EE23BTECH11054 - Sai Krishna Shanigarapu*

Exercise 9.2

13 If the sum of n terms of an A.P. is $3n^2+5n$ and its m^{th} term is 164, find the value of m.

Solution: :

$$Y(z)=\sum_{n=0}^{\infty} y(n)\, z^{-n} \tag{1}$$

$$=\frac{2\left(4-z^{-1}\right)}{\left(1-z^{-1}\right)^3}, \qquad |z|>1 \tag{2}$$

$$U(z)=\frac{1}{1-z^{-1}}, \qquad |z|>1 \tag{3}$$

$$X(z)=\frac{Y(z)}{U(z)} \tag{4}$$

$$x(n)=Z_z^{-1}\left[2\left(\frac{1}{1-z^{-1}}\right)+6\left(\frac{1}{1-z^{-1}}\right)^2\right] \tag{5}$$

The inverse $Z-$transform of $\left(1-z^{-1}\right)^{-2}$ using Contour integration is given by,

$$Z_z^{-1}=\frac{1}{2\pi j}\oint_C X(z)\, z^{n-1}dz \tag{6}$$

$$=\frac{1}{2\pi j}\oint_C \frac{z^{n+1}}{(z-1)^2} \tag{7}$$

$$=\frac{1}{(2-1)!}\left[\frac{d}{dz}\left((z-1)^2\frac{z^{n+1}}{(z-1)^2}\right)\right]_{z=1} \tag{8}$$

$$=(n+1)\left(u(n)\right) \tag{9}$$

From (5) and (9)

$$x(n)=\left(2+6(n+1)\right)\left(u(n)\right) \tag{10}$$

$$=\left(6n+8\right)\left(u(n)\right) \tag{11}$$

$$164=\left(6m+8\right)\left(u(n)\right) \tag{12}$$

$$m=26 \tag{13}$$

Symbol	Remarks
$y(n)=\left(3n^2+11n+8\right)\left(u(n)\right)$	Sum of n terms
$x(m-1)$	164
$y(n)$	$x(n)*u(n)$
$Z_z^{-1}\left(\frac{1}{1-z^{-1}}\right)$	$u(n)$

TABLE I
Parameters

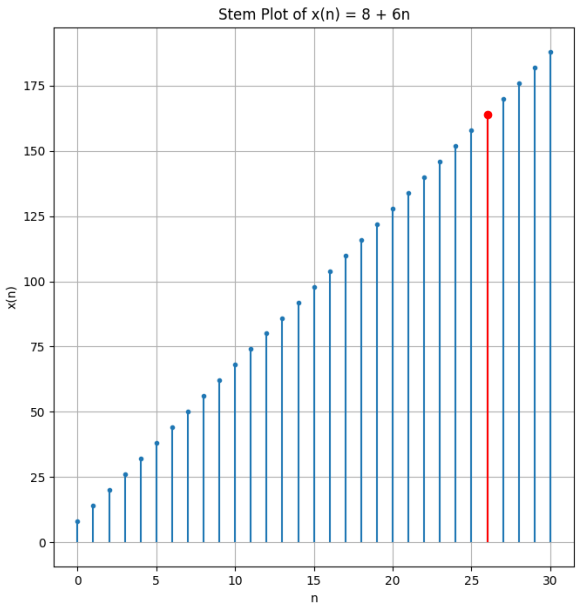

Fig. 1. Plot of x(n) vs n