

EE23BTECH11054 - Sai Krishna Shanigarapu*

EXERCISE 9.2

13 If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m .

Solution: :

$$Y(z) = \sum_{n=0}^{\infty} y(n) z^{-n} \quad (1)$$

$$= \frac{2(4 - z^{-1})}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (2)$$

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (3)$$

$$X(z) = \frac{Y(z)}{U(z)} \quad (4)$$

$$x(n) = Z_z^{-1} \left[2 \left(\frac{1}{1 - z^{-1}} \right) + 6 \left(\frac{1}{1 - z^{-1}} \right)^2 \right] \quad (5)$$

$$(6)$$

In order to compute the Z-inverse of

$$X(z) = (1 - z^{-1})^{-2} \quad (7)$$

we develop the Power series for the function $(1 - x)^{-2}$.

$$(1 - x)^{-(\beta+1)} = \sum_{n=0}^{\infty} \binom{n + \beta}{\beta} x^n \quad (8)$$

for $x = z^{-1}$ and $\beta = 1$,

$$(1 - z^{-1})^{-2} = \sum_{n=0}^{\infty} \binom{n + 1}{1} z^{-n} \quad (9)$$

Comparing with the definition of Z-transform,

$$Z_z^{-1} \left(\frac{1}{1 - z^{-1}} \right)^2 = (n + 1) (u(n)) \quad (10)$$

$$x(n) = (6n + 8) (u(n)) \quad (11)$$

$$164 = (6m + 8) (u(n)) \quad (12)$$

$$m = 26 \quad (13)$$

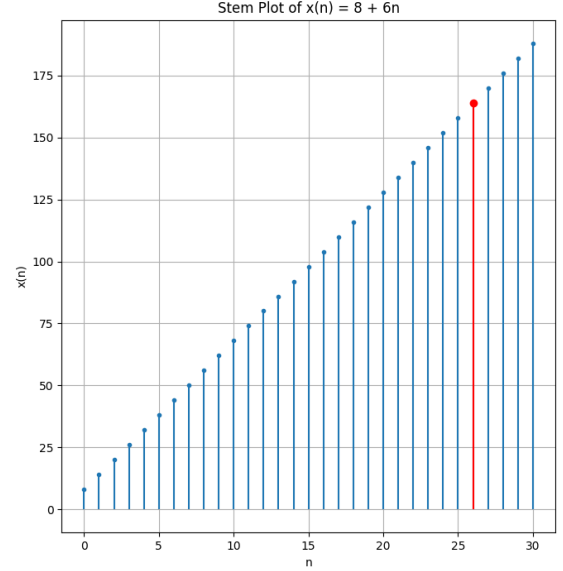


Fig. 1. Plot of $x(n)$ vs n

Symbol	Remarks
$y(n) = (3n^2 + 11n + 8) (u(n))$	Sum of n terms
$x(m - 1)$	164
$y(n)$	$x(n) * u(n)$
$Z_z^{-1} \left(\frac{1}{1 - z^{-1}} \right)$	$u(n)$

TABLE I
PARAMETERS