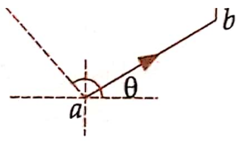
2.7. FREE BODY DIAGRAM



The force analysis of a structure is made in a simplified way by considering the equilibrium of a portion of the structure. For that, the portion is drawn separately showing applied forces, self weight and reactions at the point of contact with other bodies. The resulting diagram is known as *free body diagram* (FBD). In a free

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body diagram, all the supports (like walls, floors, hinges etc) are removed and replaced by the reactions which these supports exert on the body.

A free body diagram can be drawn for any single body of a system, for any subsystem or for the entire system irrespective of whether the system is in equilibrium, *i.e.*, at rest or in uniform motion or in a dynamic state of motion.

Further, while drawing the free diagrams, one must have consideration of internal forces and external forces.

Internal forces: the forces which hold togather the particles of the body and help it to be rigid, *i.e.*, not deform. If more than one body is involved, internal forces hold the bodies together.

Imagine a bar being pulled by two equal and opposite forces applied at the ends. The internal forces will come into play to keep the bar undeformed. These forces cause stresses and strains distributed throughout the material of the body.

External forces: the forces which act on a body or system of bodies externally, *i.e.*, applied from outside. The forces essentially denote the action of other bodies (walls, floors, hinges etc.) on the rigid body being analysed.

Consider a sphere suspended by a string. The weight of the sphere and tension induced in the string represent the external forces. The reactions developed at the contact points, if any, also constitute the external forces.

A complete isolation and systematic representation of all external (applied and reactive) forces acting on a body is an important and effective tool for the solution of problems in mechanical systems.

Given below are a few examples of systems and mechanisms together with their free body diagrams:

(*i*) A sphere resting on a frictionless plane surface (Fig. 2.44)

The forces acting on the sphere when isolated from the surface are:

(*a*) Force W equal to the weight of the sphere. This weight acts downward through the centroid of the sphere.

(*b*) Reaction R at the point of contact with the surface. This reaction acts upwards normal to the surface as it is frictionless.

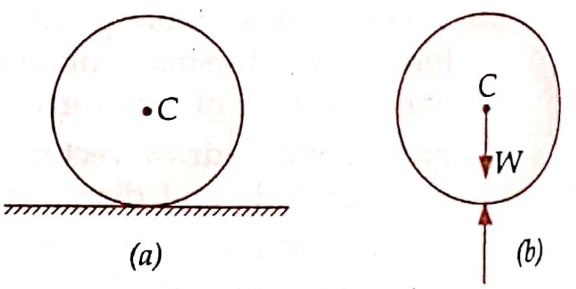

Fig. 2.44

As the sphere is in equilibrium, R and W are equal and collinear, and the free body diagram will be as shown in Fig. 2.44

(*ii*) A circular roller of weight W hangs by a string and rests against a smooth vertical wall (Fig. 2.45)

The force acting on the roller when isolated from the supports are:

(*a*) Force W equal to weight of the roller

(*b*) Wall reaction R_c at the point of contact C with the wall. The reaction will be normal to the wall as it is smooth

(*c*) Tension T in the string along BA.

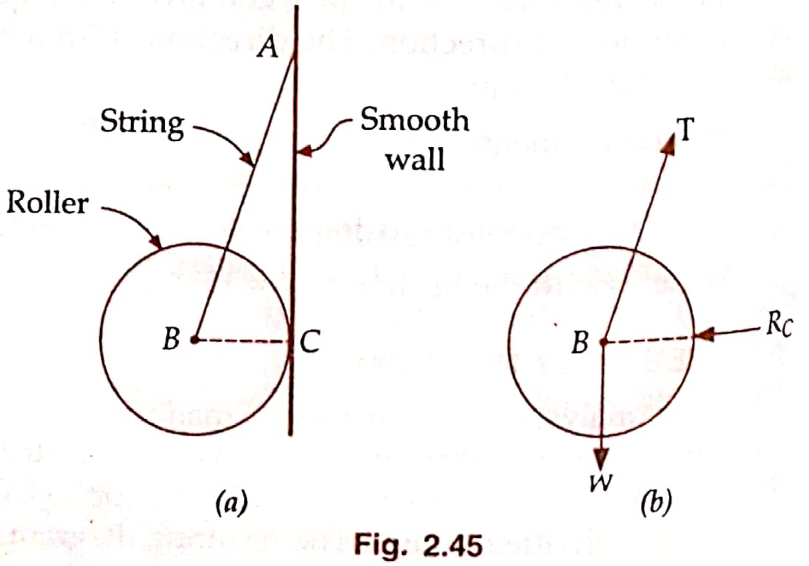

Fig. 2.45

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As the roller is in equilibrium, all the forces will be concurrent and the free body diagram will be as shown in Fig. 2.45

(iii) A sphere resting in a V-shaped groove (Fig. 2.46)

The sphere is isolated from the inclined surface forming the groove, and is considered to be acted upon by the following set of forces:

(a) Weight of the sphere acting vertically down-ward through its centre C

(b) Reaction R_a acting nor-mal to the in-clined plane at the contact point A.

(c) Reaction R_b acting normal to the in-clined plane at the contact point B.

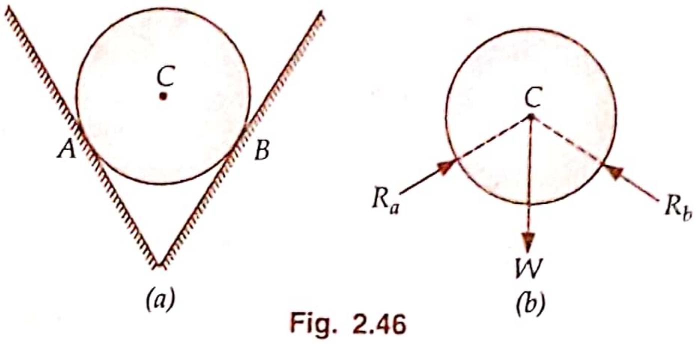

Fig. 2.46

Since the sphere is in equilibrium, all the forces meet at point C and the free body diagram will be as shown in Fig. 2.46

(iv) A drum being rolled along the horizontal comes across a stepped obstacle (Fig. 2.47)

At the instant of obstacle being overcome, the forces acting on the drum are:

(a) Weight W of the drum acting vertically downwards through its centre C

(b) Pull P required to be applied at the position shown to overcome the obstacle

(c) Reaction at A that passes through point B, the intersection of P and W.

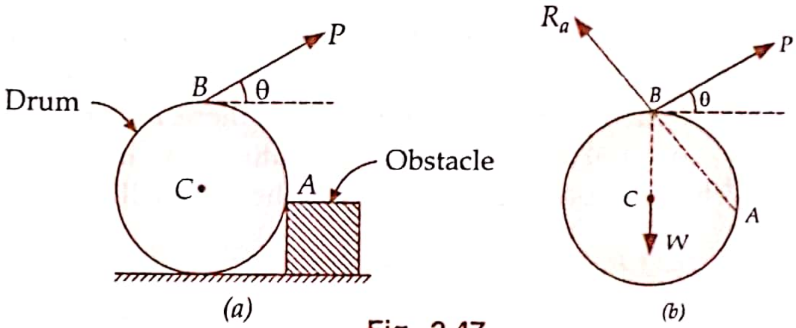

Fig. 2.47

For free body diagram of the drum, refer to Fig. 2.47.

(v) A uniform ladder of weight W leans a against a smooth wall and rests on a rough floor (Fig. 2.48)

The various forces acting on the ladder when it is isolated from the wall and floor are:

(a) Force W equal to the weight of the ladder and acting vertically downward from the mid of the ladder.

(b) Reaction R_b of the wall. This acts right angles to the wall as the wall is smooth.

(c) Reaction R_a at the ground.

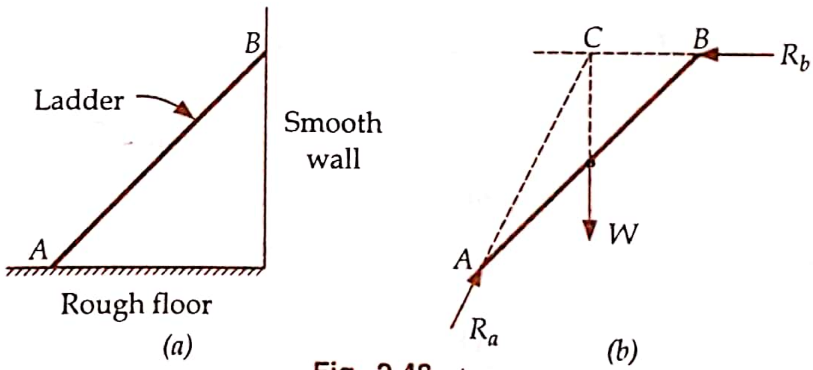

Fig. 2.48.

Since the ladder is in equilibrium, all the forces must pass though a common point. This aspect then fixes the unknown direction of reaction R_a, and the free body diagram of the ladder will be as shown in Fig. 2.48

(vi) A beam loaded and supported (Fig. 2.49)

When the beam is detached from the supports, it subjected to following set of forces:

(a) Weight W acting vertically downwards through mass centre of the beam

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(b) Reaction R_b normal to the beam at its smooth contact with the corner
(c) Horizontal applied force P and the couple M
(d) vertical and horizontal reactions (R_{av} and R_{ah}) exerted at the pin connection at B.

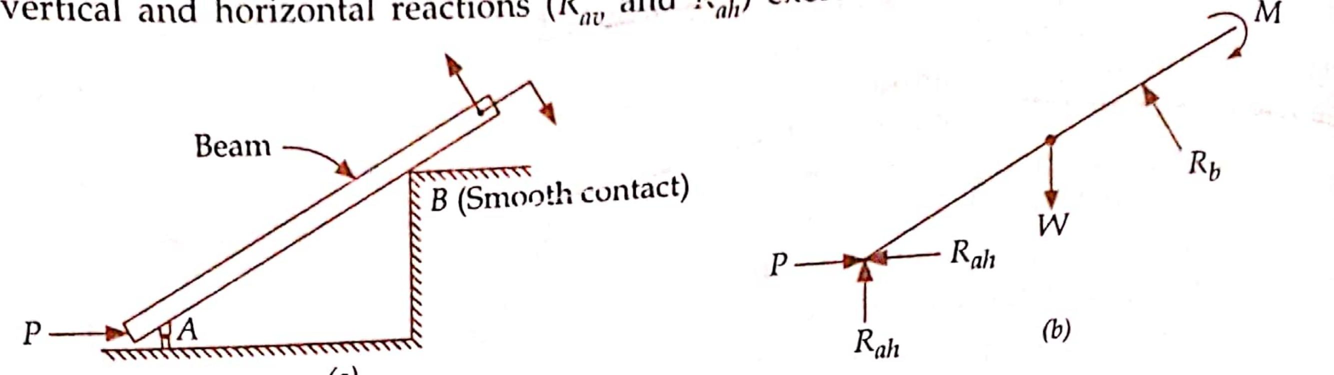


Fig. 2.49

The beam will be in equilibrium under the action of five forces and one couple, and its free body diagram will be as shown in Fig. 2.49.

(vii) Two spheres P and Q placed in a vessel (Fig. 2.50)

Forces acting on sphere P

(a) Weight W_1 of sphere acting downwards through its mass centre C_1
(b) Reaction R_a (towards right) normal to the vertical wall surface
(c) Reaction R_b (up wards) normal to the base
(d) Reaction R_d of sphere Q on sphere P at the point of contact D. This acts in a direction normal to the surface, *i.e.*, along the line C_1C_2 joining the mass centres of the spheres

The free body diagram for sphere P will be as depicted in Fig. 2.50 (b)

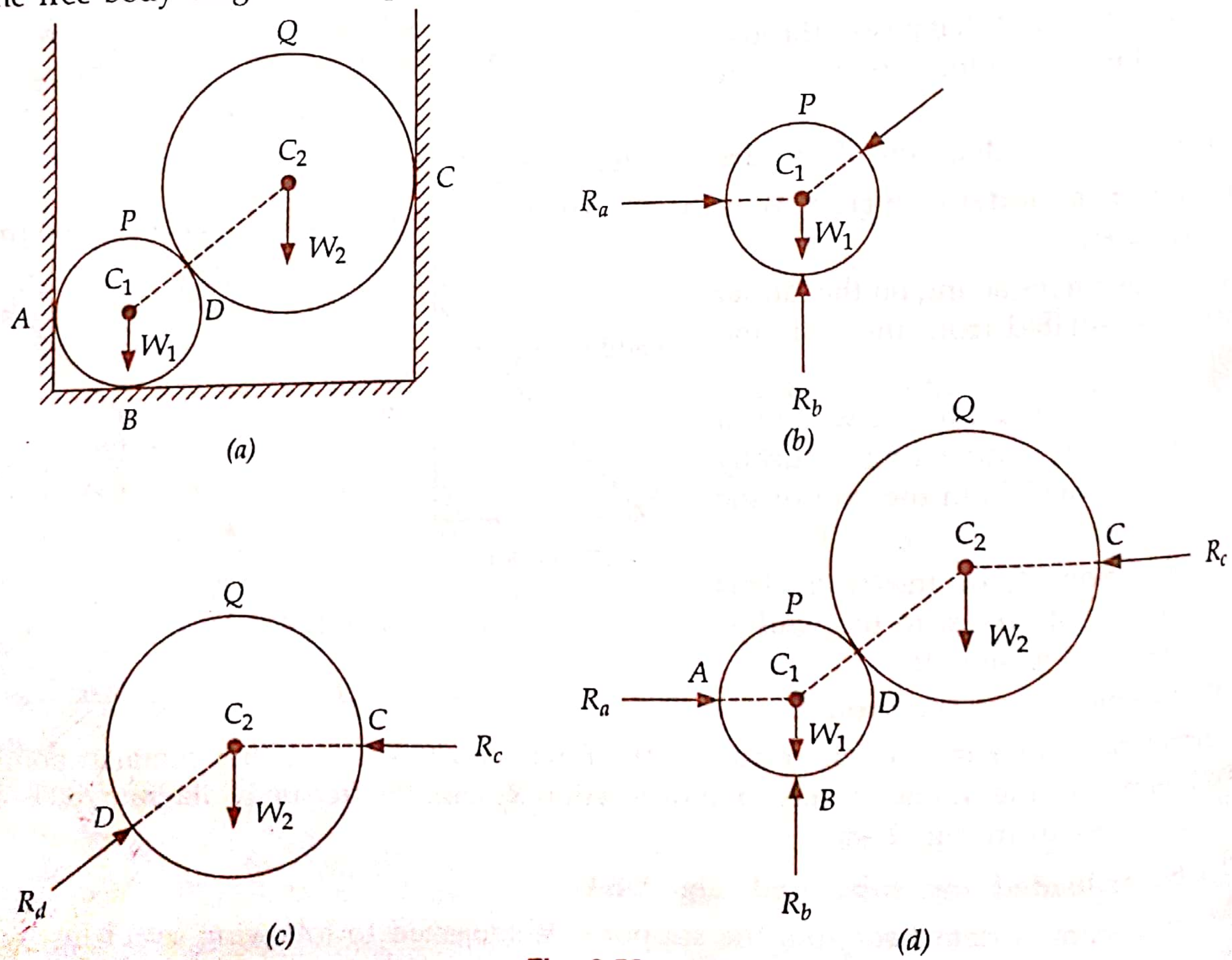


Fig. 2.50

Forces acting on sphere Q

(*a*) Weight of sphere acting downwards through its mass centre C_2

(*b*) Reaction R_c (towards left) and normal to the vertical wall surface.

(*c*) Reaction R_d of sphere *P* on sphere *Q* at the point of contact *D*. This acts in a direction normal to the surface, *i.e.*, along the line C_1C_2 joining the mass centres of the spheres.

The free body diagram for roller *Q* will be as shown in Fig. 2.50 (*c*).

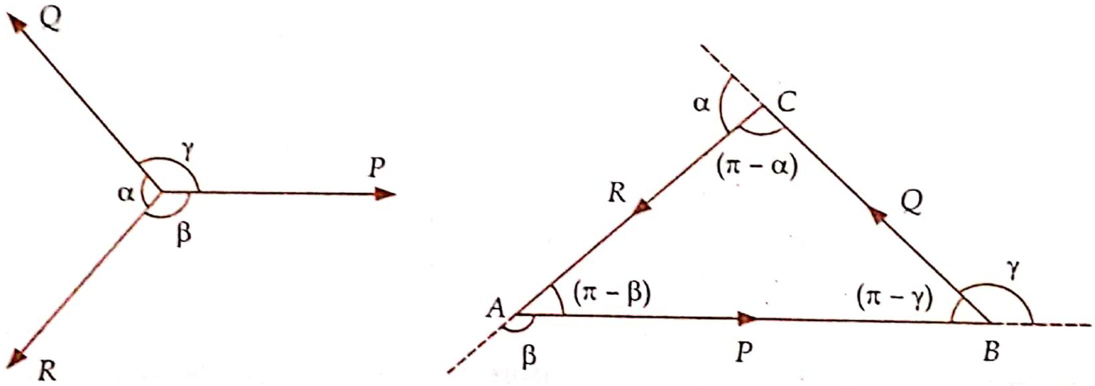
It is to be noted that

$$\begin{Bmatrix}\text{reaction of sphere } P\\ \text{on sphere } Q\end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix}\text{reaction of sphere } Q\\ \text{on sphere } P\end{Bmatrix}$$

are equal in magnitude, opposite in direction and are collinear. Obviously when both the spheres are taken together, these reactions cancel and do not appear in the free body diagram (Fig. 2.50 (*d*)).

Lami's Theorem

"If a body is in equilibrium under the action of three forces, then each force is proportional to the sine of the angle between the other two forces."



(*a*) (*b*)

Fig. 2.51

Let *P*, *Q* and *R* be the three forces acting on a body along the directions as indicated in Fig 2.51 (*a*). Since these forces are in equilibrium, they can be represented by the sides of a triangle *ABC* [Fig 2.57 (*b*)] which is drawn so as to have its sides respectively parallel to the direction of forces. Thus

$$\vec{AB} = P; \quad \vec{BC} = Q; \quad \text{and} \quad \vec{CA} = R$$

The exterior and interior angles of the triangle *ABC* of forces will be as shown in Fig 2.51 (*b*).

Applying sine rule for the triangle *ABC*

$$\frac{AB}{\sin(\pi-\alpha)} = \frac{BC}{\sin(\pi-\beta)} = \frac{CA}{\sin(\pi-\gamma)}$$

$$\therefore \quad \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

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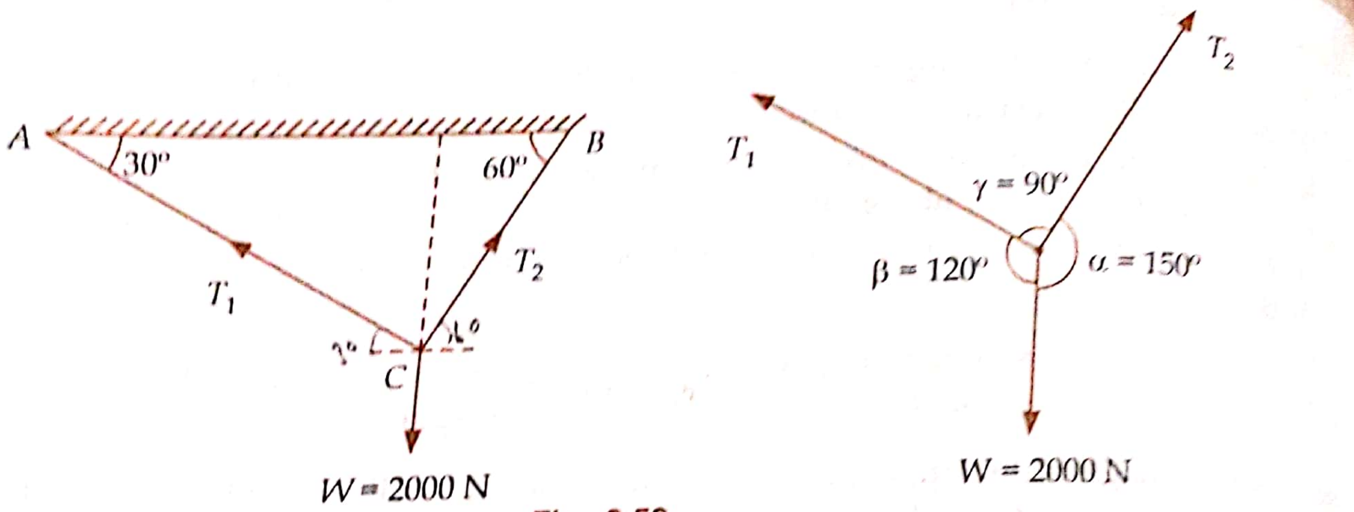

Fig. 2.52

Solution : Let T_1 and T_2 be the tensions in chains AC and BC respectively. Since the lines of action of these tensions and the weight W meet at a point, Lami's theorem can be applied. That is

$$\frac{T_1}{\sin \alpha} = \frac{T_2}{\sin \beta} = \frac{W}{\sin \gamma}$$

or
$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{2000}{\sin 90^\circ}$$

$\therefore$
$$T_1 = 2000 \times \frac{\sin 150^\circ}{\sin 90^\circ} = 2000 \times \frac{0.5}{1} = 1000 \text{ N}$$

$$T_2 = 2000 \times \frac{\sin 120^\circ}{\sin 90^\circ} = 2000 \times \frac{0.866}{1} = 1732 \text{ N}$$

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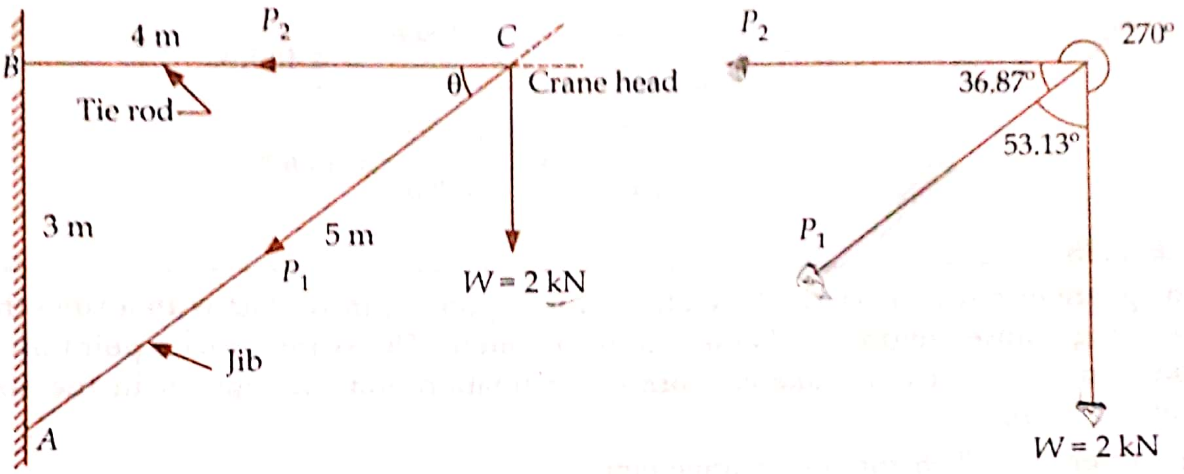
In a jib crane, the jib and the tie rod are 5 m and 4 m long respectively. The height of crane post is 3 m and the tie rod remains horizontal. Determine the forces produced in the jib and tie rod when a load of 2 kN is suspended at the crane head.

Solution : Refer Fig. 2.54 for the arrangement of the system.

$$\sin\theta = \frac{3}{5} = 0.6\ ; \quad \theta = 36.87^\circ$$



(a) (b)

Fig. 2.54

Let P_1 and P_2 be the forces developed in the jib and tie rod respectively. The three forces P_1, P_2 and W are shown in Fig. 2.54 (*b*) with the angles between the forces calculated from the given directions. The lines of action of forces P_1, P_2 and weight W meet at the point C, and therefore Lami's theorem is applicable. That gives

$$\frac{P_1}{\sin 270^\circ} = \frac{P_2}{\sin 53.13^\circ} = \frac{2}{\sin 36.87^\circ}$$

$$\therefore \quad P_1 = 2 \times \frac{\sin 270^\circ}{\sin 36.87^\circ} = 2 \times \frac{1}{0.6} = \mathbf{-\,3.33\ kN}$$

$$P_2 = 2 \times \frac{\sin 53.13^\circ}{\sin 36.87^\circ} = 2 \times \frac{0.8}{0.6} = \mathbf{2.667\ kN}$$

The – ve sign indicates that the direction of force P_1 is opposite to that shown in Fig. 2.54 (*a*). Obviously the tie rod will be under tension and the jib will in compression.

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Solution : Refer Fig. 2.55 for the arrangement. The machine is acted upon by the following set of forces:

(*i*) weight of machine $W = 5$ kN acting vertically downwards,

(*ii*) tension T_1 in the chain *OA* which goes to hook in the ceiling,

(*iii*) tension T_2 in the chain *OB* which goes to the eye bolt.

These forces are concurrent and meet at point *O*. Applying Lami's theorem

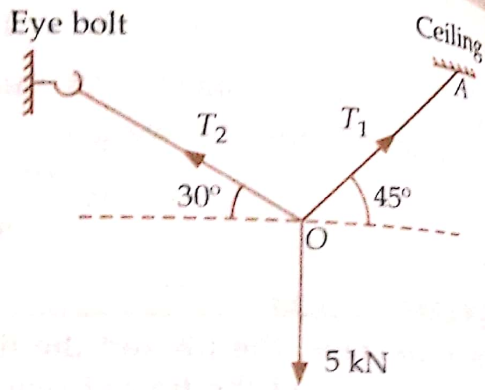

Fig. 2.55.

$$\frac{T_1}{\sin(90+30)} = \frac{T_2}{\sin(90+45)} = \frac{W}{\sin(180-30-45)}$$

or
$$\frac{T_1}{\sin 120^\circ} = \frac{T_2}{\sin 135^\circ} = \frac{W}{\sin 105^\circ}$$

$$\therefore \quad T_1 = W \times \frac{\sin 120^\circ}{\sin 105^\circ} = 5 \times \frac{0.866}{0.966} = \mathbf{4.48\ kN}$$

$$T_2 = W \times \frac{\sin 135}{\sin 105} = 5 \times \frac{0.707}{0.966} = \mathbf{3.66\ N}$$

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A smooth sphere of radius 15 cm and weight 2 N is supported in contact with a smooth vertical wall by a string whose length equals the radius of sphere. The string joins a point on the wall and a point on the surface of sphere. Workout inclination and the tension in the string and reaction of the wall.

Solution : Refer. Fig. 2.56 for the arrangement.

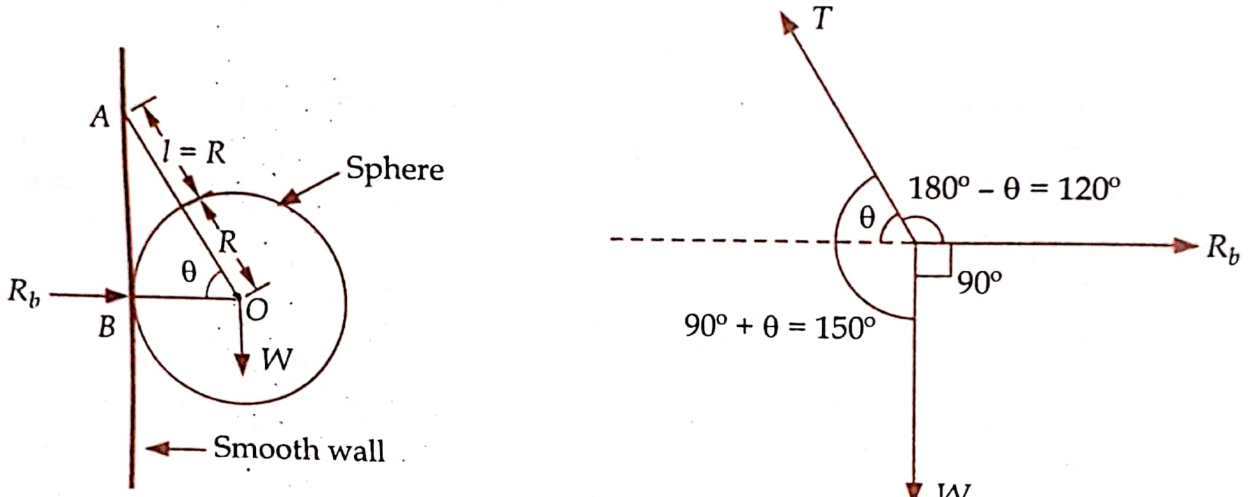

Fig. 2.56

In triangle AOB,

$$\cos\theta = \frac{OB}{OA} = \frac{R}{2R} = \frac{1}{2}; \quad \theta = 60°$$

The sphere is in equilibrium under the action of following forces:

(i) Weight $W = 2$ N of the sphere which acts vertically downwards through the centre O.

(ii) Tension T in the string

(iii) Reaction R_b; of the wall at the point of contact B. Since the wall is smooth, this reaction acts perpendicular to the wall.

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These three forces are concurrent, *i.e.*, meet at point O and as such the Lami's theorem is applicable.

$$\frac{T}{\sin 90^\circ} = \frac{W}{\sin 120^\circ} = \frac{R_b}{\sin 150^\circ}$$

$$\therefore \quad T = W\frac{\sin 90^\circ}{\sin 120^\circ} = 2 \times \frac{1}{0.866} = 2.31 \text{ N}$$

$$R_b = W\frac{\sin 150^\circ}{\sin 120^\circ} = 2 \times \frac{0.5}{0.866} = 1.15 \text{ N}$$

EXAMPLE 2.36

A roller of weight 500 N rests on a smooth inclined plane and is kept free from rolling down by a string as shown in Fig. 2.57. Work out tension in the string and reaction at the point of contact *B*.

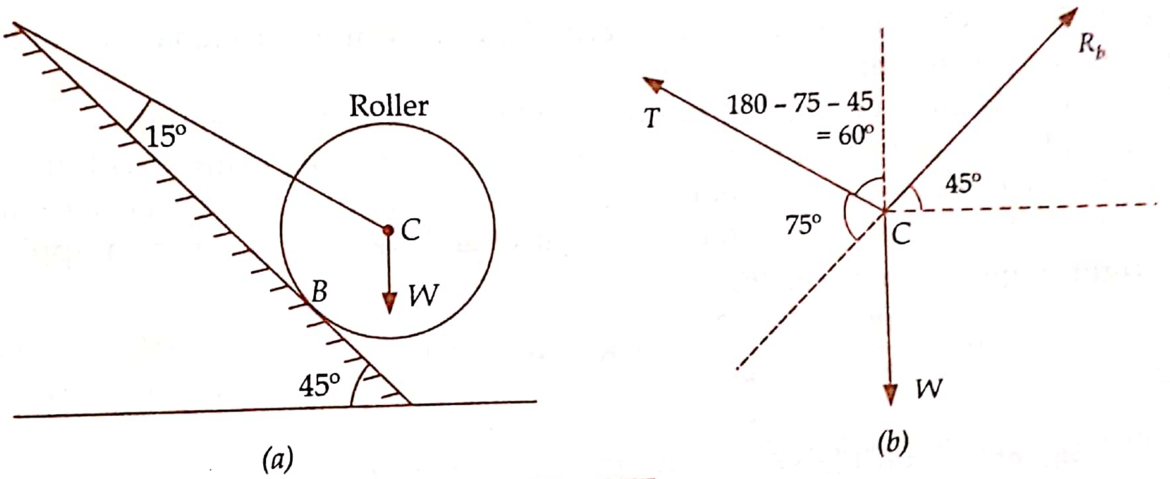

Fig. 2.57

Solution : The lines of action for tension T, weight W and reaction R_b at the contact point meet at C (the centre of the roller) and as such Lami's theorem is applicable. The angles between various segments around point C are as indicated in Fig. 2.57 (*b*).

Invoking Lami's theorem,

$$\frac{W}{\sin(60+45)} = \frac{R_b}{\sin(75+45)} = \frac{T}{\sin(90+45)}$$

$$\therefore \quad R_b = W \times \frac{\sin 120^\circ}{\sin 105^\circ} = 500 \times \frac{0.866}{0.966} = 448.24 \text{ N}$$

$$T = W \times \frac{\sin 135^\circ}{\sin 105^\circ} = 500 \times \frac{0.707}{0.966} = 365.94 \text{ N}$$

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A spherical ball of weight 100 N is attached to a string and is suspended from the ceiling as shown in Fig. 2.63. What tension would be induced in the string ?

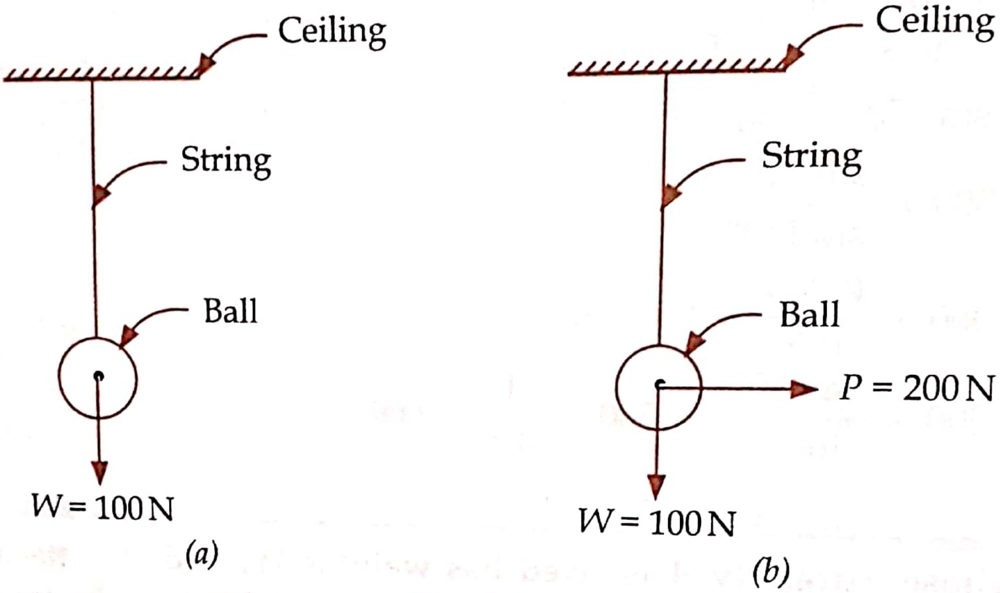

Fig. 2.63

Subsequently a horizontal force of 200 N is applied to the ball as shown in Fig. 2.70. Calculate the resultant tension in the string and the angle which the string makes with the vertical.

Solution : The ball is in equilibrium under the action of two forces namely the weight W of the ball and the tension T induced in the string

$$\therefore \quad T = W = 80 \text{ N}$$

(*ii*) When the horizontal force P is applied the ball, it will be in equilibrium under the action of three forces namely weight of the ball, horizontal force applied and the tension induced in the string.

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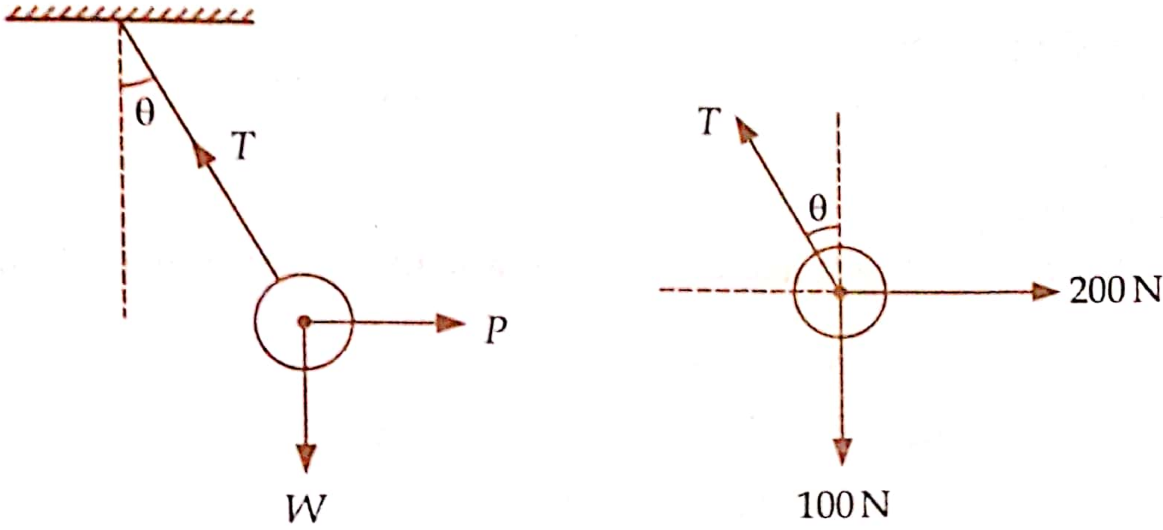

With reference to free body diagram, we have under equilibrium state

$$\Sigma F_x = 0: \quad -T\sin\theta + 200 = 0$$

$$\Sigma F_y = 0: \quad T\cos\theta - 100 = 0$$

From expressions (*i*) and (*ii*)

$$\tan\theta = \frac{200}{100} = 2\ ; \quad \theta = 63.55°$$

$$T = \frac{200}{\sin 63.55} = 223.46 \text{ N}$$

Alternatively

Invoking Lami's theorem

$$\frac{T}{\sin 90°} = \frac{100}{\sin(90+\theta)} = \frac{200}{\sin(180-\theta)}$$

or

$$\frac{T}{\sin 90} = \frac{100}{\cos\theta} = \frac{200}{\sin\theta}$$

and hence

$$\tan\theta = \frac{200}{100} = 2\ ; \quad \theta = 63.55°$$

$$T = \frac{\sin 90}{\cos 63.55} \times 100 = \frac{1}{0.4454} \times 100 = 224.52$$

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Two smooth spheres P, Q each of radius 25 cm and weighing 500 N, rest in a horizontal channel having vertical walls (Fig. 2.72). If the distance between the walls is 90 cm, make calculations for the pressure exerted on the wall and floor at points of contact A, B and C.

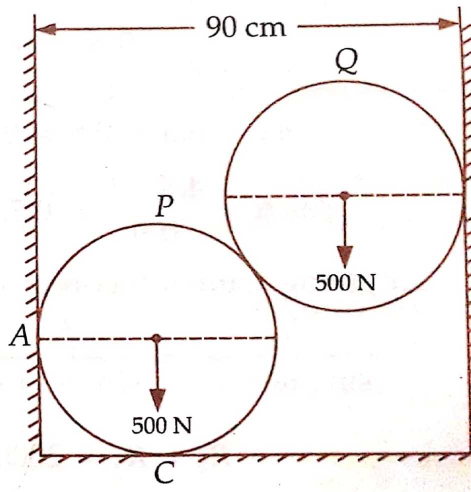

Fig. 2.72

Solution : The following points need consideration

(*i*) the spheres are smooth and as such the pressures at various points of contact would be normal to the surface.

(*ii*) at the point of contact between the two spheres, the reactions would act along the line joining their centres.

With reference to Fig. 2.73, the line $C_1 C_2$ makes an angle a with the horizontal line passing through centre C_1 of sphere *P*.

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$$\cos\alpha = \frac{b-r-r}{2r} = \frac{90-25-25}{50} = \frac{40}{50}$$

$\therefore \qquad \alpha = 36.87°$

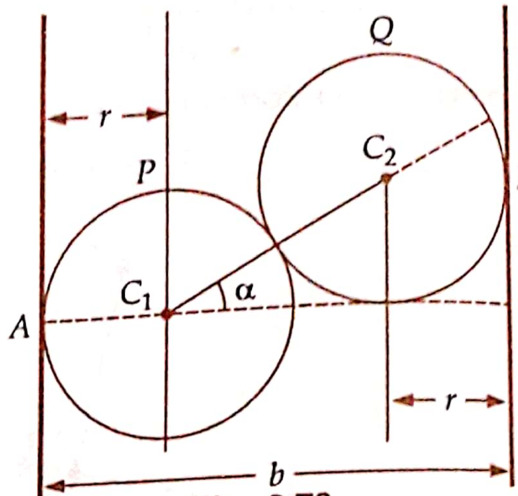


Fig. 2.73

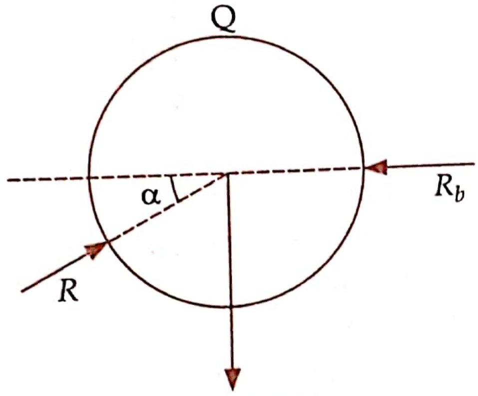
Considering the equilibrium of sphere Q

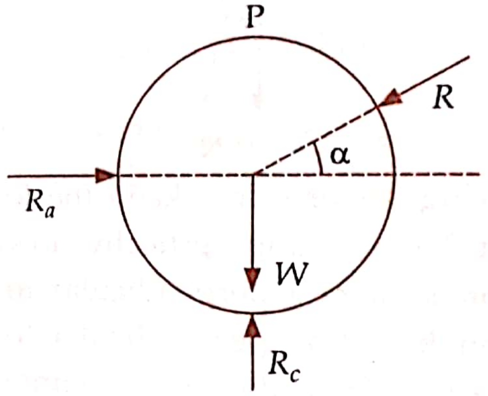
$$\Sigma F_x = 0; \quad R_b - R\cos\alpha = 0$$

$$\Sigma F_y = 0; \quad R\sin\alpha - 500 = 0$$

$$\therefore \qquad R = \frac{500}{\sin\alpha} = \frac{500}{\sin 36.87} = 833.33 \text{ N}$$

$$R_b = R\cos\alpha = 833.33 \times \cos 36.87 = 666.66 \text{ N}$$





Considering equilibrium of sphere P,

$$\Sigma F_x = 0; \quad R_a - R\cos\alpha = 0$$

$$R_a = R\cos\alpha = 833.33 \times \cos 36.87 = 666.67 \text{ N}$$

$$\Sigma F_y = 0; \quad R_c - W - R\sin\alpha = 0$$

$$R_c = W + R\sin\alpha = 500 + 833.33 \times \sin 36.87 = 1000 \text{ N}$$

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