A coplanar non-concurrent force system consists of a set of forces that lie in the same plane but the line of action of all the forces do not meet at a single point.

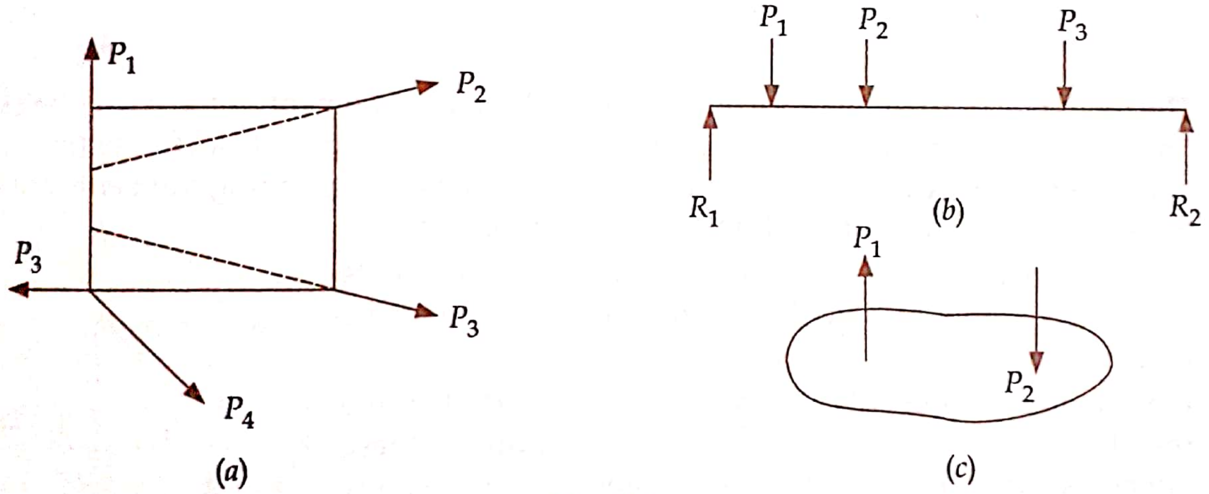

Fig. 3.1

Attention has been focussed in this chapter to study the effect of the resultant of these forces at a point which is away from the lines of action of the system of forces. Reference has also been made to the parallel forces and the effect they produce on a body. The general conditions for the equilibrium of a body have also been stated.

3.1. MOMENT OF A FORCE

Moment of force about a point is defined as the turning tendency of a force about that point. It is measured by the product of force and the perpendicular distance of the line of action of the force from that point.

With reference to Fig. 3.2,

P = force acting on a body

l = perpendicular distance between the point O and line of action of force P

Then moment of force P about point O

$$= P \times l$$

The point O is called the *moment centre* and the distance l is the *moment arm* or *arm of the force.*

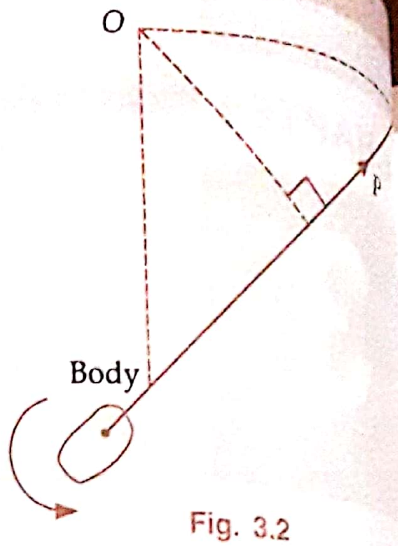

Fig. 3.2

- The moment of force about a point is a vector which is directed perpendicular to the plane containing the moment centre and the force.
- If force is measured in newton (N) and length in metres (m), then the units of force will be Nm.
- When a force acts on a body, it causes or tends to cause a change of state of rest or uniform rectilinear motion of the body. The action of moment tends to cause a rotational motion to the body.

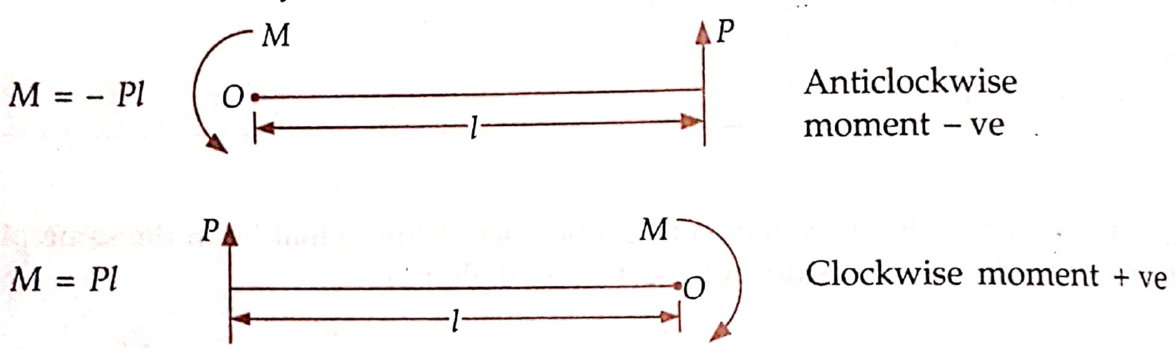

Fig. 3.3

- The tendency of rotation or turning of the body due to moment of force may be clockwise (in the same direction in which hands of the clock move) or anticlockwise (in a direction opposite to the movement of hands of a clock). The corresponding moments are referred to as clockwise moment and anticlockwise moment.

The general convention is to take the clockwise moments as positive and anticlockwise moments as negative (Fig. 3.3).

- If a number of moments act on a body (Fig. 3.4), then the resultant moment will be the algebraic sum of these moments, and the effect of resultant moment on the body will be governed by its turning tendency.

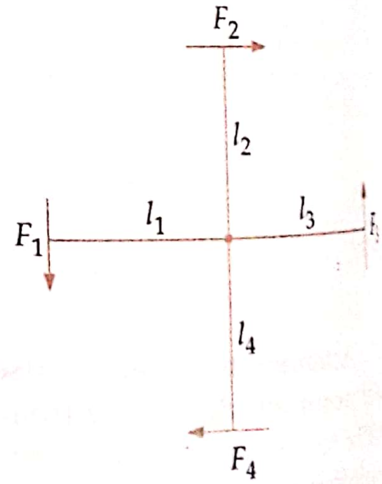

Fig. 3.4

With reference to Fig. 3.4, the resultant moment M about the point O is given by

$$M = -F_1 l_1 + F_2 l_2 - F_3 l_3 + F_4 l_4$$

3.2. GRAPHICAL REPRESENTATION OF MOMENT

Let a force P be represented in magnitude and direction by the vector AB. Further, let O be the point about which the moment is to be determined.

Moment of force P about point O,

$$= P \times OM$$

where OM is the perpendicular dropped from point O on line AB

$= AB \times OM = 2\left[\frac{1}{2} \times AB \times OM\right]$
$= 2$ (area of triangle OAB).

Hence the moment of force about any point is geometrically equal to twice the area of the triangle whose base is the line that represents the force and whose vertex is the point about which the moment is required to be found out.

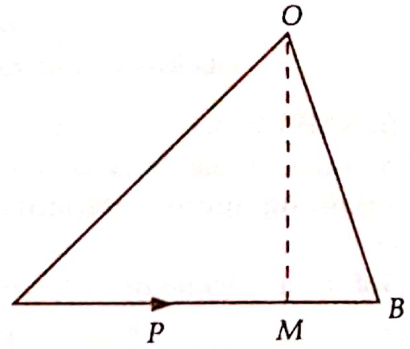

Fig. 3.5

3.3. VARIGNON'S THEOREM : LAW OF MOMENTS

"Moment of a resultant of two forces, about a point lying in the plane of the forces, is equal to the algebraic sum of moments of these two forces about the same point."

Consider two concurrent forces P and Q represented in magnitude and direction by AB and AC respectively. Let O be the point about which moment is to be taken. Through O, draw a line parallel to the direction of force P and let this line meet the line of action of force Q at point C. With AB and AC as adjacent sides, complete the parallelogram $ABCD$. The diagram AD of this parallelogram represents in magnitude and direction the resultant of forces P and Q.

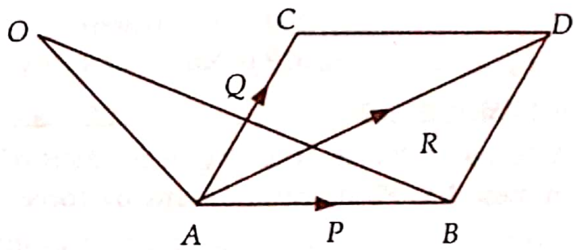

Fig. 3.6

Join O with points A and B.

Recalling that moment of force about a point is equal to twice of the area of the triangle so formed, we have

Moment of force P about $O = 2 \times$ area of triangle $AOB = 2 \times \Delta AOB$ ...(*i*)

Likewise,

moment of force Q about $O = 2 \times \Delta AOC$ ...(*ii*)

moment of force R about $O = 2 \times \Delta AOD$

From geometrical configuration of Fig. 3.6,

$$\Delta AOD = \Delta AOC + \Delta ACD = \Delta AOC + \Delta ABD$$

Further, the Δ's AOB and ABD are on the same base AB and between the same lines and as such are equal in area. Then

$$\Delta AOD = \Delta AOC + \Delta AOB$$

The moment of force R about O may then be re-written as

$$= 2 \times (\Delta AOC + \Delta AOB) \qquad ...(iii)$$

From the identities (*i*), (*ii*) and (*iii*), it is evident that

Moment of forces P and Q about pcint O

= moment of resultant R about O

This principle can be extended for any number of forces and the generalised law of moments may be stated as

"moment of resultant of a number of forces about a point lying in the plane of forces is equal to the algebraic sum of the moments of these forces about the some point."

3.4. PRINCIPLE OF MOMENTS

A body acted upon by a number of coplanar forces will be in equilibrium, if the algebraic sum of moments of all the forces about a point lying in the same plane is zero. Mathematically

$$\Sigma M = 0$$

i.e., clockwise moments = anticlockwise moments

EXAMPLE 3.1

A force of 200 N is acting at a point *B* as shown in the adjoining figure. Determine the moment of this force about *O*.

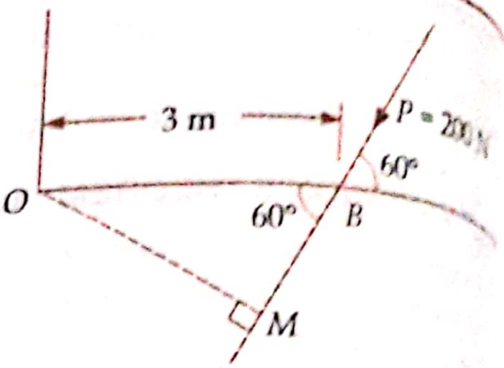

Fig. 3.7

Solution : Moment of force about point O

= force × (perpendicular distance between point O and the line of action of force)

$= P \times OM = P \times OB \cos 60^\circ$

$= 200 \times (3 \times 0.866)$

$=$ **519.6 Nm** (clockwise)

EXAMPLE 3.2

A force of 200 N is acting at point *B* of a lever *AB* which is hinged at its lower end as shown in Fig. 3.8. Find the moment of force about the hinged end.

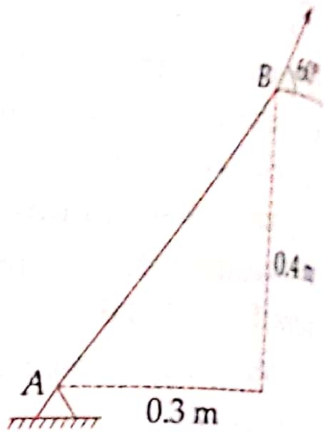

Fig. 3.8

Solution : Moment of force about point A

= force × (perpendicular distance between point A and the line of action of force)

$= P \times AM$

Now, $AB = \sqrt{0.4^2 + 0.3^2} = 0.5$ m

$$\alpha = \tan^{-1}\left(\frac{0.4}{0.3}\right) = 53.13^\circ$$

$$\gamma = 90 - 53.13 = 36.87^\circ$$

$$\beta = 36.87 - 30 = 6.87^\circ$$

$$AM = AB \sin 6.87^\circ = 0.5 \times 0.1196 = 0.0598 \text{ M}$$

$\therefore$ $M_a = 200 \times 0.0598 =$ **11.96 Nm** (anticlockwise)

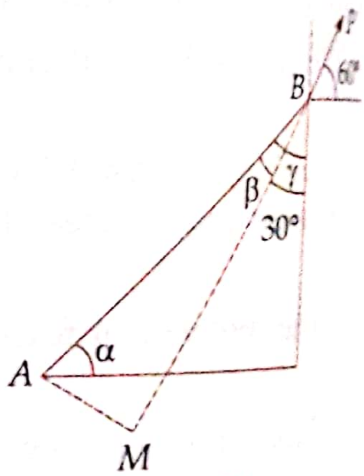

Fig. 3.9

Alternatively :

The horizontal and vertical components of the given force are:

$$P_x = 200 \cos 60^\circ = 100 \text{ N}$$

$$P_y = 200 \sin 60^\circ = 173.2 \text{ N}$$

From Varignon's theorem, moment of a given force about a specified point is equal to the sum of the moments of its components about that point.

$\therefore$ $M_a = 100 \times 0.4 - 173.2 \times 0.3$

$= 40 - 51.96 = -$ **11.96 Nm** (anticlockwise)

A uniform rodof 10 m length has a self weight of 5 N. The rod carries a weight of 30 N hung from one of its ends. From what point the rod be suspended so that it remains horizontal ?

Solution : The self weight of the rod 5 N acts at the mid point C of the rod.

Let the rod be suspended at point F which is at distance x from its end B where a load of 30 N is hung.

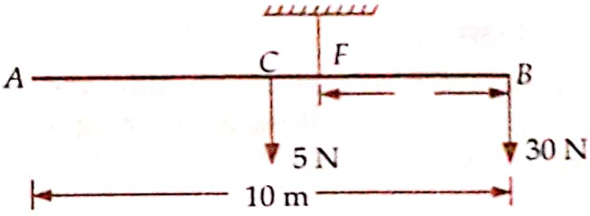

Fig. 3.11

Taking moments about point F, we get

$$5 \times CF - 30\,x = 0$$

$$5\,(BC - x) = 30\,x$$

or $$5\,(5 - x) = 30\,x$$

or $$25 - 5\,x = 30\,x$$

$$\therefore \quad x = \frac{25}{35} = \mathbf{0.7143\ m}$$

Hence for the rod to remain horizontal, it needs to be suspended from point F such that $BF = 0.7143$ m.

EXAMPLE 3.5

A uniform rod $ABCD$ weighs 8 N and is supported at B and C as shown in the figure given below. Determine the minimum force at A which will just overturn the rod.

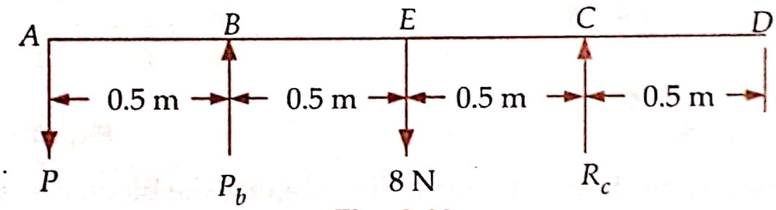

Fig. 3.12

Solution : Let P be the minimum force to be applied at end A which will just overturn the rod. At that instant reaction R_c will be zero.

Taking moments about point B,

$$8 \times 0.5 - P \times 0.5 = 0$$

$$\therefore \quad P = \frac{8 \times 0.5}{0.5} = \mathbf{8\ N}$$

EXAMPLE 3.6

A uniform rod AB of weight 100 N is hinged at A so as to be able to rotate about A in a vertical plane. It is held in horizontal position by a string attached to B and inclined at 60°

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A uniform wheel weighing 20 kN and of 600 mm diameter rests against 150 mm thick rigid block as shown in Fig. 3.17.

Considering all surfaces to be smooth, determine (a) the least pull through the centre of wheel to just turn the wheel over the corner of the block, (b) the reaction of the block.

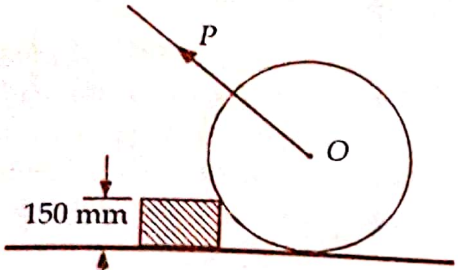

Fig. 3.17

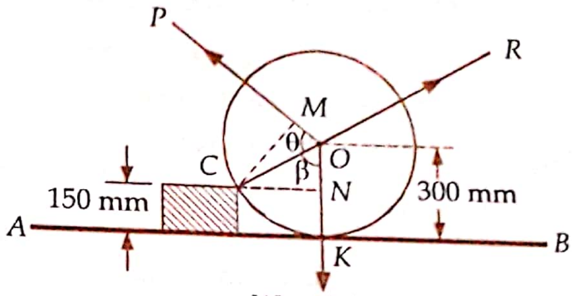

Fig. 3.18

Solution : When the wheel is about to turn, its contact with the ground will be lost. Hence the wheel has to be in equilibrium under the action of its weight W and the force P.

With reference to Fig. 3.18, CM and CN are the perpendiculars dropped on the lines of action of forces P and W respectively.

$$OC = OK = 300 \text{ mm}$$

$$ON = OK - NK = 300 - 150 = 150 \text{ mm}$$

$$CN = \sqrt{OC^2 - ON^2} = \sqrt{300^2 - 150^2} = 259.81 \text{ mm}$$

Taking moments about point C,

$$P \times CM - W \times CN = 0$$

$$P \times OC \sin\theta - 20 \times 259.81 = 0$$

$$\therefore \qquad P = \frac{20 \times 259.81}{OC \sin\theta} = \frac{20 \times 259.81}{300 \sin\theta} = \frac{17.32}{\sin\theta}$$

The force P will be minimum when $\sin\theta$ is maximum. For that

$$\sin\theta = 1 \quad \text{or} \quad \theta = 90°$$

Hence $\qquad P_{min}$ = **17.32 kN** when pull is applied perpendicular to OC.

(b) The reaction R can be determined by resolving W along OC

$$R = W \cos\beta \qquad \text{where} \quad \beta = \angle CON$$

$$\cos\beta = \frac{ON}{OC} = \frac{150}{300} = \frac{1}{2}$$

$$\therefore \qquad R = 20 \times \frac{1}{2} = \mathbf{10 \text{ kN}}$$

The force P acts perpendicular to OC and as such its resolved part along OC is zero.

EXAMPLE 3.11

A beam AB of length 2 m, hinged at A and supported at B by a cord which passes over two frictionless pulleys, carries a 50 kN load as shown in Fig. 3.21. Determine the position of 100 kN load on the beam if it is to remain in equilibrium in horizontal position. Also determine the reaction at the hinged end.

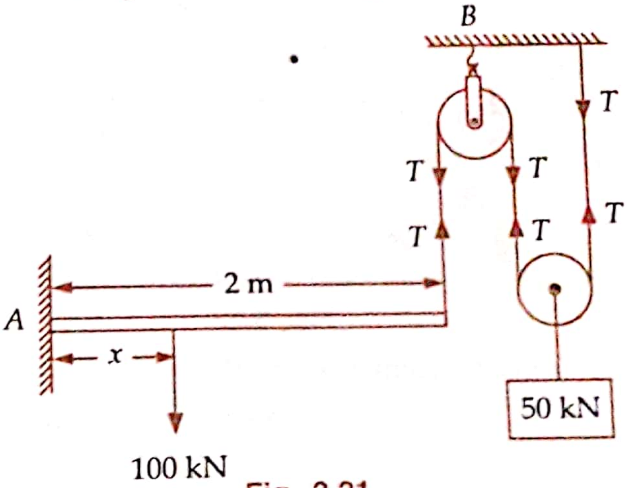

Fig. 3.21

Solution : Since the pulleys are weightless and frictionless, the tension T in the entire length of cord will be same.

For the equilibrium of pulley supporting 50 kN load (Fig. 3.21)

$$\Sigma F_y = 0; \quad T + T - 50 = 0; \quad T = 25 \text{ kN}$$

The free body diagram of the beam is shown in Fig. 3.22, and for its equilibrium

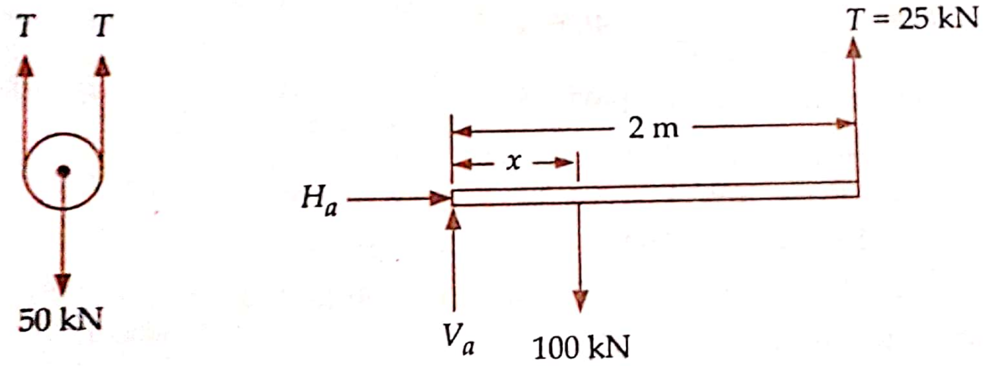

Fig. 3.22

$$\Sigma F_x = 0; \quad H_a = 0$$

$$\Sigma F_y = 0; \quad V_a - 100 + 25 = 0; \quad V_a = 75 \text{ N}$$

Therefore the reaction at the hinged end is 75 N in the vertical direction.

$$\Sigma M_a = 0; \quad 100 \times x - 25 \times 2 = 0, \quad x = 0.5 \text{ m}$$