TUTORIAL - 2 (EMT)

① An electromagnetic wave travelling in z direction in an unbounded lossless dielectric medium with relative permeability $\mu_r = 1$ and relative permitivity $\epsilon_r = 2$. It has peak electric field strength $E_0 = 5$ V/m.
Find (i) Impedence of the medium
(ii) Magnetic field intensity
(iii) Velocity of the wave

② The maximum value of electric field in an electromagnetic wave in vaccum is 800 V/m. Find the maximum value of magnetic intensity and average value of Poynting vector.

③ If the average distance between the sun and earth is 1.5×10^{11} m and the power radiated by sun is 3.8×10^{26} watt, show that the average solar energy incident on earth's surface is 2 cal / cm^2 min.

④ Assuming that all the energy from a 1000 W lamp is radiated uniformly, calculate the average values of intensities of electric and magnetic fields of radiations at a distance of 2m from the ~~lamp~~ lamb.

⑤ Find the conduction and displacement current densities in a material having conductivity 10^{-4} S/m and relative permitivity $\epsilon_r = 2.25$. The electric field in a material is $E = 5 \times 10^{-6} \sin(9 \times 10^9)t$ V/m.

⑥ The constituent parameters of aluminium are given by $\mu_r = 1$, $\epsilon_r = 1$ and $\sigma = 3.54 \times 10^7$ mho/m. Find the frequency for which the skin depth of aluminium is 0.01 mm.

7. ~~The~~ Calculate the penetration depth for 2 MHz electromagnetic wave through copper. Given $\sigma = 5.8 \times 10^{7}$ S/m. $\mu = 4\pi \times 10^{-7}$.

8. For copper $\sigma = 58$ MS m^{-1}, for teflon $\sigma = 30$ nS m^{-1} and $\epsilon = 2.1\,\epsilon_0$. Verify that 1 MHz copper is a good conductor and teflon is a good dielectric.

9. Earth is considered to be a good conductor when $\frac{\omega\epsilon}{\sigma} << 1$. Determine the highest frequency for which earth can be considered as a good conductor taking $\frac{\omega\epsilon}{\sigma} = 0.1$ ($\sigma = 5 \times 10^{-3}$ S/m)

10. After which frequency earth may be considered as perfect dielectric? Assume $\frac{\sigma}{\omega\epsilon} = \frac{1}{100}$. Given $\sigma = 5 \times 10^{-3}$ S/m, $\mu_r = 10$, $\epsilon_r = 8$.

11. Calculate the skin depth, propagation constant and wave velocity at a frequency of 1.6 MHz in aluminium where $\sigma = 38.2$ MS/m and $\mu_r = 1$.

12. Calculate the refractive index of copper at 10 MHz. Assume that conductivity of copper is 5.8×10^{7} mho/m and relative permeability and permittivity is unity.

SOLUTION OF TUTORIAL-2 (EMT)

(1) Given $\mu_r = 1$, $\epsilon_r = 2$, $E_0 = 5$ V/m

(i) Impedence of the medium $Z = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}}$

or $Z = \sqrt{\frac{\mu_r}{\epsilon_r}} \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{1}{2}} \sqrt{\frac{4\pi \times 10^{-7}}{8.86 \times 10^{-12}}}$

$= \frac{376.6}{\sqrt{2}} = 266.296$. Ohms

(ii) Peak value of magnetic field.

$Z = \frac{E_0}{H_0} \Rightarrow H_0 = \frac{E_0}{Z}$

$H_0 = \frac{5}{266.296} = 0.0187 = 0.0187$ Amp/m

(iii) The velocity of wave

$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_r \epsilon_r \mu_0 \epsilon_0}} = \frac{1}{\sqrt{\mu_r \epsilon_r}\sqrt{\mu_0 \epsilon_0}}$

$= \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{3 \times 10^8}{\sqrt{1 \times 2}} = 2.121 \times 10^8$ m/s

(2) Given $E_0 = 800$ V/m (For free space $Z = Z_0 = 377$ Ohms)

$H_0 = \frac{E_0}{Z_0} \Rightarrow H_0 = \frac{800}{377} = 2.12$ A/m

$\langle S \rangle = \frac{1}{2} E_0 H_0 = \frac{800 \times 2.12}{2} = 848$ W/m^2

(3) Let $r = 1.5 \times 10^{11}$ m, Power $P = 3.8 \times 10^{26}$ Watt

$S = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi r^2} = \frac{3.8 \times 10^{26}}{4\pi \times (1.5 \times 10^{11})^2}$

$= 1344.656$ W/m^2

Average solar energy ~~I~~ per minute is

$= \frac{1344.656 \times 60}{4.18 \times (10^{+2})^2} = 1.9 \simeq 2$ Cal/cm^2 min

④ Given $R = 2m$. , $P = 1000W$

If the total power is radiated uniformly in all directions, then the power or energy flux per unit area per sec at a distance R from the source is

$$S = \frac{P}{4\pi R^2} = \frac{1000}{4\pi(2)^2} = \frac{1000}{16\pi}$$

But from definition $|S| = |E \times H| = EH \sin 90 = EH$

$$\Rightarrow \quad EH = \frac{1000}{16\pi} \quad \text{——— ①}$$

For vaccum or free space impedance $\frac{E}{H} = Z = 377$ ohm. → ②

$$\frac{E}{H} = 377 \quad \text{——— ②}$$

$$① \times ② \Rightarrow EH \times \frac{E}{H} = \frac{1000}{16\pi} \times 377$$

$$\Rightarrow E^2 = \frac{377000}{16\pi}$$

$$\Rightarrow E = 86.59 \text{ V/m}$$

$$\text{Hence } H = \frac{E}{377} = \frac{86.59}{377} = 0.23 \text{ A/m.}$$

⑤ Given for any medium $\epsilon_R = 2.25$, $\sigma = 10^{-4}$ S/m

$$E = 5 \times 10^{-6} \sin(9 \times 10^9 t) \text{ V/m}$$

Conduction current density $J = \sigma E$

$$J = 10^{-4} \times 5 \times 10^{-6} \sin(9 \times 10^9 t)$$

Displacement current density $J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(D)$

$$D = \epsilon_0 E = \epsilon_0 \epsilon_R E$$

$$J_d = \frac{\partial}{\partial t}(\epsilon_0 \epsilon_R E) = \epsilon_0 \epsilon_R \frac{\partial E}{\partial t}$$

$$= \epsilon_0 \epsilon_r \frac{\partial}{\partial t}(5 \times 10^{-6} \sin(9 \times 10^9 t)) = \cancel{5 \times 1}$$

$$= 2.25 \times 8.8 \times 10^{-12} \times 5 \times 10^{-6} \times 9 \times 10^9 \cos(9 \times 10^9 t)$$

$$= 8.91 \times 10^{-8} \cos(9 \times 10^9 t)$$

(6)

(6) Given $\mu_r = 1$, $\epsilon_r = 1$, $\sigma = 3.54 \times 10^7$ mho/m., $\delta = 0.01$ mm $= 0.0001$

Allumainium is a good conductor. Skin depth for good conductor $\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$

$\delta = 10^{-5}$ m

$\mu = \mu_0 \mu_r$

$= 4\pi \times 10^{-7} \times 1$

$\epsilon = \epsilon_0 \epsilon_r$

$= 8.$

$$\delta^2 = \frac{2}{\mu\sigma 2\pi\nu} \Rightarrow \nu = \frac{2}{2\pi\mu\sigma\delta^2}$$

$$\nu = \frac{1}{3.14 \times (4\pi \times 10^{-7}) \times 3.54 \times 10^7 \times (10^{-6})^2}$$

$$= 7.16 \times 10^9 \text{ Hz}$$

(7)

(7) Given $\sigma = 5.8 \times 10^7$ S/m $\quad \mu = 4\pi \times 10^{-7}$, $\nu = 2$ MHz find δ

$= 2 \times 10^6$ Hz

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} = \sqrt{\frac{2}{4\pi \times 10^{-7} \times 5.8 \times 10^7 \times 2\pi \times 2 \times 10^6}} = 4.58 \times 10^{-5}$$

$$= \sqrt{1.27 \times 10^{-6}} = 0.1125 \times 10^{-3}$$

$$= 0.1125 \text{ mm}$$

(8)

(8) For copper $\sigma = 58$ MS m^{-1} $\quad$ For Teflon $\sigma = 30$ nS m^{-1}

$\epsilon = (2.1)\epsilon_0$

For good conductor $\frac{\sigma}{\epsilon\omega} >> 1$ and for good dielectric $\frac{\sigma}{\omega\epsilon} << 1$

For copper
$$\frac{\sigma}{\epsilon\omega} = \frac{58 \times 10^6}{2\pi \times (1 \times 10^6) \times (8.85 \times 10^{-12} \times 2.1)}$$

$$\cancel{= 2.57 \times 10^{-4}} << 1$$

$$= 0.496 \times 10^{12} >> 1.$$

Hence copper is a good conductor.

For Teflon
$$\frac{\sigma}{\epsilon\omega} = \frac{30 \times 10^{-9}}{2\pi \times 1 \times 10^6 \times 8.854 \times 10^{-12} \times 2.1}$$

$$= 2.57 \times 10^{-4} << 1.$$

Hence Teflon is a good dielectric.

(9) $\sigma = 5 \times 10^{-3}$ S/m

~~$\nu = 0.899$ MHz.~~

(9) Given $\frac{\omega\epsilon}{\sigma} << 1$ $\quad \frac{\omega\epsilon}{\sigma} = 0.1$ $\quad (<<1)$.

$\epsilon = 10\epsilon_0$

$$\frac{\omega\epsilon}{\sigma} = 0.1 \Rightarrow \frac{2\pi\nu\epsilon}{\sigma} = 0.1$$

$$\Rightarrow \quad \nu = \frac{0.1 \times \sigma}{2\pi\epsilon} = \frac{0.1 \times 5 \times 10^{-3}}{2\pi \times 10 \times 8.85 \times 10^{-12}}$$

$$= 0.899 \times 10^6 = 0.899 \text{ MHz}$$

(10)

(10)

After which frequence earth may be considered as perfect dielectric.

Assume $\frac{\sigma}{\omega \epsilon_r} = \frac{1}{100}$ $\sigma = 5 \times 10^{-3}$ S/m, $\mu_r = 10$, $\epsilon_r = 8$.

In case of a perfect dielectric $\frac{\sigma}{\epsilon\omega} << 1$

$\frac{\sigma}{\epsilon\omega} = \frac{1}{100} << 1 \Rightarrow \frac{\sigma}{2\pi\nu\epsilon} = \frac{1}{100}$

$\Rightarrow \nu = \frac{100\,\sigma}{2\pi\epsilon} = \frac{100 \times 5 \times 10^{-3}}{2\pi \times 8.85 \times 10^{-12} \times 8.0} = 1.123$ GHz.

After 1.123 GHz earth may be considered as perfect dielectric.

(11)

(11)

Given $\nu = 1.6$ MH $= 1.6 \times 10^{6}$ Hz, $\sigma = 38.2$ $\mu_r = 1$

$\delta = \sqrt{\frac{2}{\sigma\mu\omega}} = \frac{1}{\sqrt{\pi\nu\mu\sigma}} = \frac{1}{\pi\nu\mu_0\mu_r\sigma}$

$= \frac{1}{\sqrt{\pi \times 1.6 \times 10^{6} \times 4\pi \times 10^{7} \times 1 \times 38.2 \times 10^{6}}} = 64.40\ \mu m$.

For a good conductor $\alpha = \beta = \frac{1}{\delta} = \frac{1}{64.40 \times 10^{-6}} = 1.55 \times 10^{4}$

$K = \alpha + i\beta = 1.55 \times 10^{4} + i(1.55 \times 10^{4})$

$|K| = \sqrt{\alpha^2 + \beta^2} = 10^{4}\sqrt{(1.55)^2 + (1.55)^2} = 21.96 \times 10^{3}$.

Velocity $v = \frac{\omega}{\alpha} = \omega\delta = 2\pi\nu\delta$

$= 2\pi \times 1.6 \times 10^{6} \times 64.40 \times 10^{-6}$

$= 647.2$ m/s.

(12)

For good conductor $\alpha = \beta = \frac{1}{\delta} = \sqrt{\frac{\sigma\omega\mu}{2}}$

Velocity of wave $v_p = \frac{\omega}{K} = \frac{\omega}{\alpha} = \frac{\omega}{\sqrt{\sigma\omega\mu/2}} = \sqrt{\frac{2\omega}{\sigma\mu}}$

$v_p = \sqrt{\frac{2 \times 2\pi \times 10 \times 10^{6}}{5.8 \times 10^{7} \times 4\pi \times 10^{-7}}} = 0.131 \times 10^{4}$ m/s.

Refractive index of copper $\mu_c = \frac{c}{v_p} = \frac{3 \times 10^{8}}{0.131 \times 10^{4}} = 2.29 \times 10^{5}$.