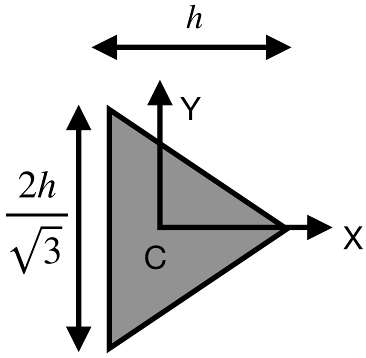
For the given cross-section:

$$F(x,y) \ = \ (x+\frac{h}{3})(x-\sqrt{3}y-\frac{2h}{3})(x+\sqrt{3}y-\frac{2h}{3})$$

$$\implies F(x,y) = x^3 - hx^2 - 3xy^2 - hy^2 + 4h^3/27$$



Cross-section
Note: Axes with reference to centroid (C)

Therefore,

$$\phi_p \ = \ BF(x,y) \ = \ B(x^3 - hx^2 - 3xy^2 - hy^2 + 4h^3/27)$$

Solving the Laplacian for above Prandtl stress function,

$$\frac{\partial^2 \phi_p}{\partial y^2} + \frac{\partial^2 \phi_p}{\partial x^2} = -2G\theta$$

We get,

$$B(6x - 2h - 6x - 2h) = -2G\theta$$

Hence,

$$B = \frac{G\theta}{2h}$$

Now, let's find the shear stresses and the corresponding shear strains:

$$\frac{-\partial \phi_p}{\partial x} = \sigma_{zy} \Rightarrow \gamma_{zy} = \frac{\sigma_{zy}}{G}$$

$$\frac{\partial \phi_p}{\partial y} = \sigma_{zx} \Rightarrow \gamma_{zx} = \frac{\sigma_{zx}}{G}$$

Note: $J = \frac{h^3}{15\sqrt{3}}$ for the given cross-section
$J \equiv Torsional\ Rigidity$

Owing to the above equations we get:

$$\sigma_{zy} \ - \frac{G\theta}{2h}(3x^2 - 3y^2 - 2hx) \Rightarrow \gamma_{zy} \ = -\frac{\theta}{2h}(3x^2 - 3y^2 - 2hx)$$

$$\sigma_{zx} \ = -\frac{G\theta}{2h}(3x+h) \Rightarrow \gamma_{zx} \ = -\frac{\theta y}{2h}(3x+h)$$

Now, we need to satisfy the compatibility equations for the strains and the equilibrium equations for the stresses (assuming no body forces are acting on the body):

Compatibility Equation:

$$\frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \gamma_{zy}}{\partial x} = -2\theta \ \Rightarrow \frac{-3\theta x}{h} - \theta - \left(\frac{-3\theta x}{h} + \theta\right) = -2\theta \qquad \text{(Satisfied!)}$$

Equilibrium Equation:

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} \ = \ 0 \ \Rightarrow \frac{3G\theta y}{h} - \frac{3G\theta y}{h} = 0 \qquad \text{(Satisfied!)}$$

The equations are trivially satisfied and no other constraint exists. Thus, the Prandtl stress function, the shear stresses and shear strains take the form:

$$\sigma_{zy} \ = \ \frac{2b^2 G\theta y}{h^2 + b^2} \Rightarrow \gamma_{zy} \ = \ \frac{2b^2\theta y}{h^2+b^2}$$

$$\phi_p \ = -\frac{h^2 b^2 (G\theta)}{h^2+b^2}(\frac{x^2}{h^2} + \frac{y^2}{b^2} - 1)$$

$$\sigma_{zx} \ = \ \frac{-2h^2 G\theta y}{h^2+b^2} \Rightarrow \gamma_{zx} \ = \ \frac{-2h^2\theta y}{h^2+b^2}$$

Remarks: The above problem has been solved by taking the centroid of the origin as the origin and the function $F(x,y)$ has been formulated by multiplying the equations of the line that make the boundary of the triangular cross section. The vertices are points of discontinuity since there are two lines that pass through a vertex (two tangents at one point!). However, the domain of the resulting Prandtl function from these three is taken to be continuous over the entire plane of cross-section.