

# $SP_N$ Update

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$$\hat{\Omega} \cdot \vec{\nabla} \psi(\vec{r}, \hat{\Omega}, E) + \sigma(\vec{r}, E) \psi(\vec{r}, \hat{\Omega}, E) = \int \int \sigma_s(\vec{r}, E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega}) \psi(\vec{r}, \hat{\Omega}', E') d\Omega' dE' + q(\vec{r}, \hat{\Omega}, E) \quad (1)$$

- $S_N$  transport is expensive
  - Difficult to parallelize: sweeps in space, pipelining in angle, energy decoupling
  - Large storage requirements
  - Ray effects
- $P_N$  equations still expensive
  - Complicated system:  $(N+1)^2$  equations in 3D
  - Coupling of equations through both angular moments and spatial derivatives



- Ad-hoc generalization of planar  $P_N$  equations by Gelbard in the 1960's
- Rigorous formulation through asymptotic and variational analysis in 1990's and 2000's
- Simpler system  $(N + 1)/2$  equations in  $3D$
- Yields elliptic, diffusion-like equations
- Applicable when diffusion theory is applicable: reasonable flux gradients, full-core transport
- Typically does not converge to transport solution as  $N \rightarrow \infty$
- **Can build the full linear operator**
- **Parallelism through the linear solver**

## $SP_N$ Equations

$$\begin{aligned} -\nabla \cdot \left[ \frac{n}{2n+1} \frac{1}{\Sigma_{n-1}} \nabla \left( \frac{n-1}{2n-1} \phi_{n-2} + \frac{n}{2n-1} \phi_n \right) \right. \\ \left. + \frac{n+1}{2n+1} \frac{1}{\Sigma_{n+1}} \nabla \left( \frac{n+1}{2n+3} \phi_n + \frac{n+2}{2n+3} \phi_{n+2} \right) \right] \\ + \Sigma_n \phi_n = q \delta_{n0} \quad n = 0, 2, 4, \dots, N, \quad (2) \end{aligned}$$

- Monte Carlo methods for have strong restrictions on the eigenvalues of the operator for convergence
- MCSA has the same restrictions on the outer stationary iteration
- We need to compute these eigenvalues for various forms of the  $SP_N$  equations to verify convergence of these methods.

We need eigenvalues for  $\mathbf{A}$ ,  $\mathbf{H}_J$ , and  $\mathbf{H}_{GS}$  with:

$$\mathbf{H}_J = \mathbf{I} - \mathbf{D}^{-1}\mathbf{A} \quad (3)$$

where  $\mathbf{D} = \text{diag}(\mathbf{A})$  and

$$\mathbf{H}_{GS} = (\mathbf{L} + \mathbf{D})^{-1}\mathbf{U} \quad (4)$$

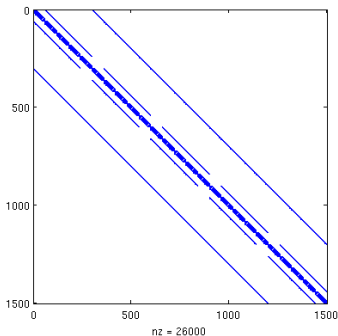


Figure: Linear operator sparsity pattern

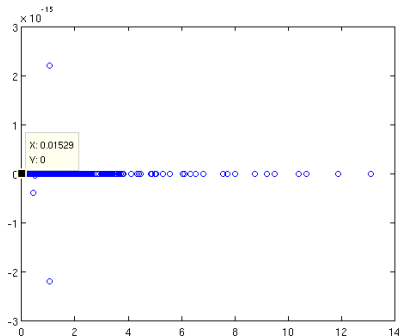


Figure: Linear operator eigenvalues

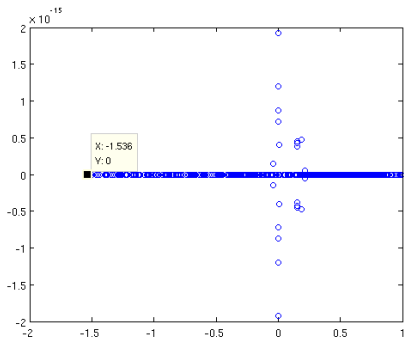


Figure: Jacobi iteration matrix eigenvalues

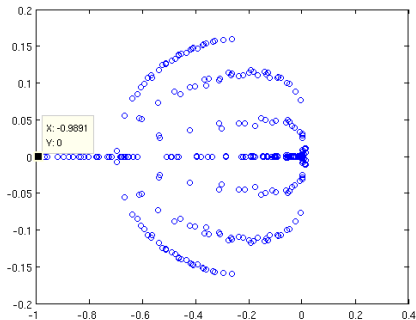
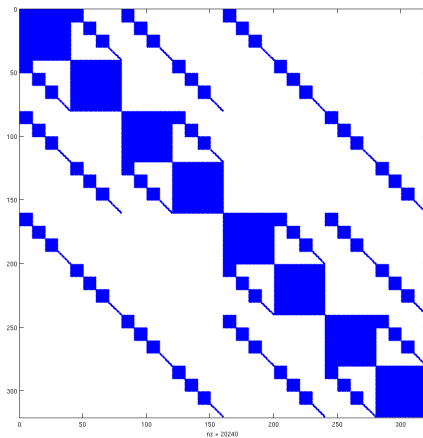


Figure: Gauss-Seidel iteration matrix eigenvalues

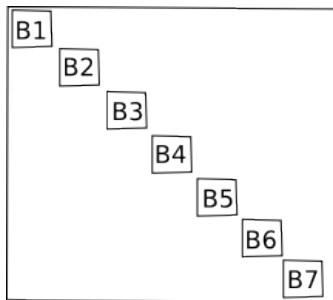
- The Jacobi method won't converge - all that stuff I said in my prelim won't work



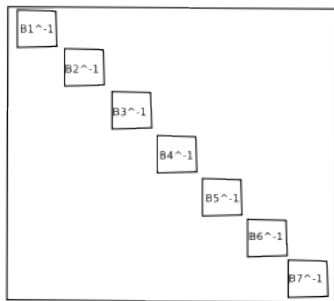
- Bug fix in Denovo  $SP_N$  implementation
- A new kind of preconditioning...



**Figure:** Linear operator sparsity pattern



$M$



$M^{-1}$

Figure: Block Jacobi Preconditioner

# Point Jacobi Results



## Base Parameters

Delta 1.00  
 Mesh elements 4x4x4  
 Num Materials 1  
 Reflecting boundaries

Source 1.00  
 Sigma<sub>t</sub> 5.00  
 Sigma<sub>ig</sub> 0.25  
 Sigma<sub>down</sub> 1.00  
 Sigma<sub>up</sub> 0.10

## POINT JACOBI SPECTRAL RADIUS

### 1 Energy Group

		SPN Order			
		1	3	5	7
PN Order	0	0.0635	0.6722	1.3144	1.976
	1	0.0666	0.6728	1.3141	1.9755
	3	0.0666	0.6822	1.3198	1.9762
	5	0.0666	0.6822	1.3278	1.9847
	7	0.0666	0.6822	1.3278	1.9914

### 10 Energy Groups without Upscatter

		SPN Order			
		1	3	5	7
PN Order	0	0.0655	0.677	1.32	1.983
	1	0.071	0.6777	1.319	1.982
	3	0.071	0.687	1.327	1.9872
	5	0.071	0.687	1.336	1.997
	7	0.071	0.687	1.336	1.9995

### 10 Energy Groups with Upscatter

		SPN Order			
		1	3	5	7
PN Order	0	0.7283	0.81	1.47	2.1446
	1	0.7317	0.8	1.46	2.1368
	3	0.7317	0.91	1.526	2.2274
	5	0.7317	0.91	1.5344	2.2562
	7	0.7317	0.91	1.5345	2.2842

<b>BLOCK JACOBI SPECTRAL RADIUS: <math>B = N_g * (N+1)/2</math></b>					
<b><i>1 Energy Group</i></b>					
		<b>SPN Order</b>			
		<b>1</b>	<b>3</b>	<b>5</b>	<b>7</b>
<b>PN Order</b>	<b>0</b>	0.0635	0.1269	0.1444	0.1513
	<b>1</b>	0.0666	0.1315	0.1474	0.1534
	<b>3</b>	0.0666	0.1365	0.154	0.1592
	<b>5</b>	0.0666	0.1365	0.1562	0.163
	<b>7</b>	0.0666	0.1365	0.1562	0.164
<b><i>10 Energy Groups without Upscatter</i></b>					
		<b>SPN Order</b>			
		<b>1</b>	<b>3</b>	<b>5</b>	<b>7</b>
<b>PN Order</b>	<b>0</b>	0.0647	0.1275	0.1449	0.1514
	<b>1</b>	0.0686	0.1338	0.1484	0.1547
	<b>3</b>	0.0687	0.1399	0.1582	0.1625
	<b>5</b>	0.0692	0.1402	0.1626	0.1657
	<b>7</b>	0.0678	0.1393	0.1624	0.166
<b><i>10 Energy Groups with Upscatter</i></b>					
		<b>SPN Order</b>			
		<b>1</b>	<b>3</b>	<b>5</b>	<b>7</b>
<b>PN Order</b>	<b>0</b>	0.1887	0.2267	0.2285	0.2286
	<b>1</b>	0.4535	0.5044	0.5045	0.5045
	<b>3</b>	0.4535	0.6453	0.6506	0.6506
	<b>5</b>	0.4535	0.6453	0.6802	0.6818
	<b>7</b>	0.4535	0.6453	0.6802	0.6927

- Block Jacobi preconditioning is a simple and appropriate solution for preconditioning the  $SP_N$  equations
- Implementation is general - slides right into ANA framework
- Implementation is scalable - local operations only, fast LAPACK operations
- Implementation works - can move on with research