

# EE23BTECH11054 - Sai Krishna Shanigarapu\*

## EXERCISE 9.2

**13** If the sum of  $n$  terms of an A.P. is  $3n^2 + 5n$  and its  $m^{th}$  term is 164, find the value of  $m$ .

**Solution:** :

$$Y(z) = \sum_{n=0}^{\infty} y(n) z^{-n} \quad (1)$$

$$= \frac{2(4 - z^{-1})}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (2)$$

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (3)$$

$$X(z) = \frac{Y(z)}{U(z)} \quad (4)$$

$$x(n) = Z_z^{-1} \left[ 2 \left( \frac{1}{1 - z^{-1}} \right) + 6 \left( \frac{1}{1 - z^{-1}} \right)^2 \right] \quad (5)$$

The inverse  $Z$ -transform of  $(1 - z^{-1})^{-2}$  using Contour integration is given by,

$$Z_z^{-1} = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \quad (6)$$

$$= \frac{1}{2\pi j} \oint_C \frac{z^{n+1}}{(z - 1)^2} \quad (7)$$

$$= \frac{1}{(2 - 1)!} \left[ \frac{d}{dz} \left( (z - 1)^2 \frac{z^{n+1}}{(z - 1)^2} \right) \right]_{z=1} \quad (8)$$

$$= (n + 1) (u(n)) \quad (9)$$

$$x(n) = (6n + 8) (u(n)) \quad (10)$$

$$164 = (6m + 8) (u(n)) \quad (11)$$

$$m = 26 \quad (12)$$

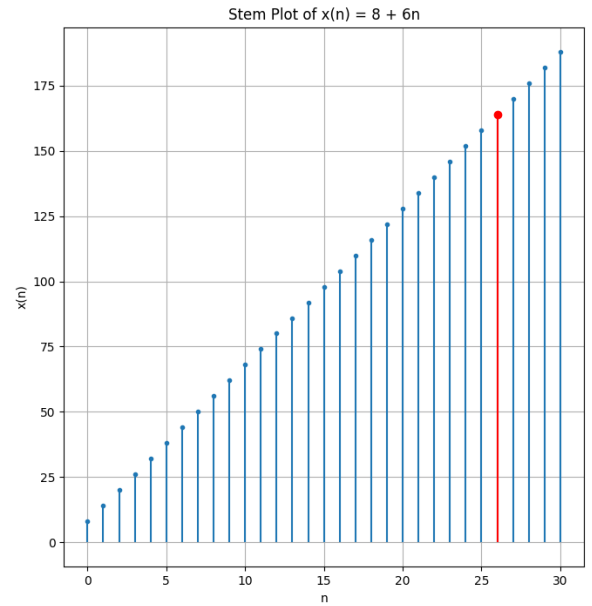


Fig. 1. Plot of  $x(n)$  vs  $n$

Symbol	Remarks
$y(n) = (3n^2 + 11n + 8) (u(n))$	Sum of $n$ terms
$x(m - 1)$	164
$y(n)$	$x(n) * u(n)$
$Z_z^{-1} \left( \frac{1}{1 - z^{-1}} \right)$	$u(n)$

TABLE I  
PARAMETERS