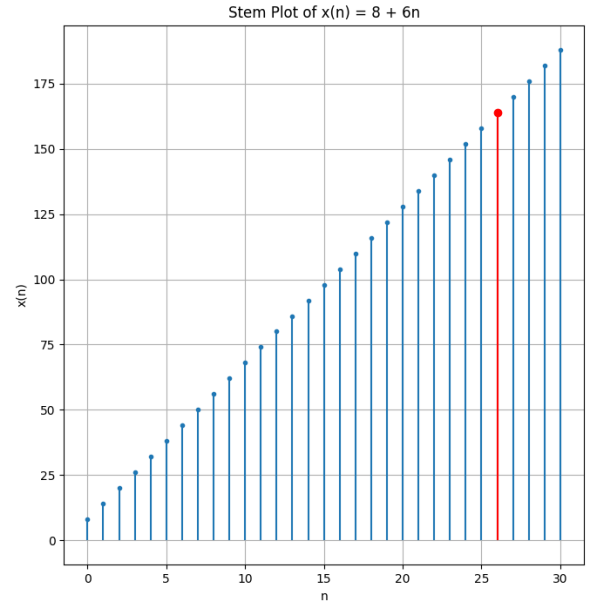


Fig. 1. Plot of  $x(n)$  vs  $n$