

EE23BTECH11054 - Sai Krishna Shanigarapu*

EXERCISE 9.2

13 If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m .

Solution: :

$$Y(z) = \sum_{n=0}^{\infty} y(n) z^{-n} \quad (1)$$

$$= \frac{2(4 - z^{-1})}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (2)$$

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (3)$$

$$X(z) = \frac{Y(z)}{U(z)} \quad (4)$$

$$x(n) = Z_z^{-1} \left[2 \left(\frac{1}{1 - z^{-1}} \right) + 6 \left(\frac{1}{1 - z^{-1}} \right)^2 \right] \quad (5)$$

The inverse Z -transform of $(1 - z^{-1})^{-2}$ using Contour integration is given by,

$$Z_z^{-1} = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \quad (6)$$

$$= \frac{1}{2\pi j} \oint_C \frac{z^{n+1}}{(z - 1)^2} \quad (7)$$

$$= \frac{1}{(2 - 1)!} \lim_{z \rightarrow 1} \left[\frac{d}{dz} (z - 1)^2 \frac{z^{n+1}}{(z - 1)^2} \right] \quad (8)$$

$$= (n + 1) (u(n)) \quad (9)$$

From (5) and (9)

$$x(n) = (2 + 6(n + 1)) (u(n)) \quad (10)$$

$$= (6n + 8) (u(n)) \quad (11)$$

$$164 = (6m + 8) (u(n)) \quad (12)$$

$$m = 26 \quad (13)$$

| Symbol | Remarks |
|---------------------------------|------------------|
| $y(n) = (3n^2 + 11n + 8)(u(n))$ | Sum of n terms |
| $y(n)$ | $x(n) * u(n)$ |
| $Z_z^{-1} (1 - z^{-1})^{-2}$ | $u(n)$ |
| $x(m - 1)$ | 164 |

TABLE I
PARAMETERS

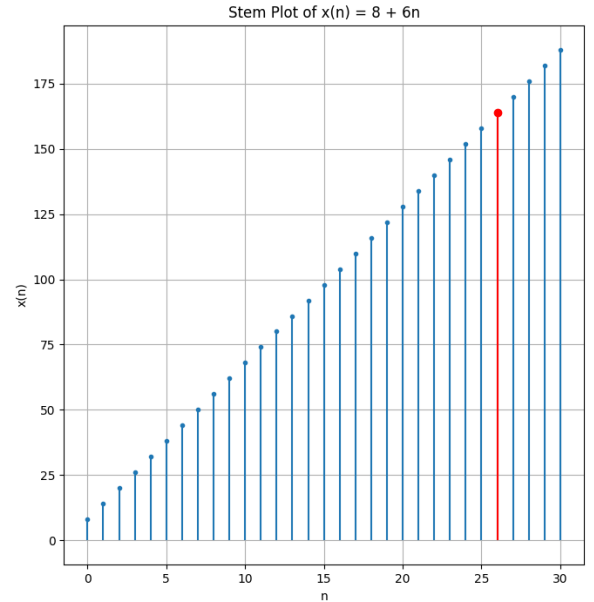


Fig. 1. Plot of $x(n)$ vs n