A block weighing 600 N has been subjected to the load system as shown in Fig. 4.12. Work-out the frictional force for the block and state whether it is in equilibrium or in motion. What additional force needs to be added to 120 N force so that the block just moves to the left?

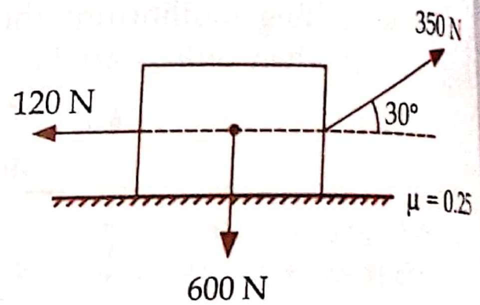

Fig. 4.12

Solution : In the limiting condition, the forces are balanced. That is

$\Sigma F_x = 0$; $-120 + 350 \cos 30 - F = 0$

where F is the frictional force

$F = -120 + 350 \times 0.866 = 183.1$ N

$\Sigma F_y = 0$; $R + 350 \sin 30° - 600 = 0$

where R is the normal reaction

$R = 600 - 350 \times 0.5 = 425$ N

Maximum value of frictional force is given by

$$F_{max} = \mu \times R = 0.25 \times 425 = 106.25 \text{ N}$$

Since $F_{max} < F$, the block moves towards right and friction force = 106.25 N acts towards left.

(*b*) Let P be the force which needs to be added to 120 N force to just move the block to the left. When the block moves towards left, the friction force will act towards right. Then from the relation $\Sigma F_x = 0$,

$$-(120 + P) + 350 \cos 30° + F_{max} = 0$$

$$-(120 + P) + 350 \times 0.866 + 106.25 = 0$$

$\therefore$ $$P = -120 + 350 \times 0.866 + 106.25 = \mathbf{289.35\ N}$$

Two blocks A and B weighing 50 kg and 80 kg respectively are in equilibrium in the position shown in Fig. 4.16. Calculate the force P required to move the lower block B and tension in the cable. Take coefficient of friction at all contact surfaces to be 0.3.

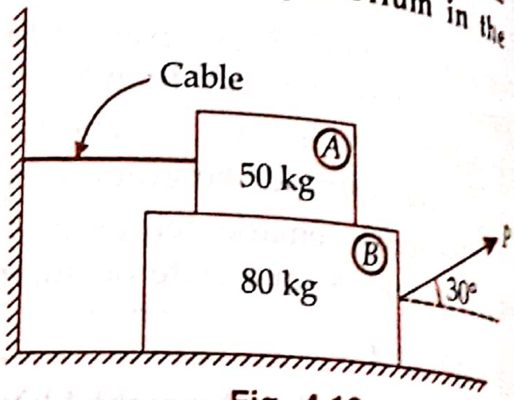

Fig. 4.16

Solution : Weight of block $A = 50 \times 9.81$

$= 490.5$ N

Weight of block $B = 80 \times 9.81$

$= 784.8$ N

As the block B tends to move towards right, there will be rubbing action between the surfaces of blocks A and B. The rubbing action will cause the force of friction to act between the two surfaces. The block A is tied to a cable, the other end of which is fixed to a vertical plane. When the block B tends to move towards right, the block A moves towards left with respect to block B. Hence the force of friction $F = \mu R_1$ on the lower surface of block A will act towards right as shown in free body diagram of block A (Fig. 4.17)

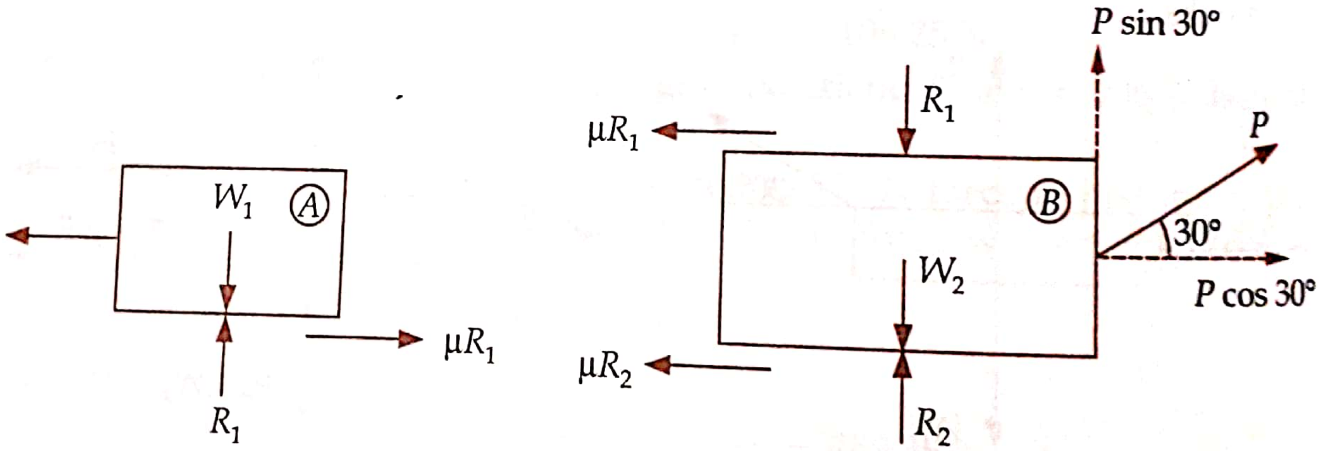

Fig. 4.17

For equilibrium of block A,

$$\Sigma F_x = 0; \quad -T + \mu R_1 = 0; \quad T = \mu R_1 = 0.3R_1$$

$$\Sigma F_y = 0; \quad R_1 - W_1 = 0; \quad R_1 = W_1 = 490.5 \text{ N}$$

$\therefore$ Tension in the cable $T = 0.3R_1 = 0.3 \times 490.5 = 147.15$ N

Considering free body diagram of block B (Fig. 4.17),

$$\Sigma F_x = 0; \quad -\mu R_1 - \mu R_2 + P \cos 30° = 0$$

or $$-0.3 \times 490.5 - 0.3R_2 + 0.866\,P = 0 \quad \text{...(i)}$$

$$\Sigma F_y = 0; \quad R_2 - W_2 - R_1 + P \sin 30° = 0$$

or $$R_2 - 784.8 - 490.5 + 0.5P = 0$$

or $$R_2 = 1275.3 - 0.5P$$

Then from identity (i)

$$-0.3 \times 490.5 - 0.3(1275.3 - 0.5P) + 0.866P = 0$$

or $$-147.15 - 382.59 + 0.15P + 0.866P = 0$$

$\therefore$ Force required to move the lower block,

$$P = \frac{382.59 + 147.15}{0.15 + 0.866} = \mathbf{521.4\ N}$$

EXAMPLE 4.11

Two blocks A and B weighing 50 N and 80 N respectively are positioned as shown in Fig. 4.18. The coefficient of friction between ground and block B is 0.1 and that between block B and block A is 0.28. State whether B is stationary with respect to ground and A moves or B is stationary with respect to A. Proceed to determine the minimum value of weight W in the pan so that motion starts.

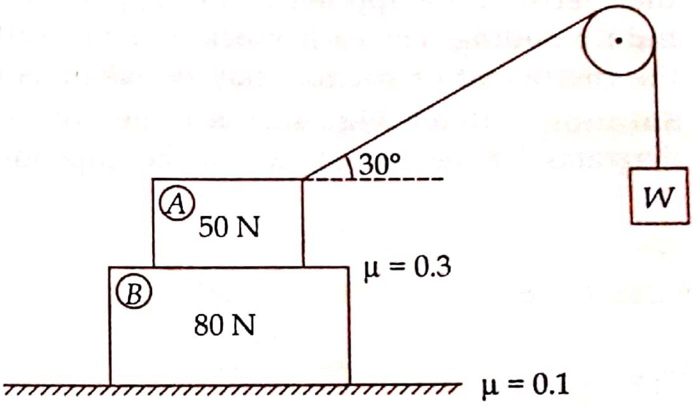

Fig. 4.18

Solution : The coefficient of friction between the block B and ground ($\mu = 0.1$) is much less than that between the blocks A and B ($\mu = 0.28$). Obviously when weight W is put in the pan, motion will start of both the blocks together. As such the block B will be stationary with respect to block A.

With reference to free body diagram of the whole system (Fig. 4.19), the conditions for limiting equilibrium are :

$$\Sigma F_x = 0 = -0.1\,R + W\cos 30^\circ$$

$$\therefore \quad R = \frac{W\cos 30^\circ}{0.1} = 8.66\,W$$

$$\Sigma F_y = 0 = R - (50 + 80) + W\sin 30^\circ$$
$$= 8.66\,W - 130 + 0.5\,W$$

$$\therefore \quad W = \frac{130}{8.66 + 0.5} = \mathbf{14.21\ N}$$

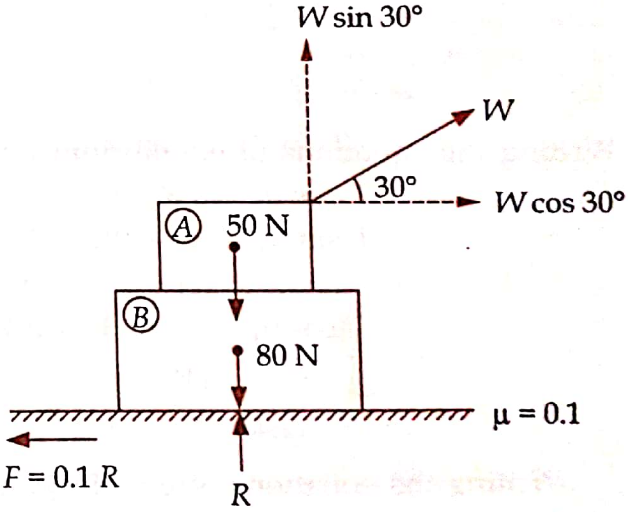

Fig. 4.19

This is the minimum weight in the pan required to start the motion.

(b) When the weight in the pan exceeds beyond a certain limit, the block A may move with respect to B. The block B will then remain stationary with respect to ground.

Considering free body diagram of block A, the conditions of limiting equilibrium are :

$$\Sigma F_x = 0 = W\cos 30^\circ - 0.3R$$

$$R = \frac{W\cos 30^\circ}{0.3} = 2.887\,W$$

$$\Sigma F_y = 0 = R - 50 + W\sin 30^\circ$$

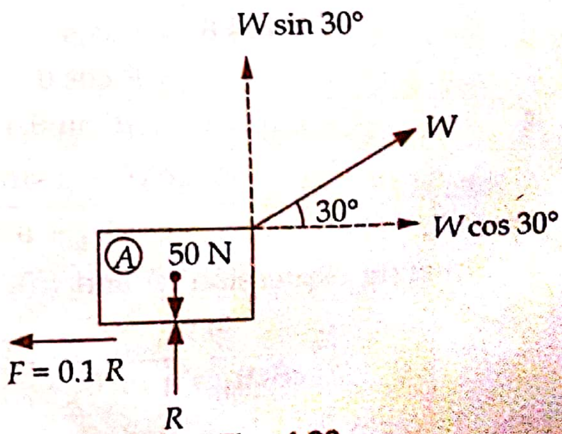

Fig. 4.20

$$= 2.887\,W - 50 + 0.5\,W$$

$$\therefore \quad W = \frac{50}{2.887 + 0.5} = \mathbf{14.76\ N}$$

EXAM...

A block, in the shape of a rectangular prism rests on a rough inclined plane, as shown in Fig. 4.37. The block is tied up by a horizontal string which has a tension of 10 N. If the block weighs 35 N, determine:

(i) the frictional force on the block,

(ii) the normal reaction of the inclined plane, and

(iii) the coefficient of friction between the contacting surfaces.

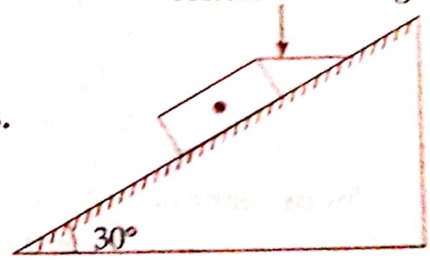

Fig. 4.37

Solution : The various forces acting on the block are:

(i) weight W = 35 N of the block,

(ii) tension T = 10 N in the string,

(iii) normal reaction R at the contact surface, and

(iv) friction force $F = \mu R$

Refer to Fig. 4.38 for the free body diagram of the force system under equilibrium conditions,

$$\Sigma F_x = 0 \text{ (along the plane)}$$

$$\mu R - W \sin 30° + T \cos 30° = 0$$

$$\mu R = W \sin 30° - T \cos 30°$$

$$= 35 \sin 30° - 10 \cos 30° = 35 \times 0.5 - 10 \times 0.866$$

$$= 17.5 - 8.66 = 8.84 \text{ N} \quad \ldots(i)$$

$$\Sigma F_y = 0 \text{ (perpendicular to the plane)}$$

$$R - W \cos 30° - T \sin 30° = 0$$

$$R = W \cos 30° + T \sin 30° = 35 \times 0.866 + 10 \times 0.5$$

$$= 30.31 + 5 = 35.31 \text{ N} \quad \ldots(ii)$$

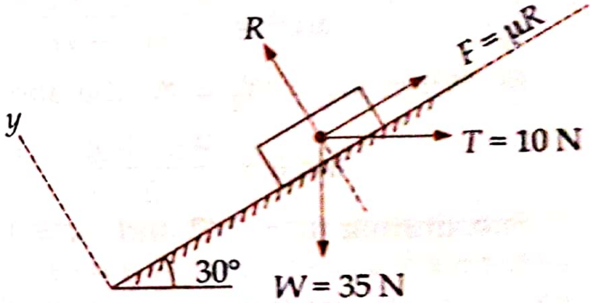

Fig. 4.38

From identities (i) and (ii)

$$\mu = \frac{8.84}{35.31} = 0.25$$

Two blocks of weight 50 N and 200 N and connected by a cord rest on two inclined planes as shown in Fig. 4.44. Determine the maximum tension in the cord when limiting friction conditions develop for both the blocks.

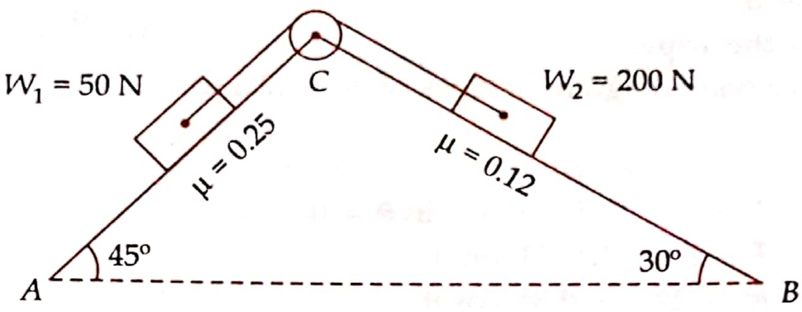

Fig. 4.44

Solution : Refer to Fig. 4.45 for the free body diagram of the two blocks

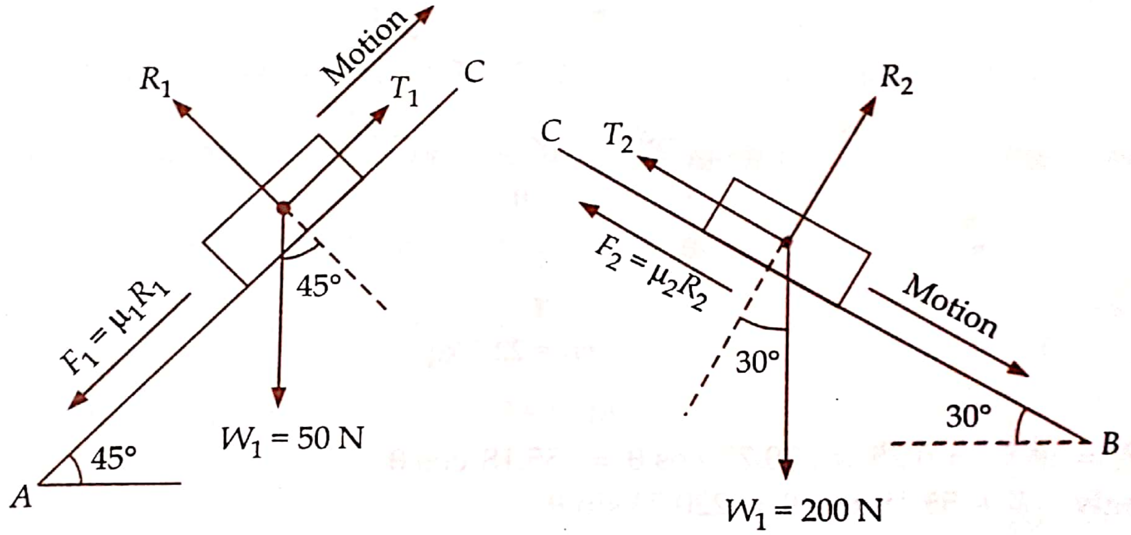

Fig. 4.45

Considering equilibrium of block resting on the plane of 45° inclination:

Resolving forces along and perpendicular to the plane *AC*

$$\mu_1 R_1 + W_1 \sin 45^\circ - T_1 = 0$$

$$R_1 - W_1 \cos 45^\circ = 0 \; ; \quad R_1 = 50 \cos 45^\circ = 35.35 \text{ N}$$

$$\therefore \quad T_1 = \mu R_1 + W_1 \sin 45^\circ = 0.25 \times 35.35 + 50 \sin 45^\circ$$

$$= 8.84 + 35.35 = \mathbf{44.19 \text{ N}}$$

Considering equilibrium of block resting on the plane of 30° inclination:

Resolving forces along and perpendicular to plane *BC*

$$T_1 + \mu_2 R_2 - W_2 \sin 30^\circ = 0$$

$$R_2 - W_2 \cos 30^\circ = 0 \; ; \quad R_2 = 200 \cos 30^\circ = 173.2 \text{ N}$$

$$\therefore \quad T_2 = W_2 \sin 30^\circ - \mu_2 R_2 = 200 \sin 30^\circ - 0.12 \times 173.2$$

$$= 100 - 20.78 = \mathbf{79.22} \text{ N}$$

Hence cord will be subjected to a maximum tension of **79.22 N**

EXAMPLE 4.27

A block A weighing W newtons is placed on a rough inclined plane having μ = 0.2 and is held in position by means of a horizontal rod hinged to the block B which presses against a rough vertical wall having μ = 0.4, as shown in Fig. 4.51. If the block B weighs 500 N, find the minimum value of W consistent with equilibrium.

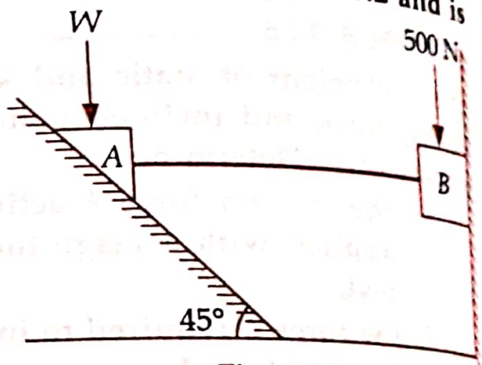

Fig. 4.51

Solution : When the block B just slides down, the frictional force $F = \mu R_b$ at the wall surface acts upwards. Under equilibrium conditions,

$$\Sigma F_x = 0 ; \qquad P - R_b = 0 ; \quad P = R_b$$

$$\Sigma F_y = 0 ; \qquad \mu R_b - 500 = 0$$

$$R_b = \frac{500}{0.4} = \mathbf{1250\ N}$$

Hence horizontal force P in the rod is 1250 N.

With block B sliding downwards, the block A has a tendency to move up the plane. The following relationship then holds good between W and P,

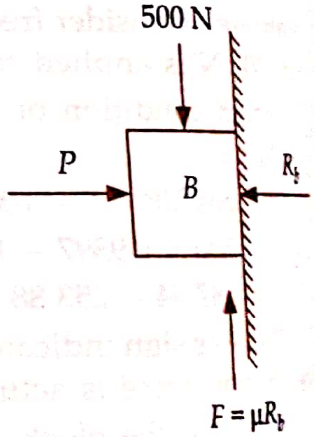

Fig. 4.52

$$P = W \tan(\alpha + \phi)$$

where α is the inclination of the plane and ϕ is the angle of friction for the block

$$\tan\phi = \mu = 0.2 ; \quad \phi = 11.30°$$

$$\therefore \quad 1250 = W \tan(45 + 11.30) = 1.5\,W$$

That gives :
$$W = \frac{1250}{1.5} = \mathbf{833.3\ N}$$

Thus the minimum value of W consistent with equilibrium is 833.3 N.

4.9. INCLINED PLANE

Fig. 4.69 shows an inclined plane AB making an angle α with the horizontal. A body of weight W is placed at point C and it is connected to an inextensible string which passes over a pulley positioned at B. An effort is applied at the free end of the string.

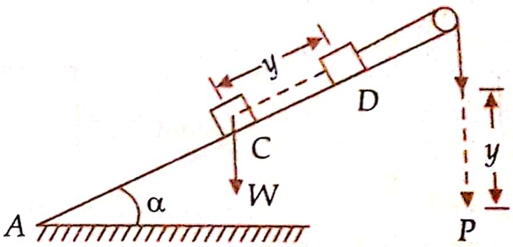

Fig. 4.69. Inclined plane

$$MA = \frac{W}{P}$$

Let the effort P move downward through a vertical distance y. Since the string is inextensible, the weight W would also move upward along the plane through the same distance y. The corresponding vertical movement of the load is $y \sin \alpha$

$$\therefore \qquad VR = \frac{\text{distance moved by the effort}}{\text{distance moved by the load}} = \frac{y}{y \sin \alpha} = \text{cosec } \alpha$$

EXAMPLE 4.36