1.7. SYSTEM OF FORCES

When several forces of different magnitude and direction act upon a body, they constitute a force system.

Considering the plane in which forces are applied and depending upon the position of line of action, forces may be classified as shown follows:

System of forces
- Coplanar forces
  - Concurrent
  - Parallel
    - Like
    - Unlike
  - Non-concurrent Non-parallel
- Non-coplanar or spatial forces
  - Concurrent
  - Parallel
  - Non-concurrent Non-parallel

- *Collinear forces:* The lines of action of all forces lie along the same straight line.

Fig. 1.8

Example: Force on a rope in a tug of war.

- *Coplanar parallel forces:* The lines of action of all forces are parallel to each other and lie in a single plane.

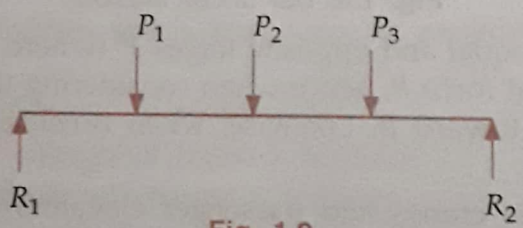


Fig. 1.9

Example: System of vertical loads (including reactions) acting on a beam.

- *Coplanar concurrent forces:* All forces lie in the same plane, have different directions but their lines of action act at one point (pass through a single point). The point where the lines of action of the forces meet is known as the point of concurrency of the force system.

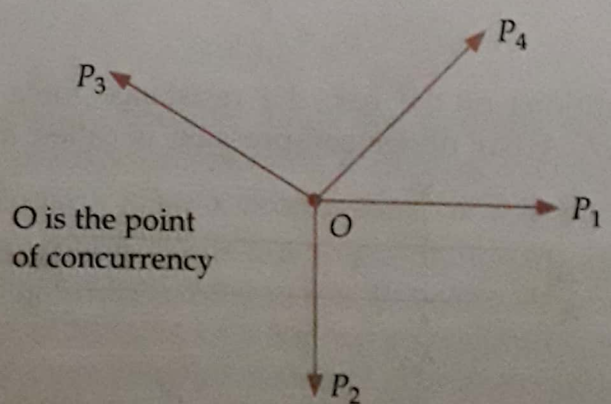


Fig. 1.10

Example: Forces on a rod resting against a wall.

- *Coplanar non-concurrent forces:* All forces lie in the same plane but their lines of action do not pass through a single point.

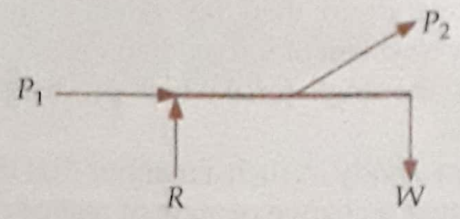


Fig. 1.11

Example: Forces on a ladder resting against wall and a person standing on a rung which is not at its centre of gravity.

- *Non-coplanar concurrent forces:* All forces do not lie in the same plane but their lines of action pass through a single point.

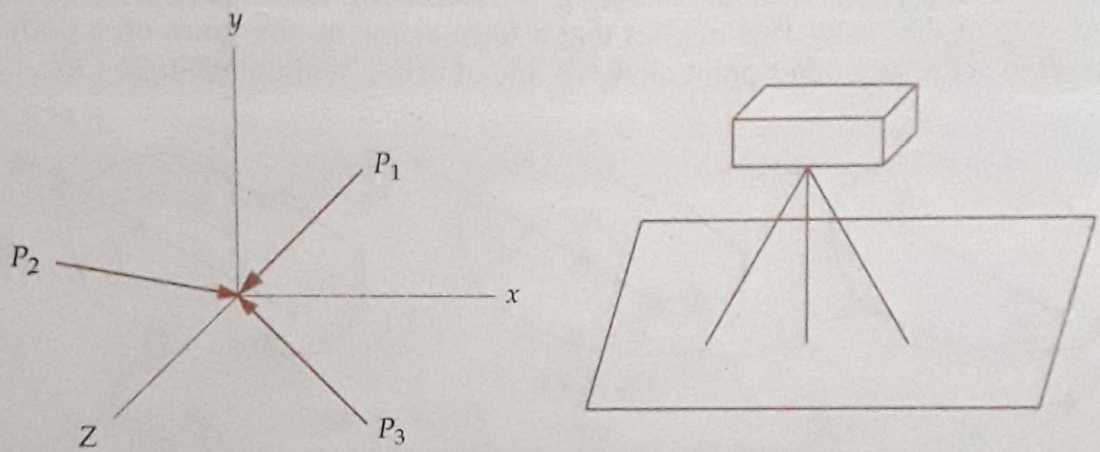


Fig. 1.12

Example: forces on a tripod carrying a camera.

- *Non-coplanar and non-concurrent forces:* All forces do not lie in the same plane, and their lines of action do not meet at a single point.

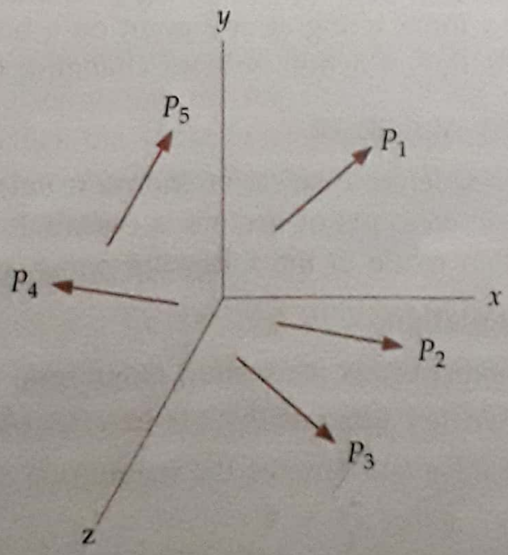


Fig. 1.13

Example: Forces acting on a moving bus.

1.8. EQUILIBRIUM, RESULTANT AND EQUILIBRANT

When two or more than two forces act on a body in such a way that the body remains in a state of rest or of uniform motion (no acceleration), then the system of forces is said to be in *equilibrium*.

When a body is acted upon by a system of forces, then vectorial sum of all the forces is known as *resultant*. Hence resultant refers to the single force which produces the same effect as is done by the combined effect of several forces.

A number of forces may act on a body in such a manner that the body is not in equilibrium. The resultant of several forces may cause a change of state of rest or of uniform motion. A single force may have to be applied to the body to bring it in equilibrium state. That single force is known as *equilibrant*. Equilibrant is equal and opposite to the resultant of several forces acting on the body.

1.9. PRINCIPLE OF TRANSMISSIBILITY

When the point of application of a force acting on a body is shifted to any other point on the line of action of the force without changing its direction, there occurs no change in the equilibrium state of the body. This implies that a force acting at any point on a body may also be considered to act at any other point along its line of action without changing its effect on the body.

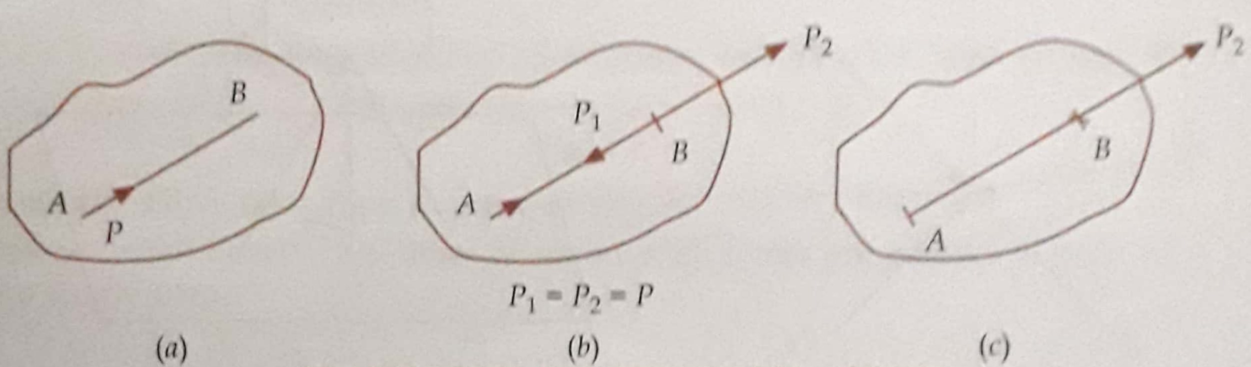


Fig 1.14. Principle of transmissibility of forces

Consider a force P acting at point A on rigid body (Fig 1.14). B is another point on the line of action of force P. At point B, apply two oppositely directed forces (P_1 and P_2) equal to and collinear with P. Such an application will in no way alter the action of given force P. At point A, forces P and P_1 are equal but opposite and accordingly cancel each other. That leaves a force $P_2 = P$ at B. This implies that a force acting at any point on a body may also be considered to act at any other point along its line of action without changing its effect on the body.

2.4. RESULTANT OF COPLANAR-CONCURRENT FORCES

2.4.1. Analytical Method (Principle of resolved parts)

The resultant of a number of a coplanar-concurrent forces acting on a body is worked out analytically by adopting the step-by-step procedure outlined blew:

(i) Find the components of each force in the system in two mutually perpendicular X and Y directions.

(ii) Make algebraic addition of components in each direction to get two components ΣF_x and ΣF_y.

(iii) Obtain the resultant both in magnitude and direction by combining the two component forces ΣF_x and ΣF_y which are mutnally prependicular.

$$\text{Resultant, } R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and its inclination θ to X-axis is given by

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

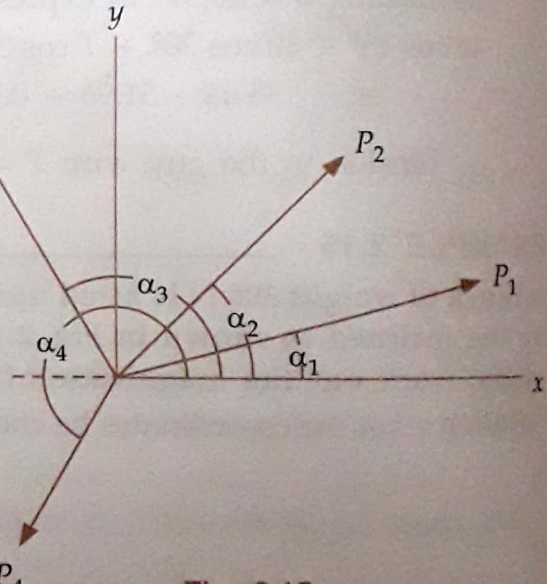


Fig. 2.15

This analytical method is based on theorem of resolved parts which states that *"the algebraic sum of the resolved parts of two forces in a given direction is equal to the resolved part of their resultant in the same direction"*.

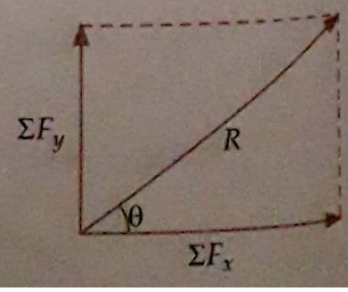
With reference to Fig. 2.15, P_1, P_2, P_3 and P_4 are the concurrent forces meeting at point O and making angles α_1, α_2, α_3 and α_4 with OX.

Resolving along x-axis and y-axis, we get

$$\Sigma F_x = P_1\cos\alpha_1 + P_2\cos\alpha_2 + P_3\cos\alpha_3 + P_4\cos\alpha_4$$

$$\Sigma F_y = P_1\sin\alpha_1 + P_2\sin\alpha_2 + P_3\sin\alpha_3 + P_4\sin\alpha_4$$

$$\text{Resultant, } R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$



and its inclination θ to x-axis is $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x}$

2.4.3. Equilibrium of a Particle

A particle will be in equilibrium when resultant of all the forces acting on it is zero

$$\text{Resultant} \quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

For a particle to be in equilibrium:

$$R = 0; \quad (\Sigma F_x)^2 + (\Sigma F_y)^2 = 0$$

Now $(\Sigma F_x)^2$ and $(\Sigma F_y)^2$ are positive quantities and their sum cannot be zero unless each of then is zero.

$$\therefore \qquad \Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0 \qquad \text{...(2.11)}$$

Hence, if any number of forces acting at a particle are in equilibrium, then the algebraic sum of their resolved parts in any two perpendicular directions are separately zero.

Equations 2.11 are called the equations of equilibrium of a particle in plane.

Conversely, if the sum of the resolved parts in any two perpendicular directions are separately zero, then the forces acting at any point are in equilibrium.

Equilibrium of a two-force and three-force body.

A **two-force body** is subjected to only two forces. For its equilibrium, the resultant force must be zero. Accordingly the two forces must be collinear and must have the same magnitude but opposite direction. These aspects have been shown below in Fig. 2.16 (*a*) and (*b*).

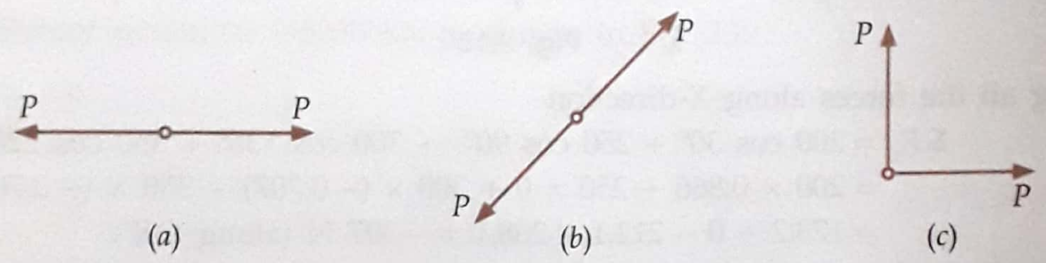


Fig. 2.16

The particle subjected to two forces of equal magnitude (Fig. 2.16 (*c*)) shall not be in equilibrium because the forces have different lines of action.

A **three-force body** is subjected to only three forces. For the equilibrium, the acting forces must be concurrent and must form a closed triangle. This aspect has been illustrated in Fig. 2.17.

Law of sines can then be used to obtain solution to the problem

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

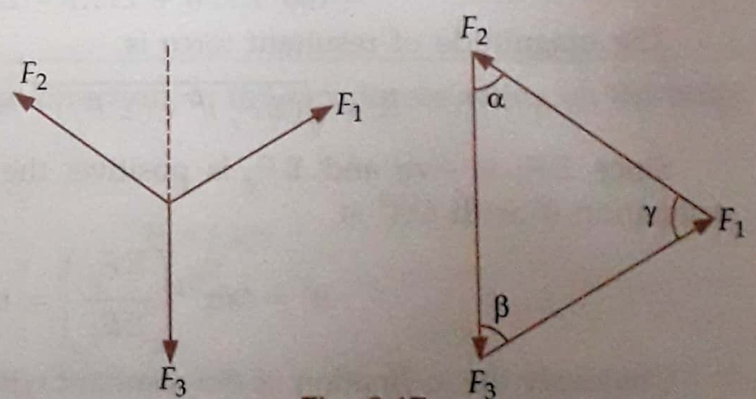


Fig. 2.17

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Determine the resultant, both in magnitude and direction, of the four forces acting on the body as shown in the figure given below:

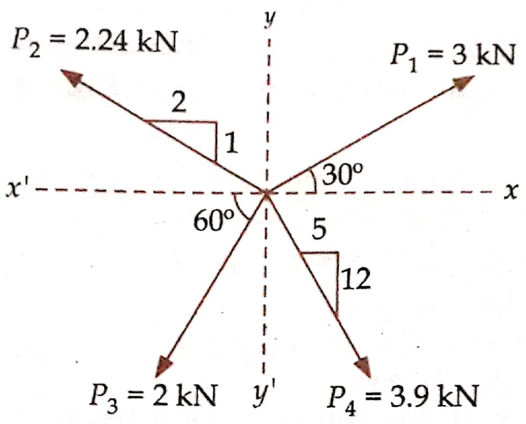

Fig. 2.22

Solution : Inclination of force 2.24 kN with OX′ = $\tan^{-1}(1/2) = 26.56°$

Inclination of force 3.9 kN with OY′ = $\tan^{-1}(5/12) = 22.62°$

Measuring all angles from axis O-X in the anticlockwise direction, the inclination of different forces will be as depicted in Fig. 2.23.

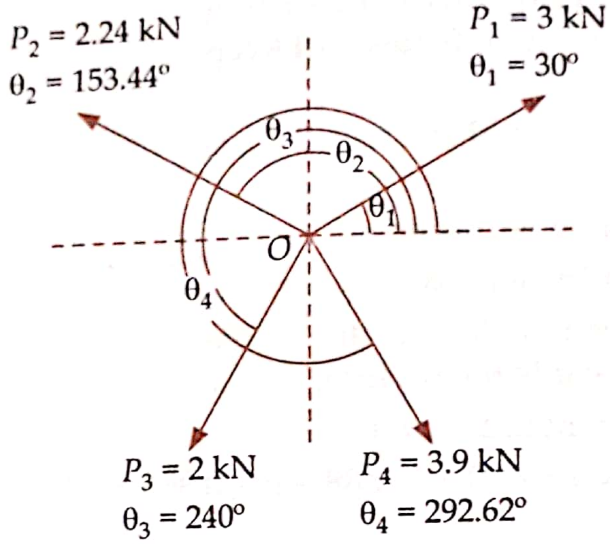

Fig. 2.23

Resolving all the forces in horizontal direction,

$$\Sigma F_x = P_1 \cos\theta_1 + P_2 \cos\theta_2 + P_3 \cos\theta_3 + P_4 \cos\theta_4$$
$$= 3\cos 30° + 2.24 \cos 153.44° + 2\cos 240° + 3.9 \cos 292.62°$$
$$= 2.598 - 2.004 - 1.0 + 1.50 = 1.094 \text{ kN}$$

Resolving all the forces in vertical direction,

$$\Sigma F_y = P_1 \sin\theta_1 + P_2 \sin\theta_2 + P_3 \sin\theta_3 + P_4 \sin\theta_4$$
$$= 3\sin 30° + 2.24 \sin 153.44° + 2\sin 240° + 3.9 \sin 292.62°$$
$$= 1.50 + 1.00 - 1.73 - 3.60 = -2.83 \text{ kN}$$

The magnitude of the resultant force is

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(1.094)^2 + (-2.83)^2} = \sqrt{1.197 + 8.009} = \mathbf{3.034 \text{ kN}}$$

and the inclination of resultant with the horizontal is

$$\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{-2.83}{1.094}\right) = \mathbf{68.86°}$$

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A particle positioned at point O is acted upon by three forces as shown in Fig. 2.20. Determine the value of force P such that the resultant of these three forces is horizontal. What should be the magnitude and direction of a fourth force which acting at point O along with the given three forces will keep the particle in equilibrium state?

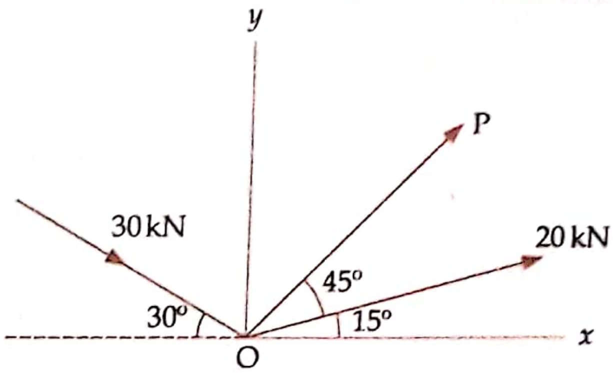

Fig. 2.20

Solution : Angle between force P and the axis 0 - x is $(15 + 45) = 60°$

Resolving all the forces along y-direction,

$\Sigma F_y = -30 \sin 30° + 20 \sin 15° + P \sin 50°$

The negative sign with force of 30 N is due to the fact that this force is acting towards the particle.

Since the resultant is horizontal, $\Sigma F_y = 0$

$\therefore$ $$0 = -30 \times 0.5 + 20 \times 0.2588 + P \times 0.866 = -15 + 5.176 + 0.866\,P$$

That gives:

$$P = \frac{15 - 5.176}{0.866} = 11.344 \text{ N}$$

Resolving all forces along X-direction,

$$\Sigma F_x = 30 \cos 30° + 20 \cos 15° + 11.344 \cos 60° = 25.98 + 19.32 + 5.67 = 50.97 \text{ kN}$$

The equilibrant would be -50.97 kN as shown in Fig. 2.21.

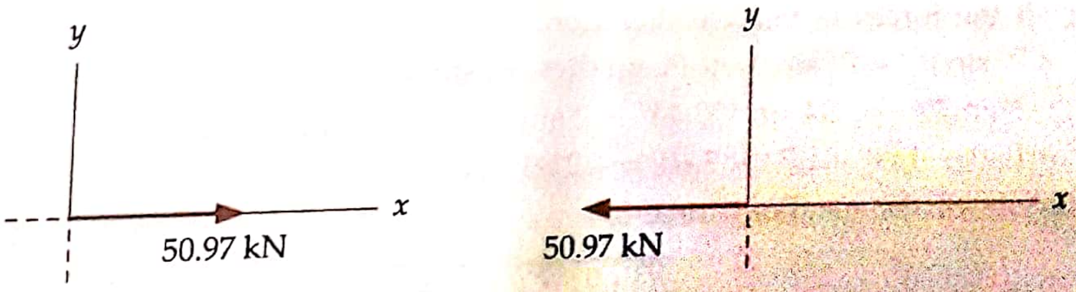

Fig. 2.21

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Calculate the tensile force in the cables AB and BC as shown in Fig. 2.27. Presume the pulleys to be frictionless.

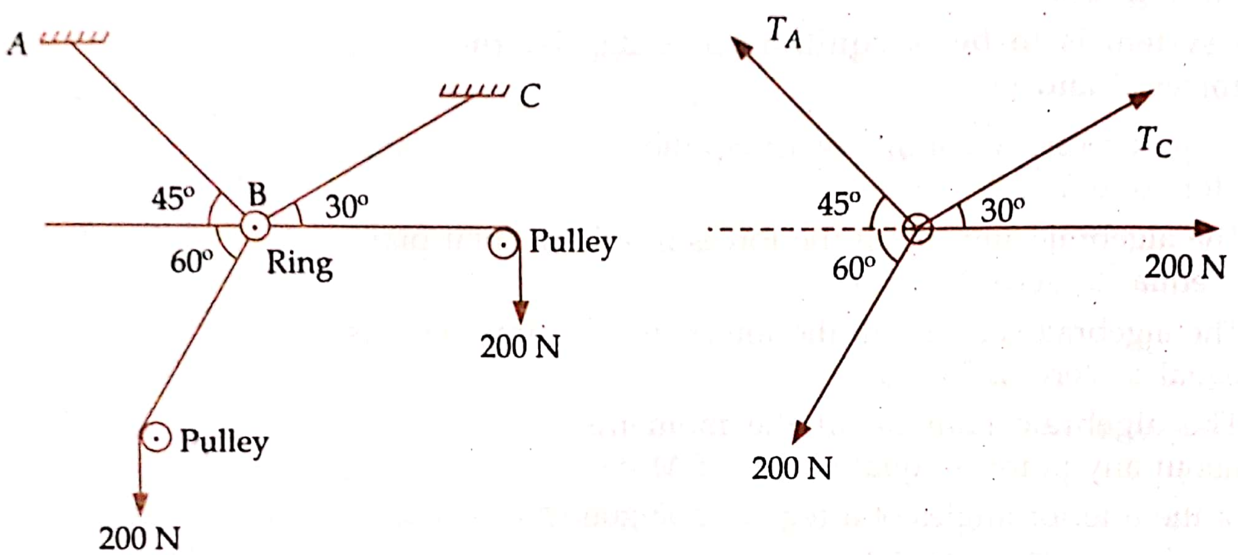

Fig. 2.27

Solution : Since the system is in equilibrium,

$\Sigma F_x = 0\,;\quad 0 = -T_A \cos 45° + T_C \cos 30° + 200 - 200 \cos 60°$

$\quad\quad = -0.707\, T_A + 0.866\, T_C + 200 - 100$

or $\quad -T_A + 1.22\, T_C + 141.0 = 0$ ...(i)

$\Sigma F_y = 0\,;\quad 0 = T_A \sin 45° + T_C \sin 30° - 200 \sin 60° = 0.707\, T_A + 0.5\, T_C - 173$

or $\quad T_A + 0.707\, T_C - 244 = 0$ ...(ii)

Adding expressions (*i*) and (*ii*),

$$1.927\,T_C - 103 = 0\,; \quad T_C = \frac{103}{1.927} = 53.45 \text{ N}$$

Then from expression (*ii*)

$$T_A + 0.707 \times 53.45 - 244 = 0$$

$$\therefore \quad T_A = 244 - 0.707 \times 53.45 = 244 - 37.79 = 206.21 \text{ N}$$