Mini Project 1

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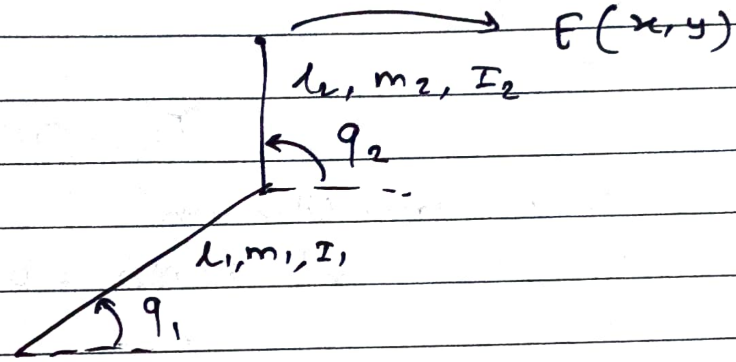
Elbow (2R) Manipulator

Task (1)



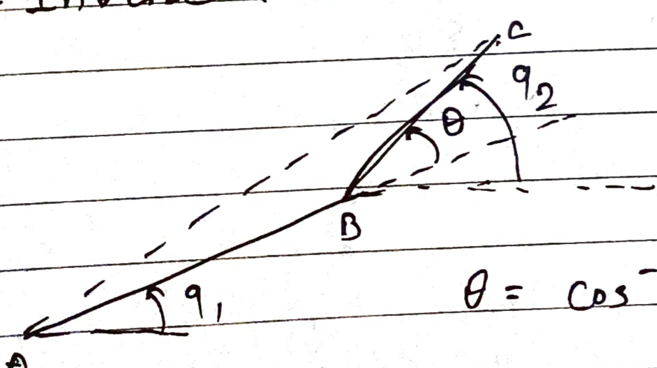
$$\left.\begin{aligned} x &= l_1 \cos q_1 + l_2 \cos q_2 \\ y &= l_1 \sin q_1 + l_2 \sin q_2 \end{aligned}\right\} \quad (1)$$

differentiating (1)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad (2)$$

End effector velocity relation.

Inverse Kinematics.



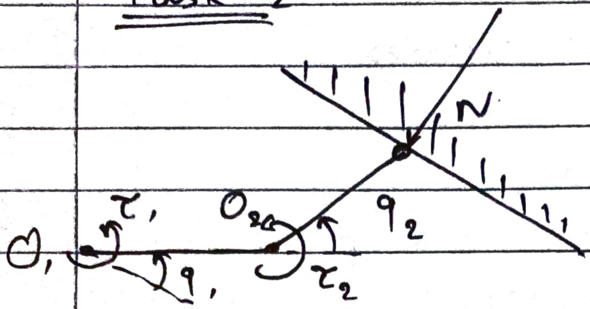
cosine rule on triangle ABC

$$\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

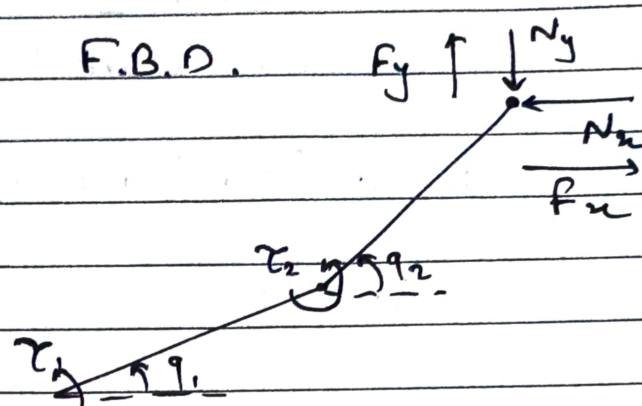
$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right)$$

$$q_2 = \theta + q_1 \quad (3)$$

Task 2



F.B.D.

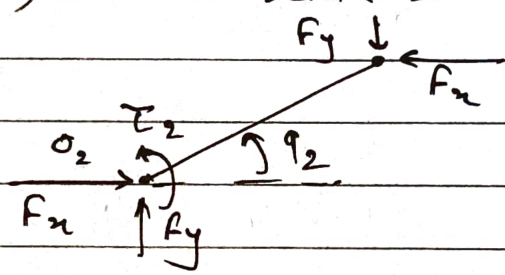


* Case of static equilibrium.

$\therefore \; \Sigma M_{O_1} = 0$ and $\Sigma M_{O_2} = 0$

FBD of each link Separarely

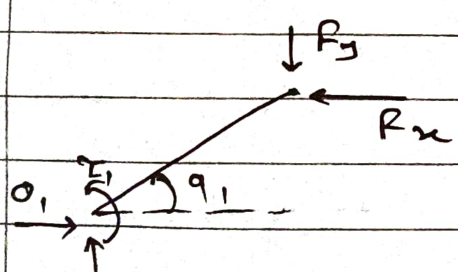
1) FBD of link 2



$\Sigma M_{O_2} = 0$

$\Rightarrow F_y l_2 \cos q_2 - F_x l_2 \sin q_2 = \tau_2$

2) FBD of link 1



$\Sigma M_{O_1} = 0$

$\Rightarrow F_y l_1 \cos q_1 - F_x l_1 \sin q_1 = \tau_1$

$$\left.\begin{aligned} F_y l_1 \cos q_1 - F_x l_1 \sin q_1 &= \tau_1 \\ F_y l_2 \cos q_2 - F_x l_2 \sin q_2 &= \tau_2 \end{aligned}\right\} \longrightarrow (4)$$

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Dynamics of robot.

Lagrangian : $\mathcal{L} = \underset{\text{K.E.}}{K} - \underset{\text{P.E.}}{V}$

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i} \quad \text{— (5)}$$

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{pure rotation about } O_1} + \underbrace{\frac{1}{2} m_2 v_{c_2}^2}_{\text{Translation of } L_2} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2}_{\text{rotation of } L_2 \text{ about C.G.}}$$

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2}\dot{q}_2 \cos(q_2 - q_1)$$

$$V = m g \frac{l_1}{2} \sin q_2 + m_2 g\left(l_1 \sin(q_1) + \frac{l_2}{2}\sin q_2\right)$$

After solving differential equations.

$$\tau_1 = \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1)$$
$$- m_2 \frac{l_1 l_2}{2} \dot{q}_2(\dot{q}_2 - \dot{q}_1)\sin(q_2 - q_1) + m_1 g \frac{l_1}{2}\cos q_2 + m_2 g l_1 \cos q_1$$

$$\tau_2 = \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4}\ddot{q}_2 + m_2 \frac{l_1 l_2}{2}\ddot{q}_1 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2}\dot{q}_1(\dot{q}_2 - \dot{q}_1)\sin(q_2 - q_1)$$
$$+ m_2 g \frac{l_2}{2}\sin q_2.$$

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Task 3

F_x, F_y (Spring force)

$$F_x = K_x = K(x - x_d)$$
$$F_y = K_y = K(y - y_d)$$

From equation (1)

$$F_x = K(l_1 \cos q_1 + l_2 \cos q_2)$$
$$F_y = K(l_2 \sin q_2 + l_1 \sin q_1)$$

from equation (4)

$$\tau_{2s} = K(l_1 \sin q_1 + l_2 \sin q_2) l_2 \cos q_2 - K(l_1 \cos q_1 + l_2 \cos q_2) l_2 \sin q_2$$

$$\tau_{1s} = K(l_1 \sin q_1 + l_2 \sin q_2) l_1 \cos q_1 - K(l_1 \cos q_1 + l_2 \cos q_2) l_1 \sin q_1 \quad \text{--- (7)}$$

If in vertical plane we have to consider gravity.

$$\therefore \tau = \tau_1 + \tau_{1s}$$
$$\tau_2 + \tau_{2s}$$

If in Horizontal plane $\rightarrow$ only τ_{1s} and τ_{2s}