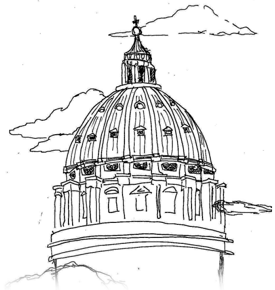
Problem D
Dome Construction



The world's largest indoor water park is built inside a hemispherical dome that was once used as an aircraft hangar. The park attracts more than $10\,000$ visitors per day and is big enough that it even has its own tropical micro-climate with clouds forming inside.

Management would like to expand business operations by opening another branch in the dome of your local cathedral. The micro-climate is a key selling point, so to really capitalise on the cathedral they asked you to expand the dome's radius so that it contains at least a given number of clouds. A cloud is contained if its centre is on or inside the boundary of the dome.

You are a cloud engineer by trade, and hence a competent meteorologist. You already identified several potential clouds close by and plotted them in three dimensions relative to the centre of the current structure. In order to capture enough of them, how large do you need to make the radius of the dome?

Input

- The first line contains the number of clouds you found, n, and the number that must be contained, k, respectively ($1 \le k \le n \le 10^5$).
- The next n lines each contain three real numbers x_i, y_i, z_i, the coordinates of the ith cloud relative to the centre of the dome ($0 \le |x_i|, |y_i|, |z_i| \le 10^6$). Every cloud has a non-negative y-coordinate.

Output

Output the minimum radius of the dome required to enclose at least k points. Your answer must be accurate to an absolute or relative error of 10^{-6}.

Sample Input 1	Sample Output 1
5 3 -4 2 1 2.1 3 5 1.2 1 -1 -2.2 3 2 1 0 2.1	4.22374242

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B • Running Steps

The coach wants his team members to run up the stadium steps taking either one or two steps with each stride so that:

a) The number of two step strides taken by each leg is the same.
b) The number of one step strides taken by each leg is the same.
c) The number of two step strides is no smaller than the number of one step strides.
d) Start with the left leg.

The coach wants to know for a given (necessarily even) number of steps how many different ways there are to run the steps and satisfy his rules.

For example, with six steps (three for each leg), there are 4 possibilities:

2***2***1***1***, 2***1***1***2***, 1***2***2***1***, 1***1***2***2*** (right leg strides are in ***highlighted*** type)

With eight steps (four for each leg) there is only one possibility since there must be at least as many two step strides as one step strides:

2***2***2***2***

For this problem, you will write a program that calculates the number of different ways there are to run the steps that satisfy the coach's four criteria.

Input

The first line of input contains a single integer P, ($1 \le P \le 10000$), which is the number of data sets that follow. Each data set should be processed identically and independently.

Each data set consists of a single line of input. It contains the data set number, K, followed by an even integer which is the total number of steps to be run, S, ($2 \le S \le 100$).

Output

For each data set there is a single line of output. The single output line consists of the data set number, K, followed by a single space followed by the number of different ways of running the steps that satisfy the coach's four criteria.

Sample Input	Sample Output
5 1 6 2 8 3 10 4 12 5 60	1 4 2 1 3 9 4 37 5 40197719157

Problem F
Feeding Seals

You are in charge of feeding the seals in the Welsh Mountain Zoo. This involves purchasing buckets of fish and allocating them to volunteers to trek into the enclosure and distribute fairly to the blubbery residents.

The buckets of fish are already set out. Each volunteer can be assigned to carry either one or two of these buckets, as long as the combined weight of the buckets is small enough.

How many volunteers will you need to distribute all of the fish in one trip?

Input

- The first line contains the number of buckets to be delivered, n ($1 \leq n \leq 10^5$), and the integer carrying capacity of a volunteer, c ($1 \leq c \leq 10^9$).
- The second line contains the integer weights of each of the n buckets, $w_1 \ldots w_n$ ($1 \leq w \leq c$).

Output

Output the minimum number of volunteers required to deliver all of the buckets of fish.

Sample Input 1	Sample Output 1
4 100 44 35 66 67	3

Sample Input 2	Sample Output 2
1 10 7	1

Sample Input 3	Sample Output 3
3 12 10 5 6	2

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D • Farey Sequence Length

Given a positive integer, ***N***, the sequence of all fractions ***a*** / ***b*** with (***0*** < ***a*** ≤ ***b***), (***1*** < ***b*** ≤ ***N***) and ***a*** and ***b*** relatively prime, listed in increasing order, is called the *Farey Sequence of order **N***.

For example, the *Farey Sequence of order **6*** is:

0/1, **1/6**, **1/5**, **1/4**, **1/3**, **2/5**, **1/2**, **3/5**, **2/3**, **3/4**, **4/5**, **5/6**, **1/1**

For this problem, you will write a program to compute the length of the *Farey sequence of order **N*** (input).

Input

The first line of input contains a single integer P, ($1 \le P \le 10000$), which is the number of data sets that follow. Each data set should be processed identically and independently.

Each data set consists of a single line of input. It contains the data set number, K**,** followed by the order N**,** N ($2 <= N <= 10000$), of the *Farey Sequence* whose length is to be found.

Output

For each data set there is a single line of output. The single output line consists of the data set number, K, followed by a single space followed by the length of the *Farey Sequence* as a decimal integer.

Sample Input	Sample Output
4 1 6 2 15 3 57 4 9999	1 13 2 73 3 1001 4 30393487

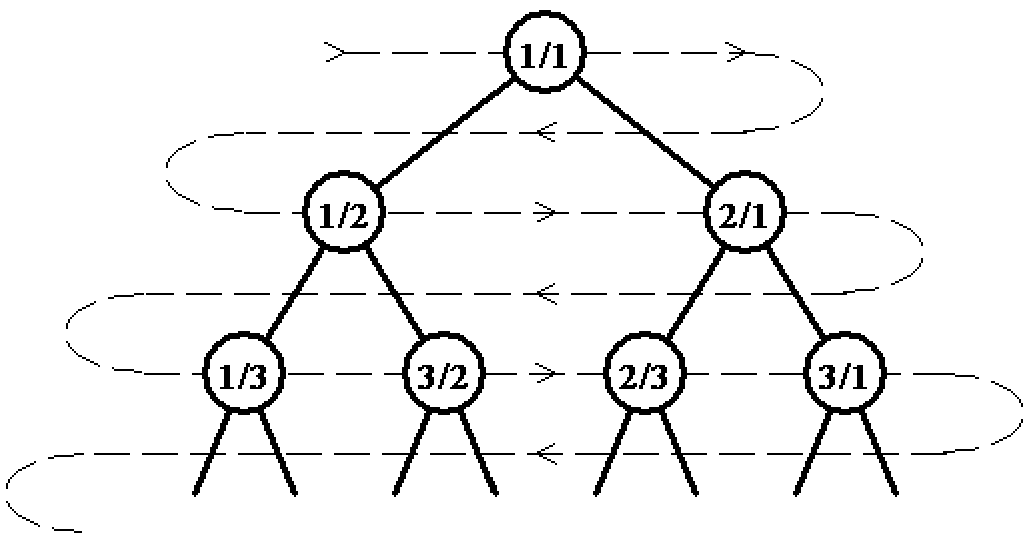
E • A Rational Sequence

A sequence of positive rational numbers is defined as follows:

An infinite full binary tree labeled by positive rational numbers is defined by:

- The label of the root is `1/1`.
- The left child of label `p/q` is `p/(p+q)`.
- The right child of label `p/q` is `(p+q)/q`.

The top of the tree is shown in the following figure:



The sequence is defined by doing a level order (breadth first) traversal of the tree (indicated by the light dashed line). So that:

`F(1) = 1/1, F(2) = 1/2, F(3) = 2/1, F(4) = 1/3, F(5) = 3/2, F(6) = 2/3, …`

Write a program which finds the value of ***n*** for which `F(`***n***`)` is **p/q** for inputs **p** and **q**.

Input

The first line of input contains a single integer ***P***, (**1 ≤ *P* ≤ 1000**), which is the number of data sets that follow. Each data set should be processed identically and independently.

Each data set consists of a single line of input. It contains the data set number, ***K,*** a single space, the numerator, **p**, a forward slash (/) and the denominator, **q**, of the desired fraction.

Output

For each data set there is a single line of output. It contains the data set number, ***K***, followed by a single space which is then followed by the value of ***n*** for which F(**n)** is **p/q**. Inputs will be chosen so n will fit in a 32-bit integer.

Sample Input	Sample Output
4	1 1
1 1/1	2 4
2 1/3	3 11
3 5/2	4 1431655765
4 2178309/1346269	

F • Robots

While you weren't watching, your ***N*** robots have developed a life of their own and spread throughout your hometown. Each of your hometown's ***N*** intersections (numbered ***0,…,N-1***) contains exactly one robot. On each intersection i, there is exactly one red signpost pointing to an intersection, $r_i \neq i$, and exactly one green signpost pointing to an intersection $g_i \neq i$. When you press the red button on your remote control, each robot will move to the intersection indicated by the red signpost (robots at intersection i move to r_i). When you press the green button, each robot will move to the intersection indicated by the green signpost (robots at intersection i move to g_i). Write a program that determines whether you can make the robots all meet at the same intersection at the same time via some sequence of commands on your remote control.

Input

The first line of input contains a single decimal integer P, ($1 \leq P \leq 500$), which is the number of data sets that follow. Each data set should be processed identically and independently.

Each data set consists three lines of input as follows:

- The first line contains the data set number, ***K,*** followed by a single integer ***N*** which is the number of intersections.
- The second line contains ***N*** space separated integers $r_0, \ldots, r_{N-1}$ ($0 \leq r_i \leq N-1$ and $r_i \neq i$).
- The third line contains ***N*** space separated integers $g_0, \ldots, g_{N-1}$ ($0 \leq g_i \leq N-1$ and $g_i \neq i$).

On some intersections, both signposts might point the same way (i.e. $r_i = g_i$).

Output

For each data set there is one line of output. The single output line consists of the string "**YES**" if you can make all robots meet or "**NO**" otherwise.

Sample Input	Sample Output
2 1 4 1 2 3 0 3 0 1 0 2 4 1 2 3 0 2 2 1 2	1 NO 2 YES

Note: For the second case, the button press sequence GREEN, RED, RED, GREEN makes all robots meet at intersection 2.

G • Compositions

A *composition* of an integer ***n*** is an ordered set of integers which sum to ***n***. Two *compositions* with the same elements but in different orders are considered different (this distinguishes *compositions* from *partitions*). For example, all the *compositions* of the first few integers are:

```
1: {1}
2: {1+1, 2}
3: {1+1+1, 1+2, 2+1, 3}
4: {1+1+1+1, 1+1+2, 1+2+1, 1+3, 2+1+1, 2+2, 3+1, 4}
```

Note that 1+2 and 2+1 each count as distinct compositions of 3. As you may have suspected, there are $2^{(n-1)}$ *compositions* of ***n***.

In this problem, we set conditions on the elements of the *compositions* of ***n***. A *composition* misses a set ***S*** if no element of the composition is in the set ***S***. For example, the *compositions* of the first few integers which miss the set of even integers are:

```
1: {1}
2: {1+1}
3: {1+1+1, 3}
4: {1+1+1+1, 1+3, 3+1}
```

No odd integer can have a *composition* missing the set of odd integers and any *composition* of an even integer consisting of only even integers must be 2 times a composition of $n/2$.

For this problem you will write a program to compute the number of *compositions* of an input integer ***n*** which miss the elements of the arithmetic sequence $\{m + i*k \mid i = 0, 1, \ldots\}$.

Input

The first line of input contains a single decimal integer ***P***, ($1 \le P \le 10000$), which is the number of data sets that follow. Each data set should be processed identically and independently.

Each data set consists of a single line of input. It contains the data set number, ***K***, followed by the three space separated integers ***n***, ***m*** and ***k*** with ($1 \le n \le 30$) and ($0 \le m < k < 30$).

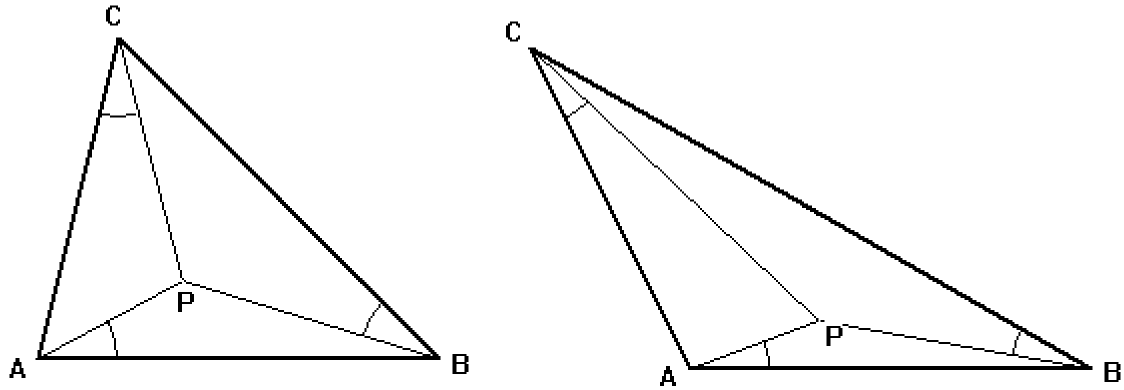
Output

For each data set there is one line of output. The single output line consists of the data set number, ***K***, followed by a single space followed by the number of *compositions* of ***n*** which miss the specified sequence.

Sample Input	Sample Output
3	1 55
1 10 0 2	2 235
2 15 1 4	3 18848806
3 28 3 7	

H • Brocard Point of a Triangle

The *Brocard point* of a triangle ABC is a point P in the triangle chosen so that:
$\angle PAB = \angle PBC = \angle PCA$ (see figure below).



The common angle is called the *Brocard angle*. The largest *Brocard angle* is $\pi/6$ which is the *Brocard angle* for an equilateral triangle (the *Brocard point* is the centroid of the triangle).

Write a program to compute the coordinates of the *Brocard point* of a triangle given the coordinates of the vertices.

Input

The first line of input contains a single integer P, $(1 \le P \le 10000)$, which is the number of data sets that follow. Each data set should be processed identically and independently.

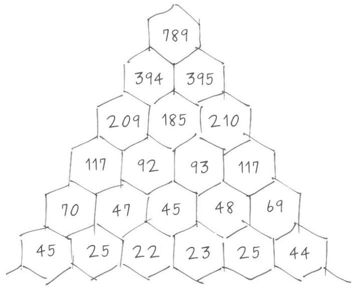
Each data set consists of a single line of input. It contains the data set number, ***K,*** followed by the six space separated coordinate values `Ax`, `Ay`, `Bx`, `By`, `Cx`, `Cy` of the vertices of the triangle. The vertices will always be specified so going from `A` to `B` to `C` and back to `A` circles the triangle counter-clockwise. Input coordinates are floating point values.

Output

For each data set there is a single line of output. The single output line consists of the data set number, ***K***, followed by a single space followed by the ***x*** coordinate of the *Brocard point*, followed by a single space followed by the ***y*** coordinate of the *Brocard point*. Coordinates should be rounded to five decimal places.

Sample Input	Sample Output
3	1 1.40456 0.82890
1 0 -1.3 3.4 0.5 1.1 2.3	2 1.56047 0.74902
2 0 0 3 0 0 4	3 3.87699 0.40167
3 3.1 0.2 4.3 0.4 0 0.8	

Problem I
Integral Pyramid



Pascal's triangle is a marvel of the combinatorical world, and what's more you can easily build one for yourself at home.

The lowest row has n numbers. The next row is staggered and has $n - 1$ numbers, where the ith is the sum of the ith and the $i + 1$th on the previous row.

You can choose any positive integers for the lowest row, but the single cell on the top row needs to be equal to a given x. Is this possible?

Input

- The only line contains the number of rows, n ($1 \leq n \leq 20$), and the value needed at the top, x ($1 \leq x \leq 10^9$).

Output

If a pyramid can be constructed, output all of the numbers on each row, starting from the top. Every number must be greater than or equal to 1.

Otherwise, output `impossible`.

Sample Input 1	Sample Output 1
3 15	15 8 7 3 5 2

Sample Input 2	Sample Output 2
6 789	789 394 395 209 185 210 117 92 93 117 70 47 45 48 69 45 25 22 23 25 44

Sample Input 3	Sample Output 3
20 1	impossible

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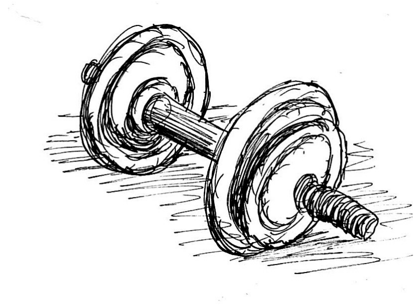
Problem J
Jammed Gym



You are at the fitness centre to run through your exercise programme. You must use the kinds of exercise machine in an order precisely dictated by the programme, although there may be more than one instance of a machine.

You start at the centre of a unit circle around which the exercise stations are arranged. You can walk directly between any two points in the circle, and you may also visit the same point multiple times. See Figure J.1 below for an example.

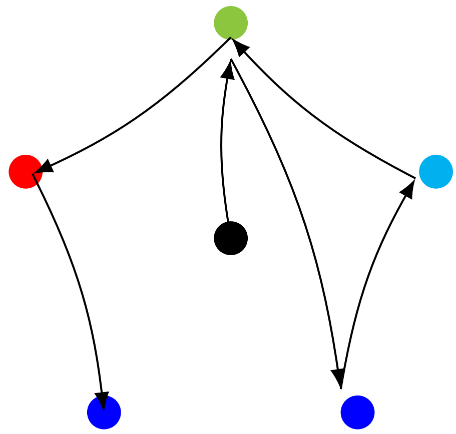


Figure J.1: Illustration of Sample Input 1. Types of machine: $[1, 2, 4, 1, 3, 2]$

Exercise is an important and noble endeavour, but in today's busy world we must strive for efficiency in everything we do. Find the most efficient way of visiting exercise stations that matches the order given.

Input

- The first line of input contains the number of exercises in the programme, n ($1 \le n \le 100$).
- The second line of input contains n space-separated integers each denoting the type of an item on the programme t ($1 \le t_i \le 100$). There will always be at least one station for each programme in this list.
- The third line of input contains the number of stations, m ($1 \le m \le 100$).
- The fourth line of input contains m space-separated integers each denoting the type of a station q ($1 \le q_i \le 100$).

Output

Output the minimum distance you will need to walk. Your answer must be accurate to an absolute or relative error of 10^{-6}.

Sample Input 1

```
6
1 2 4 1 3 2
5
1 4 2 2 3
```

Sample Output 1

```
7.604395
```

Sample Input 2

```
5
4 2 1 3 1
6
1 2 1 3 1 4
```

Sample Output 2

```
5.732051
```