Data Science HW 1 — Sanat Konda

1) $P(A) = 0.5$, $P(B) = 0.2$, $P(C) = 0.1$, $P(\overline{A \cup B}) = 0.45$

a) $P(A \cup B) = 1 - P(\overline{A \cup B})$

$= 1 - 0.45$

$= \boxed{0.55}$

b) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.5 + 0.2 - 0.55$

$= \boxed{0.15}$

c) $P(A) - P(A \cap B) = 0.5 - 0.15 = \boxed{0.35}$

d) NO, they are not mutally exclusive

2) a) $p = 1/6$, $q = 5/6$

$P(x \geq 1) = 1 - P(x = 0)$

$= 1 - \binom{4}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{4-0}$

$= 1 - \left(\frac{5}{6}\right)^4$

$= \boxed{0.5177}$

b) $P(x \geq 1) = 1 - P(x = 0)$

$= 1 - \binom{20}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{20-0}$

$= 1 - \left(\frac{5}{6}\right)^{20}$

$= \boxed{0.9739}$

c) $P(x \geq 1) = 1 - \binom{n}{0}\left(\frac{1}{6}\right)^n\left(\frac{5}{6}\right)^{n-0}$

$0.9 = 1 - \left(\frac{5}{6}\right)^n$

$\left(\frac{5}{6}\right)^n = 0.1$

$\boxed{n = 12.629 \approx 13 \text{ rolls}}$

3) $3p + 3(2p) = 1$

$p = \frac{1}{9}$

$P(1) + P(2) + P(3) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \boxed{\frac{4}{9}}$

4) Total: $3 + 2 + 2 + 3 = 10$

$\binom{10}{3} = \boxed{120 \text{ ways}}$

b) $\binom{3}{1}\binom{3}{1}\binom{2}{1} = 18$ ways

$\frac{18}{120} = \boxed{0.15}$

5) a) $P(A) = P(B) = P(C) = 1/3$

$P(\text{defective}) = 0.002(P(A)) + 0.02(P(B)) + 0.001(P(C))$

$= 0.002(1/3) + 0.02(1/3) + 0.001(1/3)$

$= \boxed{0.00767}$

b) $P(A|\text{defective}) = \frac{0.002(1/3)}{0.00767} = \boxed{0.087}$

$P(B|\text{defective}) = \frac{0.02(1/3)}{0.00767} = \boxed{0.872}$

$P(C|\text{defective}) = \frac{0.001(1/3)}{0.00767} = \boxed{0.043}$

c) $P(A) = 0.5$, $P(B) = 0.1$, $P(C) = 0.4$

$P(D) = 0.002(0.5) + 0.02(0.1) + 0.001(0.4) = \boxed{0.003}$

d) $P(A|D) = \frac{0.002(0.5)}{0.003} = \boxed{0.333}$

$P(B|D) = \frac{0.02(0.1)}{0.003} = \boxed{0.667}$

$P(C|D) = \frac{0.001(0.4)}{0.003} = \boxed{0.133}$

6) a) $P(\text{Ace}) = \frac{4}{52} = \boxed{\frac{1}{13}}$

b) $P(\text{Jack of spades}) = \boxed{\frac{1}{52}}$

c) $P(\text{JS or 6D}) = \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \boxed{\frac{1}{26}}$

d) $P(\overline{\text{S or H}}) = \frac{26}{52} = \boxed{\frac{1}{2}}$

7) a) $P(H_1) = \frac{13}{52} = \boxed{\frac{1}{4}}$

b) $P(H_2) = P(H_2|H_1)P(H_1) + P(H_2|S_1)P(S_1) + P(H_2|D_1)P(D_1) + P(H_2|C_1)P(C_1)$

$P(H_2) = \frac{12}{51}\left(\frac{1}{4}\right) + \frac{13}{51}\left(\frac{1}{4}\right) + \frac{13}{51}\left(\frac{1}{4}\right) + \frac{13}{51}\left(\frac{1}{4}\right) = \frac{51}{204} = \boxed{\frac{1}{4}}$