2. laboratorijska vježba

Multivarijatna analiza podataka

ak. god. 2021/2022

1. Uvod i upute za predaju

Cilj ove laboratorijske vježbe je primijeniti osnovne koncepte multivarijatne analize podataka, istražiti podatke te ispitati hipoteze. Preduvjet za rješavanje vježbe je osnovno znanje programskog jezika *R* i rad s *R Markdown* dokumentima. Sama vježba je koncipirana kao projekt u kojem istražujete i eksperimentirate koristeći dane podatke - ne postoji nužno samo jedan točan način rješavanja svakog podzadatka.

Rješavanje vježbe svodi se na čitanje uputa u tekstu ovog dokumenta, nadopunjavanje blokova kôda (možete dodavati i dodatne blokove kôda ukoliko je potrebno) i ispisivanje rezultata (u vidu ispisa iz funkcija, tablica i grafova). Vježbu radite samostalno, a svoje rješenje branite na terminima koji su vam dodijeljeni u kalendaru. Pritom morate razumjeti teorijske osnove u okviru onoga što je obrađeno na predavanjima i morate pokazati da razumijete sav kôd koji ste napisali.

Vaše rješenje potrebno je predati u sustav *Moodle* u obliku dvije datoteke:

1. Ovaj .Rmd dokument s Vašim rješenjem (naziva IME_PREZIME_JMBAG.rmd),

2. PDF ili HTML dokument kao izvještaj generiran iz vašeg .Rmd rješenja (također naziva IME_PREZIME_JMBAG).

Rok za predaju je **15. svibnja 2022. u 23:59h**. **Jedan od uvjeta za prolaz predmeta je minimalno ostvarenih 50% bodova na svim laboratorijskim vježbama. Nadoknade laboratorijskih vježbi neće biti organizirane.** Za sva dodatna pitanja svakako se javite na email adresu predmeta: *map@fer.hr*.

2. Podatkovni skup

U laboratorijskoj vježbi razmatra se dinamika cijena vrijednosnica na financijskim tržištima. Dane su povijesne tjedne cijene ETF-ova (eng. exchange traded fund) koji prate određene dioničke, obvezničke ili druge indekse. Konkretno, radi se o sljedećim fondovima:

- `AGG` (iShares Core U.S. Aggregate Bond ETF) - obveznice s američkog tržišta,
- `IEF` (iShares 7-10 Year Treasury Bond ETF) - srednjeročne državne obveznice,
- `LQD` (iShares iBoxx $ Investment Grade Corporate Bond ETF) - korporativne obveznice,
- `SHY` (iShares 1-3 Year Treasury Bond ETF) - kratkoročne državne obveznice,
- `TIP` (iShares TIPS Bond ETF) - državne obveznice zaštićene od inflacije,
- `TLT` (iShares 20+ Year Treasury Bond ETF) - dugoročne državne obveznice,
- `DBC` (Invesco DB Commodity Index Tracking Fund) - sirovine i roba,
- `GLD` (SPDR Gold Trust) - zlato,
- `USO` (United States Oil Fund) - nafta,
- `IJH` (iShares Core S&P Mid-Cap ETF) - dionice tvrtki s američkog tržišta,
- `IWM` (iShares Russell 2000 ETF) - dionice američkih tvrtki s malim kapitalom,
- `SPY` (SPDR S&P 500 ETF Trust) - dionice tvrtki s američkog tržišta,
- `VTV` (Vanguard Value ETF) - dionice tvrtki s američkog tržišta,
- `XLB` (Materials Select Sector SPDR Fund) - dionice tvrtki za materijale,
- `XLE` (Energy Select Sector SPDR Fund) - dionice tvrtki energetskog sektora,
- `XLF` (Financial Select Sector SPDR Fund) - dionice tvrtki financijskog sektora,

- `XLI` (Industrial Select Sector SPDR Fund) - dionice tvrtki industrijskog sektora,
- `XLK` (Technology Select Sector SPDR Fund) - dionice tvrtki iz tehnološkog sektora,
- `XLP` (Consumer Staples Select Sector SPDR Fund) - dionice tvrki za necikličku potrošačku robu,
- `XLU` (Utilities Select Sector SPDR Fund) - dionice tvrtki komunalnih djelatnosti,
- `XLV` (Health Care Select Sector SPDR Fund) - dionice tvrtki iz zdravstvenog sektora,
- `XLY` (Consumer Discretionary Select Sector SPDR Fund) - dionice tvrtki za cikličku potrošačku robu,
- `IYR` (iShares U.S. Real Estate ETF) - dionice tvrtki iz područja nekretnina,
- `VNQ` (Vanguard Real Estate Index Fund) - dionice tvrtki iz područja nekretnina.

Pri modeliranju zajedničkog kretanja i rizika vrijednosnica, najčešće se koriste povrati: $R(t) = \frac{S(t)-S(t-1)}{S(t-1)}$, gdje je $S(t)$ cijena vrijednosnice u tjednu t.

2.1. Učitavanje podataka i korelacijska analiza

Podaci se nalaze u datoteci "ETFprices.csv". Učitajte ih, provjerite ispravnost, izračunajte tjedne povrate te vizualizirajte matricu korelacije povrata - razmislite o grupama i korelacijskim strukturama koje u njoj vidite. U ostatku laboratorijske vježbe također koristite povrate, a ne cijene.

```r
#  Vaš kôd ovdje

# ucitaj
ETFprices = read.csv(file = 'ETFprices.csv')
summary(ETFprices)
```

```
##      Time                AGG              IEF              LQD        
##  Length:667         Min.   : 64.93   Min.   : 56.54   Min.   : 54.42  
##  Class :character   1st Qu.: 78.45   1st Qu.: 74.56   1st Qu.: 69.36  
##  Mode  :character   Median : 94.34   Median : 92.17   Median : 94.16  
##                     Mean   : 90.16   Mean   : 86.75   Mean   : 90.01  
##                     3rd Qu.:101.10   3rd Qu.:100.00   3rd Qu.:105.13  
##                     Max.   :106.69   Max.   :108.30   Max.   :117.26  
##       SHY             TIP              TLT              DBC       
##  Min.   :65.21   Min.   : 68.62   Min.   : 54.14   Min.   :11.92  
##  1st Qu.:77.13   1st Qu.: 83.69   1st Qu.: 71.00   1st Qu.:17.04  
##  Median :80.57   Median :103.62   Median : 96.30   Median :23.43  
##  Mean   :78.57   Mean   : 96.78   Mean   : 92.61   Mean   :22.68  
##  3rd Qu.:81.80   3rd Qu.:108.71   3rd Qu.:113.83   3rd Qu.:26.30  
##  Max.   :83.62   Max.   :112.45   Max.   :134.72   Max.   :45.11  
##       GLD              USO              IJH              IWM        
##  Min.   : 56.99   Min.   :  8.33   Min.   : 35.47   Min.   : 30.54  
##  1st Qu.: 93.73   1st Qu.: 14.71   1st Qu.: 69.84   1st Qu.: 62.51  
##  Median :117.84   Median : 34.58   Median : 89.81   Median : 74.99  
##  Mean   :114.75   Mean   : 34.99   Mean   :107.04   Mean   : 89.68  
##  3rd Qu.:127.50   3rd Qu.: 39.56   3rd Qu.:139.99   3rd Qu.:112.25  
##  Max.   :183.24   Max.   :117.39   Max.   :203.47   Max.   :171.99  
##       SPY             VTV              XLB             XLE       
##  Min.   : 56.2   Min.   : 22.35   Min.   :14.74   Min.   :30.90  
##  1st Qu.:105.1   1st Qu.: 43.96   1st Qu.:28.78   1st Qu.:50.52  
##  Median :123.8   Median : 52.52   Median :33.75   Median :61.05  
##  Mean   :151.0   Mean   : 61.32   Mean   :36.80   Mean   :59.53  
##  3rd Qu.:194.2   3rd Qu.: 77.25   3rd Qu.:45.13   3rd Qu.:67.59  
##  Max.   :290.3   Max.   :111.68   Max.   :62.84   Max.   :88.75  
##       XLF              XLI             XLK             XLP       
##  Min.   : 3.200   Min.   :12.55   Min.   :11.56   Min.   :14.99  
##  1st Qu.: 8.322   1st Qu.:27.42   1st Qu.:19.73   1st Qu.:20.74  
```

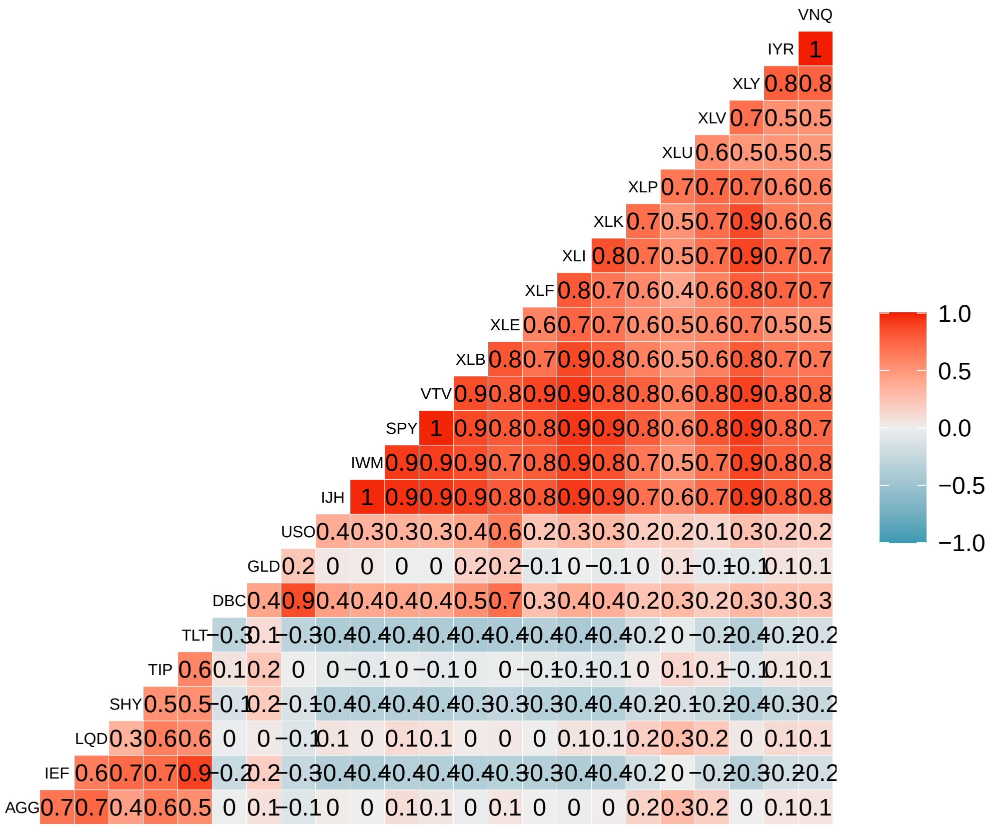
```
##  Median :12.913   Median :32.44   Median :26.26   Median :29.86
##  Mean   :13.782   Mean   :40.52   Mean   :32.07   Mean   :33.05
##  3rd Qu.:16.881   3rd Qu.:51.81   3rd Qu.:40.36   3rd Qu.:45.49
##  Max.   :29.614   Max.   :79.51   Max.   :75.02   Max.   :57.01
##       XLU             XLV              XLY               IYR
##  Min.   :15.88   Min.   :18.74   Min.   : 14.06   Min.   :14.88
##  1st Qu.:23.73   1st Qu.:27.15   1st Qu.: 29.06   1st Qu.:41.81
##  Median :28.92   Median :35.05   Median : 41.46   Median :51.47
##  Mean   :32.57   Mean   :46.27   Mean   : 52.11   Mean   :53.08
##  3rd Qu.:39.97   3rd Qu.:67.28   3rd Qu.: 74.06   3rd Qu.:66.79
##  Max.   :56.34   Max.   :95.44   Max.   :116.76   Max.   :81.89
##       VNQ
##  Min.   :13.86
##  1st Qu.:39.18
##  Median :49.90
##  Mean   :52.62
##  3rd Qu.:69.29
##  Max.   :82.43
```

```
# tjedni povrati

n = nrow(ETFprices)
p = ncol(ETFprices)

tjedan = data.matrix(ETFprices[2:n, 2:p])
tjedan_1 = data.matrix(ETFprices[1:(n-1), 2:p])

ETF_returns = (tjedan - tjedan_1) / tjedan_1

summary(ETF_returns)
```

```
##       AGG                  IEF                  LQD
##  Min.   :-0.1036303   Min.   :-0.0314219   Min.   :-0.0805767
##  1st Qu.:-0.0019141   1st Qu.:-0.0043649   1st Qu.:-0.0035478
##  Median : 0.0012097   Median : 0.0013732   Median : 0.0016093
##  Mean   : 0.0007614   Mean   : 0.0009396   Mean   : 0.0009947
##  3rd Qu.: 0.0040984   3rd Qu.: 0.0066539   3rd Qu.: 0.0060010
##  Max.   : 0.0643661   Max.   : 0.0339494   Max.   : 0.0446517
##       SHY                  TIP                  TLT
##  Min.   :-0.0094048   Min.   :-0.0636173   Min.   :-0.073630
##  1st Qu.:-0.0004741   1st Qu.:-0.0038624   1st Qu.:-0.010001
##  Median : 0.0002455   Median : 0.0009852   Median : 0.001993
##  Mean   : 0.0003732   Mean   : 0.0007413   Mean   : 0.001336
##  3rd Qu.: 0.0010639   3rd Qu.: 0.0053218   3rd Qu.: 0.012453
##  Max.   : 0.0098577   Max.   : 0.0436792   Max.   : 0.088013
##       DBC                  GLD                 USO
##  Min.   :-0.1311522   Min.   :-0.092200   Min.   :-0.1862675
##  1st Qu.:-0.0144914   1st Qu.:-0.012619   1st Qu.:-0.0266644
##  Median : 0.0010320   Median : 0.002680   Median : 0.0009747
##  Mean   :-0.0002427   Mean   : 0.001379   Mean   :-0.0017004
##  3rd Qu.: 0.0163680   3rd Qu.: 0.017228   3rd Qu.: 0.0260442
##  Max.   : 0.1116751   Max.   : 0.138054   Max.   : 0.2243987
##       IJH                 IWM                 SPY
##  Min.   :-0.168843   Min.   :-0.163476   Min.   :-0.197934
##  1st Qu.:-0.010930   1st Qu.:-0.013970   1st Qu.:-0.008683
```

```
##  Median : 0.003524   Median : 0.003564   Median : 0.003000
##  Mean   : 0.001930   Mean   : 0.001771   Mean   : 0.001790
##  3rd Qu.: 0.016032   3rd Qu.: 0.018584   3rd Qu.: 0.014454
##  Max.   : 0.167385   Max.   : 0.164862   Max.   : 0.132923
##       VTV                 XLB                 XLE
##  Min.   :-0.178200   Min.   :-0.149387   Min.   :-0.251853
##  1st Qu.:-0.008779   1st Qu.:-0.013416   1st Qu.:-0.016888
##  Median : 0.003367   Median : 0.003867   Median : 0.002611
##  Mean   : 0.001655   Mean   : 0.001664   Mean   : 0.001222
##  3rd Qu.: 0.013900   3rd Qu.: 0.017341   3rd Qu.: 0.021406
##  Max.   : 0.138295   Max.   : 0.151741   Max.   : 0.177549
##       XLF                 XLI                 XLK
##  Min.   :-0.239592   Min.   :-0.153846   Min.   :-0.146538
##  1st Qu.:-0.014824   1st Qu.:-0.011467   1st Qu.:-0.010445
##  Median : 0.002622   Median : 0.003507   Median : 0.003362
##  Mean   : 0.001677   Mean   : 0.001908   Mean   : 0.002259
##  3rd Qu.: 0.018631   3rd Qu.: 0.017953   3rd Qu.: 0.017086
##  Max.   : 0.325243   Max.   : 0.139882   Max.   : 0.105578
##       XLP                 XLU                 XLV
##  Min.   :-0.133309   Min.   :-0.198008   Min.   :-0.185835
##  1st Qu.:-0.007672   1st Qu.:-0.009280   1st Qu.:-0.009010
##  Median : 0.003355   Median : 0.003659   Median : 0.003373
##  Mean   : 0.001882   Mean   : 0.001809   Mean   : 0.002183
##  3rd Qu.: 0.012660   3rd Qu.: 0.014628   3rd Qu.: 0.015080
##  Max.   : 0.057895   Max.   : 0.085099   Max.   : 0.082548
##       XLY                 IYR                 VNQ
##  Min.   :-0.147206   Min.   :-0.172507   Min.   :-0.182961
##  1st Qu.:-0.010074   1st Qu.:-0.013418   1st Qu.:-0.014621
##  Median : 0.002984   Median : 0.003719   Median : 0.003658
##  Mean   : 0.002432   Mean   : 0.001642   Mean   : 0.001866
##  3rd Qu.: 0.018136   3rd Qu.: 0.017805   3rd Qu.: 0.018790
##  Max.   : 0.183316   Max.   : 0.218693   Max.   : 0.234321
```

```r
# vizualiziraj matricu korelacije povrata

library(ggplot2)
library(GGally) # za ggcorr

cor_matrix = cor(ETF_returns)
ggcorr(ETF_returns, label = TRUE, label_size=3, cex=2)
```



3. Analiza glavnih komponenti

Cilj ovog zadatka je analizirati kretanje danih ETF-ova i izračunati glavne komponente koje objašnjavaju njihovu dinamiku.

3.1. Glavne komponente

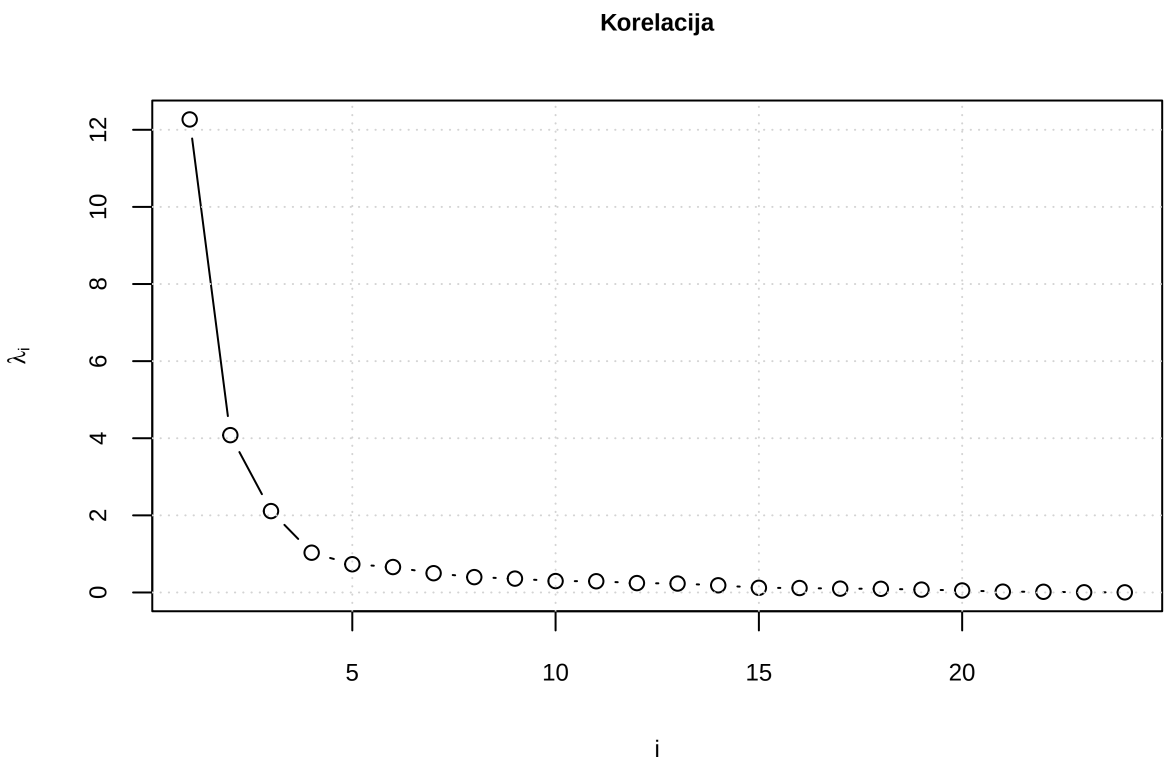
Izračunajte glavne komponente matrice korelacije i izračunajte koliki udio varijance objašnjavaju. Odredite broj glavnih komponenti koje ćete zadržati u analizi. Grafički prikažite i usporedite koeficijente prvih nekoliko komponenti.

```
#  Vaš kôd ovdje

# pca standardiziranih varijabli (korelacija)

#center = TRUE i scale = TRUE kombinacija odgovaraju dekompoziciji korelacije
pca.cor <- prcomp(ETF_returns, center = TRUE,scale = TRUE)

summary(pca.cor)
```

```
## Importance of components:
##                           PC1    PC2     PC3     PC4     PC5    PC6     PC7
## Standard deviation     3.5026 2.0201 1.45277 1.01573 0.85603 0.8124 0.70740
## Proportion of Variance 0.5112 0.1700 0.08794 0.04299 0.03053 0.0275 0.02085
## Cumulative Proportion  0.5112 0.6812 0.76914 0.81213 0.84266 0.8702 0.89101
##                           PC8     PC9    PC10    PC11    PC12    PC13    PC14
## Standard deviation     0.6313 0.59983 0.54444 0.53993 0.49353 0.48162 0.43129
## Proportion of Variance 0.0166 0.01499 0.01235 0.01215 0.01015 0.00966 0.00775
```

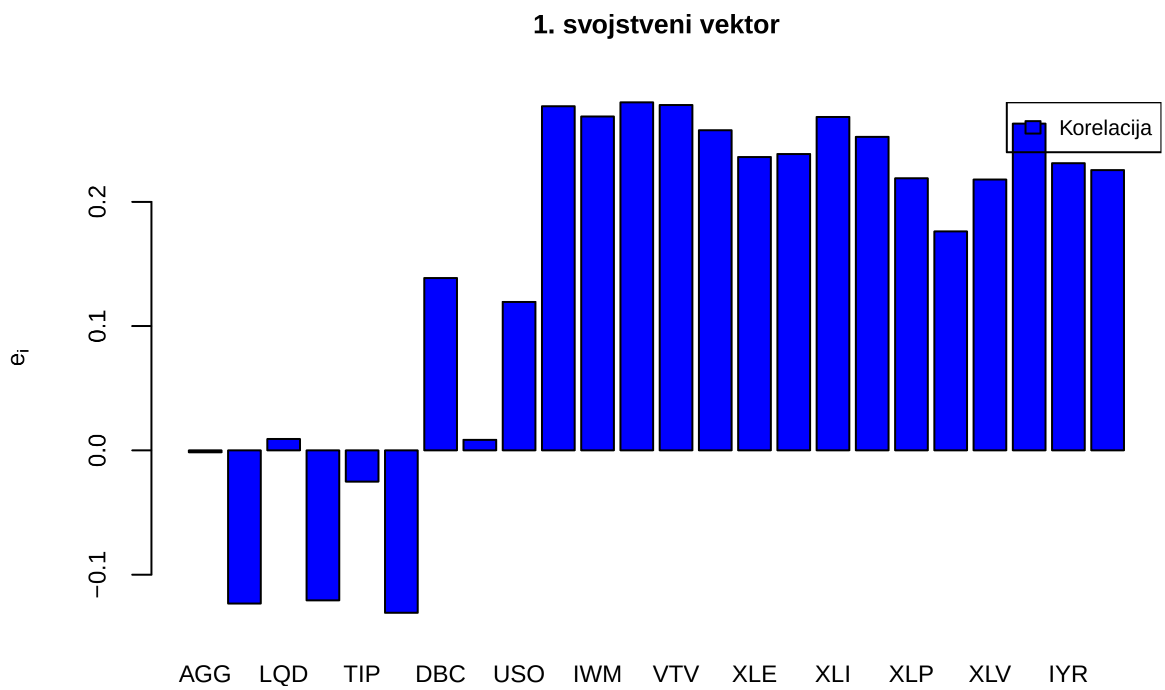
```
## Cumulative Proportion  0.9076 0.92260 0.93495 0.94710 0.95725 0.96691 0.97466
##                           PC15    PC16    PC17    PC18    PC19    PC20    PC21
## Standard deviation     0.34917 0.33995 0.31615 0.30848 0.27417 0.22747 0.14648
## Proportion of Variance 0.00508 0.00482 0.00416 0.00396 0.00313 0.00216 0.00089
## Cumulative Proportion  0.97974 0.98456 0.98872 0.99269 0.99582 0.99797 0.99887
##                           PC22    PC23    PC24
## Standard deviation     0.13519 0.07108 0.06184
## Proportion of Variance 0.00076 0.00021 0.00016
## Cumulative Proportion  0.99963 0.99984 1.00000
```

```
# odabir broja komponenti

#scree plot za glavne komponente kovarijance
plot(pca.cor$sdev^2, type = "b", cex.lab=0.75, cex.main=0.75, cex.axis=0.75, xlab="i", ylab=expression(l
grid()
```

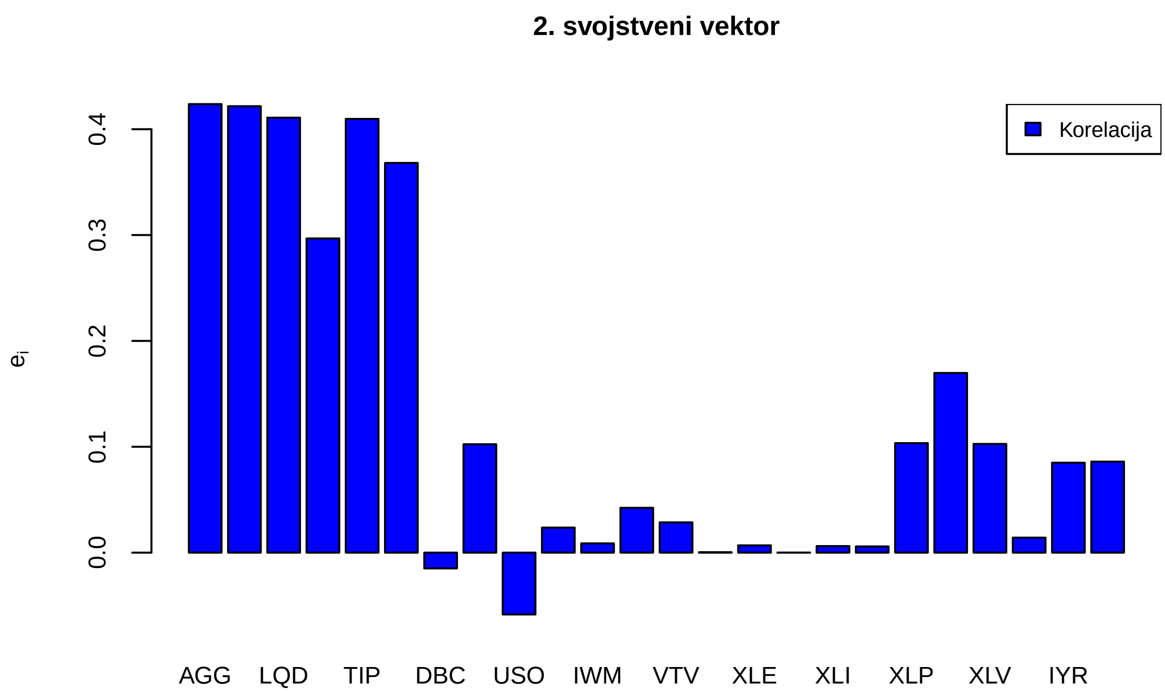



```
# priprema podataka za vizualizaciju

#prije vizualizacije rotirajmo koeficijente tako da im je suma pozitivna
#također izračunajmo korelacije između komponenti i originalnih varijabli
for (i in 1:dim(pca.cor$rotation)[1]){
  pca.cor$rotation[,i] = pca.cor$rotation[,i]*sign(sum(pca.cor$rotation[,i]))
}


# vizualizacija

#barplot koeficijenata prve glavne komponente - za korelaciju
barplot((pca.cor$rotation[,1]), beside=TRUE, col=c("blue"), main="1. svojstveni vektor",ylab=expression
legend("topright",
       legend = c("Korelacija"),
```

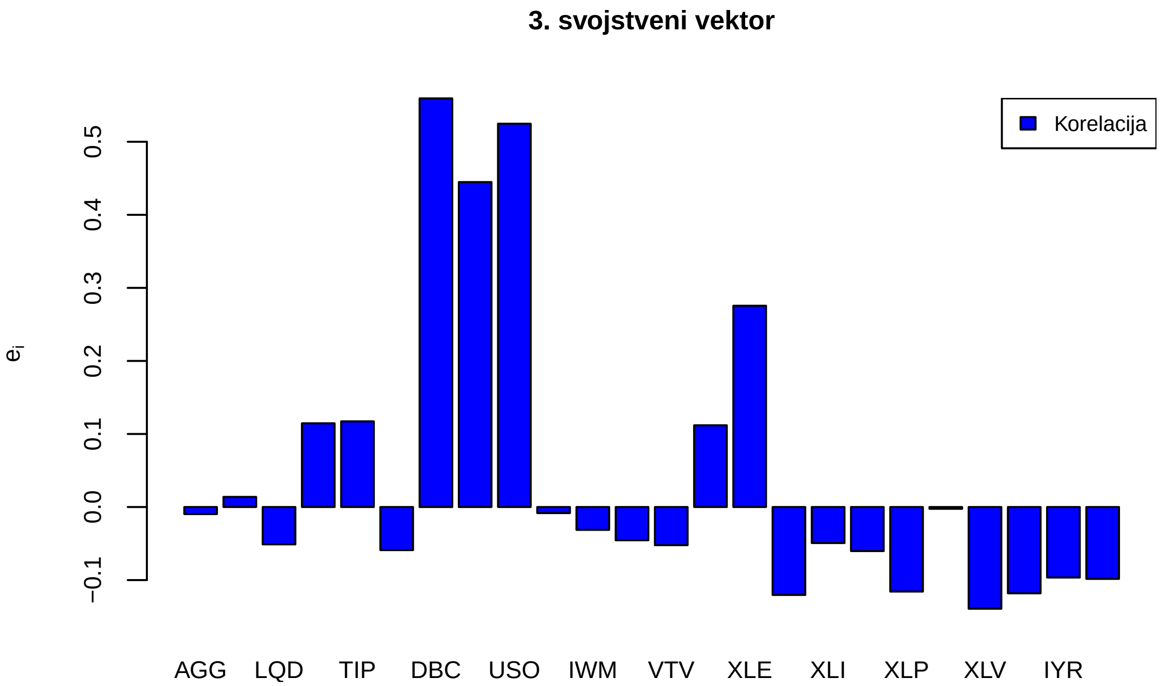
```
        fill = c("blue"),
        cex = 0.65)
```




```
#barplot koeficijenata druge glavne komponente - za korelaciju
barplot((pca.cor$rotation[,2]), beside=TRUE, col=c("blue"), main="2. svojstveni vektor",ylab=expression
legend("topright",
        legend = c("Korelacija"),
        fill = c("blue"),
        cex = 0.65)
```

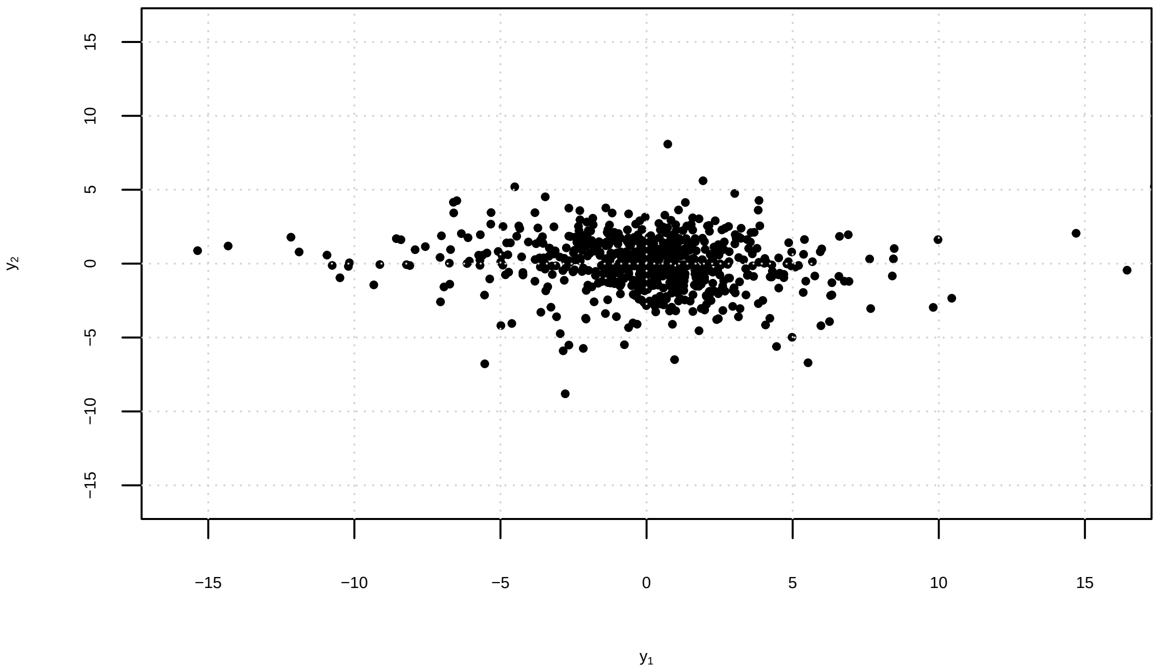


```
#barplot koeficijenata trece glavne komponente - za korelaciju
barplot((pca.cor$rotation[,3]), beside=TRUE, col=c("blue"), main="3. svojstveni vektor",ylab=expression
legend("topright",
       legend = c("Korelacija"),
       fill = c("blue"),
       cex = 0.65)
```



Prikažite graf raspršenja prve dvije glavne komponente i proučite možete li primijetiti neke grupe fondova.

```
#  Vaš kôd ovdje

Y = as.matrix(scale(ETF_returns))%*%pca.cor$rotation

plot(Y[,1],Y[,2], pch = 20, cex=0.7, cex.lab=0.5, cex.axis=0.5, xlab=expression("y"["1"]), ylab=express
grid()
```



3.2. Svojstveni portfelji

U primjeni PCA i svojstvenoj dekompoziciji kovarijance u financijama, svojstveni vektori se često zovu i tzv. svojstveni portfelji. Općenito, portfelj je vektor $w = [w_1, ..., w_N]$ u kojem svaki element predstavlja težinu ili udio kapitala u određenoj vrijednosnici. Često je dobro pomnožiti njihove težine s predznakom njihove sume - na taj način zapravo samo "okrećemo" predznak svojstvenog vektora tako da mu je suma pozitivna (konačni PCA rastav je i dalje isti ako svojstveni vektor pomnožimo s -1). Također, dobro je i skalirati svojstvene portfelje sa sumom njihovih apsolutnih vrijednosti: $\tilde{w}_i = \frac{w_i}{\sum_j^N |w_j|}$. Na taj način se osigurava da visoke magnitude pojedinih elemenata ne uzrokuju velike razlike u volatilnostima svojstvenih portfelja. Ukoliko znamo povrate $R \in \mathbb{R}^{T \times N}$ (gdje je $R_i \in \mathbb{R}^T$ vektor povrata za vrijednosnicu i) za N vrijednosnica u nekom vremenskom periodu od T dana, povrate portfelja w u tom istom periodu možemo izračunati kao: $R_p = \sum R_i w_i = R \cdot w$. Izračunajte skalirane svojstvene portfelje $\tilde{w}$ koji proizlaze iz prve dvije glavne komponente. Za ta dva svojstvena portfelja izračunajte povijesne povrate kroz razmatrani period. Grafički prikažite vremensko kretanje njihovih vrijednosti tako da njihove povrate "vratite" natrag u cijene, s tim da početna cijena bude jednaka za oba portfelja, npr. $V_0 = 100$. Vrijednost portfelja u trenutku t možemo izračunati po formuli: $V_t = V_{t-1} \cdot (1 + R_t)$.

```
#  Vaš kôd ovdje

# TODO
```

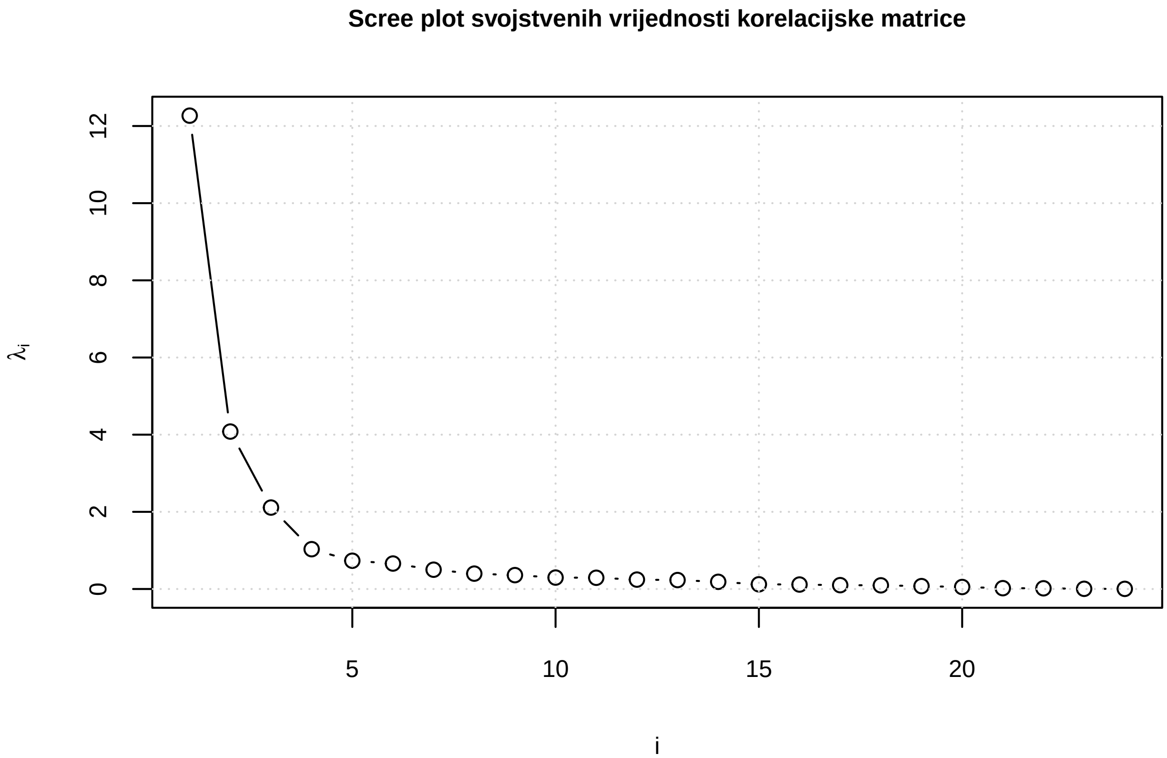
4. Faktorska analiza

4.1. Metode procjena koeficijenata modela

Na danim podacima odredite broj faktora te procijenite faktorski model pomoću metode glavnih komponenti i metode najveće izglednosti. Usporedite procjene ove dvije metode. Koja Vam se čini bolja? Što možete zaključiti iz vrijednosti faktora? Pronađite procjenu vrijednosti faktora koja daje najbolju interpretabilnost.

```
#  Vaš kôd ovdje

# svojstvene vrijednosti i vektori korelacije
R = cor(ETF_returns)
```

```
ev_R = eigen(R)

lambda_R = ev_R$values
e_R = ev_R$vectors


#scree plot za glavne komponente kovarijance
plot(lambda_R, type = "b", cex.lab=0.75, cex.main=0.75, cex.axis=0.75, xlab="i", ylab=expression(lambda
grid()
```




```
# procjena koeficijenata modela koristeci matricu korelacije i metodu glavnih komponenti}

L = sqrt(lambda_R[1])*e_R[,1]
L = cbind(L,sqrt(lambda_R[2])*e_R[,2])
L = cbind(L,sqrt(lambda_R[3])*e_R[,3])

h = rowSums(L^2)

psi = 1-h

cbind(data.matrix(names(data.frame(ETF_returns))), L)
```

```
##             L
## [1,] "AGG" "-0.00553118305290133" "-0.85593142691024"
## [2,] "IEF" "-0.431522254808382"   "-0.851916249134718"
## [3,] "LQD" "0.0317791551921248"   "-0.829994657614043"
## [4,] "SHY" "-0.422689195456655"   "-0.599531418663184"
## [5,] "TIP" "-0.087879073807552"   "-0.827644136059827"
## [6,] "TLT" "-0.457689588827854"   "-0.743658212386812"
## [7,] "DBC" "0.485698785620215"    "0.030100427208844"
```

```
##  [8,] "GLD" "0.0299825753356696"   "-0.206872610885008"
##  [9,] "USO" "0.418825917812505"    "0.117986561927518"
## [10,] "IJH" "0.96980772485188"     "-0.0478616829061607"
## [11,] "IWM" "0.940870001742236"    "-0.0177036971269365"
## [12,] "SPY" "0.980678430640717"    "-0.0855238359014252"
## [13,] "VTV" "0.973462337062699"    "-0.057764592320914"
## [14,] "XLB" "0.902105962212203"    "-0.000887699831981365"
## [15,] "XLE" "0.826887805821322"    "-0.0138717602945772"
## [16,] "XLF" "0.83538691912535"     "-0.000149514768984158"
## [17,] "XLI" "0.939862497385112"    "-0.0125122788188008"
## [18,] "XLK" "0.883757521838607"    "-0.0118619535835571"
## [19,] "XLP" "0.766541826953338"    "-0.209025076101476"
## [20,] "XLU" "0.616949639130344"    "-0.342820705784434"
## [21,] "XLV" "0.763266784556813"    "-0.207422181783988"
## [22,] "XLY" "0.920928762507662"    "-0.0286089745596864"
## [23,] "IYR" "0.809191381021429"    "-0.171633076225232"
## [24,] "VNQ" "0.789963020225876"    "-0.173716815291829"
##
##  [1,] "0.0140462481020162"
##  [2,] "-0.0201262088160327"
##  [3,] "0.0742416234049966"
##  [4,] "-0.166498139934523"
##  [5,] "-0.170370771811292"
##  [6,] "0.085770865310839"
##  [7,] "-0.812662242690749"
##  [8,] "-0.646212453384298"
##  [9,] "-0.762311587451594"
## [10,] "0.0120584142662634"
## [11,] "0.0454483731146156"
## [12,] "0.066152888587039"
## [13,] "0.0758854098525766"
## [14,] "-0.162564003910758"
## [15,] "-0.400332254450613"
## [16,] "0.174875420356319"
## [17,] "0.0716261224162236"
## [18,] "0.0876050105806218"
## [19,] "0.168125803068158"
## [20,] "0.00355776853648878"
## [21,] "0.202321888165077"
## [22,] "0.171627656225254"
## [23,] "0.140245300147539"
## [24,] "0.142933292949524"
```

```
# rezidualna matrica

residual = R - L%*%t(L) - diag(psi)
print(residual)
```

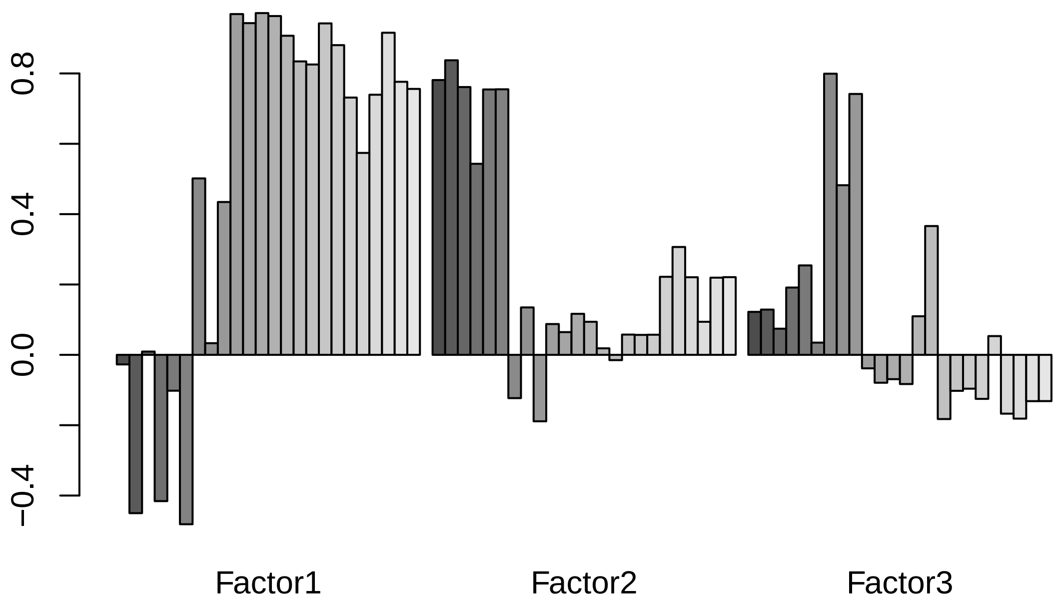
```
##              AGG          IEF           LQD          SHY          TIP
## AGG  0.000000000 -0.056719267  3.193390e-02 -0.067619900 -0.065269081
## IEF -0.056719267  0.000000000 -6.713355e-02  0.029549043 -0.027624183
## LQD  0.031933899 -0.067133550  1.110223e-16 -0.131576429 -0.050928719
## SHY -0.067619900  0.029549043 -1.315764e-01  0.000000000 -0.033974250
## TIP -0.065269081 -0.027624183 -5.092872e-02 -0.033974250  0.000000000
## TLT -0.090336275  0.063694320 -5.690214e-02 -0.092146764 -0.056753332
```

```
## DBC  0.024934548 -0.010826870  4.565981e-02 -0.049783088 -0.006671587
## GLD -0.091427423  0.008929224 -9.605691e-02 -0.014929611 -0.044987338
## USO  0.018056293  0.011828323  3.812358e-02 -0.013133904 -0.002281907
## IJH -0.009748408  0.015820660 -6.637150e-03  0.030428526  0.005639130
## IWM -0.014738947  0.016960064 -1.039817e-02  0.043781365  0.009607482
## SPY  0.028373782 -0.002589231 -1.974281e-04  0.016415346 -0.007779944
## VTV  0.007926296  0.003871668 -8.847886e-03  0.024023305 -0.003333958
## XLB -0.013870154  0.015034593  3.463563e-03  0.022620879  0.012026942
## XLE  0.057282478 -0.006172538  2.995365e-02 -0.012834197 -0.026071827
## XLF  0.001216807  0.017970815 -3.519624e-02  0.039188753  0.043861599
## XLI -0.024735972  0.008721184  2.032726e-02  0.033497051  0.017905728
## XLK  0.005085419  0.013297498  9.207030e-03  0.013406504 -0.008224306
## XLP -0.012875667 -0.018268328 -2.721198e-02 -0.002620395 -0.046758818
## XLU  0.013267601 -0.040432528 -9.837481e-03 -0.091164260 -0.092978156
## XLV  0.014180624 -0.032632248  2.553724e-03  0.008046153  0.004407525
## XLY -0.015799746  0.027053740 -2.487285e-02  0.034695993  0.013296849
## IYR -0.087336145  0.026318676 -6.727686e-02  0.004367051  0.010844137
## VNQ -0.088437103  0.026047280 -7.373960e-02  0.007117932  0.011004913
##              TLT           DBC           GLD           USO          IJH
## AGG -0.090336275  0.0249345485 -0.0914274231  1.805629e-02 -0.009748408
## IEF  0.063694320 -0.0108268695  0.0089292237  1.182832e-02  0.015820660
## LQD -0.056902142  0.0456598105 -0.0960569066  3.812358e-02 -0.006637150
## SHY -0.092146764 -0.0497830877 -0.0149296109 -1.313390e-02  0.030428526
## TIP -0.056753332 -0.0066715875 -0.0449873380 -2.281907e-03  0.005639130
## TLT  0.000000000  0.0116604981  0.0207283947  4.371722e-02  0.006396951
## DBC  0.011660498  0.0000000000 -0.1155917037  2.882911e-02 -0.015878981
## GLD  0.020728395 -0.1155917037  0.0000000000 -2.512614e-01  0.011459916
## USO  0.043717222  0.0288291071 -0.2512613526  1.110223e-16 -0.020714396
## IJH  0.006396951 -0.0158789807  0.0114599164 -2.071440e-02  0.000000000
## IWM  0.008529473 -0.0201580416  0.0244192903 -2.074990e-02  0.055759391
## SPY -0.012344610 -0.0041261384 -0.0146284157 -1.073198e-03 -0.007064487
## VTV -0.006771922 -0.0088475260 -0.0077478553  3.073763e-03 -0.012346926
## XLB -0.002710786 -0.0317217617  0.0348914144 -7.206870e-02  0.026570447
## XLE -0.014776083 -0.0251517152 -0.0804780675 -1.394517e-02 -0.001716447
## XLF  0.006472815  0.0002149989  0.0105298577  1.998519e-02 -0.004180798
## XLI -0.003097692 -0.0179388256  0.0109425572 -2.318876e-02  0.012292096
## XLK  0.018301444  0.0031784886 -0.0226052307  5.181076e-03  0.005800029
## XLP -0.006231246  0.0149362759  0.0180272686  2.645222e-02 -0.058284133
## XLU -0.017334810  0.0146727598 -0.0026385145 -1.543962e-02 -0.051129471
## XLV -0.049853960  0.0002676074  0.0098565042  4.429978e-03 -0.033179229
## XLY  0.027272234  0.0028457516 -0.0009043537  1.706668e-02  0.012495812
## IYR  0.058094277  0.0024700748  0.0983985542 -7.609252e-04  0.003674188
## VNQ  0.060071413  0.0052955328  0.0963464576  1.448317e-03  0.002294977
##              IWM           SPY          VTV          XLB           XLE
## AGG -0.014738947  0.0283737817  0.007926296 -0.013870154  0.0572824781
## IEF  0.016960064 -0.0025892314  0.003871668  0.015034593 -0.0061725380
## LQD -0.010398173 -0.0001974281 -0.008847886  0.003463563  0.0299536474
## SHY  0.043781365  0.0164153463  0.024023305  0.022620879 -0.0128341975
## TIP  0.009607482 -0.0077799438 -0.003333958  0.012026942 -0.0260718271
## TLT  0.008529473 -0.0123446101 -0.006771922 -0.002710786 -0.0147760830
## DBC -0.020158042 -0.0041261384 -0.008847526 -0.031721762 -0.0251517152
## GLD  0.024419290 -0.0146284157 -0.007747855  0.034891414 -0.0804780675
## USO -0.020749902 -0.0010731983  0.003073763 -0.072068699 -0.0139451659
## IJH  0.055759391 -0.0070644869 -0.012346926  0.026570447 -0.0017164467
```

```
## IWM  0.000000000 -0.0088235457 -0.014987622  0.013653484 -0.0107260997
## SPY -0.008823546  0.0000000000  0.015423796 -0.002793842  0.0212637169
## VTV -0.014987622  0.0154237964  0.000000000 -0.010790728  0.0224235021
## XLB  0.013653484 -0.0027938418 -0.010790728  0.000000000  0.0081780600
## XLE -0.010726100  0.0212637169  0.022423502  0.008178060  0.0000000000
## XLF -0.002081632 -0.0038218238  0.053069509 -0.028302009 -0.0242501898
## XLI  0.007335493  0.0058791786  0.010371155  0.034991693 -0.0003715195
## XLK  0.008000486  0.0358494704 -0.024283156  0.007541397 -0.0112617706
## XLP -0.068335223  0.0102353558 -0.001316395 -0.050239927 -0.0146134173
## XLU -0.079552932 -0.0114565577 -0.003315881 -0.057842579  0.0218090292
## XLV -0.027218359  0.0355464637  0.017588875 -0.027803382  0.0231741747
## XLY  0.010280529  0.0042678335 -0.013716314 -0.003433009 -0.0293751744
## IYR  0.005062756 -0.0642393575 -0.035670791 -0.018786080 -0.0783154686
## VNQ  0.006326213 -0.0662301585 -0.037317673 -0.023003242 -0.0802497853
##               XLF           XLI          XLK          XLP          XLU
## AGG  0.0012168071 -0.0247359721  0.005085419 -0.012875667  0.013267601
## IEF  0.0179708153  0.0087211842  0.013297498 -0.018268328 -0.040432528
## LQD -0.0351962397  0.0203272600  0.009207030 -0.027211983 -0.009837481
## SHY  0.0391887527  0.0334970513  0.013406504 -0.002620395 -0.091164260
## TIP  0.0438615993  0.0179057282 -0.008224306 -0.046758818 -0.092978156
## TLT  0.0064728153 -0.0030976921  0.018301444 -0.006231246 -0.017334810
## DBC  0.0002149989 -0.0179388256  0.003178489  0.014936276  0.014672760
## GLD  0.0105298577  0.0109425572 -0.022605231  0.018027269 -0.002638514
## USO  0.0199851931 -0.0231887633  0.005181076  0.026452215 -0.015439623
## IJH -0.0041807981  0.0122920964  0.005800029 -0.058284133 -0.051129471
## IWM -0.0020816322  0.0073354935  0.008000486 -0.068335223 -0.079552932
## SPY -0.0038218238  0.0058791786  0.035849470  0.010235356 -0.011456558
## VTV  0.0530695092  0.0103711554 -0.024283156 -0.001316395 -0.003315881
## XLB -0.0283020087  0.0349916933  0.007541397 -0.050239927 -0.057842579
## XLE -0.0242501898 -0.0003715195 -0.011261771 -0.014613417  0.021809029
## XLF  0.0000000000  0.0028874981 -0.091883765 -0.102389476 -0.096893904
## XLI  0.0028874981  0.0000000000  0.001100412 -0.031047721 -0.049669861
## XLK -0.0918837652  0.0011004121  0.000000000  0.008460979 -0.031816585
## XLP -0.1023894762 -0.0310477214  0.008460979  0.000000000  0.112233762
## XLU -0.0968939044 -0.0496698606 -0.031816585  0.112233762  0.000000000
## XLV -0.0652220573 -0.0259283528  0.011771340  0.068321736  0.018051053
## XLY -0.0060000058  0.0100131588  0.035656538 -0.023642747 -0.085066640
## IYR  0.0450708894 -0.0433685994 -0.088445369 -0.073150587 -0.047087105
## VNQ  0.0454424184 -0.0487420192 -0.094181386 -0.074337529 -0.046161155
##               XLV           XLY           IYR          VNQ
## AGG  0.0141806237 -0.0157997455 -8.733615e-02 -0.088437103
## IEF -0.0326322484  0.0270537403  2.631868e-02  0.026047280
## LQD  0.0025537243 -0.0248728457 -6.727686e-02 -0.073739598
## SHY  0.0080461533  0.0346959932  4.367051e-03  0.007117932
## TIP  0.0044075254  0.0132968495  1.084414e-02  0.011004913
## TLT -0.0498539602  0.0272722341  5.809428e-02  0.060071413
## DBC  0.0002676074  0.0028457516  2.470075e-03  0.005295533
## GLD  0.0098565042 -0.0009043537  9.839855e-02  0.096346458
## USO  0.0044299775  0.0170666804 -7.609252e-04  0.001448317
## IJH -0.0331792292  0.0124958117  3.674188e-03  0.002294977
## IWM -0.0272183595  0.0102805291  5.062756e-03  0.006326213
## SPY  0.0355464637  0.0042678335 -6.423936e-02 -0.066230159
## VTV  0.0175888749 -0.0137163138 -3.567079e-02 -0.037317673
## XLB -0.0278033823 -0.0034330091 -1.878608e-02 -0.023003242
```

```
## XLE  0.0231741747 -0.0293751744 -7.831547e-02 -0.080249785
## XLF -0.0652220573 -0.0060000058  4.507089e-02  0.045442418
## XLI -0.0259283528  0.0100131588 -4.336860e-02 -0.048742019
## XLK  0.0117713398  0.0356565383 -8.844537e-02 -0.094181386
## XLP  0.0683217365 -0.0236427472 -7.315059e-02 -0.074337529
## XLU  0.0180510526 -0.0850666404 -4.708710e-02 -0.046161155
## XLV  0.0000000000 -0.0465704621 -1.427728e-01 -0.145363332
## XLY -0.0465704621  0.0000000000  4.397416e-03  0.002070827
## IYR -0.1427728044  0.0043974157 -1.110223e-16  0.305587230
## VNQ -0.1453633321  0.0020708266  3.055872e-01  0.000000000
```

```
# metoda najvece izgledanosti

fa = factanal(factors = 3, covmat = R,rotation="none", lower = 0.1)

barplot(fa$loadings, beside=TRUE)
```



4.2. Specifične varijance faktora

Izračunajte specifične varijance faktora za model s dva faktora i model s tri faktora. Pomoću stupčastog dijagrama prikažite i usporedite dobivene vrijednosti.

```
#  Vaš kôd ovdje

rowSums(fa$loadings^2)
```

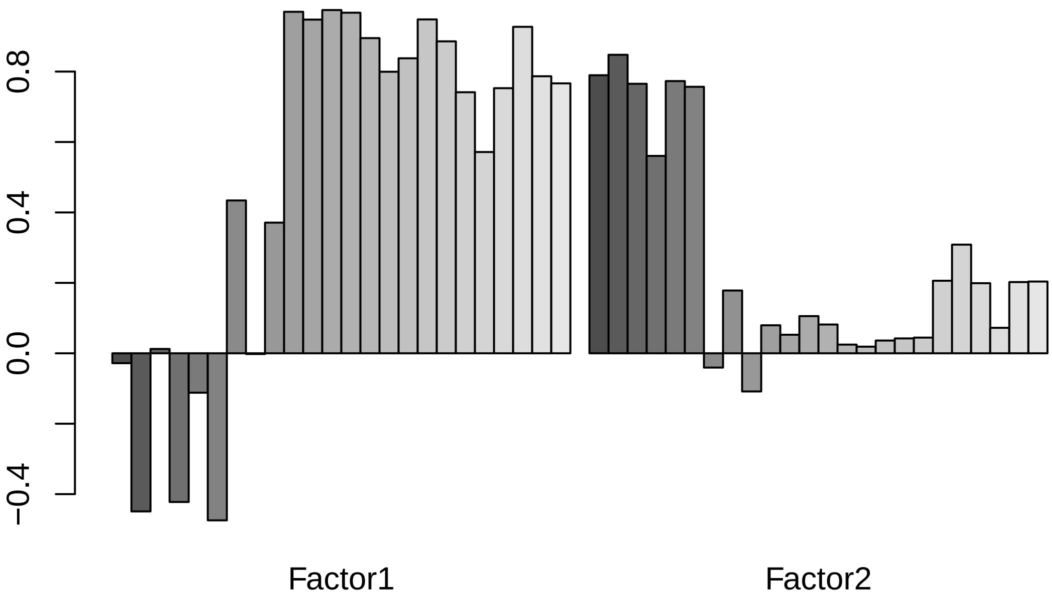
```
##       AGG       IEF       LQD       SHY       TIP       TLT       DBC       GLD
## 0.6259774 0.9204605 0.5851768 0.5045331 0.6443064 0.8030059 0.9055429 0.2517743
##       USO       IJH       IWM       SPY       VTV       XLB       XLE       XLF
## 0.7746147 0.9476366 0.8998253 0.9622944 0.9433534 0.8352429 0.8302764 0.7179810
##       XLI       XLK       XLP       XLU       XLV       XLY       IYR       VNQ
## 0.9014973 0.7876993 0.5998560 0.4264158 0.6234644 0.8802750 0.6681069 0.6376117
```

```
fa$uniquenesses
```

```
##       AGG       IEF       LQD       SHY       TIP       TLT       DBC       GLD
## 0.3740338 0.1000000 0.4148478 0.4954758 0.3556954 0.1969931 0.1000000 0.7482554
```

```
##       USO       IJH       IWM       SPY       VTV       XLB       XLE       XLF
## 0.2253855 0.1000000 0.1001757 0.1000000 0.1000000 0.1647590 0.1697232 0.2820202
##       XLI       XLK       XLP       XLU       XLV       XLY       IYR       VNQ
## 0.1000000 0.2123007 0.4001492 0.5736068 0.3765393 0.1197184 0.3319107 0.3623803
```

```
fa = factanal(factors = 2, covmat = R,rotation="none", lower = 0.1)

barplot(fa$loadings, beside=TRUE)
```




```
rowSums(fa$loadings^2)
```

```
##        AGG        IEF        LQD        SHY        TIP        TLT        DBC
## 0.62432329 0.92020270 0.58589854 0.49253453 0.61022293 0.79780472 0.19005420
##        GLD        USO        IJH        IWM        SPY        VTV        XLB
## 0.03178785 0.14945013 0.94681177 0.90084069 0.96124397 0.94219588 0.80173234
##        XLE        XLF        XLI        XLK        XLP        XLU        XLV
## 0.63967069 0.70323717 0.90078457 0.78685586 0.59215088 0.42191705 0.60650783
##        XLY        IYR        VNQ
## 0.86480610 0.65992766 0.62917265
```

```
fa$uniquenesses
```

```
##       AGG       IEF       LQD       SHY       TIP       TLT       DBC       GLD
## 0.3756758 0.1000000 0.4141025 0.5074483 0.3897753 0.2021919 0.8099615 0.9682132
##       USO       IJH       IWM       SPY       VTV       XLB       XLE       XLF
## 0.8505354 0.1000000 0.1000000 0.1000000 0.1000000 0.1982767 0.3603295 0.2967601
##       XLI       XLK       XLP       XLU       XLV       XLY       IYR       VNQ
## 0.1000000 0.2131339 0.4078493 0.5781100 0.3934928 0.1351928 0.3400725 0.3708269
```

5. Diskriminantna analiza

Financijska tržišta su od listopada 2007. do srpnja 2009. godine bila u krizi. U datoteci "crisis.csv" za svaki tjedan iz prethodno učitanih povijesnih tjednih cijena možete pronaći je li tržište tada bilo u krizi ili ne - 1 predstavlja krizu, 0 predstavlja period bez krize. Učitajte nove podatke te ih spojite s tablicom povrata.

```
#  Vaš kôd ovdje
```

```
crisis = read.csv(file = "crisis.csv")
crisis = data.matrix(crisis[2:nrow(crisis), 2])

ETF_crisis = cbind(crisis, as.data.frame(ETF_returns))
```

5.1. Diskriminantna analiza pomoću povrata

Provedite diskriminantnu analizu koja tjedne odvaja na krizne i one bez krize pomoću povrata fondova. Pomoću stupčastog dijagrama prikažite vektore srednjih vrijednosti u krizi i izvan nje. Također, na isti način prikažite korelaciju fonda AGG (Aggregate Bond ETF-a) s ostalim fondovima u krizi i izvan krize. Usporedite rezultate linearne diskriminantne analize (funkcija u R-u: `lda`) i kvadratne diskriminantne analize (funkcija u R-u: `qda`) pomoću tablica konfuzije i mjere APER (eng. apparent error rate). Razmislite o tome koji je razlog razlike u rezultatima ove dvije metode.

```
#  Vaš kôd ovdje
ETF_crisis_scaled=ETF_crisis
ETF_crisis_scaled[, 2:ncol(ETF_crisis)] <- scale(ETF_crisis[, 2:ncol(ETF_crisis)])

cov(ETF_crisis_scaled[, 2:ncol(ETF_crisis)])
```

```
##              AGG        IEF          LQD        SHY          TIP         TLT
## AGG  1.000000000  0.6745668  0.743219450  0.4455372  0.641230551  0.54992048
## IEF  0.674566754  1.0000000  0.624744770  0.7260504  0.718809999  0.89300584
## LQD  0.743219450  0.6247448  1.000000000  0.3402376  0.620570167  0.55215298
## SHY  0.445537180  0.7260504  0.340237648  1.0000000  0.527736365  0.53287945
## TIP  0.641230551  0.7188100  0.620570167  0.5277364  1.000000000  0.58433952
## TLT  0.549920484  0.8930058  0.552152981  0.5328795  0.584339515  1.00000000
## DBC -0.014930698 -0.2297039 -0.024221650 -0.1378221  0.064187105 -0.30272595
## GLD  0.076398646  0.1852350  0.028623215  0.2040169  0.233690439  0.10542200
## USO -0.095956334 -0.2540766 -0.103089957 -0.1339803 -0.006863213 -0.30110084
## IJH  0.026023102 -0.3621415  0.064802698 -0.3528118 -0.042028635 -0.40084695
## IWM -0.004151541 -0.3748789  0.037570016 -0.3508674 -0.066166015 -0.40503329
## SPY  0.097081209 -0.3542461  0.106863329 -0.3578469 -0.034448273 -0.39191642
## VTV  0.053050332 -0.3685157  0.075666084 -0.3654518 -0.054001056 -0.40284963
## XLB -0.020383471 -0.3702162  0.020799499 -0.3310908 -0.038818440 -0.42887840
## XLE  0.058958920 -0.3431183  0.038023614 -0.2873796 -0.019052165 -0.41725502
## XLF -0.000819553 -0.3459094  0.004458782 -0.3429471 -0.059221345 -0.36076468
## XLI -0.018218793 -0.3876325  0.065898060 -0.3681968 -0.066535801 -0.41781468
## XLK  0.011580735 -0.3597213  0.053641394 -0.3676227 -0.090995955 -0.36984998
## XLP  0.164157118 -0.1743601  0.183119698 -0.2293049  0.030232852 -0.18720595
## XLU  0.303336129 -0.0146771  0.294572146 -0.1470028  0.135932288 -0.04445965
## XLV  0.190339883 -0.1893645  0.213989706 -0.2239086  0.074534364 -0.22758869
## XLY  0.006004474 -0.3494293  0.040880704 -0.3659944 -0.073195803 -0.35823132
## IYR  0.057064133 -0.1794710  0.111305135 -0.2581206  0.057890557 -0.17259889
## VNQ  0.057890825 -0.1697239  0.106160387 -0.2464403  0.061007743 -0.16004099
##             DBC         GLD          USO         IJH          IWM         SPY
## AGG -0.01493070  0.076398646 -0.095956334  0.02602310 -0.004151541  0.09708121
## IEF -0.22970394  0.185235021 -0.254076608 -0.36214150 -0.374878917 -0.35424606
## LQD -0.02422165  0.028623215 -0.103089957  0.06480270  0.037570016  0.10686333
## SHY -0.13782212  0.204016880 -0.133980284 -0.35281184 -0.350867366 -0.35784694
## TIP  0.06418710  0.233690439 -0.006863213 -0.04202863 -0.066166015 -0.03444827
## TLT -0.30272595  0.105421997 -0.301100840 -0.40084695 -0.405033285 -0.39191642
## DBC  1.00000000  0.417896304  0.855305637  0.44391538  0.399354310  0.41585393
## GLD  0.41789630  1.000000000  0.229503180  0.04264622  0.026922101 -0.01028143
```

```
## USO  0.85530564  0.229503180  1.000000000  0.37062691  0.336576220  0.34914057
## IJH  0.44391538  0.042646223  0.370626910  1.00000000  0.969617751  0.94889604
## IWM  0.39935431  0.026922101  0.336576220  0.96961775  1.000000000  0.91838800
## SPY  0.41585393 -0.010281432  0.349140569  0.94889604  0.918388000  1.00000000
## VTV  0.40055400 -0.015649132  0.346121247  0.93540414  0.905385404  0.98003760
## XLB  0.53851292  0.167173399  0.429576346  0.89952200  0.855045368  0.87120386
## XLE  0.70138405  0.205883534  0.635918113  0.79604229  0.749318963  0.80687798
## XLF  0.26384226 -0.077398835  0.236539686  0.80809977  0.791859310  0.82700541
## XLI  0.37996678 -0.004575189  0.314372306  0.92524056  0.895100727  0.93339043
## XLK  0.36086811 -0.050265440  0.307139765  0.86449901  0.843692932  0.90934121
## XLP  0.24432347 -0.024393258  0.194673401  0.69714568  0.664222548  0.79096503
## XLU  0.30111414  0.084480365  0.199794511  0.56364393  0.507147364  0.62312734
## XLV  0.20032250 -0.055091747  0.145400539  0.71941005  0.703783802  0.81518945
## XLY  0.30980327 -0.078282053  0.268566289  0.90905847  0.885061458  0.92120322
## IYR  0.27635506  0.067538119  0.210988384  0.79834002  0.775819113  0.75327353
## VNQ  0.26759417  0.063603561  0.202849349  0.77844514  0.759150237  0.73278192
##             VTV         XLB         XLE          XLF          XLI         XLK
## AGG  0.05305033 -0.02038347  0.05895892 -0.000819553 -0.018218793  0.01158074
## IEF -0.36851569 -0.37021616 -0.34311828 -0.345909437 -0.387632549 -0.35972131
## LQD  0.07566608  0.02079950  0.03802361  0.004458782  0.065898060  0.05364139
## SHY -0.36545180 -0.33109076 -0.28737961 -0.342947065 -0.368196783 -0.36762271
## TIP -0.05400106 -0.03881844 -0.01905217 -0.059221345 -0.066535801 -0.09099596
## TLT -0.40284963 -0.42887840 -0.41725502 -0.360764676 -0.417814679 -0.36984998
## DBC  0.40055400  0.53851292  0.70138405  0.263842259  0.379966778  0.36086811
## GLD -0.01564913  0.16717340  0.20588353 -0.077398835 -0.004575189 -0.05026544
## USO  0.34612125  0.42957635  0.63591811  0.236539686  0.314372306  0.30713977
## IJH  0.93540414  0.89952200  0.79604229  0.808099766  0.925240563  0.86449901
## IWM  0.90538540  0.85504537  0.74931896  0.791859310  0.895100727  0.84369293
## SPY  0.98003760  0.87120386  0.80687798  0.827005410  0.933390430  0.90934121
## VTV  1.00000000  0.85509049  0.79778956  0.879566341  0.931450043  0.84335465
## XLB  0.85509049  1.00000000  0.81921041  0.696877196  0.871214534  0.79055343
## XLE  0.79778956  0.81921041  1.00000000  0.596514870  0.748288639  0.68459998
## XLF  0.87956634  0.69687720  0.59651487  1.000000000  0.800563853  0.66171744
## XLI  0.93145004  0.87121453  0.74828864  0.800563853  1.000000000  0.83813419
## XLK  0.84335465  0.79055343  0.68459998  0.661717445  0.838134191  1.00000000
## XLP  0.76971575  0.61411637  0.55482404  0.567401862  0.704053774  0.70310619
## XLU  0.61733424  0.49843733  0.53528840  0.419171177  0.534722265  0.51779550
## XLV  0.78793528  0.62803801  0.57619150  0.607813168  0.708524330  0.70649895
## XLY  0.89744977  0.79946724  0.66381836  0.793349572  0.888210552  0.86490906
## IYR  0.77260343  0.68854381  0.53703115  0.745609902  0.729352781  0.64100570
## VNQ  0.75256281  0.66654551  0.51814995  0.730388685  0.706125949  0.61853667
##             XLP         XLU         XLV          XLY         IYR         VNQ
## AGG  0.16415712  0.30333613  0.19033988  0.006004474  0.05706413  0.05789083
## IEF -0.17436006 -0.01467710 -0.18936450 -0.349429280 -0.17947101 -0.16972387
## LQD  0.18311970  0.29457215  0.21398971  0.040880704  0.11130514  0.10616039
## SHY -0.22930488 -0.14700278 -0.22390857 -0.365994351 -0.25812056 -0.24644034
## TIP  0.03023285  0.13593229  0.07453436 -0.073195803  0.05789056  0.06100774
## TLT -0.18720595 -0.04445965 -0.22758869 -0.358231321 -0.17259889 -0.16004099
## DBC  0.24432347  0.30111414  0.20032250  0.309803275  0.27635506  0.26759417
## GLD -0.02439326  0.08448036 -0.05509175 -0.078282053  0.06753812  0.06360356
## USO  0.19467340  0.19979451  0.14540054  0.268566289  0.21098838  0.20284935
## IJH  0.69714568  0.56364393  0.71941005  0.909058471  0.79834002  0.77844514
## IWM  0.66422255  0.50714736  0.70378380  0.885061458  0.77581911  0.75915024
## SPY  0.79096503  0.62312734  0.81518945  0.921203221  0.75327353  0.73278192
```

```
## VTV  0.76971575  0.61733424  0.78793528  0.897449772  0.77260343  0.75256281
## XLB  0.61411637  0.49843733  0.62803801  0.799467236  0.68854381  0.66654551
## XLE  0.55482404  0.53528840  0.57619150  0.663818360  0.53703115  0.51814995
## XLF  0.56740186  0.41917118  0.60781317  0.793349572  0.74560990  0.73038868
## XLI  0.70405377  0.53472227  0.70852433  0.888210552  0.72935278  0.70612595
## XLK  0.70310619  0.51779550  0.70649895  0.864909060  0.64100570  0.61853667
## XLP  1.00000000  0.65740774  0.73076962  0.717122700  0.60658292  0.59154411
## XLU  0.65740774  1.00000000  0.56077665  0.493518388  0.51148156  0.50126849
## XLV  0.73076962  0.56077665  1.00000000  0.697002041  0.53883130  0.52254046
## XLY  0.71712270  0.49351839  0.69700204  1.000000000  0.77858525  0.75907166
## IYR  0.60658292  0.51148156  0.53883130  0.778585251  1.00000000  0.99467977
## VNQ  0.59154411  0.50126849  0.52254046  0.759071659  0.99467977  1.00000000
```

```r
lda.fit <- lda(crisis ~ ., data = ETF_crisis_scaled[,2:ncol(ETF_crisis)])
lda.fit
```

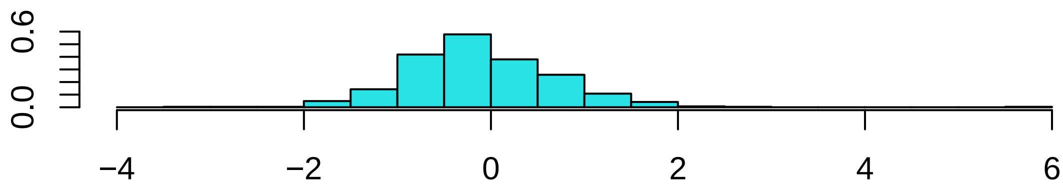
```
## Call:
## lda(crisis ~ ., data = ETF_crisis_scaled[, 2:ncol(ETF_crisis)])
##
## Prior probabilities of groups:
##         0         1
## 0.8618619 0.1381381
##
## Group means:
##           AGG         IEF         LQD         SHY          TIP          TLT
## 0 -0.01495071 -0.01805876  0.00342670 -0.06070373 -0.006304978 -0.005849677
## 1  0.09327943  0.11267094 -0.02137963  0.37873850  0.039337583  0.036496898
##            DBC          GLD         USO        IJH         IWM         SPY
## 0  0.008796049 -0.009665649  0.01096275  0.0338521  0.03220038  0.04271126
## 1 -0.054879699  0.060305243 -0.06839805 -0.2112077 -0.20090234 -0.26648112
##           VTV         XLB         XLE         XLF         XLI         XLK
## 0  0.04594869  0.03053749  0.02206441  0.03696749  0.04332299  0.03674437
## 1 -0.28667990 -0.19052737 -0.13766271 -0.23064501 -0.27029778 -0.22925292
##           XLP         XLU         XLV         XLY         IYR         VNQ
## 0  0.02946121  0.03462193  0.03541497  0.03647374  0.03967429  0.03769996
## 1 -0.18381235 -0.21601071 -0.22095862 -0.22756442 -0.24753307 -0.23521498
##
## Coefficients of linear discriminants:
##              LD1
## AGG  0.650379594
## IEF -0.949495234
## LQD  0.024281819
## SHY  0.972299637
## TIP -0.148880617
## TLT -0.051219655
## DBC  0.009703912
## GLD  0.007619750
## USO -0.284333846
## IJH  0.757909617
## IWM  0.115154968
## SPY -4.475550763
## VTV -0.925981528
## XLB  0.277040261
## XLE  1.102714015
## XLF  0.985254454
```

```
## XLI -0.080281196
## XLK  0.918081589
## XLP  0.681512409
## XLU -0.014047291
## XLV  0.417343951
## XLY  0.841061298
## IYR -1.725097993
## VNQ  1.487767942
```

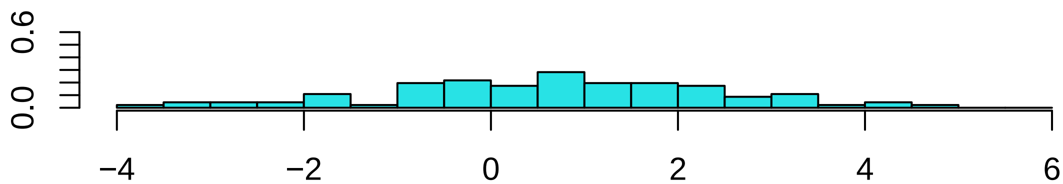
```r
predictions <- predict(lda.fit, ETF_crisis_scaled[,2:ncol(ETF_crisis)])
lapply(predictions, head)
```

```
## $class
## [1] 0 0 0 0 0 0
## Levels: 0 1
##
## $posterior
##           0          1
## 2 0.9341382 0.06586185
## 3 0.9013584 0.09864161
## 4 0.8310224 0.16897760
## 5 0.8909312 0.10906877
## 6 0.8861293 0.11387074
## 7 0.8451192 0.15488085
##
## $x
##           LD1
## 2 -0.79818670
## 3 -0.22282894
## 4  0.58790915
## 5 -0.07610080
## 6 -0.01264435
## 7  0.45189918
```

```r
ldahist(data = predictions$x[,1], g = ETF_crisis_scaled$crisis)
```



group 0



group 1

```
mistakes <- 0
df <- ETF_crisis_scaled
for (i in 1:nrow(df)) {
  holdout <- df[i, ]
  df.tmp <- df[-i, ]
  lda.fit <- lda(crisis ~ ., data = df.tmp)
  if (predict(lda.fit, holdout)$class != holdout$crisis) mistakes <- mistakes + 1
}

str_c("APER: ",mistakes / nrow(df) * 100, "%")
```

```
## [1] "APER: 13.2132132132132%"
```

5.2. Diskriminantna analiza pomoću glavnih komponenti

Provedite diskriminantnu analizu kao u prošlom podzadatku, no ovaj put koristeći glavne komponente izračunate u 3. zadatku kao varijable. Provjerite i usporedite uspješnost klasifikacije koristeći tablice konfuzije i APER za različit broj komponenti.

```
#  Vaš kôd ovdje
```