Spectral Estimation of Distorted Signals Using Prony Method

Tadeusz Łobos, Jacek Rezmer

Abstract — **Modern frequency power converters generate a wide spectrum of harmonic components. Large converter systems can also generate non-characteristic harmonics and interharmonics. Standard tools of harmonic analysis based on the Fourier transform assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable and very long. In the case of frequency converters the periods of the main component are unknown. The Prony method as applied for signal analysis in power frequency converter was tested in the paper. The method does not show the disadvantages of the traditional tools and allow exact estimation the frequencies of all or dominant components, even when the periodicity intervals are unknown. To investigate the appropriateness of the method several experiments were performed. For comparison, similar experiments were repeated using the FFT. The comparison proved the superiority of the Prony method.**

I. Introduction

The quality of voltage waveforms is nowadays an issue of the utmost importance for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The liberalization of European energy market will strengthen the competition and is expected to drive down the energy prices. This is reason for the requirements concerning the power quality. The voltage waveform is expected to be a pure sinusoidal with a given frequency and amplitude. Modern frequency power converters generate a wide spectrum of harmonic components witch deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. In some cases, large converters systems generate not only characteristic harmonics typical for the ideal converter operation, but also considerable amount of non-characteristic harmonics and interharmonics which may strongly deteriorate the quality of the power supply voltage [1]. Interharmonics are defined as non-integer harmonics of the main fundamental under consideration. The estimation of the components is very important for control and protection tasks. The design of harmonics filters relies on the measurement of distortions in both current and voltage waveforms.

There are many different approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, SVD, higher-order spectra, etc [2,3,4,5]. Most of them operate adequately only in the narrow range of frequencies and at moderate noise levels. The linear methods of system spectrum estimation (Blackman-Tukey), based on the Fourier transform, suffer from the major problem of resolution. Because of some invalid assumptions (zero data or repetitive data outside the duration of observation) made in these methods, the estimated spectrum can be a smeared version of the true spectrum [6, 7].

These methods usually assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable and very long [1]. It is very important to develop better tools of parameter estimation of signal frequency components.

In the case of power frequency converter the periodicity intervals are unknown. Identification of some power converter faults can be a difficult task, especially in under-load- conditions. Different faults cause specific additional distortions of voltage and current waveforms. Detection of additional frequency components can be used for fault identification.

In this paper the frequencies of signal components are estimated using the Prony model. Prony method is a technique for modelling sampled data as a linear combination of exponentials. Although it is not a spectral estimation technique, Prony method has a close relationship to the least squares linear prediction algorithms used for AR and ARMA parameter estimation. Prony method seeks to fit a deterministic exponential model to the data in contrast to AR and ARMA methods that seek to fit a random model to the second-order data statistics.

II. Prony Method

Prony method is a technique for extracting sinusoid or exponential signals from time series data, by solving a set of linear equations.

Assuming the N complex data samples $x[1],\dots,x[N]$ the investigated function can be approximated by M exponential functions:

$$y[n]=\sum_{k=1}^{M} A_k e^{(\alpha_k+j\omega_k)(n-1)T_p+j\psi_k} \tag{1}$$

where $n=1,2,\dots,N$

T_p – sampling period,

A_k - amplitude

α_k – damping factor,

ω_k – angular velocity

ψ_k – initial phase.

Authors are with the Institute of Electrical Engineering Fundamentals, Wroclaw University of Technology, 50-370 Wroclaw Poland, e-mail: lobos@ipee.pwr.wroc.pl

The discrete-time function may be concisely expressed in the form

$$y[n]=\sum_{k=1}^{M} h_k z_k^{n-1} \tag{2}$$

where $h_k = A_k e^{j\psi_k}$; $z_k = e^{(\alpha_k + j\omega_k)T_p}$

The estimation problem bases on the minimization of the squared error over the N data values:

$$\delta = \sum_{n=1}^{N} |\varepsilon[n]|^2 \tag{3}$$

where $\varepsilon[n] = x[n] - y[n] = x[n] - \sum_{k=1}^{M} h_k z_k^{n-1}$ (4)

This turns out to be a difficult nonlinear problem. It can be solved using the Prony method that utilizes linear equation solutions.

If as many data samples are used as there are exponential parameters, then an exact exponential fit to the data may be made.

Consider the M-exponent discrete-time function:

$$x[n]=\sum_{k=1}^{M} h_k z_k^{n-1} \tag{5}$$

The M equations of (5) may be expressed in matrix from as:

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_M^0 \\ z_1^1 & z_2^1 & \cdots & z_M^1 \\ \vdots & \vdots & & \vdots \\ z_1^{M-1} & z_2^{M-1} & \cdots & z_M^{M-1} \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[M] \end{bmatrix} \tag{6}$$

The matrix equation represents a set of linear equations that can be solved for the unknown vector of amplitudes.

Prony proposed to define the polynomial that has the z_k exponents as its roots:

$$\begin{aligned} F(z) &= \prod_{k=1}^{M} (z - z_k) = \\ &= (z-z_1)(z-z_2)\ldots(z-z_M) \end{aligned} \tag{7}$$

The polynomial may be represented as the sum:

$$\begin{aligned} F(z) &= \sum_{m=0}^{M} a[m] z^{M-m} = \\ &= a[0]z^M + a[1]z^{M-1} + \ldots + a[M-1]z + a[M] \end{aligned} \tag{8}$$

Shifting the index on (5) from n to n-m and multiplying by the parameter $a[m]$ yield:

$$a[m]x[n-m] = a[m]\sum_{k=1}^{M} h_k z_k^{n-m-1} \tag{9}$$

The (9) can be modified into:

$$\begin{aligned} &\sum_{m=0}^{M} a[m]x[n-m] = \\ &= \sum_{k=1}^{M} h_k z_k^{n-M} \left\{ \sum_{m=0}^{M} a[m] z_k^{M-m-1} \right\} \end{aligned} \tag{10}$$

The right-hand summation in (10) may be recognize as polynomial defined by (8), evaluated at each of its roots z_k yielding the zero result:

$$\sum_{m=0}^{M} a[m]x[n-m] = 0 \tag{11}$$

The equation can be solved for the polynomial coefficients. In the second step the roots of the polynomial defined by (8) can be calculated. The damping factors and sinusoidal frequencies may be determined from the roots z_k.

For practical situations, the number of data points N usually exceeds the minimum number needed to fit a model of exponentials, i.e. $N > 2M$. In the overdetermined data case, the linear equation (11) must be modified to:

$$\sum_{m=0}^{M} a[m]x[n-m] = e[n] \tag{12}$$

The estimation problem bases on the minimization of the total squared error:

$$E = \sum_{n=M+1}^{N} |e[n]|^2 \tag{13}$$

When estimating the parameters of the signal components, in the case of heavy distorted waveforms, we identify apart from some sinusoidal components, also some components with relativ great damping factors. In reality they do not exist, but are caused by noise and computation errors. To eliminate the components we divide the estimated amplitudes by damping factors and choose the components with the biggest results of the division.

The investigations have been carried out using also the modified Prony algorithm [8], which maximises the likelihood of the fitting problem. For different numbers of the estimated exponential components M *(5...N/2, changed by step of 5)* the square error of the approximation was calculated. As optimal was chosen the number which assures the minimal error. From the calculated components, the components with the smallest relation A/α has been neglected.

III. INDUSTRIAL FREQUENCY CONVERTER

The investigated drive represents a typical configuration of industrial drives, consisting of a three-phase asynchronous motor and a power converter, composed of a single-phase half-controlled bridge rectifier and a voltage source converter (Fig. 1) [10].

The currents waveforms at the converter input (I_input), at the converter output (I_output)as well as in the intermediate circuit (I_intermed), during normal conditions and under fault conditions (capacitor or switch failure) were investigated using the Prony and FFT methods. The main frequency of the waveform at the converter input was *50 Hz*, and at the converter output *40 Hz*.

Figs 2, 3 and 4 show the current waveforms in the intermediate circuit (I_intermed). Under fault free nor-

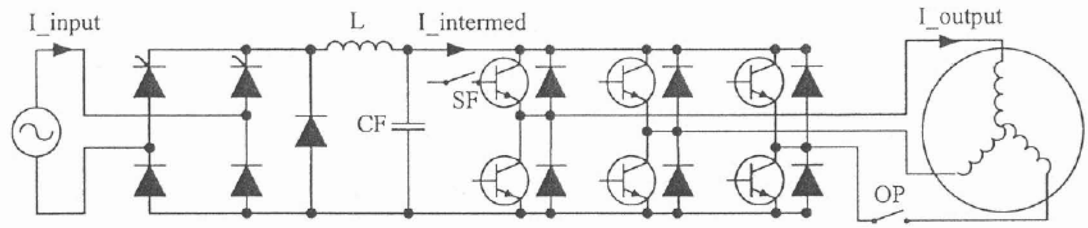

Fig. 1 Industrial converter drive

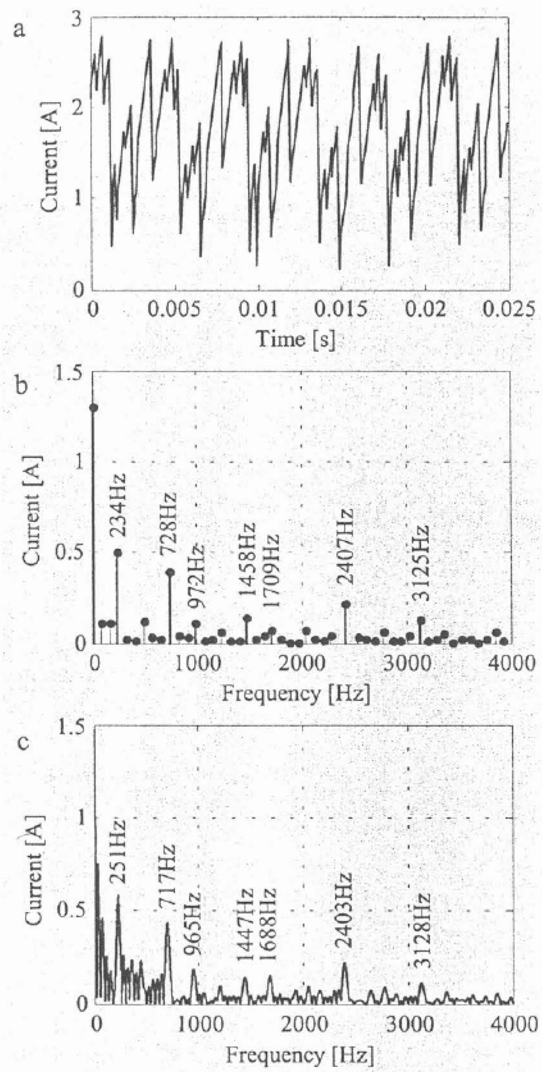

Fig. 2. Current in the intermediate circuit (I_intermed), under normal conditions (a), investigation results: Prony N=250, M=120 (b); FFT N=250 (c), f_p=10 kHz

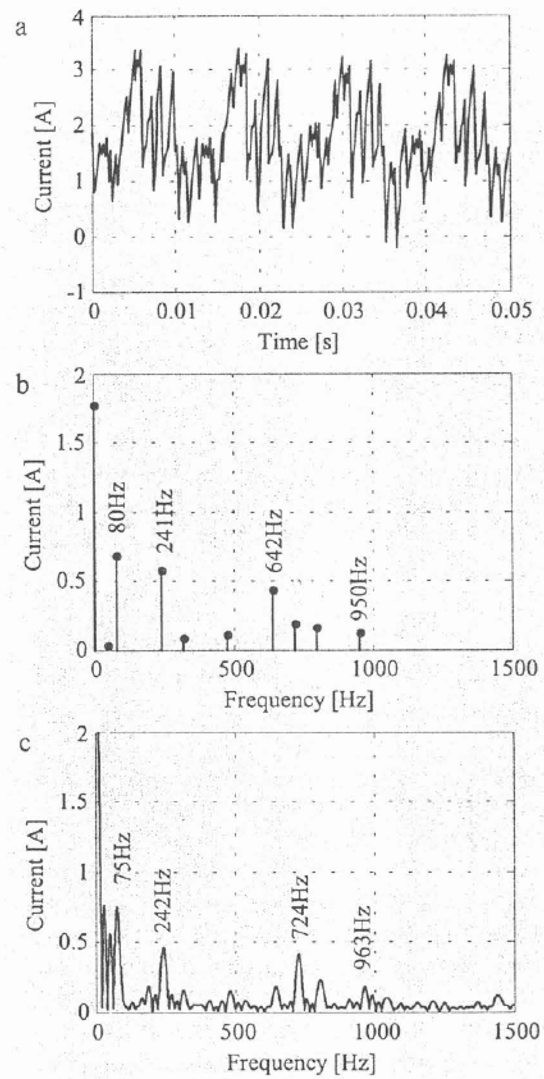

Fig. 3. Current in the intermediate circuit (I_intermed), after capacitor failure (a), investigation results: Prony N=250, M=105 (b); FFT N=250 (c), f_p=5 kHz

mal conditions (Fig. 2), apart from dc component, the following frequencies have been detected, when applying the sampling window equal to 25 ms.,: *ca. 240, 720, 970, 1460, 1700, 2400, 3100 Hz.*

Capacitor or switch failure changes significantly the current waveform in the intermediate circuit (Figs. 3, 4, 5). Figs. 3 and 5 show the investigation results for the sampling window equal to 50 ms. For comparison, Fig. 4 shows the results for the sampling window equal to 30 ms. In the first case the results are more exact. Additional frequency component have been detected. In the case of capacitor failure (Fig. 3) 80 and 640 Hz and in the case of switch failure (Fig. 5) 40 Hz. Detection of the component indicates a fault.

IV. CONCLUSIONS

It has been shown that a high-resolution spectrum estimation method, such as Prony method could be effectively used for estimation of the frequencies of signal components. The accuracy of the estimation depends on the signal distortion, the sampling window and on number of samples taken into the estimation process.

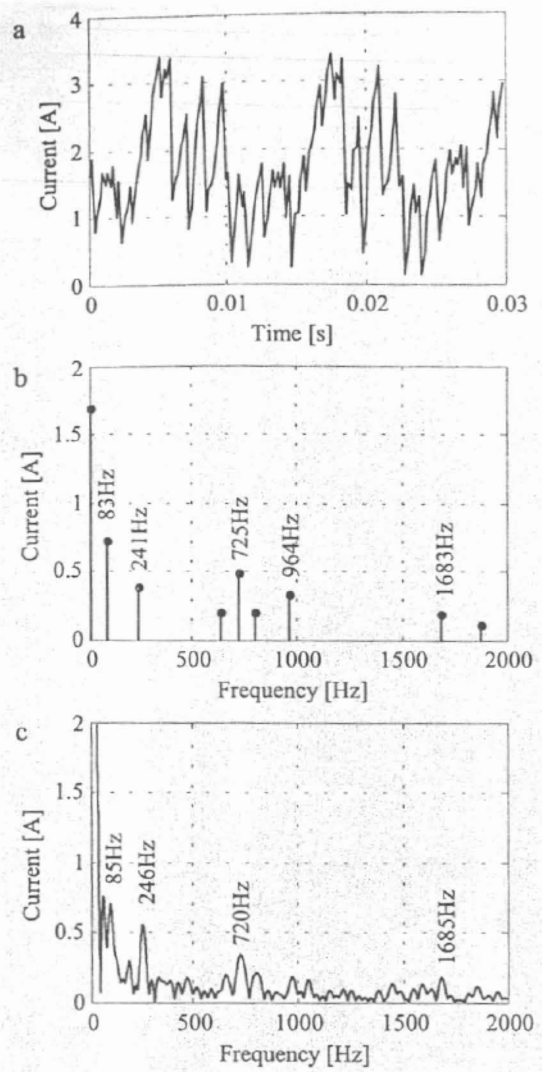

Fig. 4. Current in the intermediate circuit (I_intermed), after capacitor failure (a), investigation results: Prony N=150, M=75 (b); FFT N=150 (c), f_p=5 kHz

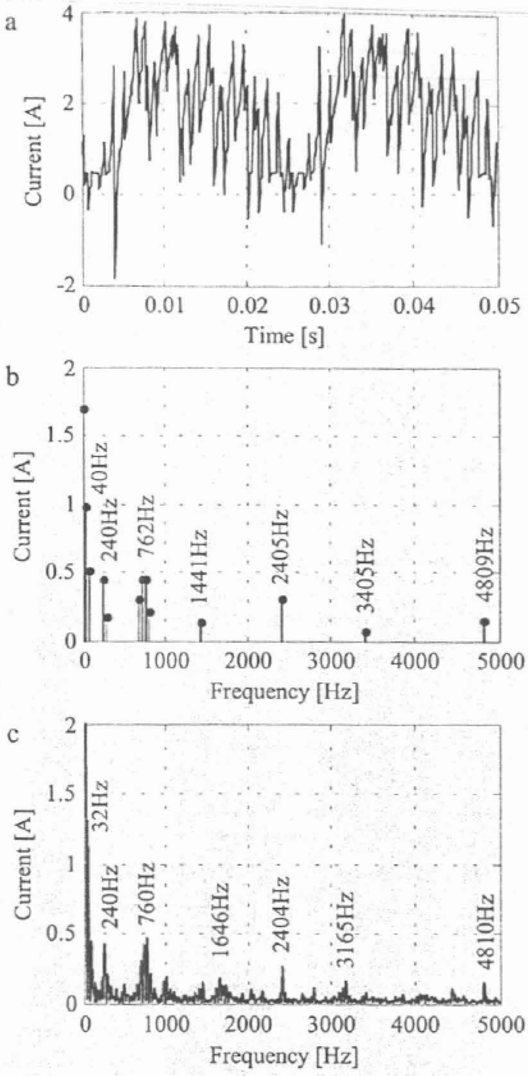

Fig. 5. Current in the intermediate circuit (I_intermed), after switch failure (a), investigation results: Prony N=500, M=250 (b); FFT N=500 (c), f_p=10 kHz

The proposed method was investigated under different conditions and found to be a variable and efficient tool for detection of all higher harmonic existing in a signal. It also makes it possible the estimation of frequencies of interharmonics.

Detection of some non-characteristic signal components can be used for identification of frequency converter faults.

V. REFERENCES

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