EE23BTECH11054 - Sai Krishna Shanigarapu*

Exercise 9.2

13 If the sum of n terms of an A.P. is $3n^2+5n$ and its m^{th} term is 164, find the value of m.

Solution: :

$$Y(z)=\sum_{n=0}^{\infty} y(n)\, z^{-n} \tag{1}$$

$$=\frac{2\left(4-z^{-1}\right)}{\left(1-z^{-1}\right)^3}, \qquad |z|>1 \tag{2}$$

$$U(z)=\frac{1}{1-z^{-1}}, \qquad |z|>1 \tag{3}$$

$$X(z)=\frac{Y(z)}{U(z)} \tag{4}$$

$$=2\left(\frac{1}{1-z^{-1}}\right)+6\left(\frac{1}{1-z^{-1}}\right)^2 \tag{5}$$

$$=\frac{8z^2-2z}{(z-1)^2} \tag{6}$$

Using Contour Integration to find the inverse Z-transform,

$$x[n]=\frac{1}{2\pi j}\oint_C X(z)\, z^{n-1}dz \tag{7}$$

$$=\frac{1}{2\pi j}\oint_C \frac{\left(8z^{n+1}-2z^n\right)dz}{(z-1)^2} \tag{8}$$

We can observe that the pole is repeated 2 times and thus $m=2$,

$$x[n]=\frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{9}$$

$$=\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{8z^{n+1}-2z^n}{(z-1)^2}\right) \tag{10}$$

$$=\lim_{z\to 1}\left(8(n+1)z^n-2nz^{n-1}\right) \tag{11}$$

$$=6n+8 \tag{12}$$

$$\implies x(n)=(6n+8)\,(u(n)) \tag{13}$$

$$164=(6m+8)\,(u(n)) \tag{14}$$

$$\implies m=26 \tag{15}$$

Symbol	Remarks
$y(n)=\left(3n^2+11n+8\right)(u(n))$	Sum of n terms
$x(m-1)$	164
$y(n)$	$x(n)*u(n)$

TABLE I
Parameters

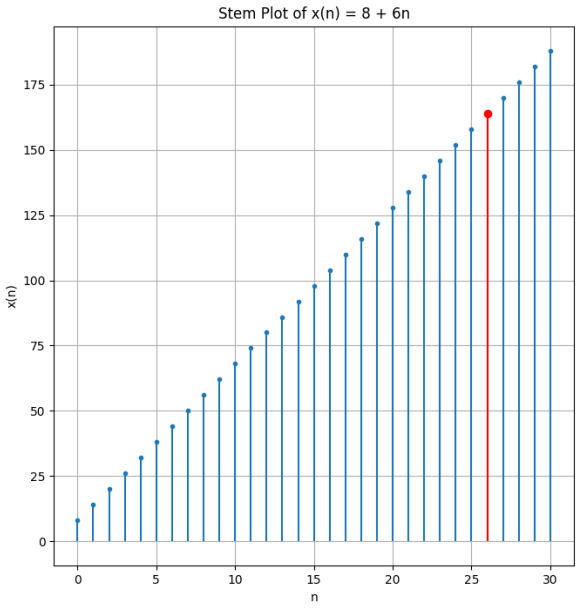

Fig. 1. Plot of x(n) vs n