When a body slides over another body, a force is exerted at the surface of contact by the stationary body on the moving body. This resisting force is called the force of friction and acts in a direction opposite to the direction of motion. The frictional forces are present throughout in nature and exist to a considerable extent in all machines; no matter how accurately they are made. Friction is quite undesirable and needs to be mitigated in some machines and processes such as

- power screws
- bearings and gears
- flow of fluids in pipes

Its presence would cause loss of power, wearing out of parts and huge economic losses. However, the working of many devices such as

— friction brakes and clutches

— belt and rope drives

— holding and fastening devices

depends on friction and there the presence of friction forces is advantageous. Obviously friction is both a liability and a necessity, and is often referred to as necessary evil.

4.1. FRICTION AND ITS CHARACTERISTICS

Consider a block resting on a horizontal plane. Experience and observations indicate that the mating surfaces are generally rough and have irregularities. The irregularities are somewhat like hills and valleys. The valleys of one surface mesh with the hills of the other surface. Obviously, there is some degree of interlocking and wedging action between the contact surfaces.

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When a force P is applied to the block in a direction parallel to the surface of contact, a force F automatically gets induced, acts in a direction opposite to that in which motion is sought and offers resistance to the movement of block. This resisting force acts tangentially to the contact surface and is known as friction force. Such forces always develop whenever there exists a tendency for the sliding of contact surfaces. The friction is attributed to the interlocking of minute projections on the surface which oppose the relative motion.

Refer Fig. 4.2 for the idealised plot of friction force F and the tractive (applied) force P. Its salient features are:

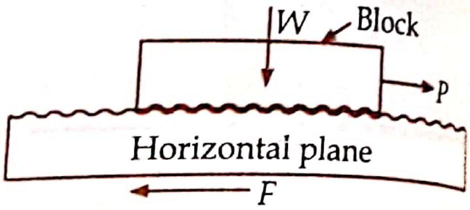

Fig. 4.1

(*i*) When static conditions prevail, *i.e.*, when there is no motion of the block, balance between applied force P and the friction force F is maintained. The frictional force increases in direct proportion to the applied force and this is depicted by line 0-1 which slopes at 45° with the axis of applied force. Line 0-1 represents the *zone of static friction* and point 1 corresponds to the maximum static friction.

(*ii*) There comes a limit beyond which frictional force cannot increase and the block is set into motion. The friction force at this instant is maximum and is called the limiting friction.

The *limiting friction* is the maximum friction force at the time of impending motion, i.e., when the motion is about to begin.

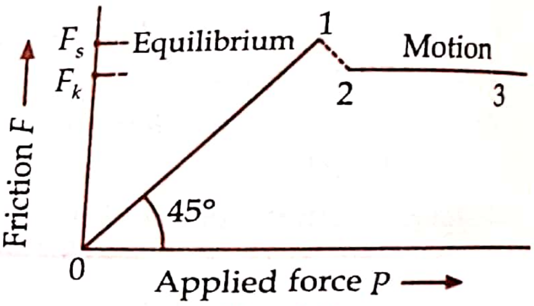

Fig. 4.2

(*iii*) When the motion of block commences, the friction force drops to somewhat low value called the kinetic friction. The decrease may be atribtuted to the fact that the chance of meshing of hills and valleys of the two surfaces becomes less. Such a stage is represented by straight line 2-3 of the graph and it is known as zone of *kinetic friction*:

Betwen points 1 and 2, the variation is uncertain and is shown by a dotted line.

The main characteristics of friction force are:

- It always acts in a direction opposite to that in which motion is intended; it always opposes the tractive force.
- It is a passive force; it exists as long as the tractive force acts.
- It is a self-adjusting force; only that much comes into play as is just sufficient to prevent motion.

4.2. NATURE OF FRICTION

There are essentially two types of friction, namely, dry friction and fluid friction.

4.2.1. Dry friction

Dry friction (also called *coulomb friction*) manifests when the contact sufaces are dry and there is tendency for relative motion.

Dry friction is further subdivided into:

- Sliding friction: friction between two surfaces when one surface slides over another.
- Rolling friction: friction between two surfaces which are separated by balls or rollers.

It may be pointed out that rolling friction is always less than sliding friction.

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4.2.2. Fluid Friction

Fluid friction manifests when a lubricating fluid is introduced between the contact surfaces of two bodies.

If the thickness of the lubricant or oil between the mating surfaces is small, then the friction between the surfaces is called *greasy* or *non-viscous friction*. The oil gets absorbd by the surfaces and the contact between them is no more a metal to metal contact. Instead the contact is through thin layer of oil and that ultimately results is less friction.

When the two surfaces are separated by a thick film of lubricant, metallic contact is entirely non-existent. The friction is due to viscosity of the oil, or the shear resistance between the layers of the oil rubbing against each other. Obviously then there occurs a great reduction in friction. This frictional force is knwon as *viscous* or *fluid friction*.

4.3. STATIC AND DYNAMIC FRICTION

The *static friction* is the frictional force that develops between mating surfaces when subjected to external forces but there is no relative motion between them.

The *dynamic friction* is the frictional force that develops between mating surfaces when subjected to external forces and there is relative motion between them. The dynamic friction is also known as *kinetic friction*.

The kinetic friction is always somewhat less than the maximum static friction force.

4.4. LAWS OF SOLID FRICTION (STATIC OR DYNAMIC)

Based upon experimental evidence, the following laws of friction have been established for dry contact surfaces.

(*i*) Friction acts tangential to the surfaces in contact and is in a direction opposite to that in which motion is to impend, i.e., take place.

(*ii*) Friction force is maximum at the instant of impending motion. Its variation from zero to maximum value (limiting friction) depends upon the resultant force tending to cause motion.

(*iii*) The magnitude of limiting friction bears a constant ratio to the normal reaction between the mating surfaces.

This ratio drops to a slightly lower value when the motion starts.

(*iv*) Limiting friction is independent of the area and shape of contact surfaces.

(*v*) Limiting friction depends upon the nature (roughness or smoothness) of the surfaces in contact.

(*vi*) At low velocities between sliding surfaces, the friction force is practically independent of the velocity. However, slight reduction in friction occurs when the speeds are high.

4.5. TERMS RELATED TO FRICTION

4.5.1. Coefficient of Friction

The frictional force F depends upon the wedging action between irregularities of contact surfaces and becomes more pronounced with increase in normal reaction R between them. Mathematically $F \propto R$. The ratio F/R is called *coefficient of friction* and is denoted by μ.

The coefficient of friction is defined as the ratio of force of friction to the normal reaction between the contact surfaces.

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The coefficient of friction would be high if the contact surfaces are rough. Obviously, the coefficient of friction measures the roughness between a pair of contact surfaces and incorporates the geometric property of the mating contours.

(*i*) When the system is in a state of impending motion, the frictional force has the limiting (maximum) value F_s and the ration F_s/R is called the coefficient of static friction μ_s. Thus $\mu_s = F_s/R$.

(*ii*) When motion starts, the maximum friction falls to a lower value F_k called kinetic friction. Experimental results indicate that the friction force developed during motions remain fairly constant. The ratio F_k/R is called the coefficient of kinetic friction μ_k. Thus $\mu_k = F_k/R$.

The value of μ_k is found to be less than μ_s for the same pair of contact surfaces.

(*iii*) The approximate values of static friction coefficients for pair of contacting surfaces are:

- wood on wood = 0.3 to 0.5
- metal on metals = 0.15 to 0.25
- rubber tyre on concrete = 0.7 to 0.9
- wood on metals = 0.2 to 0.6
- masonry on clay = 0.3 to 0.5.

4.5.2. Total Reaction

The block *A* resting on a rough surface is in equilibrium under the action of following forces:

(*i*) weight *W* of the block, acting downwards through its centre of gravity

(*ii*) normal reaction *R* between the block and surface

(*iii*) frictional force $F = \mu R$ where μ is the coefficient of friction

(*iv*) tractive force *P*

The forces *R* and *F* act at right angles to each other. Their resultant equals $\sqrt{R^2 + F^2}$ and is called the total reaction or resultant reaction.

4.5.3. Angle of Friction

It is defined as the angle which the resultant of normal reaction and limiting force of friction makes with the normal reaction.

With reference to Fig. 4.3

R = normal reaction;

F = limiting force of friction

$$S = \sqrt{R^2 + F^2}$$

= total or resultant reaction

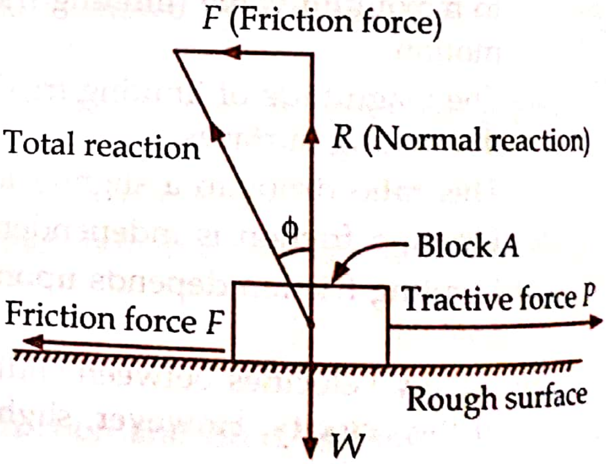

Fig. 4.3

$$\tan\phi = \frac{F}{R}$$ where ϕ is the angle of friction

The ratio F/R is also called the coefficient of friction, μ.

$$\therefore \quad \mu = \tan\phi \qquad ...(4.1)$$

The angle of friction, ϕ, is a measure of the limiting position of total reaction between the contacting surfaces.

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4.5.4. Angle of Repose

Consider a block of weight W resting on an inclined plane OA making an angle α with the horizontal. Let the angle α be increased gradually till the block is just at the point of sliding. The block is then in equilibrium state under the influence of following set of forces:

(*i*) weight W of the block acting vertically downwards,

(*ii*) normal reaction R acting at right angles to the inclined plane, and

(*iii*) limiting forces of friction, $F = \mu R$; acting up the plane as the block is to slide downwards.

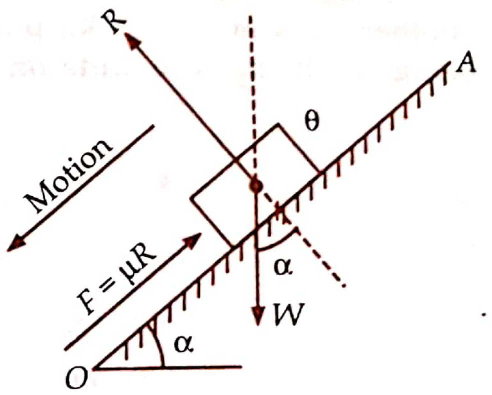

Fig. 4.4

Resolving these forces along and perpendicular to the plane

$$\mu R = W \sin \alpha$$

$$R = W \cos \alpha$$

These two expressions give : $\mu = \tan \alpha$

In terms of angle of friction ϕ, the coefficient of friction is given as: $\mu = \tan \phi$

$$\therefore \quad \tan \phi = \tan \alpha \quad \text{or} \quad \phi = \alpha \qquad \text{...(4.2)}$$

The angle α of the inclined plane at which a block resting on it is about to slide down the plane is called the angle of repose, and it is equal to the angle of friction between the block and the inclined plane.

4.5.5 Cone of Friction

With reference to Fig. 4.5, the lines of action of normal reaction R and the frictional force F meet at point O. The magnitude of total reaction is $\sqrt{F^2 + R^2}$ and it makes an angle ϕ with the normal reaction. Obviously O is the vertex of a cone whose axis is R and semi-vertex angle is ϕ. Such a right circular cone is called the cone of friction.

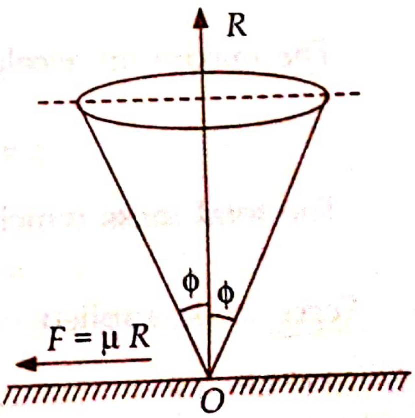

Fig. 4.5

It is worthwhile to point out that in case of impending motion, the friction force is maximum and the total reaction lies on the surface of cone. When the friction force is less than the limiting friction, the total reaction would lie within the cone. In that case, the forces acting on the body are not sufficient to cause motion. This aspect forms the working principle for self-locking mechanisms.

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EXAMPLE 4.1

(*a*) A block of weight 120 N is placed on a rough horizontal plane. If a horizontal force of 50 N just causes the body to slide over the horizontal plane, calculate the coefficient of friction between the block and the plane, and the angle of friction.

(*b*) A force of 10 N is required to start a uniform plank of wood slide over a bench for which the coefficient of friction is constant. The plank is cut into three equal pieces and these pieces are stacked one above the other. What force will be now needed to start movement of the pile?

(*c*) A block of mass 80 kg rests on a floor and the static coefficient of friction for the contact surface is 0.15. It is acted upon by a horizontal force of 50 N as shown in Fig. 4.6. What frictional force will be developed along the contact surface?

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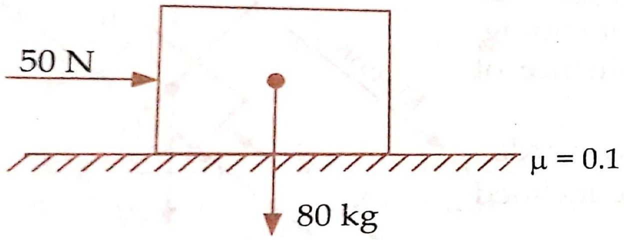
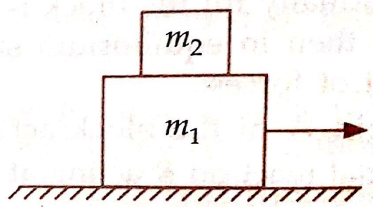
(*d*) **Refer Fig. 4.7 which shows a mass m_1 = 0.5 kg resting on a frictionless surface and another mass m_2 = 0.25 kg placed above m_1; the coefficient of friction between m_1 and m_2 being 0.3. If m_2 is to slide on m_1, what amount of force is needed to pull mass m_1.**





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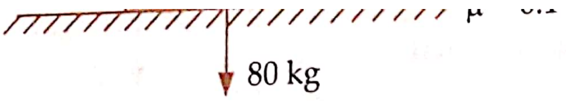

Fig. 4.6

Fig. 4.7

Solution : (*a*) $\mu = \frac{F}{R} = \frac{50}{120} = 0.4167$

Also $\mu = \tan \phi$, where ϕ is the angle of friction

$\phi = \tan^{-1}(\mu) = \tan^{-1}(0.4167) = 22.62°$

(*b*) $F = \mu R$. The coefficient of friction μ and the normal reaction $R = mg$ remain constant. As such the same force of 10 N would be needed to start the movement of pile system.

(*c*) Maximum value of static friction $= \mu R = \mu mg$

$= 0.15 \times (80 \times 9.81) = 117.72$ N

When the applied force is less than this maximum value, the friction force is obtained by the equations of equilibrium. $\Sigma F_y = 0$ and that gives $F = 50$ N.

(*d*) Frictional force between mass m_1 and $m_2 = \mu R = \mu \; (m_2 g)$

$= 0.3 \times (0.25 \times 9.81) = 0.736$ N

The maximum accelaration with which it can move without sliding

$$a = \frac{0.736}{0.25} = 2.944 \text{ N} \quad (F = m\,a)$$

The total mass which the applied force has to move

$= 0.25 + 0.5 = 0.75$ kg

Force to be applied $= 0.75 \times 2.944 = 2.208$ N

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A block of mass 25 kg is at rest on a horizontal floor. To slide the block, a force P is applied along the horizontal direction. The coefficient of static friction and dynamic friction between the block and floor are stated to be 0.36 and 0.3 respectively. For each value of P given below, work-out the corresponding value of frictional force F and complete the table.

Applied force P (N)	20	40	60	80	88.3	90
Frictional Force F (N)						

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Solution : Weight of the block $W = 25 \times 9.81 = 245.25$ N

Normal reaction $R = W = 245.25$ N

Limiting value of force of friction $F = \mu_s R = 0.36 \times 245.25 =$ **88.3 N**

The block will remain at rest till the applied force is less than 88.3 N

When the applied force exceeds μR, the block starts sliding. The dynamic conditions then prevail and the frictional force reduces to

$$F = \mu_d R = 0.3 \times 245.25 = \mathbf{73.57\ N}$$

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Applied force (N)	20	40	60	80	88.3	90
Frictional Force (N)	20	40	60	80	88.3	73.57

PLE 4.3

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EXAMPLE ...

A body weighing 300 N is resting on a rough horizontal table. A pull of 100 N applied at an angle of 15 degree with the horizontal just causes the body to slide over the table. Make calculations for the normal reaction and the coefficient of friction.

Solution : In the limiting equilibrium, the forces are balanced. That is

$$\Sigma F_x = 0 ; \quad F - 100 \cos 15^\circ = 0$$

$$F = 100 \cos 15^\circ$$

$$= 96.59 \text{ N}$$

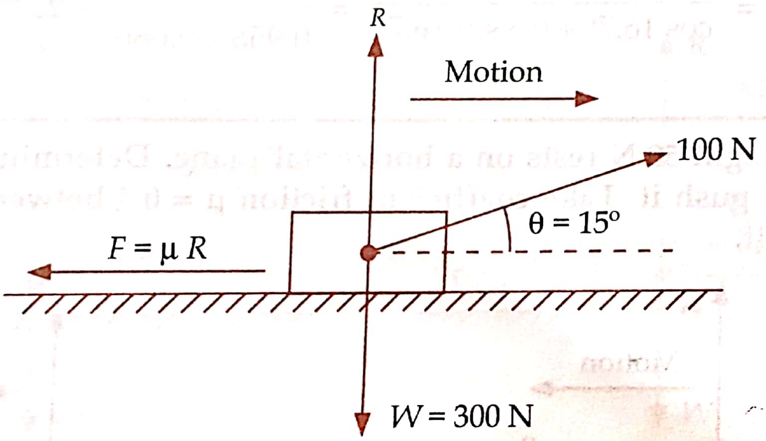

Fig. 4.8

$$\Sigma F_y = 0 ; \quad R + 100 \sin 15^\circ - 300 = 0$$

$$R = 300 - 100 \sin 15^\circ$$

$$= 300 - 25.88 = \mathbf{274.12 \text{ N}}$$

$$\mu = \frac{F}{R} = \frac{96.59}{274.12} = \mathbf{0.352}$$

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A wooden block of weight 50 N rests on a horizontal plane. Determine the force required to just (*a*) pull it, and (*b*) push it. Take coefficient friction μ = 0.4 between the mating surfaces. Comment on the result.

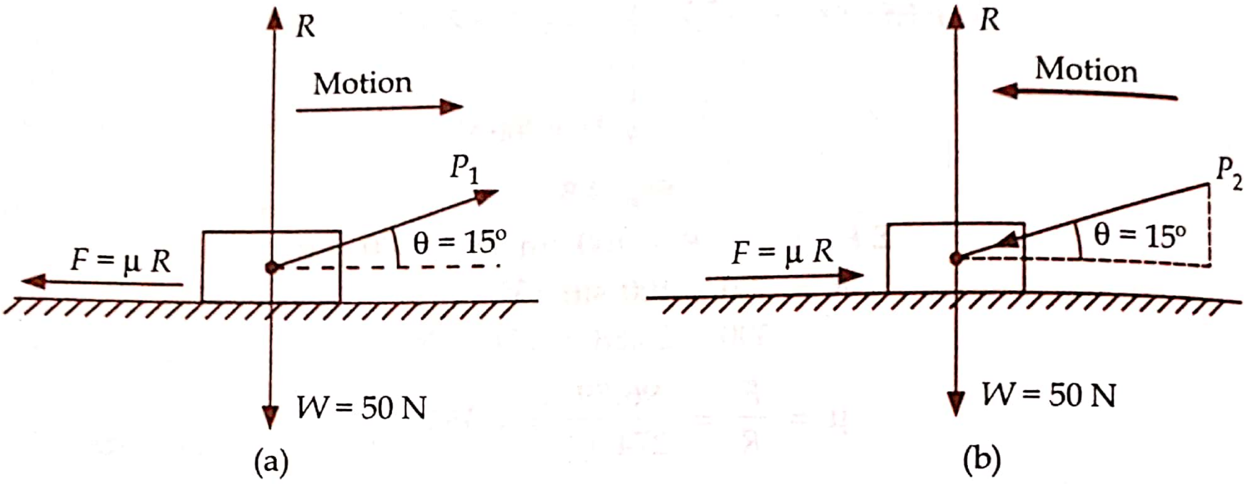

Fig. 4.10

Solution : Let P_1 be the force required to just pull the block. In the limiting equilibrium, the forces are balanced. That gives

$$\Sigma F_x = 0; \quad F = P_1 \cos\theta$$

$$\Sigma F_y = 0; \quad R = W - P_1 \sin\theta.$$

Also $\quad F = \mu R \quad \therefore \quad \mu (W - P_1 \sin\theta) = P_1 \cos\theta$

or $$P_1 = \frac{\mu W}{\cos\theta + \mu\sin\theta} = \frac{0.4 \times 50}{\cos 15° + 0.4 \sin 15°} = \frac{0.4 \times 50}{0.966 + 0.103} = 18.70 \text{ N}$$

(*b*) Let P_2 be the force required to just push the block. With reference to the free body diagram [Fig. 4.10 (*b*)],

Let us write the equations of equilibrium,

$$\Sigma F_x = 0; \quad F = P_2 \cos\theta$$

$$\Sigma F_y = 0; \quad R = W + P_2 \sin\theta$$

Also $\quad F = \mu R$

$\therefore \quad \mu (W + P_2 \sin\theta) = P_2 \cos\theta$

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or $$P_2 = \frac{\mu W}{\cos\theta - \mu \sin\theta} = \frac{0.4 \times 50}{\cos 15^\circ - 0.4 \sin 15^\circ} = \frac{0.4 \times 50}{0.966 - 0.103} = \mathbf{23.17\ N}$$

Comments. It is easier to pull the block than push it.

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