4.12. ROPE AND BELT FRICTION

The transmission of power in factories from one rotating shaft to another that lies at a considerable distance is achieved through belts and ropes. The shafts are fitted with pulleys, the belt is wrapped round the pulleys and its ends are connected to form an endless connector. The belts and the pulley remain in contact by frictional grip.

With reference to Fig. 4.76, the pulley A which is connected to the rotating shaft is called the *driver*. The pulley B that needs to be driven is termed as *follower*. When the driver rotates, it carries the belt because of friction that exists between the pulley and the belt. The frictional

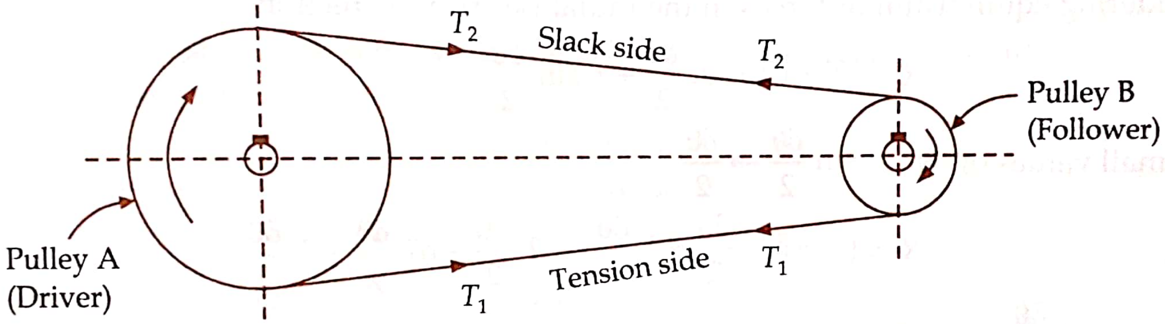

Fig. 4.76. Belt drive-open system

resistance develops all along the contact surfaces. That makes the belt carry the follower which too starts rotating. The driving pulley pulls the belt from one side (called tension side) and delivers it to the other side (called slack side). The tension T_1 in the belt on the tension side is more than tension T_2 on the slack side.

4.13. RATIO OF TENSIONS

Fig. 4.77 shows a flexible belt resting over the flat rim of a stationary pulley. The tensions T_1 and T_2 are such that the motion is impending (just to take place) between the belt and the pulley.

Considering the impending motion to be clockwise relative to the drum, the tension T_1 is more than T_2. It is to be noted that only a part of the belt is in contact with the pulley. The angle subtended at the centre of the pulley by the position of belt in contact with it is called the *angle of contact* or the *angle of lap*

Angle of contact $\theta = \angle AOB$

Let attention be focused on small element EF of the belt which subtends an angle $\delta\theta$ at the centre. The segment EF is acted upon by the following set of forces :

- tension T in the belt acting tangentially at F,
- tension $(T + \delta T)$ in the belt acting tangentially at E
- normal reaction R exerted by the pulley rim, and
- friction force $F = \mu R$ which acts against the tendency to slip and is perpendicular to normal reaction R.

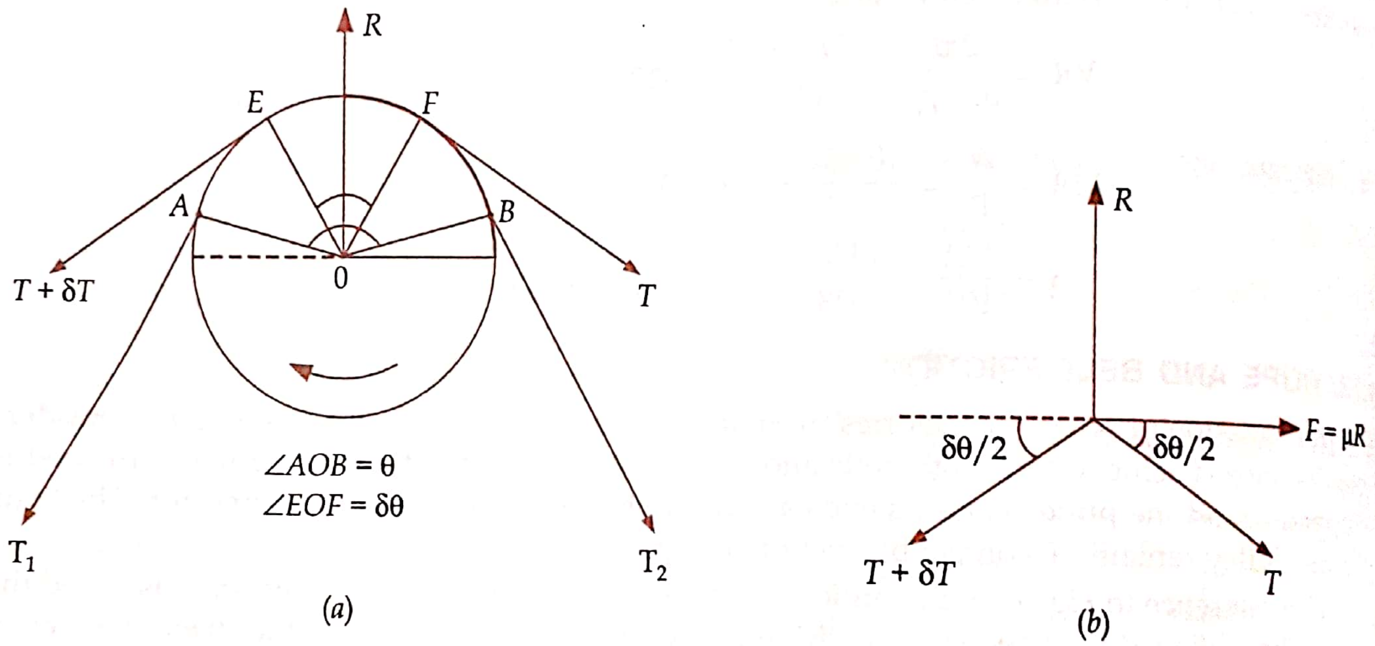

Fig. 4.77. Ratio of tension on tight and block side

Considering equilibrium of forces in the radial (vertical) direction,

$$R = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

For small values of $\delta\theta$; $\quad \sin\frac{\delta\theta}{2} \to \frac{\delta\theta}{2}$

$$R = (T + \delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = T\frac{\delta\theta}{2} + \delta T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2}$$

The term $\delta T\frac{\delta\theta}{2}$ is small in magnitude and can be neglected

$$R = T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = T\,\delta\theta \qquad ...(i)$$

Considering equilibrium of forces in tangential (horizontal) direction,

$$\mu R = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

For small values of $\delta\theta$; $\quad \cos\frac{\delta\theta}{2} \to 1$

$$\mu R = (T + \delta T) - T = \delta T$$

$$R = \frac{\delta T}{\mu} \quad ...(ii)$$

From expressions (*i*) and (*ii*)

$$T\,\delta\theta = \frac{\delta T}{\mu}$$

Separating the variables and integrating between the limit $T = T_2$ at $\theta = 0$ and $T = T_1$ at $\theta = \theta$, we get,

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta \;;\quad \log_e \frac{T_1}{T_2} = \mu\theta \quad \therefore \quad \frac{T_1}{T_2} = e^{\mu\theta} \quad ...(4.20)$$

When two pulley of unequal diameters are connected by open belt drive, the slip occurs first on the smaller pulley where the force of friction is less. Accordingly, the angle of contact on smaller pulley is taken into account while using the equation 4.20.

(*ii*) ***V-belt and rope drive :*** A V-belt or a rope runs into a grooved pulley.

Refer Fig. 4.78 (*a*) which shows a V-belt of trapezoidal section resting in a grooved pulley.

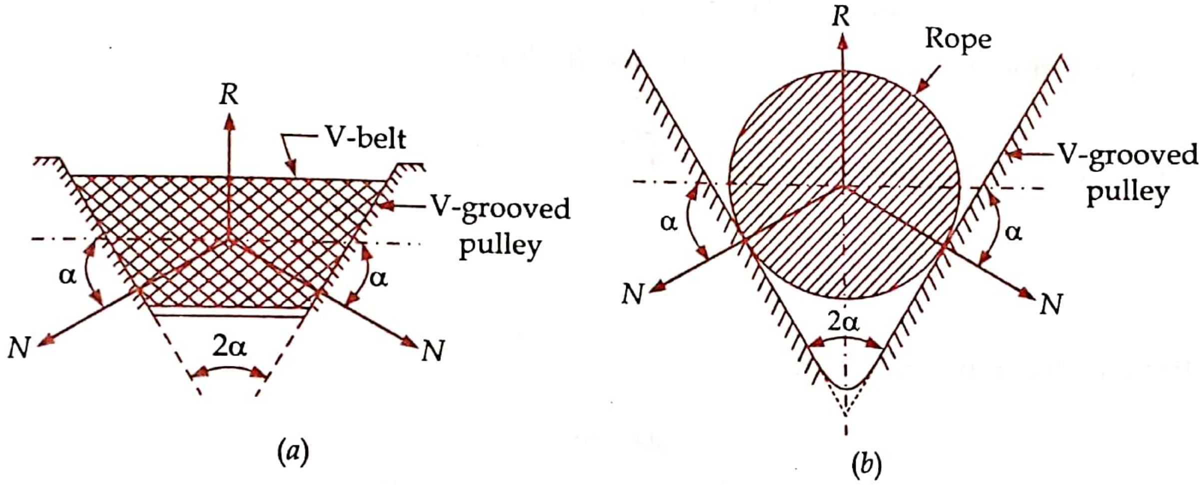

Fig. 4.78. V-belt and rope drive

Let 2α = angle of groove

N = normal reaction between belt and sides of the V-grooved pulley

R = total reaction in the plane of groove

μ = coefficient of friction between belt and groove

Considering equilibrium between R and N, we have

$$R = N \sin\alpha + N \sin\alpha = 2N \sin\alpha$$

$$\therefore \quad N = \frac{R}{2\sin\alpha} = \frac{R}{2}\operatorname{cosec}\alpha$$

$$\text{Friction resistance} = \mu N + \mu N = 2\,\mu N = 2\mu \times \frac{R}{2}\operatorname{cosec}\alpha = \mu R \operatorname{cosec}\alpha$$

Then with reference to Fig. 4.79

$$R = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

$$= (T + \delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} \qquad (\because \delta\theta \text{ is small})$$

$$= T\frac{\delta\theta}{2} + \frac{\delta T\,\delta\theta}{2} + T\frac{\delta\theta}{2}$$

The term $\frac{\delta T\,\delta\theta}{2}$ is the product of two small quantities and hence neglected. That gives

$$R = T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = T\,\delta\theta \quad ...(a)$$

Also : $$\mu R \operatorname{cosec}\alpha = (T + \delta\theta)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} = (T + \delta T) - T = \delta T \quad ...(b)$$

This is because $\frac{\delta\theta}{2}$ is small and $\cos\frac{\delta\theta}{2} \to 1$

From expressions (*a*) and (*b*),

$$\mu\,(T\,\delta\theta)\operatorname{cosec}\alpha = \delta T \quad ...(c)$$

or $$(\mu \operatorname{cosec}\alpha)\,\delta\theta = \frac{\delta T}{T}$$

Upon integration with in appropriate limits, we have

$$\mu \operatorname{cosec}\alpha \int_{0}^{\theta} \delta\theta = \int_{T_2}^{T_1} \frac{\delta T}{T}$$

or $$(\mu \operatorname{cosec}\alpha) \times \theta = \log_e \frac{T_1}{T_2}$$

∴ Ratio of tensions for V-belt,

$$\frac{T_1}{T_2} = e^{(\mu \operatorname{cosec}\alpha)\,\theta} \quad ...(4.21)$$

Equations 4.21 is equally valid for a rope drive (Fig 4.78 *b*).

(iii) Power transmitted :

Let T_1 = tension on the tight side

T_2 = tension on the slack side

V = velocity of the belt

Then, effective turning force = $(T_1 - T_2)$

work done = force × distance moved = $(T_1 - T_2)\,V$ in Nm/s

and power = $(T_1 - T_2)\,V$ in watts ...(4.22)

where values of T_1 and T_2 are in N and V is the velocity of belt in m/s.

EXAMPLE 4.44

Find out the number of turns a hauling rope must be wound round a rotating capstan in order to haul a load of 3 MN up to a gradient of 1 in 30. Presume the following data :

Resistance due to rolling = 0.00375 per newton load

Friction coefficient between the rope and drum = 0.35

Pull on the free end of the rope = 250 N

Solution : The hauling is required up an inclined plane and for that T_1 has to overcome the frictional resistance as well as effort component of the incline. That is

$$T_1 = (3 \times 10^6 \times 0.00375) + \left(3 \times 10^6 \times \frac{1}{30}\right) = 111250 \text{ N}$$

$$T_2 = 250 \text{ N}$$

Invoking the relation $\frac{T_1}{T_2} = e^{\mu\theta}$, we have

$$e^{\mu\theta} = \frac{111250}{250} = 445$$

$$\mu\theta = \log_e 445 \; ; \; \theta = \frac{\log_e 445}{0.35} = 41742 \text{ radians}$$

$$\text{Number of turns} = \frac{17.42}{2\pi} = 2.77$$

Hence three turns need to be given.

EXAMPLE 4.45

Determine the minimum value of weight W required to cause motion of a block which rests on a horizontal plane. The block weighs 300 N and the coefficient of friction between the block and plane is 0.6. Angle of wrap over the pulley is 90° and the coefficient of friction between the pulley and rope is 0.3.

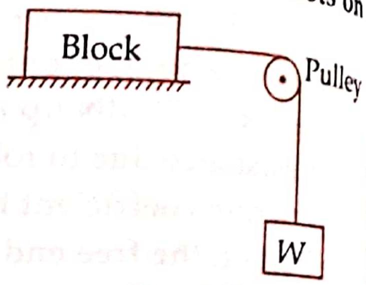

Fig. 4.79

Solution : Since the weight W impends vertical motion in the downward direction, the tension on the two sides of the pulley will be as shown in Fig. 4.70.

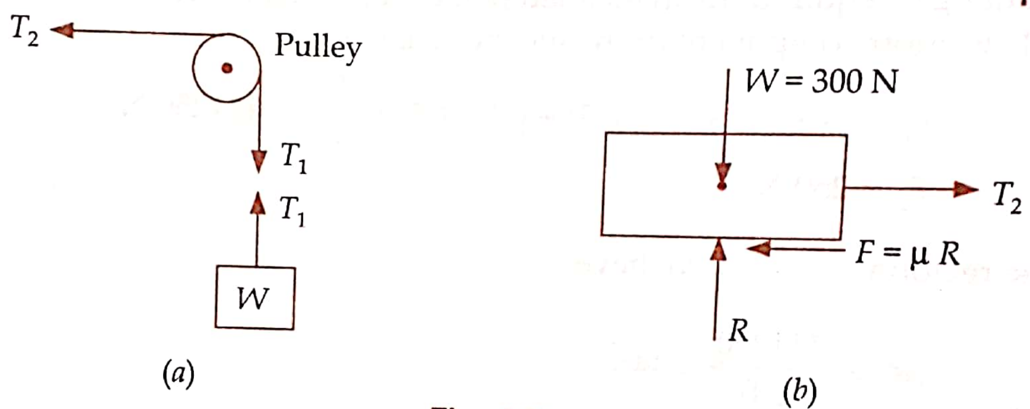

Fig. 4.80.

$$T_1 = W;\ \mu = 0.3;\ \theta = 90° = \frac{\pi}{2} \text{ radians}$$

Invoking the relation

$$\frac{T_1}{T_2} = e^{\mu\theta};\ \frac{W}{T_2} = e^{0.3 \times \pi/2} = (2.718)^{0.471} = 1.60$$

Considering equilibrium of block,

$$\Sigma F_x = 0;\ T_2 = F = \mu R$$

$$\Sigma F_y = 0;\ R = 300 \text{ N}$$

$\therefore$ $\quad T_2 = 0.6 \times 300 = 180$ N

Hence $\quad W = 1.6 \times 180 = 288$ N