SEM - II

APPLIED PHYSICS - II

UNIT - I

ELECTROMAGNETIC THEORY

SCALAR AND VECTOR FIELDS

We know all physical quantities are of two types. Scalars and Vectors.

scalar quantities require only magnitude for its specify.

Vector quantities require magnitude and direction for its complete specification

A physical ~~valu~~ quantity has different values at different points in space. Hence a physical quantity has to be represented by different co-ordinates at different points in space i.e the physical quantity may be expressed as a continuous function of the position of the point in that region. The region in which the function represents the physical quantity is called the field of the physical quantity.

Scalar field → The field which require only magnitude for complete identification are called scalar fields. A scalar function of which has values through out a region constitutes a field known as scalar field. Ex. density, temperature, electric potential.

Vector Field → The fields which requires magnitude and direction at each point for their complete identification are called vector fields

A vector function which has values throughout a region constitutes a field known as a vector field.

From any scalar field a vector field can be derived
Also from any vector field a scalar field can be derived
and a vector field can be derived from another vector field.
This can be done by using an operation called del operator "$\vec{\nabla}$".

Del Operator ($\vec{\nabla}$)

$\vec{\nabla}$ is a vector operator which is represented as

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}.$$

① Gradient of a scalar (ϕ) – (grad ϕ) (= $\vec{\nabla}\phi$)

Let $\phi(x, y, z)$ is a continuously differentiable scalar function.
The gradient of a scalar function is defined as

$$\text{grad } \phi = \vec{\nabla}\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\phi$$

$$= \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}.$$

$=$ vector quantity.

Gradient of a scalar function is ~~a~~ a vector function whose magnitude is equal to max rate of change of scalar function ϕ w.r.t. space variables and whose direction is along that change.

Example – ① Temperature (T) is a scalar.
Change in temperature is $\vec{\nabla}T = \hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}$
is a vector quantity called "Temperature Gradient".

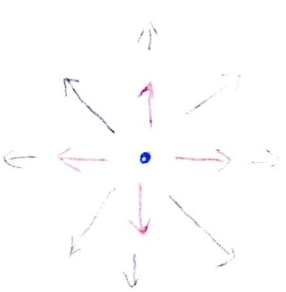
② Electric potential V (Scalar)

$$\vec{\nabla}V = \vec{E} = \text{Electric field}.$$

② DIVERGENCE OF A VECTOR $\vec{A}$ ($\vec{\nabla}\cdot\vec{A}$) (div $\vec{A}$)

When $\vec{\nabla}$ operates on any vector $\vec{A}$ by dot product ($\vec{\nabla}\cdot\vec{A}$) is called divergence of $\vec{A}$ or $\vec{\nabla}\cdot\vec{A}$

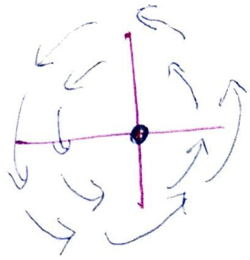
Divergence of any vector measures how much the vector $\vec{A}$ spreads out or ~~divergence~~ diverges from the point in question.



③ CURL OF A VECTOR $\vec{A}$ $(\vec{\nabla} \times \vec{A})$

When $\vec{\nabla}$ operates on any vector by cross product then the operation is known as curl $\vec{A}$ $(\vec{\nabla} \times \vec{A})$
curl $\vec{A}$ $(\vec{\nabla} \times \vec{A})$ measures how much the vector $\vec{A}$ curls around the point in question (or rotates the vector at the point in question).

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$



If $\boxed{\vec{\nabla} \times \vec{A} = 0}$, then the vector $\vec{A}$ is said to be **irrotational vector**

$\vec{\nabla}.\vec{A}$ measures how much the vector $\vec{A}$ spreads out or divergence from the point.

In terms of fluid, divergence of a fluid is represented by the excess of outward flow over the inward flow.

~~If +ve const~~. It is represented by $(\vec{\nabla}.\vec{A})$ net amount of flux coming out of the volume element.

When inward flow is more than outward flow then it is said to be -ve divergence i.e convergence $= -\vec{\nabla}.\vec{A}$.

When inward flux = Outward flux

$\Rightarrow \boxed{\vec{\nabla}.\vec{A} = 0}$.

In this case the vector $\vec{A}$ is said to be $\boxed{\text{solenoidal}}$.

Let $\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$.

$$\vec{\nabla}.\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \text{scalar quantity.}$$

Second Derivative of $\vec{\nabla}$

① Divergence of gradient. $\nabla \cdot (\nabla T)$

$$\nabla \cdot (\nabla T) = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right)$$

$$= \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \nabla^2 T. \quad \nabla^2 = \text{Laplacian.}$$

② Curl curl $\vec{A}$ $= \nabla \times (\nabla \times \vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2 \vec{A}$.

$= \cancel{\text{div}}$ grad div $\vec{A}$ $- \nabla^2 \vec{A}$

③ The curl of a gradient is alway zero

$\nabla \times (\nabla T) = 0$. Curl grad $T = 0$

Any vector which is irrotational i.e curl $\vec{A} = 0 \Rightarrow \vec{A}$ is irrotational can always be expressed in terms of gradient of a scalar.

If $\nabla \times \vec{A} = 0 \Rightarrow \vec{A} = \text{grad}\,\phi$.

④ Divergence of a curl is always zero.

$$\nabla \cdot (\vec{\nabla} \times \vec{A}) = 0.$$

div curl $\vec{A}$ $= 0$

⑤ Gradient of divergence $\vec{A}$ = Grad div $\vec{A}$ $= \vec{\nabla}(\vec{\nabla}\cdot\vec{A})$.

① Gauss Divergence Theorem.

① The volume integral of the divergence of a vector $\vec{A}$ taken over any volume V bounded by a closed surface S is equal to the surface integral of A taken over the surface S.

$$\boxed{\iiint_V \text{div}\, A \, dV = \iint_S A \cdot ds.}$$ Important

② STOKE's THEOREM.

The surface integral of the curl of a vector field $\vec{A}$ taken over any surface S is equal to the line integral of $\vec{A}$ around a closed curve forming the periphery of the surface.

$$\boxed{\iint_S (\nabla \times \vec{A})\, ds = \oint A \cdot dl.}$$ Important.

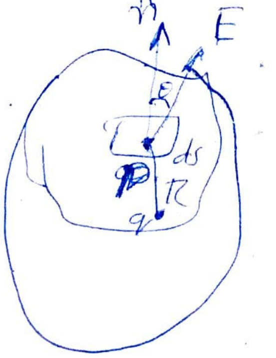
GAUSS THEOREM

In free space the total outward flux of an electric field vector over a closed surface is equal to $1/\epsilon_0$ times the total charge enclosed in a volume enclosed by the surface.

In integral form $\iint E.ds = q/\epsilon_0$.

E = Electric field intensity, ds = Surface element, q = charge enclosed.

ϵ_0 = Permittivity of free space.



PROOF →

Consider a closed surface S enclosing a charge $+q$ and small elemental area ds on the surface.

n' is the unit vector in the direction of the outward normal to the surface ds.

Let the point P is at a distance r from the charge.

So $\vec{E}$ at the point P is $\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$

The flux over dS is $d\phi = E.dS$.

Total flux over the whole surface $\phi = \int d\phi = \int E.ds = \int E ds \cos\theta$

$\oint E.ds = \oint \frac{q}{4\pi\epsilon_0 r^2} ds\cos\theta = \frac{q}{4\pi\epsilon_0}\int \frac{ds\cos\theta}{r^2} = \frac{q}{4\pi\epsilon_0}\int d\omega = \frac{q}{4\pi\epsilon_0} 4\pi$

$= \frac{q}{\epsilon_0}$.

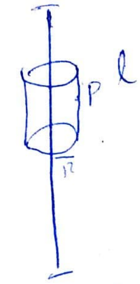
$d\omega$ = solid angle subtended at the point due to surface ds.

$\oint E.ds = \frac{q}{\epsilon_0}$.

$\int(\nabla.E)dV = q/\epsilon_0 = \int \frac{\rho dV}{\epsilon_0}$

$\boxed{\nabla.E = \rho/\epsilon_0}$ Differential form of Gauss law.

$\boxed{\nabla.D = \rho}$ $D = E\epsilon_0$ = Electric displacement vector.



APPLICATIONS OF GAUSS LAW →

① Electric field due to a line charge.

Let us consider a conductor of length l.

Let charge contained in the conductor = q.

So line charge density $\lambda = \frac{q}{l} \Rightarrow q = \lambda l$.

Let us find the electric field at a point P which is r distance from conductor.

For this we have to consider a Gaussian surface i.e a coaxial cylinder of radius r & length l.

Using Gauss law $\int E.ds = q/\epsilon_0 \Rightarrow 2\pi r l E = \lambda l/\epsilon_0 \Rightarrow \boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$

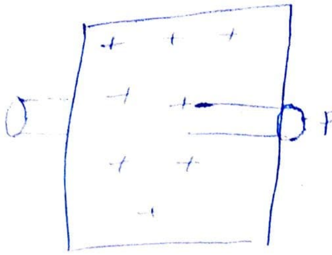
(2) Electric field due to a sheet of charge →

Let us consider a thin sheet of +ve charge.

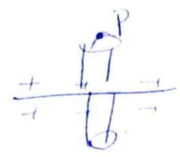
Let σ = surface charge density.

So charge contained = σS.

Electric field at any point P due to this sheet.

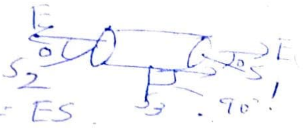


Let us consider a small cylinder of cross-section area S such that the point P lies on one face of the cylinder while the other face lies on the other side of the sheet over an area S.



Gauss law $\int_S E \cdot ds = \frac{q}{\epsilon_0}$

$$\int_{S_1} E \cdot ds + \int_{S_2} E \cdot ds + \int_{S_3} E \cdot ds = \frac{\sigma S}{\epsilon_0}$$



For S_1 & S_2, E & angle between E & S = 0, So $E \cdot ds = ES$.

For S_3, $E \perp S$, So $E \cdot S = 0$

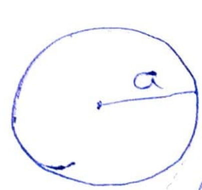
$$ES + ES = \frac{\sigma S}{\epsilon_0} \Rightarrow 2ES = \frac{\sigma S}{\epsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

(3) Electric field due to a spherical charge distribution (non-conducting sphere)

Let us consider a spherical charge distribution of radius a.

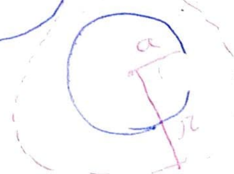
Let q is charge contained.

ρ = volume charge density = $q / \frac{4}{3}\pi a^3$.



$$q = \frac{4}{3}\pi a^3 \rho.$$

(i) Electric field on the surface (At any point)



$$\int E \cdot ds = q/\epsilon_0 \Rightarrow E \, 4\pi r^2 = \frac{\frac{4}{3}\pi a^3 \rho}{\epsilon_0}$$

$$\Rightarrow E = \frac{\rho a^3}{3\epsilon_0 r^2}.$$

(ii) Electric Field at a inside Point →

Let us consider a Gaussian surface of radius r which contain P.

Charge enclosed in the volume $\frac{4}{3}\pi r^3 = \frac{q \frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} = \frac{q r^3}{a^3} = q'$. $\boxed{q' = \frac{q r^3}{a^3}}$

$$E \cdot ds = q/\epsilon_0 \Rightarrow E \, 4\pi r^2 = \frac{q r^3}{\epsilon_0 a^3}$$

$$\Rightarrow \boxed{E = \frac{q r}{4\pi\epsilon_0 a^3}} = \cancel{\frac{4}{3}\pi a}$$

(iii) Electric field at a outside Point P

Let us consider an Gaussian surface of radius r at including point P.

$$E \cdot ds = q/\epsilon_0 \Rightarrow E \, 4\pi r^2 \cdot q/\epsilon_0 \Rightarrow \boxed{E = \frac{q}{4\pi\epsilon_0 r^2}}$$

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