

Parallel Monte Carlo Synthetic Acceleration Methods for Discrete Transport Problems

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- Monte Carlo Synthetic Acceleration Methods
- Application to Neutron Transport
- Application to Fluid Flow
- Parallelization of MCSA
- Summary



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The goal of this work is to improve the iterative performance and parallel scalability of solutions to discrete linear and nonlinear transport problems by researching and developing a new set of domain decomposed Monte Carlo Synthetic Acceleration methods.

- Modern hardware is moving in two directions (Kogge,2011):
 - Lightweight machines
 - Heterogeneous machines
 - Both characterized by low power and high concurrency
- Some issues:
 - Higher potential for both soft and hard failures (DOE,2012)
 - Memory restrictions are expected with a continued decrease in memory/FLOPS
- Potential resolution from Monte Carlo:
 - Soft failures buried within the tally variance
 - Hard failures are high variance events
 - Memory savings over conventional methods

- Development of a linear solver for discrete systems leveraging Monte Carlo Synthetic Acceleration
 - Application to neutron transport
 - Research is required to explore general solver development
 - Performance is of concern
- Development of a nonlinear solver for discrete systems leveraging Monte Carlo Synthetic Acceleration
 - Monte Carlo is a more natural fit
 - Application to fluid flow
 - Convergence of the linear model is of concern
- Parallelization of Monte Carlo Synthetic Acceleration
 - Parallel strategies taken from modern reactor physics methods
 - Research is required to explore varying parallel strategies
 - Scalability is of concern

- **Monte Carlo Synthetic Acceleration Methods**
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- First proposed by J. Von Neumann and S.M. Ulam in the 1940's
- Earliest published reference in 1950
- General lack of published work
- Modern work by Evans and others has yielded new applications

Thomas Evans and Scott Mosher, "A Monte Carlo Synthetic Acceleration method for the non-linear, time-dependent diffusion equation", American Nuclear Society - International Conference on Mathematics, Computational Methods and Reactor Physics, 2009.

- Split the linear operator

$$\mathbf{Ax} = \mathbf{b} \quad \rightarrow \quad \mathbf{x} = \mathbf{Hx} + \mathbf{b}$$

$$\mathbf{H} = \mathbf{I} - \mathbf{A}$$

- Generate the *Neumann series*

$$\mathbf{A}^{-1} = (\mathbf{I} - \mathbf{H})^{-1} = \sum_{k=0}^{\infty} \mathbf{H}^k$$

- Require $\rho(\mathbf{H}) < 1$ for convergence

$$\mathbf{A}^{-1}\mathbf{b} = \sum_{k=0}^{\infty} \mathbf{H}^k \mathbf{b} = \mathbf{x}$$

- Expand the Neumann series

$$x_i = \sum_{k=0}^{\infty} \sum_{i_1}^N \sum_{i_2}^N \cdots \sum_{i_k}^N h_{i,i_1} h_{i_1,i_2} \cdots h_{i_{k-1},i_k} b_{i_k}$$

- Define a sequence of state transitions

$$\nu = i \rightarrow i_1 \rightarrow \cdots \rightarrow i_{k-1} \rightarrow i_k$$

- Use the adjoint Neumann-Ulam decomposition

$$\mathbf{H}^T = \mathbf{P} \circ \mathbf{W}$$

$$p_{ij} = \frac{|h_{ji}|}{\sum_j |h_{ji}|}, \quad w_{ij} = \frac{h_{ji}}{p_{ij}}$$

The Hadamard product $\mathbf{A} = \mathbf{B} \circ \mathbf{C}$ is defined element-wise as $a_{ij} = b_{ij}c_{ij}$.

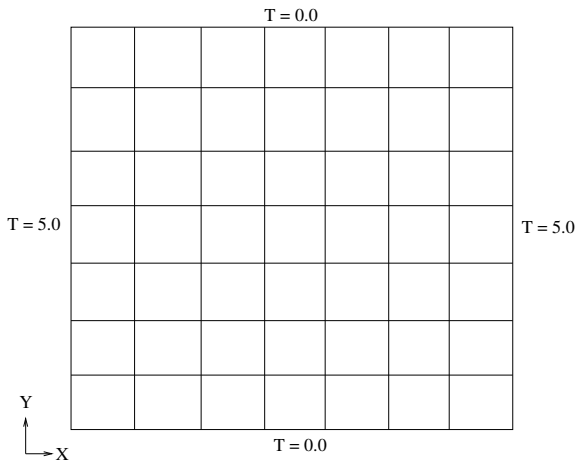


Figure: Poisson Problem. *Distributed source of 1.0 in the domain.*

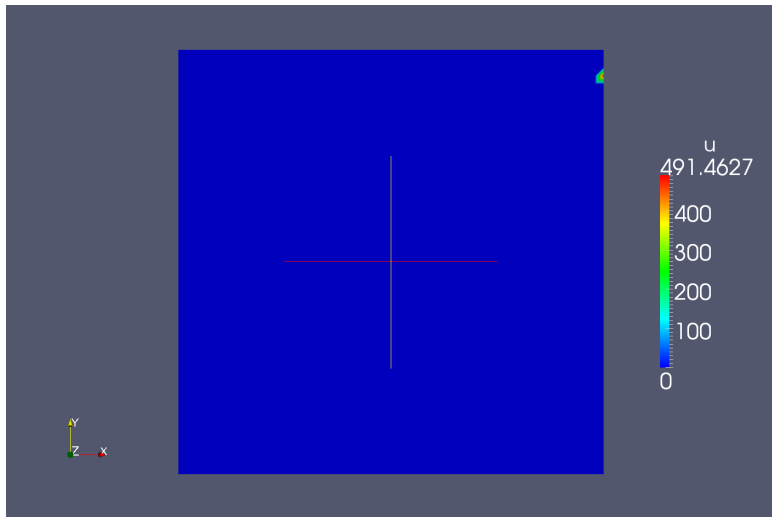


Figure: Adjoint solution to Poisson Equation. 1×10^0 total histories, 0.286 seconds CPU time.

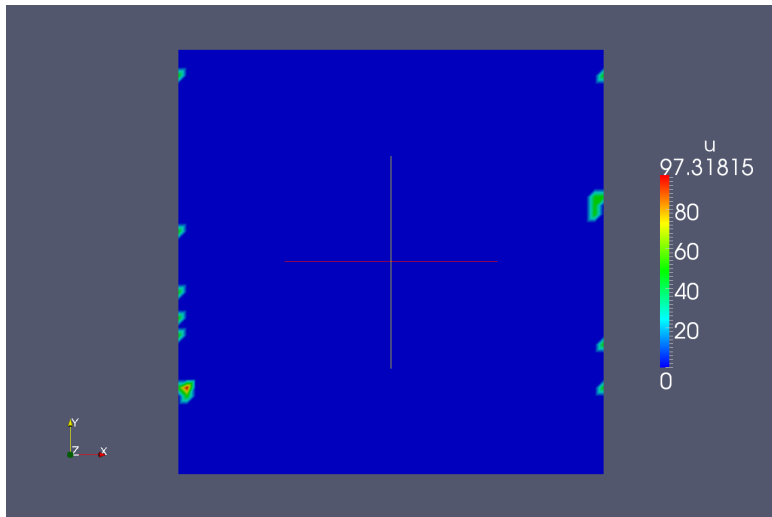


Figure: Adjoint solution to Poisson Equation. 1×10^1 total histories, 0.278 seconds CPU time.

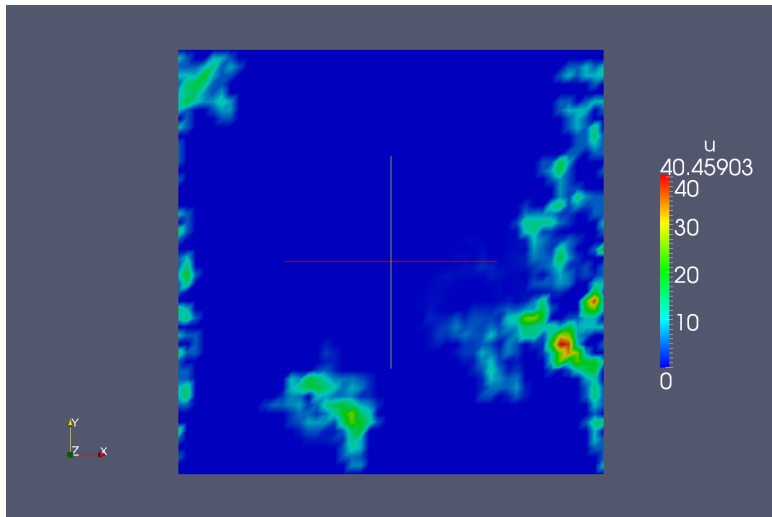


Figure: Adjoint solution to Poisson Equation. 1×10^2 total histories, 0.275 seconds CPU time.

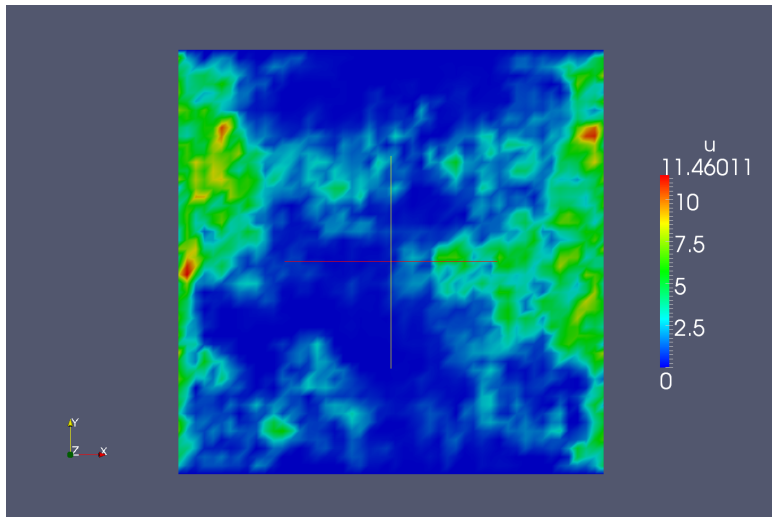


Figure: Adjoint solution to Poisson Equation. 1×10^3 total histories, 0.291 seconds CPU time.

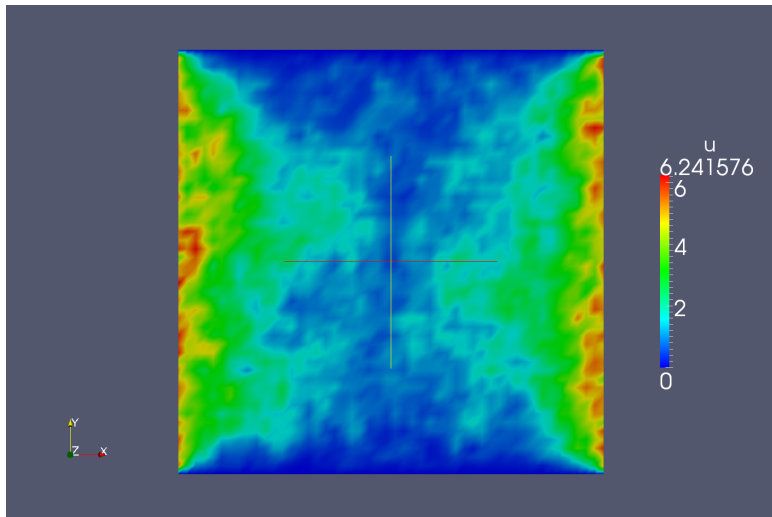


Figure: Adjoint solution to Poisson Equation. 1×10^4 total histories, 0.428 seconds CPU time.

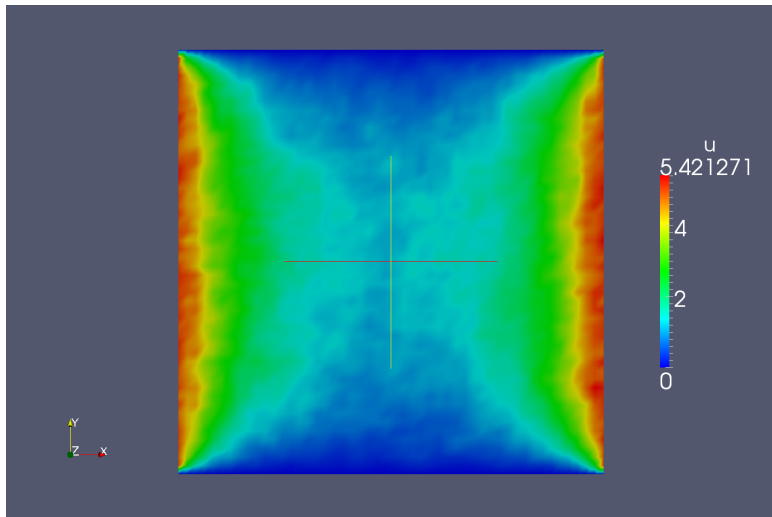


Figure: Adjoint solution to Poisson Equation. 1×10^5 total histories, 1.76 seconds CPU time.

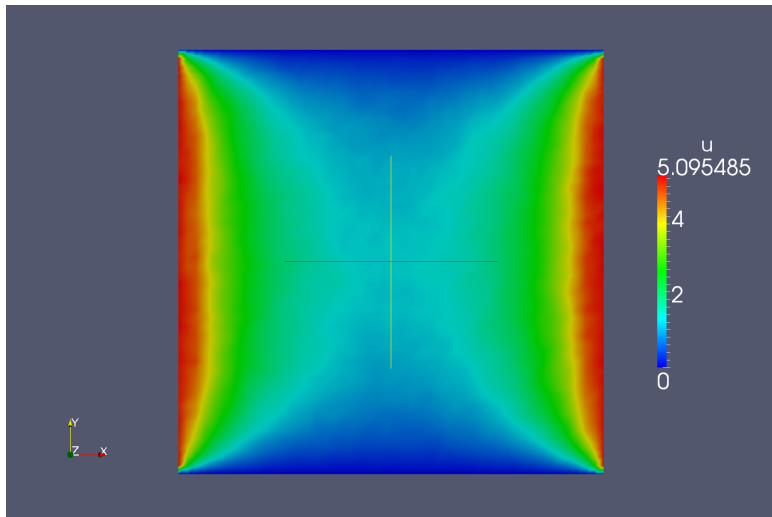


Figure: Adjoint solution to Poisson Equation. 1×10^6 total histories, 15.1 seconds CPU time.

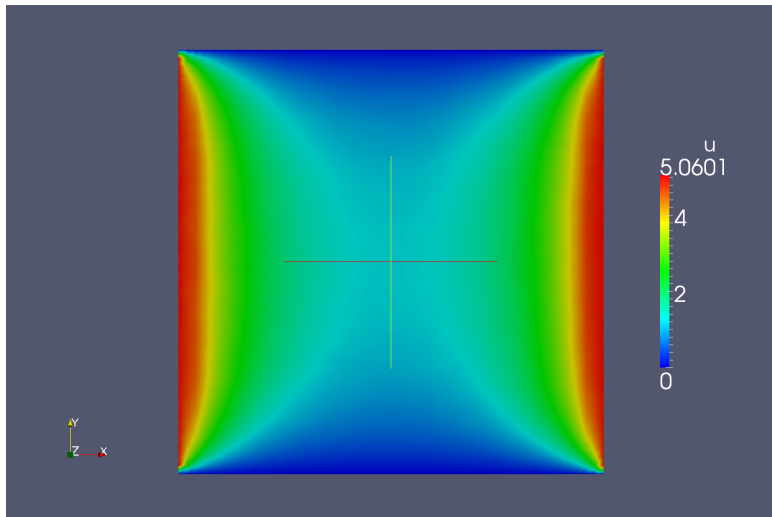


Figure: Adjoint solution to Poisson Equation. 1×10^7 total histories, 149 seconds CPU time.

MCSA Iteration

$$\mathbf{r}^k = \mathbf{b} - \mathbf{A}\mathbf{x}^k$$

$$\mathbf{x}^{k+1/2} = \mathbf{x}^k + \mathbf{r}^k$$

$$\mathbf{r}^{k+1/2} = \mathbf{b} - \mathbf{A}\mathbf{x}^{k+1/2}$$

$$\hat{\mathbf{A}}\delta\mathbf{x}^{k+1/2} = \mathbf{r}^{k+1/2}$$

$$\mathbf{x}^{k+1} = \mathbf{x}^{k+1/2} + \delta\mathbf{x}^{k+1/2}$$

- Neumann-Ulam methods bound by the Central Limit Theorem
- Build on Halton's 1962 Sequential Monte Carlo method
- Neumann-Ulam Monte Carlo solver computes the correction
- Decouples MC error from solution error, exponential convergence



- Designed to be easily incorporated with production physics codes
- General asynchronous MSOD MCSA implementation
 - Forward and adjoint Monte Carlo with method of expected values
 - Parallel row matrix/vector interface
 - General fixed point iteration strategy
 - Explicit algebraic preconditioner suite
- Implemented in C++
- Heavy use of the Trilinos scientific computing libraries
- Open-source BSD 3-clause license
- <https://github.com/sslattery/MCLS>



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$$\hat{\Omega} \cdot \vec{\nabla} \psi(\vec{r}, \hat{\Omega}, E) + \sigma(\vec{r}, E) \psi(\vec{r}, \hat{\Omega}, E) = \iint \sigma_s(\vec{r}, E' \rightarrow E, \hat{\Omega}' \cdot \hat{\Omega}) \psi(\vec{r}, \hat{\Omega}', E') d\Omega' dE' + q(\vec{r}, \hat{\Omega}, E), \quad (1)$$

$$\begin{aligned} -\nabla \cdot \left[\frac{n}{2n+1} \frac{1}{\Sigma_{n-1}} \nabla \left(\frac{n-1}{2n-1} \phi_{n-2} + \frac{n}{2n-1} \phi_n \right) \right. \\ \left. + \frac{n+1}{2n+1} \frac{1}{\Sigma_{n+1}} \nabla \left(\frac{n+1}{2n+3} \phi_n + \frac{n+2}{2n+3} \phi_{n+2} \right) \right] \\ + \Sigma_n \phi_n = q \delta_{n0} \quad n = 0, 2, 4, \dots, N, \quad (2) \end{aligned}$$

$$-\nabla \cdot \mathbb{D}_n \nabla \mathbb{U}_n + \sum_{m=1}^4 \mathbb{A}_{nm} \mathbb{U}_m = \frac{1}{k} \sum_{m=1}^4 \mathbb{F}_{nm} \mathbb{U}_n \quad n = 1, 2, 3, 4. \quad (3)$$

Algorithm 1 Power Iteration MCSA Scheme

k_0 = initial guess

Φ_0 = initial guess

$n = 0$

while $\left| \frac{k^n - k^{n-1}}{k^n} \right| < \epsilon$ **do**

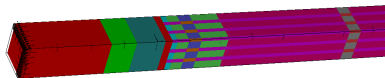
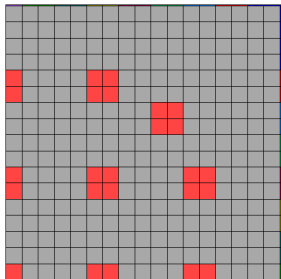
$\mathbf{M}\Phi^{n+1} = \frac{1}{k^n} \mathbf{F}\Phi^n$ {Solve for the new flux state with MCSA}

$$k^{n+1} = k^n \frac{\int \mathbf{F}\Phi^{n+1} d\mathbf{r}}{\int \mathbf{F}\Phi^n d\mathbf{r}}$$

$n = n + 1$

end while

- Swap in MCSA as the solver at each eigenvalue operation
- Use Thyra/Stratimikos interfaces for plug and play - Thanks Ross!
- Current Exnihilo implementation gives the full operator

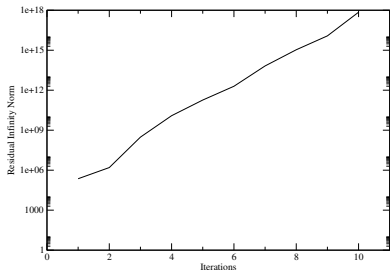


- CASL Problem 3: 17×17 quarter symmetry HZP LWR fuel assembly
- SP_N criticality calculation
- MCLS leveraged by the Exnihilo code base (ORNL)
- Comparison to Trilinos Aztec Krylov solvers with ILUT

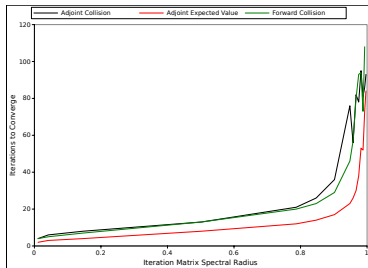
Parameter	Value
Power Level	0 MW
Inlet Temperature	326.85C
Fuel Temperature	600C
Boron Concentration	1300 ppm
Moderator Density	0.743 g/cc
Helium Density	1.79×10^{-4} g/cc
Zirconium Density	6.56 g/cc
Stainless Steel Density	8.0 g/cc
Inconel Density	8.19 g/cc
UO2 Density	10.257 g/cc
Fuel Pin Radius (w/o clad)	0.4096 cm

		SP_N Order			
P_N Order	0	1	3	5	7
	0	0.0647	0.1275	0.1449	0.1514
	1	0.0686	0.1338	0.1484	0.1547
	3	0.0687	0.1399	0.1582	0.1625
	5	0.0692	0.1399	0.1582	0.1657
	7	0.0678	0.1393	0.1624	0.166

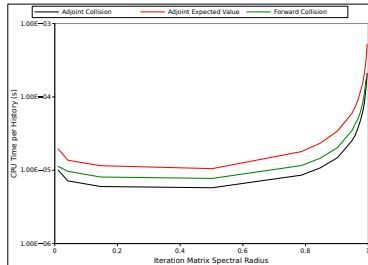
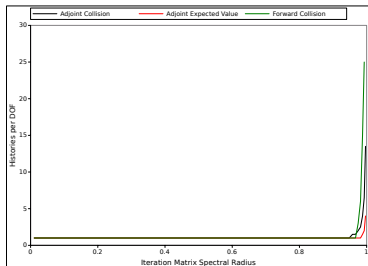
Table: Spectral radius results for the block Jacobi preconditioned iteration matrix with 10 energy groups and full downscatter for sample problem.



- Light water moderator creates a lot of scattering and $\rho(\mathbf{H}) \approx 1$
- Intractable number of histories required for MCSA convergence



- As $\rho(\mathbf{H}) \rightarrow 1$ terrible things happen...
- A more robust set of preconditioners is required for the SP_N equations



$$\mathbf{M}_L^{-1} \mathbf{A} \mathbf{M}_R^{-1} \mathbf{M}_R \mathbf{x} = \mathbf{M}_L^{-1} \mathbf{b} \quad \rightarrow \quad \mathbf{M}_L^{-1} \mathbf{A} \mathbf{M}_R^{-1} \mathbf{u} = \mathbf{M}_L^{-1} \mathbf{b}$$

$$\mathbf{x} = \mathbf{M}_R^{-1} \mathbf{u}$$

Left/Right Preconditioned MCSA Iteration

$$\mathbf{r}^k = \mathbf{M}_L^{-1} (\mathbf{b} - \mathbf{A} \mathbf{M}_R^{-1} \mathbf{u}^k)$$

$$\mathbf{u}^{k+1/2} = \mathbf{u}^k + \mathbf{r}^k$$

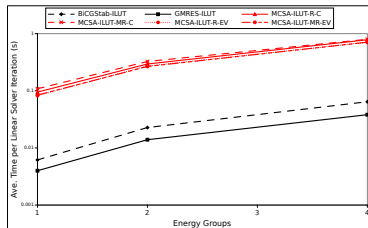
$$\mathbf{r}^{k+1/2} = \mathbf{M}_L^{-1} (\mathbf{b} - \mathbf{A} \mathbf{M}_R^{-1} \mathbf{u}^{k+1/2})$$

$$\mathbf{M}_L^{-1} \mathbf{A} \mathbf{M}_R^{-1} \delta \mathbf{u}^{k+1/2} = \mathbf{r}^{k+1/2}$$

$$\mathbf{u}^{k+1} = \mathbf{u}^{k+1/2} + \delta \mathbf{u}^{k+1/2}$$

Solver	1 Group	2 Groups	4 Groups
BiCGStab-ILUT	11.6	11.6	12.4
GMRES-ILUT	18.1	17.9	18.9
MCSA-ILUT-R-C	14.6	15.4	17.6
MCSA-ILUT-MR-C	16.0	17.1	23.7
MCSA-ILUT-R-EV	18.3	19.4	16.8
MCSA-ILUT-MR-EV	19.6	22.4	17.5
Richardson-ILUT	60.9	60.4	63.4

Table: Average number of linear solver iterations per eigenvalue iteration.



- MCLS generates the same k-eigenvalue and neutron flux in all groups and spatial locations
- MCSA converged in fewer iterations than GMRES, more iterations than BiCGStab
- Explicit preconditioning strategy destroys sparsity and elevates CPU times (Ifpack ILUT)
- Spectral radius and memory limitations combined to prevent solutions at finer discretizations



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Monte Carlo Synthetic Acceleration for Nonlinear Problems

- Many physics problems of interest are nonlinear
- Significant research on Newton methods since the 1980's
- Newton methods often leverage Krylov solvers
 - Robust implementations
 - No operator required
- Monte Carlo methods need the full operator
- Automatic construction of the linear model is available
 - Operator overloading for nonlinear residual differentiation
 - Ideal for Monte Carlo
 - Potential scaling improvements
 - Resiliency benefits

- Seek solutions of the general nonlinear problem

$$\mathbf{F}(\mathbf{u}) = \mathbf{0}$$

$$\mathbf{u} \in \mathbb{R}^n, \mathbf{F} : \mathbb{R}^N \rightarrow \mathbb{R}^N$$

- Interpret the exact solution \mathbf{u} to be the roots of $\mathbf{F}(\mathbf{u})$

$$\mathbf{F}(\mathbf{u}^{k+1}) = \mathbf{F}(\mathbf{u}^k) + \mathbf{F}'(\mathbf{u}^k)(\mathbf{u}^{k+1} - \mathbf{u}^k) + \frac{\mathbf{F}''(\mathbf{u}^k)}{2}(\mathbf{u}^{k+1} - \mathbf{u}^k)^2 + \dots$$

- Form Newton's method

$$\mathbf{J}(\mathbf{u})\delta\mathbf{u}^k = -\mathbf{F}(\mathbf{u}^k)$$

$$\mathbf{u}^{k+1} = \mathbf{u}^k + \delta\mathbf{u}^k$$

Forward-Automated Newton-MCSA

Algorithm 2 FANM

```
1:  $k := 0$ 
2: while  $\|\mathbf{F}(\mathbf{u}^k)\| > \epsilon \|\mathbf{F}(\mathbf{u}^0)\|$  do
3:    $\mathbf{J}(\mathbf{u}^k) \leftarrow AD(\mathbf{F}(\mathbf{u}^k))$  {Automatic differentiation}
4:    $\mathbf{J}(\mathbf{u}^k)\delta\mathbf{u}^k = -\mathbf{F}(\mathbf{u}^k)$  {Solve for the Newton correction with MCSA}
5:    $\mathbf{u}^{k+1} \leftarrow \mathbf{u}^k + \delta\mathbf{u}^k$ 
6:    $k \leftarrow k + 1$ 
7: end while
```

- Robustness of Newton's method (inexact)
- Accuracy and convenience of FAD
- Potential parallelism, and resiliency benefits of MCSA
- Requires only nonlinear function evaluations
- Can utilize globalization and forcing term selection methods

- Sequence of Navier-Stokes benchmarks for the nonlinear methods
 - Thermal convection cavity problem (De Vahl Davis, 1983)
 - Lid driven cavity problem (Ghia et al., 1982)
 - Backward-Facing step problem (Gartling, 1990)
- Tuning benchmark parameters varies the strength of nonlinearities
- Newton-Krylov method leveraging Aztec GMRES used for comparisons
- All problems preconditioned with algebraic multigrid (ML) and leveraged some kind of globalization (e.g. backtracking)

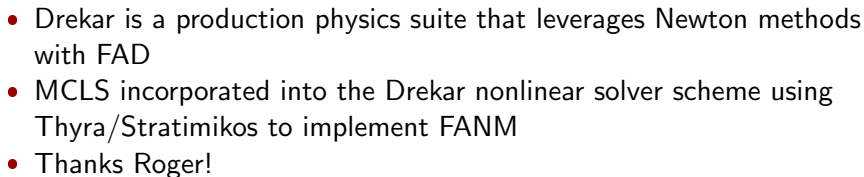
$$\rho \mathbf{u} \cdot \nabla \mathbf{u} - \nabla \cdot \mathbf{T} - \rho \mathbf{g} = \mathbf{0}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho C_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = 0$$

$$\mathbf{T} = -P\mathbf{I} + \mu[\nabla \mathbf{u} + \nabla \mathbf{u}^T]$$

$$\mathbf{q} = -k \nabla T$$



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Benchmark	NK	FANM	NR
Convection, $Ra=1 \times 10^3$	5	5	5
Convection, $Ra=1 \times 10^4$	7	7	7
Convection, $Ra=1 \times 10^5$	9	10	9
Convection, $Ra=1 \times 10^6$	11	11	11
Lid Driven, $Re=100$	6	6	7
Lid Driven, $Re=300$	9	9	9
Lid Driven, $Re=500$	11	11	11
Lid Driven, $Re=700$	14	10	12
Backward Step, $Re=200$	10	9	10
Backward Step, $Re=300$	15	14	15
Backward Step, $Re=400$	10	10	10
Backward Step, $Re=500$	19	20	21

Table: Navier-Stokes benchmark comparison for nonlinear iterations. Over all benchmarks, FANM performed better in terms of nonlinear iterations for 1 more case than the Newton-Krylov method.

Benchmark	GMRES	MCSA	Richardson
Convection, $Ra=1 \times 10^3$	32	18	38
Convection, $Ra=1 \times 10^4$	23	17	34
Convection, $Ra=1 \times 10^5$	25	20	34
Convection, $Ra=1 \times 10^6$	39	25	48
Lid Driven, $Re=100$	27	42	151
Lid Driven, $Re=300$	35	52	133
Lid Driven, $Re=500$	41	56	154
Lid Driven, $Re=700$	21	14	32
Backward Step, $Re=200$	24	13	21
Backward Step, $Re=300$	23	17	21
Backward Step, $Re=400$	18	12	14
Backward Step, $Re=500$	30	52	98

Table: Navier-Stokes benchmark comparison for total linear solver iterations. *Over all benchmarks, FANM performed better in terms of linear solver iterations for twice as many cases as the Newton-Krylov method.*

Benchmark	Newton-Krylov Speedup
Convection, $Ra=1 \times 10^3$	338
Convection, $Ra=1 \times 10^4$	336
Convection, $Ra=1 \times 10^5$	346
Convection, $Ra=1 \times 10^6$	465
Lid Driven, $Re=100$	299
Lid Driven, $Re=300$	322
Lid Driven, $Re=500$	288
Lid Driven, $Re=700$	488
Backward Step, $Re=200$	400
Backward Step, $Re=300$	593
Backward Step, $Re=400$	825
Backward Step, $Re=500$	1057

Table: Newton-Krylov speedup over FANM. *For all benchmarks the explicit MCSA preconditioning strategy caused significantly larger CPU times for FANM when compared to the Newton-Krylov solutions.*



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- No literature observed for parallel Neumann-Ulam solvers beyond history-level parallelism
- Numerous references for modern parallel Monte Carlo methods in reactor physics
- Build a strategy for applying modern methods to the Neumann-Ulam method
- MCSA iteration-level parallelism comes from parallel matrix/vector operations

- Each parallel process owns a piece of the domain (linear system)
- Random walks must be transported between adjacent domains through parallel communication
- Domain decomposition determined by the input system

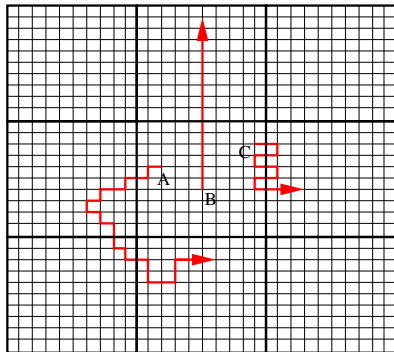
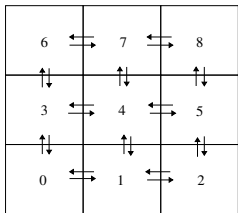


Figure: *Domain decomposition example illustrating how domain-to-domain transport creates communication costs.*

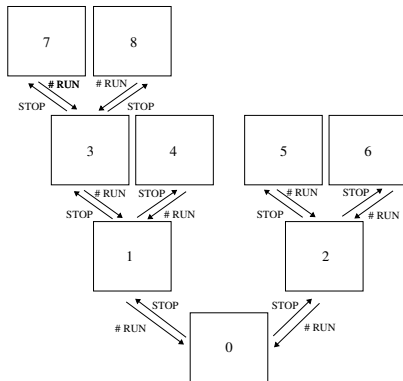
Asynchronous Monte Carlo Transport Kernel



- Developed by Brunner and Brantley in 2009
- Asynchronous nearest neighbor communication of histories
- Binary asynchronous communication tree for completing transport



- Extensible to problems where histories may be created (i.e. variance reduction)



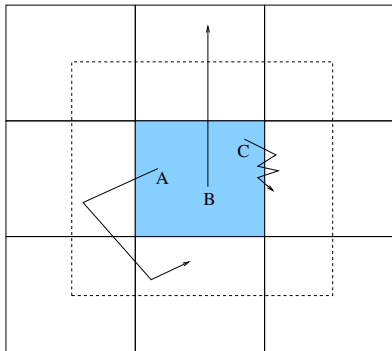


Figure: *Overlapping domain example illustrating how domain overlap can reduce communication costs.*

- Developed by Wagner and colleagues in 2010
- Each set contains the full domain
- Multiple sets replicate the domain
- Domains overlap within a set
- Redundancy for resiliency (and useful work)

Wagner et. al., "Hybrid and parallel domain-decomposition methods development to enable Monte Carlo for reactor analysis", Joint International Conference on Supercomputing in Nuclear Applications and Monte Carlo (SNA+MC 2010), 2010.

Multiple-Set Overlapping-Domain Decomposition

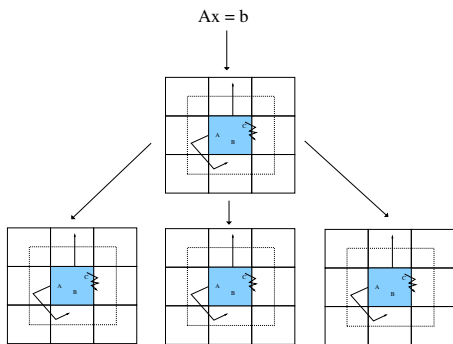


Figure: *MSOD construction.*

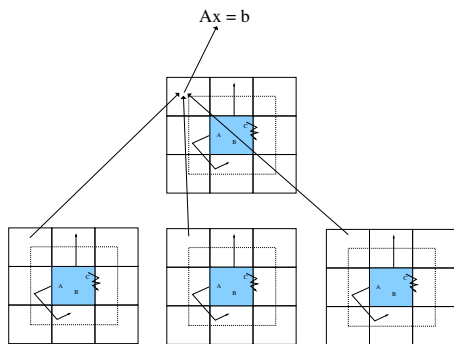


Figure: *MSOD tally reduction.*



- Simple 2D neutron diffusion problem for control - spectral radius is maintained as global problem size grows
- Comparison to Trilinos Belos Krylov solvers with Jacobi preconditioning - conjugate gradient and GMRES
- Strong scaling - Global size fixed at $1.6E7$ DOFs
- Weak scaling - Local size fixed at $4.0E4$ DOFs
- Calculations performed on the Titan Cray XK7 machine at ORNL (MPI only)
- Limited MCLS arithmetic optimization artificially inflates efficiencies

Strong Scaling Results

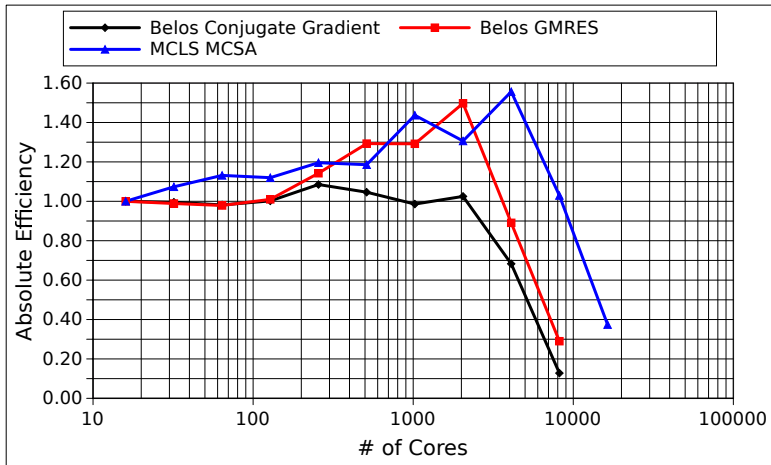
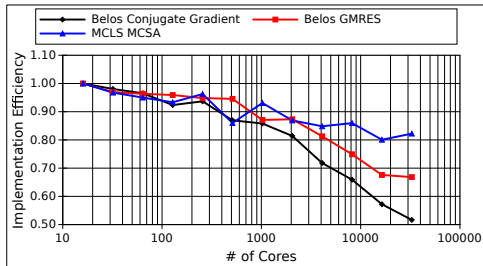
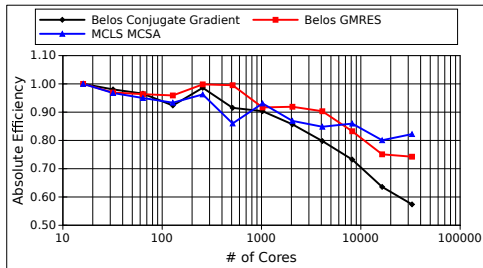


Figure: Pure domain decomposition. *Super-linear speed-up from memory thrashing in base case. MCLS is an order of magnitude slower arithmetically.*

Weak Scaling Results



- CG demonstrates poor scaling due to the cheaper iteration sequence
- Implementation efficiency is effectively the parallel efficiency of a single iteration
- Improved implementation efficiency means improved iterative performance will potentially give a better time to solution

Strong Scaling Results with Multiple Sets



Splitting: same global number of histories as the single set

Replicating: set-multiple of single set problem global histories

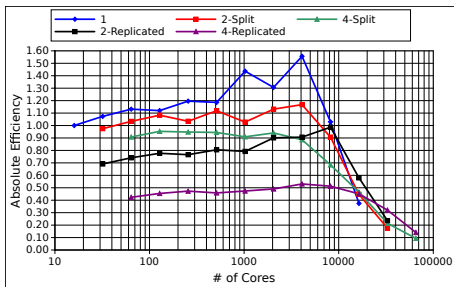


Figure: Absolute parallel efficiency relative to 16-core 1-set base case.

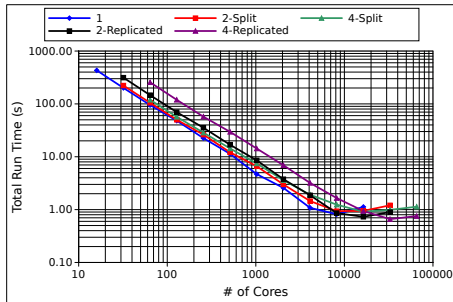


Figure: Wall time in seconds to solution for each case.

Weak Scaling Results with Multiple Sets



- Need to consider adding sets is a strong scaling exercise
- Modify the weak scaling efficiency computation to account for these extra resources
- Superposition of Monte Carlo results enhances time to solution!

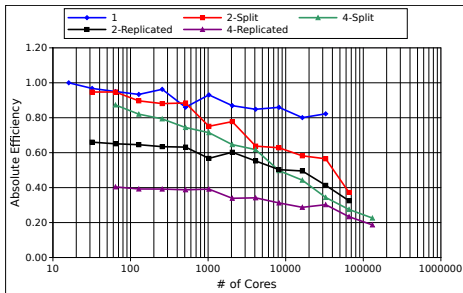


Figure: Absolute parallel efficiency relative to 16-core 1-set base case.

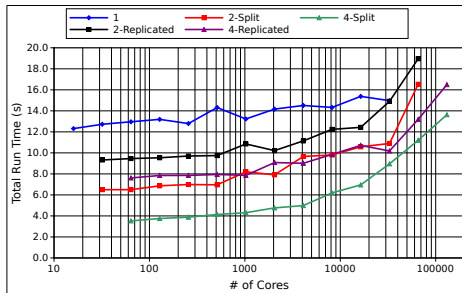


Figure: Wall time in seconds to solution for each case.

Strong Scaling Results with Overlap



- Overlap values selected based on average 'diffusion length' of a history in the system of 2.6 discrete states
- Overlap eliminates communication in the Monte Carlo sequence but simply differs it to an overlapping tally vector reduction

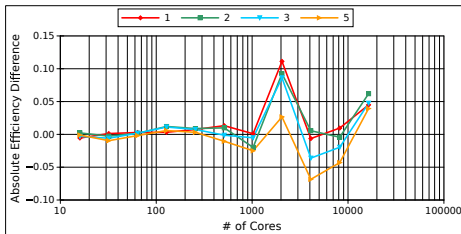


Figure: Strong scaling efficiency difference compared to the 0 overlap case.

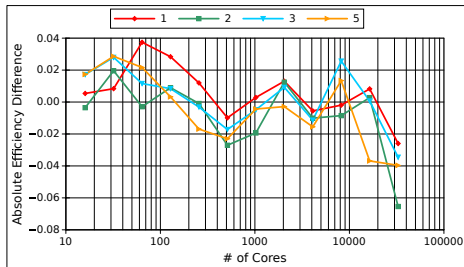


Figure: Weak scaling efficiency difference compared to the 0 overlap case.

MCSA as a Stochastic Additive Schwarz Method



- No domain-to-domain communication in Monte Carlo sequence
- Fixed point iteration acts as a smoother
- Observed to converge in the same number of iterations
- Can add overlap to preserve iterative performance for more ill-conditioned problems

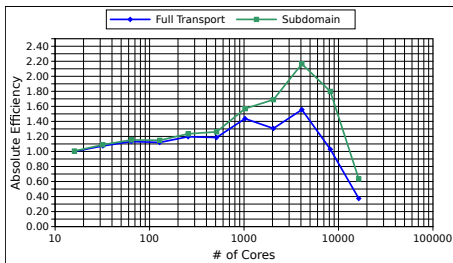


Figure: Strong scaling absolute efficiency for pure domain decomposition.

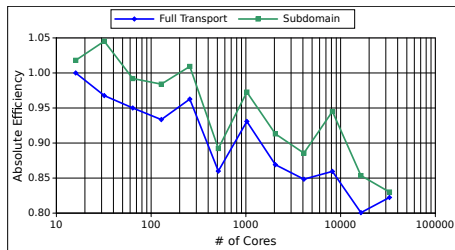


Figure: Weak scaling absolute efficiency for pure domain decomposition.

- Monte Carlo Synthetic Acceleration Methods
- Application to Neutron Transport
- Application to Fluid Flow
- Parallelization of MCSA
- **Summary**



- MCSA has been incorporated into the Exnihilo neutronics production code base developed at Oak Ridge National Laboratory
- MCSA can solve the asymmetric system generated by the SP_N equations
- Light water reactor problems are difficult to solve with MCSA as they have large spectral radii due to the neutron scattering in the moderator
- Advanced algebraic preconditioning strategies were applied to the SP_N equations to obtain convergence with ILUT chosen for subsequent investigations
- MCSA was observed to converge in fewer iterations per eigenvalue iteration than GMRES for the fuel assembly criticality problem and more than Bi-CGStab using the same preconditioning



- Forward-Automated Newton-MCSA (FANM) has been developed
- The FANM method has been incorporated into the Drekar multiphysics production code base developed at Sandia National Laboratories
- The FANM method has better iterative performance than the Newton-Krylov method for convection dominated problems, converging in fewer linear solver iterations with the same preconditioning for high and low Rayleigh numbers
- The spectral radius convergence restriction on MCSA was observed to be a significant hindrance by preventing solutions to forced flow problems at high Reynolds numbers
- More Monte Carlo histories at every FANM iteration can reduce the number of linear and nonlinear iterations required to converge the problem



- The multiple-set overlapping-domain (MSOD) parallel algorithm for domain decomposed particle transport has been adapted to parallelize MCSA
- MCSA scales favorably compared to production Krylov methods for both strong and weak scaling cases
- Overlap in small quantities can provide parallel efficiency boosts of a few percent in strong scaling cases but is ineffective in weak scaling cases
- Multiple sets offers a means to reduce time to solution by solving multiple copies of the original problem and combining the solutions using superposition
- MCSA is most efficiently used in parallel as a stochastic realization of an additive Schwarz method

- Shortcomings observed on real problems
 - Significant optimization required to determine production feasibility and true scalability
 - Explicit algebraic preconditioning methods not sufficient
 - Spectral radius limitation is severe
- Performance improvements
 - Random walk optimizations
 - Alias sampling implementations
 - Multiple set reduction analysis
 - FANM forcing term and MCSA history relationships
- Preconditioning improvements
 - Alternative left/right sampling schemes
 - Variance reduction based strategy
 - Reduced order physics/PDE models for acceleration
- Breaking away from $\rho(\mathbf{H}) < 1$
 - Monte Carlo methods of the second degree
 - Stochastic projection methods

- ① S. Slattery, T.M. Evans, P.P.H. Wilson, **A Multiple-Set Overlapping-Domain Decomposed Monte Carlo Synthetic Acceleration Method for Linear Systems**, *Joint International Conference on Supercomputing in Nuclear Applications and Monte Carlo 2013 (SNA+MC 2013), Paris, France, October 27-31, 2013. Accepted for oral presentation.*
- ② T.M. Evans, S.W. Mosher, S.R. Slattery, S.P. Hamilton, **A Monte Carlo Synthetic-Acceleration Method for Solving the Thermal Radiation Diffusion Equation**, *Journal of Computational Physics, Submitted.*
- ③ S. Slattery, T.M. Evans, P.P.H. Wilson, **A Spectral Analysis of the Domain Decomposed Monte Carlo Method for Linear Systems**, *International Conference on Mathematics and Computational Methods Applied to Nuclear Science & Engineering (M&C 2013), American Nuclear Society, Sun Valley, ID, May 5-9, 2013.*



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