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Resonant Modes Analysis in Power Systems

Algorithms and Matlab GUI

Branislav Vuksanovic
School of Engineering
University of Portsmouth
Portsmouth, UK
e-mail: branislav.vuksanovic@port.ac.uk

Hemanshu Pota
School of Engineering and Information Technology
University of New South Wales, ADFA
Canberra, Australia
e-mail: h.pota@adfa.edu.au

Abstract—**This paper describes a set of algorithms used to detect and analyse resonant modes present in the oscillatory type signals measured in power systems using Wide Area Measurement System (WAMS) technology. To enable easier analysis and extraction of the main signal parameters – frequency, amplitude and damping, described algorithms have been implemented and tested using Matlab based GUI program. Algorithms performance, operation of developed GUI and some initial results are presented in this paper indicating the potential use of this approach and described methods in large, interconnected power systems of today.**

Keywords-damping; power systems stability; oscilations; Prony algorithm; LPSVD; matrix pencil

I. INTRODUCTION

In UK, energy has been one of national priorities for a long time but has recently been put in the spotlight stronger than ever with government commitment to reduce the carbon emission and ensure that by 2020 15% of energy comes from renewable sources, as part of an EU target for 20% renewable energy. On the other side is the expected increase in energy demand from both industry and domestic customers as a result of electrification of the transport and heating sectors. Nature of new energy sources is likely to be different from the existing ones ranging from small, roof-top solar panels to large offshore wind farms. These alternative sources of energy will not be able to respond to increased fluctuations in energy demand from consumers in the way consumers have been accustomed to in the previous century. The effective integration of those sources in the system therefore needs to tackle problems during surges as well as the problems of energy waste reduction in less busy periods. Those objectives and trends emphasise the need for efficient and reliable energy supply and means of delivery placed upon the power grid in UK and Europe.

Integration of renewable energy sources in the existing electrical power system and increased interconnection of power systems can in many cases result in long distant power transfers between the parts of the grid. Such power transfer carries the potential risk of low frequency inter-area oscillations in large transmission systems.

Electromechanical oscillations [1] occur frequently in power networks when two electrical systems with large number of generators are connected by a weak-tie line. Usually, those oscillations are stable but can have low damping ratios. These days power networks are operated close to their limits and poorly damped oscillations can severely limit transfer capacities. In some cases, damping ratios can even be negative and oscillations can grow out of control leading to grid reliability issues and potentially large-scale blackouts. A number of those events have been recorded in the past.

Oscillations are usually low in frequency and can happen for various reasons. Most often they represent the response of the system perturbed in some way. Faults in the system operation can cause those oscillations but in many cases oscillations can be excited by a more or less random event in the system operation such are variations in the load demand.

To understand the exact nature and estimate damping of the system oscillations, system modelling and computer simulations can be applied. A complex power system is represented using a state-space model and linearised about an equilibrium point. Eigenvalues of the system characteristic equation can then be obtained and analysed in order to obtain the information about the system behaviour [1], [2]. Positive real eignevalues represent the unstable system modes - oscillations with negative damping. This approach, whilst providing a useful insight into system operation can be unreliable due to complexities involved in accurate modelling of large power systems. Uncertainties and inaccurate estimates of the generation and demand levels in the system can also play an important role in the accuracy of the obtained results.

With recent advances in Wide Area Monitoring Systems (WAMS) and introduction of Phasor Measurement Units (PMUs) the alternative approach taken in the recent years is to perform oscillation detection and analysis using a real-time measurement data, rather than relying on simulations and system models. Measurement based methods require less effort but can achieve higher accuracy in estimation of the oscillation modes. Results obtained using measurement methods can also be updated using a new set of measurement data. This work analyses and compares three related Digital Signal Processing (DSP) algorithms proposed for grid oscillation detection and analysis tasks in the recent years. It demonstrates the performance of much used Prony algorithm and compares it with other more advanced and accurate algorithms – LPSVD (Linear prediction singular value decomposition) and Matrix Pencil techniques. All three algorithms have been implemented using Matlab programming language and tested using both simulated and measured data. Moreover, to enable easy application and assessment of the performance of those algorithms, Graphical User Interface (GUI) has been developed and briefly described in this paper. Section II of this paper summarises the implemented algorithms while the section III presents some results achieved with each technique. Section IV describes the operation of the Matlab GUI and Section V provides some conclusions and describes directions for the future work on this project.

II. TECHNIQUES

A. Prony Analysis

Prony analysis [2] is a method of fitting a linear combination of exponential terms to observed, damped oscillatory signal $y(n)$. If such signal is represented by P complex exponentials its estimate can be given with:

$$\hat{y}(n) = \sum_{k=1}^{P} A_k e^{(\alpha_k + i\omega_k)(n-1)T + i\varphi_k} \qquad n = 1, 2, ..., N \tag{1}$$

Each of P terms, also known as the modes of the original signal $y(n)$, is defined with four elements: the magnitude A_k, the damping α_k, the angular frequency ω_k, and the initial phase φ_k.

It can be useful to recast the above equation in a slightly different form:

$$\hat{y}(n) = \sum_{k=1}^{P} h_k z_k^{n-1} \tag{2}$$

where:

$$h_k = A_k e^{i\varphi_k} \tag{3}$$

is a complex amplitude representing a time-independent parameter and:

$$z_k = e^{(\alpha_k + i\omega_k)T} \tag{4}$$

is a complex exponent that represents time-dependent parameter. Those complex exponents are usually referred to as signal poles. The problem of estimating two sets of parameters - h_k and z_k is based on the minimization of the squared error over N samples of the observed signal $y(n)$:

$$\delta = \sum_{n=1}^{N} |e(n)|^2 = \sum_{n=1}^{N} |y(n) - \hat{y}(n)|^2 = \sum_{n=1}^{N} \left| y(n) - \sum_{k=1}^{P} h_k z_k^{n-1} \right|^2 \tag{5}$$

This is a difficult nonlinear problem and can be solved using Prony method which utilizes linear equations solution.

Since we are fitting observed signal $y(n)$ to a given exponential model, i.e.

$$y(n) = \sum_{k=1}^{P} h_k z_k^{n-1} \tag{6}$$

We can write the above equation for $1 \le n \le P$ in the matrix form:

$$\begin{bmatrix} z_1^0 & z_2^0 & \dots & z_P^0 \\ z_1^1 & z_2^1 & \dots & z_P^1 \\ \dots & & & \dots \\ z_1^{P-1} & z_2^{P-1} & \dots & z_P^{P-1} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \dots \\ h_P \end{bmatrix} = \begin{bmatrix} y(1) \\ y(2) \\ \dots \\ y(P) \end{bmatrix} \tag{7}$$

From the above equation complex amplitudes h_k can be obtained providing z_k's are known. Prony method provides a way to determine z_k's without having to resort to non-linear minimization. This method proposed to define polynomial $\mathbf{A}(z)$ which has z_k's as its roots:

$$\mathbf{A}(z) = \prod_{k=1}^{P} (z - z_k) = \sum_{m=0}^{P} a(m) z^{P-m} \tag{8}$$

Shifting the index in (6) from n to n-m and multiplying by the parameter $a(m)$ yields:

$$a(m) y(n-m) = a(m) \sum_{k=1}^{P} h_k z_k^{n-m-1} \tag{9}$$

Equation (9) can be further modified by summing from m= 0 to m = P to:

$$\sum_{m=0}^{P} a(m) y(n-m) = \sum_{k=1}^{P} h_k z_k^{n-P} \sum_{m=0}^{P} a(m) h_k z_k^{P-m-1} = 0 \tag{10}$$

This is forward linear prediction equation which can be expressed in matrix form as:

$$\begin{bmatrix} y(P) & y(P-1) & \dots & y(1) \\ y(P+1) & y(P) & \dots & y(2) \\ \dots & & & \dots \\ y(2P-1) & y(2P-2) & \dots & y(P) \end{bmatrix} \begin{bmatrix} a(1) \\ a(2) \\ \dots \\ a(P) \end{bmatrix} = - \begin{bmatrix} y(P+1) \\ y(P+2) \\ \dots \\ y(2P) \end{bmatrix} \tag{11}$$

Thus, $a(k)$ coefficients can be determined using Prony method which decouples the problem of determining h_k and z_k parameters. The whole procedure is usually split into three steps:

1. Solve (11) to determine coefficients $a(k)$.
2. Determine the complex exponents, roots z_k of polynomial $\mathbf{A}(z)$ defined in (8). At this point damping and frequency of each mode can be determined from (4) as:

$$\alpha_k = \mathrm{Re}\left(\log(z_k) \frac{1}{T} \right) \tag{12}$$

$$\omega_k = \mathrm{Im}\left(\log(z_k) \frac{1}{T} \right) \tag{13}$$

3. Determine complex amplitudes h_k from (7) and then use (3) to find amplitudes A_k and initial phase φ_k as:

$$A_k = |h_k| \tag{14}$$

$$\varphi_k = \mathrm{Im}\left(\ln \frac{h_k}{|h_k|} \right) \tag{15}$$

Thus Prony analysis can estimate the damping factor of each mode in the signal and identify the potentially unstable or highly oscillatory modes in the signal.

B. Extended Prony Method

It is clear that the method proposed by Prony reduces the non-linear fitting problem to two numerical tasks of solving linear systems of equations (step 1) and finding the roots of polynomial (step 2). The length of data record should and usually does exceed the number of data points needed to fit a model, i.e. $N>2P$. In practical situations N should be at least three times larger than the model order P. In this case matrix form of linear prediction equation (11) is modified to:

$$\begin{bmatrix} y(P) & y(P-1) & \dots & y(1) \\ y(P+1) & y(P) & \dots & y(2) \\ \dots & & & \dots \\ y(N-1) & y(N-2) & \dots & y(N-P) \end{bmatrix} \begin{bmatrix} a(1) \\ a(2) \\ \dots \\ a(P) \end{bmatrix} = - \begin{bmatrix} y(P+1) \\ y(P+2) \\ \dots \\ y(N) \end{bmatrix} \quad (16)$$

The equation can be written in the condensed, matrix form:

$$\mathbf{Ya} = -\mathbf{y} \quad (17)$$

and solved in the least-squares sense:

$$\mathbf{a} = -\left(\mathbf{Y}^H \mathbf{Y}\right)^{-1} \mathbf{Y}^H \mathbf{y} \quad (18)$$

This method is known as "least-squares Prony method" or "extended Prony method".

C. LPSVD - Linear Prediction Signular Value Decomposition Method (Kumaresan-Tufts)

One of the problems Prony method faces are large variance and bias when analyzing noisy signals. Influence of noise can be reduced by setting the number of exponential components to be estimated to L, where $L>P$. Using singular value decomposition (SVD) we can then write matrix $\mathbf{Y}$ from equation (17) as:

$$\mathbf{Y} = \mathbf{USV}^H \quad (19)$$

where $\mathbf{S}$ is a $(N-L)\times L$ matrix with the singular values on the diagonal arranged in decreasing order. Noise can be reduced by considering the reduced rank approximation

$$\hat{\mathbf{Y}} = \mathbf{U}\hat{\mathbf{S}}\mathbf{V}^H \quad (20)$$

with

$$\hat{\mathbf{S}} = \begin{bmatrix} \mathbf{S}_p & 0 \\ 0 & 0 \end{bmatrix}_{(N-L)\times L} \quad (21)$$

where $\mathbf{S}_P$ is the top-left $P\times P$ minor of $\mathbf{S}$. An estimate for the coefficients $a(m)$ is then:

$$\hat{\mathbf{a}} = -\mathbf{Y}^{\dagger}\mathbf{y} \quad (22)$$

where $\mathbf{Y}^{\dagger}$ is the pseudoinverse of $\hat{\mathbf{Y}}$. The use of truncated SVD improves the SNR, providing a better estimate of the vector $\hat{\mathbf{a}}$ and consequently of the exponential parameters. This approach is known as Kumaresan-Tufts algorithm [3] but is sometimes also referred to as Linear Prediction Signular Value Decomposition Method.

D. Matrix Pencil Method

Another method to estimate exponential parameters from noisy signals is the Matrix Pencil (MP) method [4]. It is generally more robust than LPSVD having a lower variance on the estimated parameters but a slightly larger bias. Summary of this method is as follows. First, two matrices $\mathbf{Y}_0$ and $\mathbf{Y}_1$ are defined as:

$$\mathbf{Y}_0 = \begin{bmatrix} y(0) & y(1) & \dots & y(L-1) \\ y(1) & y(2) & \dots & y(L) \\ \dots & & & \dots \\ y(N-L-1) & y(N-L) & \dots & y(N-2) \end{bmatrix}; \quad \mathbf{Y}_1 = \begin{bmatrix} y(1) & y(2) & \dots & y(L) \\ y(2) & y(3) & \dots & y(L+1) \\ \dots & & & \dots \\ y(N-L) & y(N-L+1) & \dots & y(N-1) \end{bmatrix} \quad (23)$$

here L is the so called pencil parameter which plays the role of the prediction order parameter in the LPSVD/KT method. Matrices $\mathbf{Y}_0$ and $\mathbf{Y}_1$ can now be decomposed as:

$$\mathbf{Y}_0 = \mathbf{Z}_l \mathbf{H} \mathbf{Z}_r \quad (24)$$

$$\mathbf{Y}_1 = \mathbf{Z}_l \mathbf{H} \mathbf{Z} \mathbf{Z}_r \quad (25)$$

where:

$$\mathbf{Z}_l = \begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_P \\ \dots & & & \dots \\ z_1^{N-L-1} & z_2^{N-L-1} & \dots & z_P^{N-L-1} \end{bmatrix}; \quad \mathbf{Z}_r = \begin{bmatrix} 1 & z_1 & \dots & z_1^{L-1} \\ 1 & z_2 & \dots & z_2^{L-1} \\ \dots & & & \dots \\ 1 & z_P & \dots & z_P^{L-1} \end{bmatrix} \quad (26)$$

$$\mathbf{H} = \operatorname{diag}\left(h_1, h_1, \dots, h_P\right) \quad (27)$$

$$\mathbf{Z} = \operatorname{diag}\left(z_1, z_1, \dots, z_P\right) \quad (28)$$

Matrix pencil $\mathbf{Y}_1 - z\mathbf{Y}_0$ can be rewritten using decomposition of two matrices as:

$$\mathbf{Y}_1 - z\mathbf{Y}_0 = \mathbf{Z}_l \mathbf{H}\left(\mathbf{Z} - z\mathbf{I}_p\right)\mathbf{Z}_r \quad (29)$$

When $z \neq z_i$, the matrix $\mathbf{Z} - z\mathbf{I}_p$ is of rank P. However, for $z = z_i$ it is of rank P-1. Therefore, the poles of the signal reduce the rank of the matrix pencil for $P \le L \le N-P$. This is equivalent to saying that the poles z_i are the generalized eigenvalues of $\left(\mathbf{Y}_1, \mathbf{Y}_0\right)$, in the sense that $\left(\mathbf{Y}_1 - z\mathbf{Y}_0\right)\mathbf{v} = 0$, with $\mathbf{v}$ an eigenvector of $\mathbf{Y}_1 - z\mathbf{Y}_0$. To find the poles z_i, one can use the fact that $\mathbf{Y}_0^{\dagger}\mathbf{Y}_1$ has P eigenvalues equal to the poles z_i and L-P null eigenvalues. Here $\mathbf{Y}_0^{\dagger}$ denotes a pseudo-inverse (Moore-Penrose) of $\mathbf{Y}_0$.

In practice we do not have access to the noiseless signal, therefore we must work directly with the noisy data. SVD is again necessary to select the singular values due to the signal. The basic steps of the MP method can be summarized as follows:

1. Build the matrices $\mathbf{Y}_0$ and $\mathbf{Y}_1$ as in (23).
2. Perform SVD of $\mathbf{Y}_1$ i.e. $\mathbf{Y}_1 = \mathbf{USV}^T$
3. Estimate the signal subspace of $\mathbf{Y}_1$ by considering the P largest singular values of $\mathbf{S}$: $\tilde{\mathbf{Y}}_1 = \mathbf{U}_P\mathbf{S}_P\mathbf{V}_P^H$, where $\mathbf{U}_P$ and $\mathbf{V}_P$ are built from the first P columns of $\mathbf{U}_P$ and $\mathbf{V}_P$ and $\mathbf{S}_P$, is the top-left $P \times P$ minor of $\mathbf{S}$.
4. The matrix $\mathbf{Z}_L = \mathbf{Y}_1^{\dagger}\mathbf{Y}_0 = \mathbf{V}_P\mathbf{S}_P^{-1}\mathbf{U}_P^T\mathbf{Y}_0$ has P eigenvalues which provide estimates of the inverse poles $1/z_i$; the other L-P eigenvalues are zero. Since $\boldsymbol{Z}_L$ has only P non-zero eigenvalues, it is convenient to restrict attention to a $P \times P$ matrix $\mathbf{Z}_P = \mathbf{S}_P^{-1}\mathbf{U}_P^T\mathbf{Y}_0\mathbf{V}_P$.

The MP technique exploits the matrix pencil structure of the underlying signal, rather than the prediction equations satisfied by it. Nevertheless, there are strong similarities between the MP and LPSVD methods.

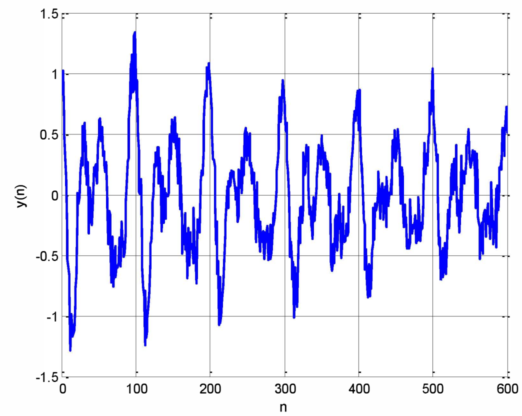
III. Algorithm Tests and Model Order Estimation

A. Simulated Signals

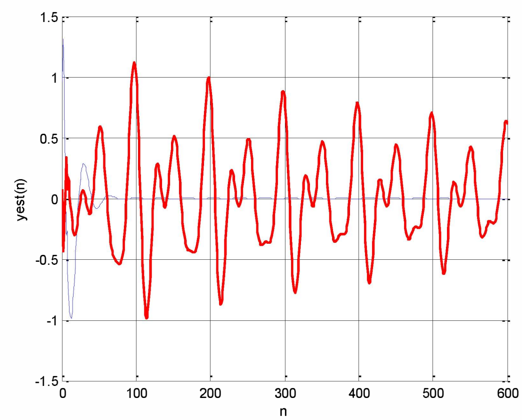
In this section, performance of three previously described signal modeling algorithms is demonstrated using artificial – computer generated resonant signals with a number of resonant modes and various levels of noise. Analysis results for the signal sampled at 100Hz, containing three resonant modes shown in Fig. 1a) have been used to illustrate the process. All three modes present in the signal are stable, i.e. three related damping coefficients are positive. Analysis was performed on a noise-free signal (SNR=∞) and on signals containing various levels of noise (SNR=80 dB, SNR=10 dB). Reconstructed and residual signals using each algorithm are shown in Fig. 1b) – 1d). Powers of residual signals shown on these plots are: 0.1639 (EP), 0.0013 (LPSVD) and 0.000537 (MP) indicating inferior performance of the EP algorithm and its sensitivity to noise compared to other two methods – LPSVD and MP in particular.

B. Measured Signals and Model Order Estimation

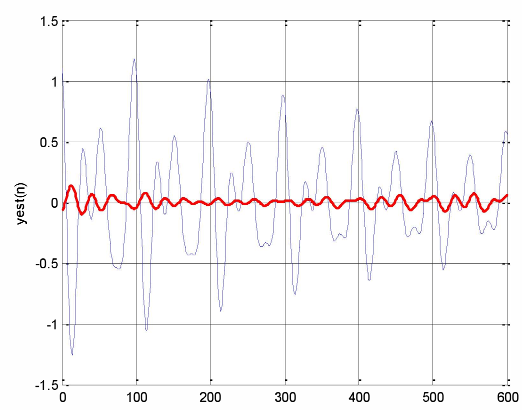
Another important issue arising when performing the analysis of the real, PMU signals measured in the system is that of model order estimation. Unlike the analysis process undertaken in the previous section, where the model order of simulated signal was known a priori, for most real signals, measured in the grid, model order is unknown. Although experienced user may have some idea of the inherent model order for particular signals, if the selected order is too low, some modes will not be estimated, while too high order specified for the analysis might introduce superfluous components not present in the original signal. The model order selection is therefore a tradeoff between increased resolution and decreased variance in the estimated spectrum. There are several methods and criteria to estimate model order described in literature although guidelines on the usage of some of those criteria in practical situations are rather limited.



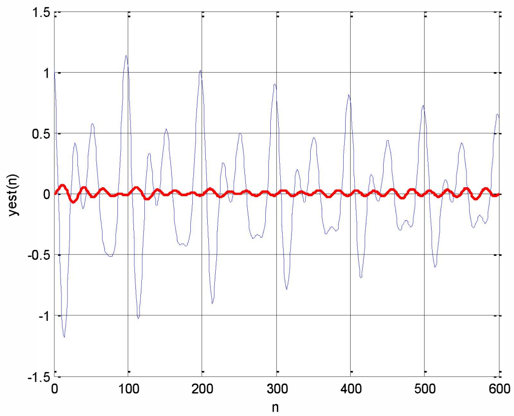
(a)



(b)



(c)



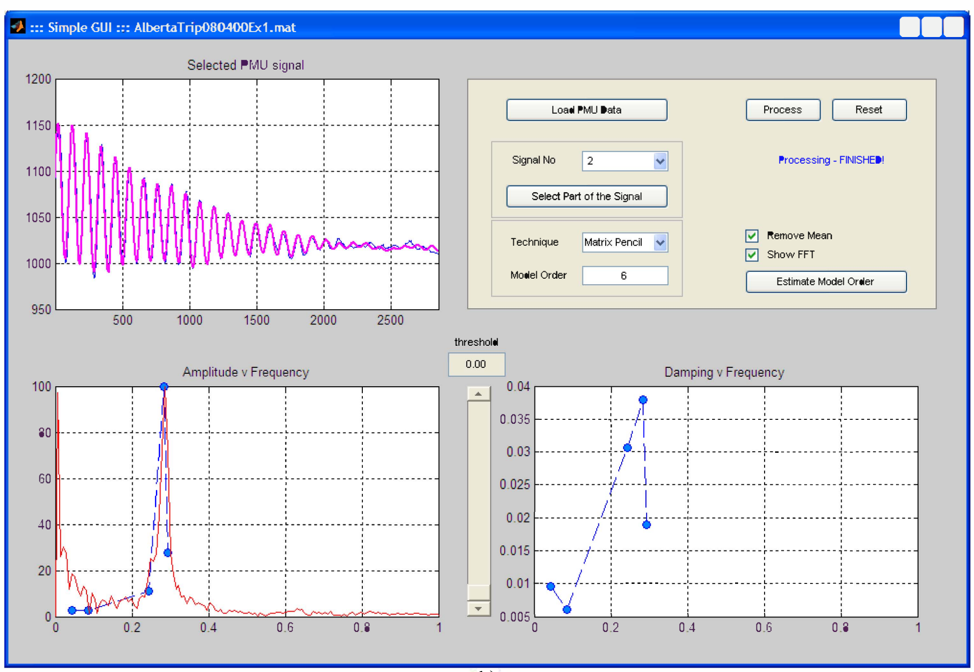
(d)

Figure 1. Performance of studied algorithms for very noisy signal (SNR = 10dB), a) original signal, b) EP, c) LPSVD and d) MP reconstructed signals and residuals

Some of the better known criteria investigated in recent decades include Akaike Information Criterion [5] and Minimum Descriptive Length [6] criterion but both of those criteria are known to perform poorly for closely spaced modes [5]. In this work two simple approaches for model order estimation – minimization of the residue signal power and the singular entropy technique [7] have been used and investigated. In the first method, power of the residue signal – error between the produced model and the measured signal is estimated for various model orders and the order resulting in minimum residue power adopted. As an additional check, FFT based analysis of the PMU signal was also performed and the power spectrum of the measured signal obtained and compared to that of the signal estimated using adopted model order. Singular entropy approach relies on the position of the inflection point in the singular entropy curve obtained via Singular Value Decomposition (SVD) of the PMU signal Hankel matrix. Briefly, following the SVD of the signal Henkel matrix, resulting matrix of singular values D, is used to obtain the singular entropy increment curve. When the effective signal saturates, the singular entropy increment converges to a bounded value rapidly forming an inflection point on the curve which corresponds to the signal modal order. With the help of differential of singular entropy increment, it is then easy to extract the order of the corresponding inflection point, namely, the modal order *n*.

IV. MATLAB GUI

To enable easy testing and evaluation of described algorithms for the extraction of poles from the measured PMU signals, next step taken in this project was to design a simple graphical user interface (GUI) based Matlab program. Another objective for the program development was to facilitate further research into this topic by enabling easy addition of new algorithms and comparison with the techniques described in this work. Finally, GUI can present a first step towards the application of developed methods in practice and allow near-real-time analysis of the recently obtained PMU signals. Main functions available in this program at the moment include: loading and display of the simulated or measured set of PMU data, selection of the oscillatory signal from the set, extraction of the signal segment to be analysed, choice of analysis technique (EP, LPSVD or MP at the moment), application of various pre-processing and post-processing steps as well as model order estimation (residual power or singular entropy techniques).



b)

Figure 2. Basic layout of developed GUI containing the PMU signal with the indicated section of the signal and estimated model (top left) as well as the results of the analysis (bottom of the GUI)

Fig. 2 shows the basic layout of developed GUI with a selected segment of the PMU signal in the top left panel of the GUI as well as initial results of the analysis. Reconstructed signal is in this figure plotted over the original PMU signal (top left panel). Overlap between the two signals is high indicating accurate analysis of the original resonant PMU signal. Comparison between the results of the employed estimation method (MP in Fig. 1) and the FFT analysis has also been indicated in this figure (bottom left panel). While the agreement is very good for midd-frequencies, there seems to be some disagreement in the very-low frequency range due to applied mean-removal pre-processing step before MP modeling is applied to signal.

V. CONCLUSION

In this work three related but different methods for the analysis of oscillatory signals measured in power grid have been discussed, implemented and tested. Performance of each technique has been initially evaluated using simulated signals containing various amounts of noise in order to assess the influence of noise on its performance. Following this preliminary investigation step, it was established that the Extended Prony Method performs poorly even when a moderate amount of noise is present in the signal while both LPSVD and Matrix Pencil techniques perform much better on noisy signals and are able to extract all signal parameters – frequency, amplitude and damping with satisfactory accuracy in most situations. Those two techniques are then used for the analysis of real – PMU measured grid signals.

One of frequently encountered problems in signal modeling and analysis tasks is the accurate estimation of model order. In this work, this problem has been addressed in two different ways. Differential singular entropy increment and minimal residual power characteristics have been calculated for each signal and the model order estimated by the visual inspection of those characteristics. For low model orders it is hard to accurately obtain the correct order from the residual power characteristics but differential singular entropy curve can be used in those cases yielding accurate results which are also in agreement with traditional FFT based frequency techniques as no additional peaks revealing undetected modes could be detected in the FFT estimated power spectrum results. The problem of detecting and tracking oscillation modes is however persistent in power grids and further investigation into alternative techniques might be needed in order to enable near-real time tracking and estimation of oscillation modes.

Finally, to test and evaluate described techniques a Matlab based GUI program has been developed and described in the conclusion of this paper. This enabled easier evaluation of all described algorithms for signal modeling and visualization of relevant oscillatory parameters – frequency and damping and modeled signal. Described model order estimation techniques have also been implemented in the same GUI and their performance evaluated.

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