SP_N Update

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Neutron Transport Solution Methods

$$\hat{\Omega} \cdot \vec{\nabla}\psi(\vec{r}, \hat{\Omega}, E) + \sigma(\vec{r}, E)\psi(\vec{r}, \hat{\Omega}, E) = \int\int \sigma_s(\vec{r}, E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega})\psi(\vec{r}, \hat{\Omega}', E')d\Omega' dE' + q(\vec{r}, \hat{\Omega}, E) \quad (1)$$

- S_N transport is expensive
  - Difficult to parallelize: sweeps in space, pipelining in angle, energy decoupling
  - Large storage requirements
  - Ray effects
- P_N equations still expensive
  - Complicated system: $(N+1)^2$ equations in 3D
  - Coupling of equations through both angular moments and spatial derivatives

SP_N Approximation

- Ad-hoc generalization of planar P_N equations by Gelbard in the 1960's
- Rigorous formulation through asymptotic and variational analysis in 1990's and 2000's
- Simpler system $(N+1)/2$ equations in $3D$
- Yields elliptic, diffusion-like equations
- Applicable when diffusion theory is applicable: reasonable flux gradients, full-core transport
- Typically does not converge to transport solution as $N \to \infty$
- **Can build the full linear operator**
- **Parallelism through the linear solver**

SP_N Approximation

SP_N Equations

$$
\begin{aligned}
-\nabla\cdot\Bigg[&\frac{n}{2n+1}\frac{1}{\Sigma_{n-1}}\nabla\Big(\frac{n-1}{2n-1}\phi_{n-2}+\frac{n}{2n-1}\phi_n\Big)\\
&+\frac{n+1}{2n+1}\frac{1}{\Sigma_{n+1}}\nabla\Big(\frac{n+1}{2n+3}\phi_n+\frac{n+2}{2n+3}\phi_{n+2}\Big)\Bigg]\\
&+\Sigma_n\phi_n=q\delta_{n0}\qquad\qquad n=0,2,4,\cdots,N\,, \quad (2)
\end{aligned}
$$

SP_N Numerical Spectral Analysis

- Monte Carlo methods for have strong restrictions on the eigenvalues of the operator for convergence
- MCSA has the same restrictions on the outer stationary iteration
- We need to compute these eigenvalues for various forms of the SP_N equations to verify convergence of these methods.

We need eigenvalues for $\mathbf{A}$, $\mathbf{H_J}$, and $\mathbf{H_{GS}}$ with:

$$\mathbf{H_J} = \mathbf{I} - \mathbf{D}^{-1}\mathbf{A} \tag{3}$$

where $\mathbf{D} = diag(\mathbf{A})$ and

$$\mathbf{H_{GS}} = (\mathbf{L} + \mathbf{D})^{-1}\mathbf{U} \tag{4}$$

SP_7, P_3, 3 groups, reflecting boundaries

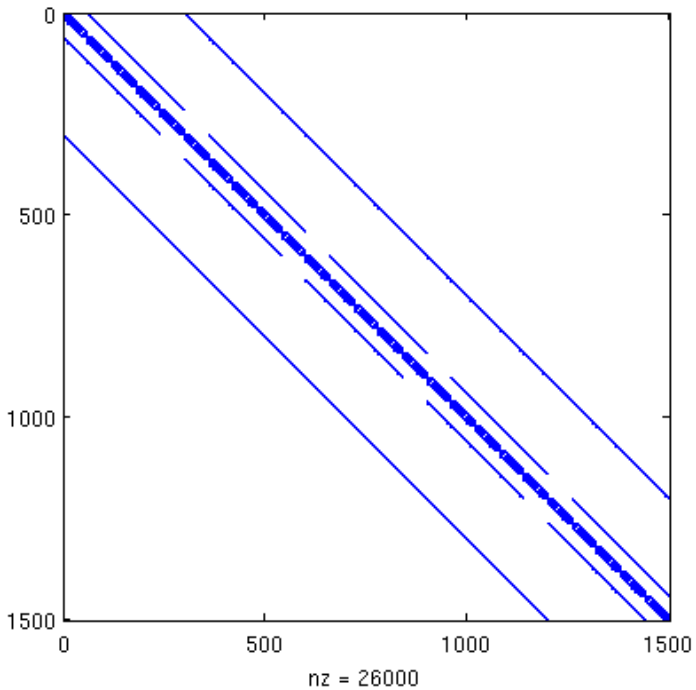

Figure: **Linear operator sparsity pattern**

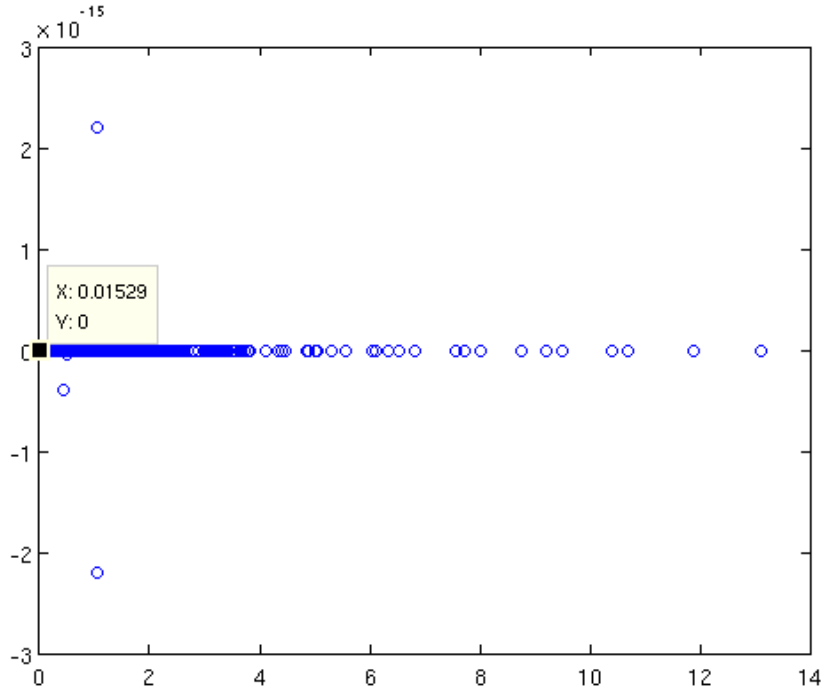

Figure: **Linear operator eigenvalues**

SP_7, P_3, 3 groups, reflecting boundaries

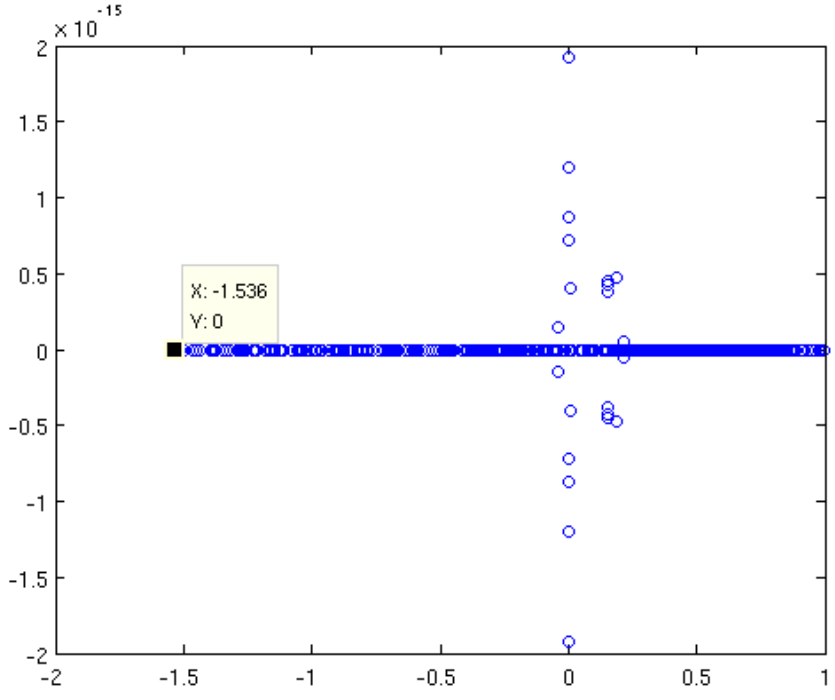

Figure: **Jacobi iteration matrix eigenvalues**

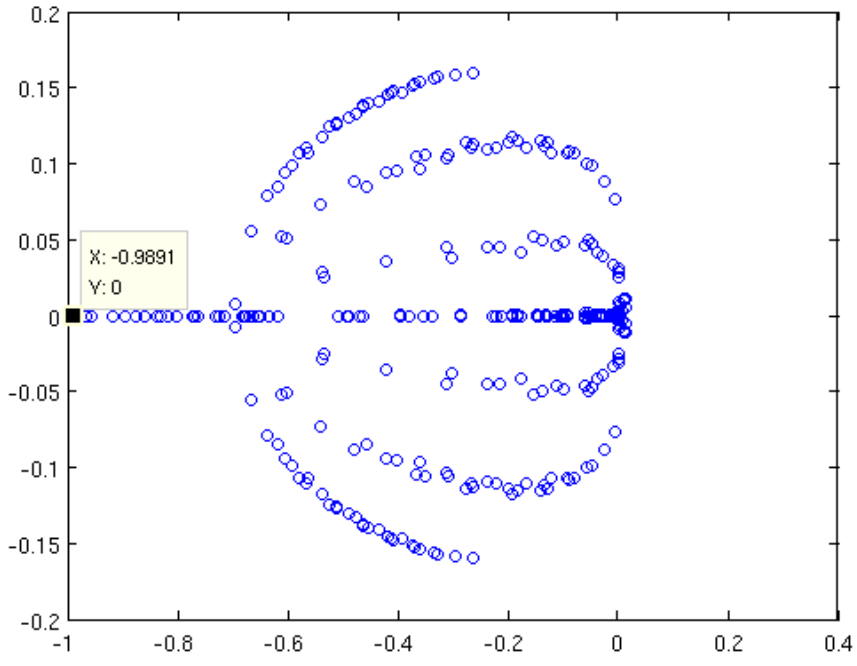

Figure: **Gauss-Seidel iteration matrix eigenvalues**

Oh No!

- The Jacobi method won't converge - all that stuff I said in my prelim won't work

Solution

- Bug fix in Denovo SP_N implementation
- A new kind of preconditioning $\cdots$

Multigroup Matrix Pattern

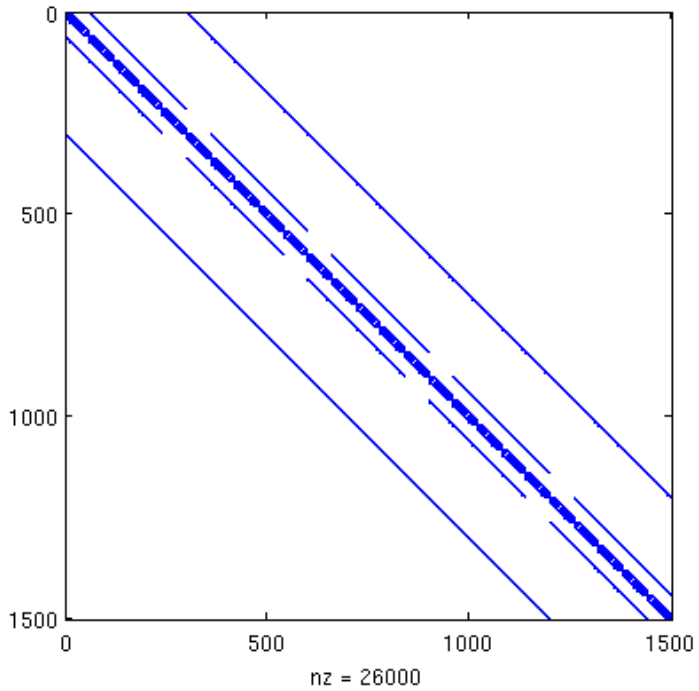

Figure: **Linear operator sparsity pattern**

Block Jacobi Preconditioning

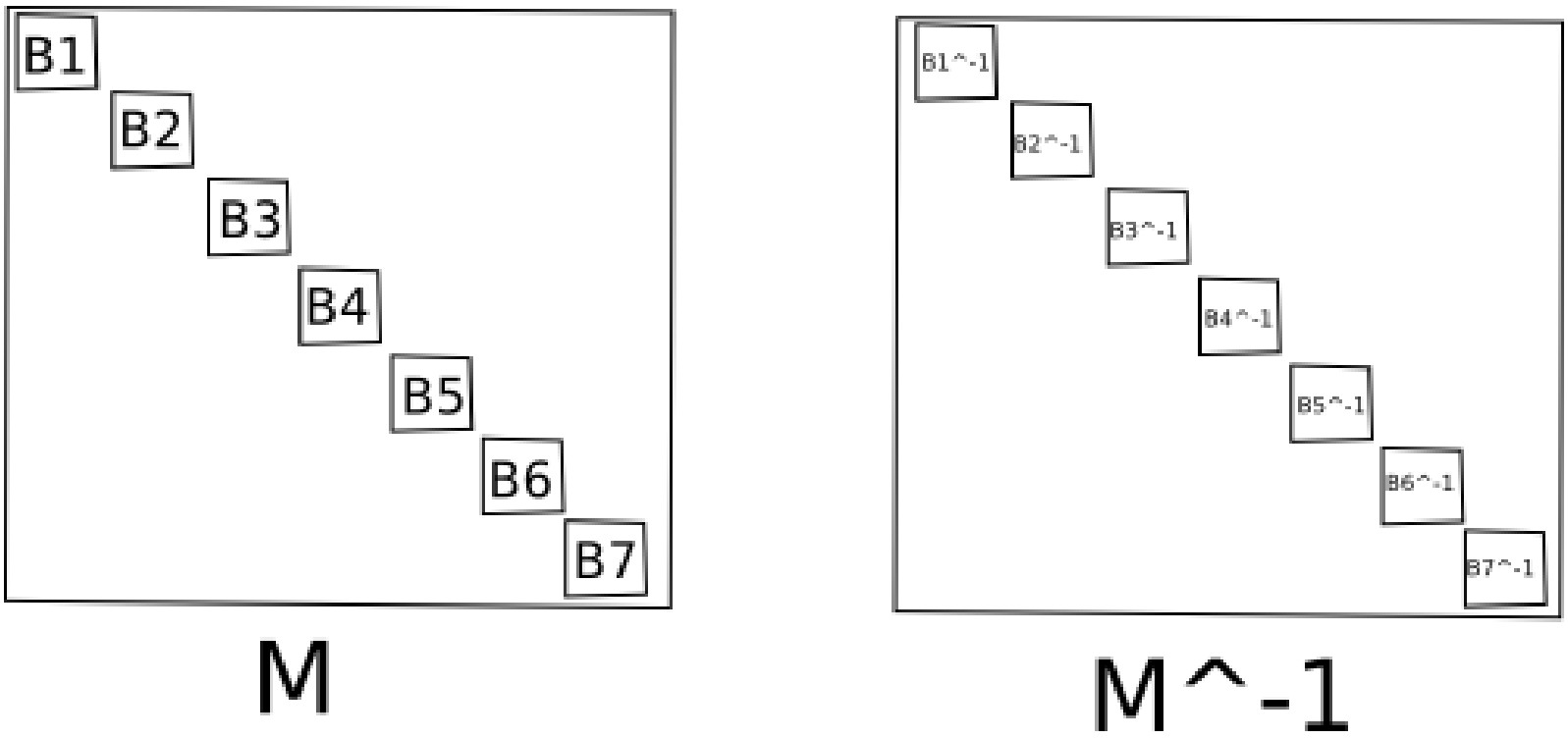

Figure: **Block Jacobi Preconditioner**

Point Jacobi Results

Base Parameters					
Delta	1.00		*Source*	1.00
Mesh elements	4x4x4		*Sigma_t*	5.00
Num Materials		1	*Sigma_ig*	0.25
Reflecting boundaries			*Sigma_down*	1.00
			Sigma_up	0.10

POINT JACOBI SPECTRAL RADIUS

1 Energy Group

		SPN Order			
		1	**3**	**5**	**7**
	0	0.0635	0.6722	1.3144	1.976
	1	0.0666	0.6728	1.3141	1.9755
PN Order	**3**	0.0666	0.6822	1.3198	1.9762
	5	0.0666	0.6822	1.3278	1.9847
	7	0.0666	0.6822	1.3278	1.9914

10 Energy Groups without Upscatter

		SPN Order			
		1	**3**	**5**	**7**
	0	0.0655	0.677	1.32	1.983
	1	0.071	0.6777	1.319	1.982
PN Order	**3**	0.071	0.687	1.327	1.9872
	5	0.071	0.687	1.336	1.997
	7	0.071	0.687	1.336	1.9995

10 Energy Groups with Upscatter

		SPN Order			
		1	**3**	**5**	**7**
	0	0.7283	0.81	1.47	2.1446
	1	0.7317	0.8	1.46	2.1368
PN Order	**3**	0.7317	0.91	1.526	2.2274
	5	0.7317	0.91	1.5344	2.2562
	7	0.7317	0.91	1.5345	2.2842

Block Jacobi Results

BLOCK JACOBI SPECTRAL RADIUS: B = N_g * (N+1)/2

1 Energy Group

		SPN Order			
		1	3	5	7
PN Order	0	0.0635	0.1269	0.1444	0.1513
	1	0.0666	0.1315	0.1474	0.1534
	3	0.0666	0.1365	0.154	0.1592
	5	0.0666	0.1365	0.1562	0.163
	7	0.0666	0.1365	0.1562	0.164

10 Energy Groups without Upscatter

		SPN Order			
		1	3	5	7
PN Order	0	0.0647	0.1275	0.1449	0.1514
	1	0.0686	0.1338	0.1484	0.1547
	3	0.0687	0.1399	0.1582	0.1625
	5	0.0692	0.1402	0.1626	0.1657
	7	0.0678	0.1393	0.1624	0.166

10 Energy Groups with Upscatter

		SPN Order			
		1	3	5	7
PN Order	0	0.1887	0.2267	0.2285	0.2286
	1	0.4535	0.5044	0.5045	0.5045
	3	0.4535	0.6453	0.6506	0.6506
	5	0.4535	0.6453	0.6802	0.6818
	7	0.4535	0.6453	0.6802	0.6927

Conclusions

- Block Jacobi preconditioning is a simple and appropriate solution for preconditioning the SP_N equations
- Implementation is general - slides right into ANA framework
- Implementation is scalable - local operations only, fast LAPACK operations
- Implementation works - can move on with research