Parallel Monte Carlo Synthetic Acceleration Methods for Discrete Transport Problems

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Outline

- Monte Carlo Synthetic Acceleration Methods
- Application to Neutron Transport
- Application to Fluid Flow
- Parallelization of MCSA

Acknowledgments

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Statement of Work

The goal of this work is to improve the iterative performance and parallel scalability of solutions to discrete linear and nonlinear transport problems by researching and developing a new set of domain decomposed Monte Carlo Synthetic Acceleration methods.

Hardware-Based Motivation

- Modern hardware is moving in two directions (Kogge,2011):
  - Lightweight machines
  - Heterogeneous machines
  - Both characterized by low power and high concurrency
- Some issues:
  - Higher potential for both soft and hard failures (DOE,2012)
  - Memory restrictions are expected with a continued decrease in memory/FLOPS
- Potential resolution from Monte Carlo:
  - Soft failures buried within the tally variance
  - Hard failures are high variance events
  - Memory savings over conventional methods

Research Outline

- Development of a linear solver for discrete systems leveraging Monte Carlo Synthetic Acceleration
  - Application to neutron transport
  - Research is required to explore general solver development
  - Performance is of concern
- Development of a nonlinear solver for discrete systems leveraging Monte Carlo Synthetic Acceleration
  - Application to fluid flow
  - Potential memory benefits
  - Convergence of the linear model is of concern
- Parallelization of Monte Carlo Synthetic Acceleration
  - Parallel strategies taken from modern reactor physics methods
  - Research is required to explore varying parallel strategies
  - Scalability is of concern

Outline

- **Monte Carlo Synthetic Acceleration Methods**
- Application to Neutron Transport
- Application to Fluid Flow
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Monte Carlo Methods for Discrete Linear Systems

- First proposed by J. Von Neumann and S.M. Ulam in the 1940's
- Earliest published reference in 1950
- General lack of published work
- Modern work by Evans and others has yielded new applications

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Thomas Evans and Scott Mosher, "A Monte Carlo Synthetic Acceleration method for the non-linear, time-dependent diffusion equation", American Nuclear Society - International Conference on Mathematics, Computational Methods and Reactor Physics, 2009.

Monte Carlo Linear Solver Preliminaries

- Split the linear operator

$$\mathbf{Ax} = \mathbf{b} \quad \rightarrow \quad \mathbf{x} = \mathbf{Hx} + \mathbf{b}$$

$$\mathbf{H} = \mathbf{I} - \mathbf{A}$$

- Generate the *Neumann series*

$$\mathbf{A}^{-1} = (\mathbf{I} - \mathbf{H})^{-1} = \sum_{k=0}^{\infty} \mathbf{H}^k$$

- Require $\rho(\mathbf{H}) < 1$ for convergence

$$\mathbf{A}^{-1}\mathbf{b} = \sum_{k=0}^{\infty} \mathbf{H}^k\mathbf{b} = \mathbf{x}$$

Monte Carlo Linear Solver Preliminaries

- Expand the Neumann series

$$x_i = \sum_{k=0}^{\infty} \sum_{i_1}^{N} \sum_{i_2}^{N} \cdots \sum_{i_k}^{N} h_{i,i_1} h_{i_1,i_2} \cdots h_{i_{k-1},i_k} b_{i_k}$$

- Define a sequence of state transitions

$$\nu = i \to i_1 \to \cdots \to i_{k-1} \to i_k$$

- Use the adjoint Neumann-Ulam decomposition

$$\mathbf{H}^T = \mathbf{P} \circ \mathbf{W}$$

$$p_{ij} = \frac{|h_{ji}|}{\sum_j |h_{ji}|}, \quad w_{ij} = \frac{h_{ji}}{p_{ij}}$$

The Hadamard product $\mathbf{A} = \mathbf{B} \circ \mathbf{C}$ is defined element-wise as $a_{ij} = b_{ij} c_{ij}$.

Evolution of a Solution

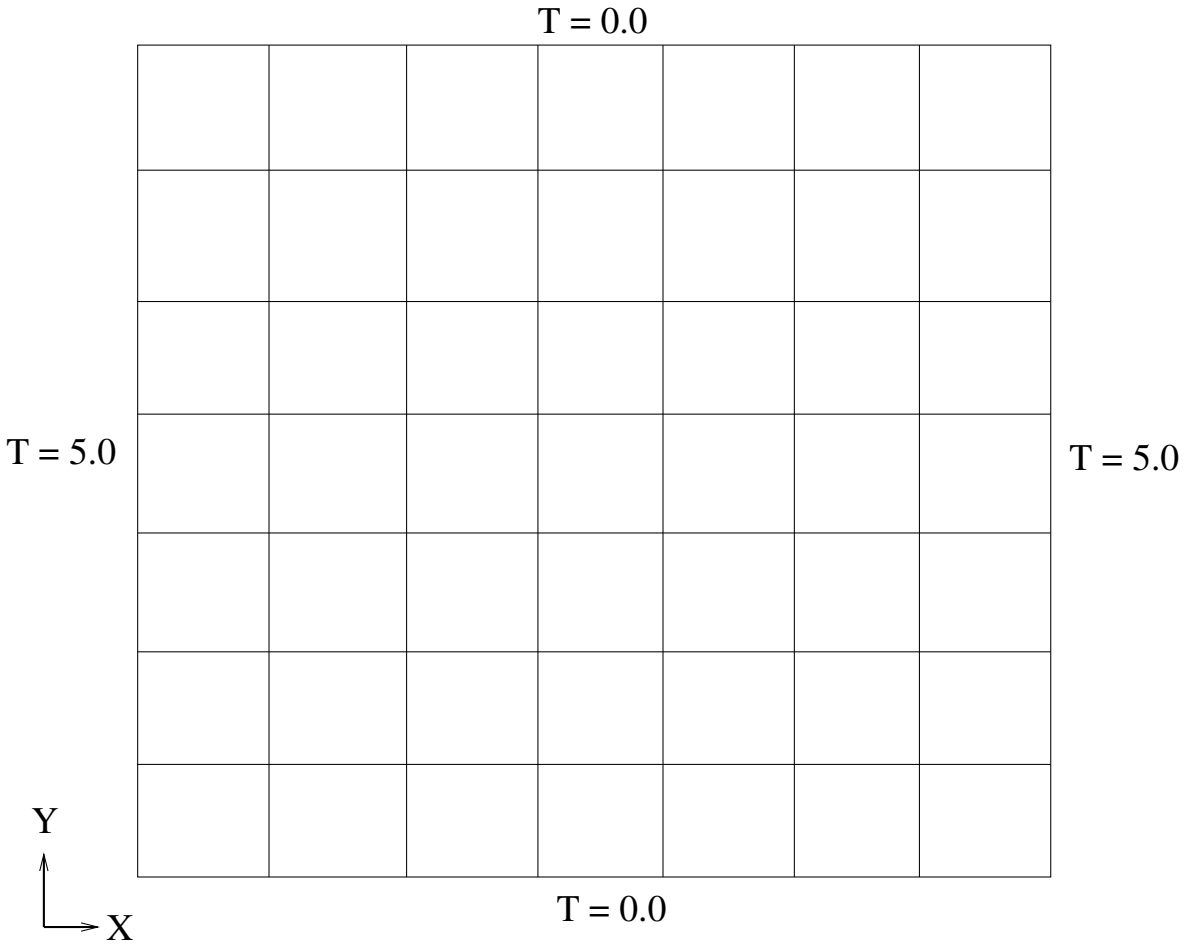

Figure: **Poisson Problem.** *Distributed source of 1.0 in the domain.*

Evolution of a Solution

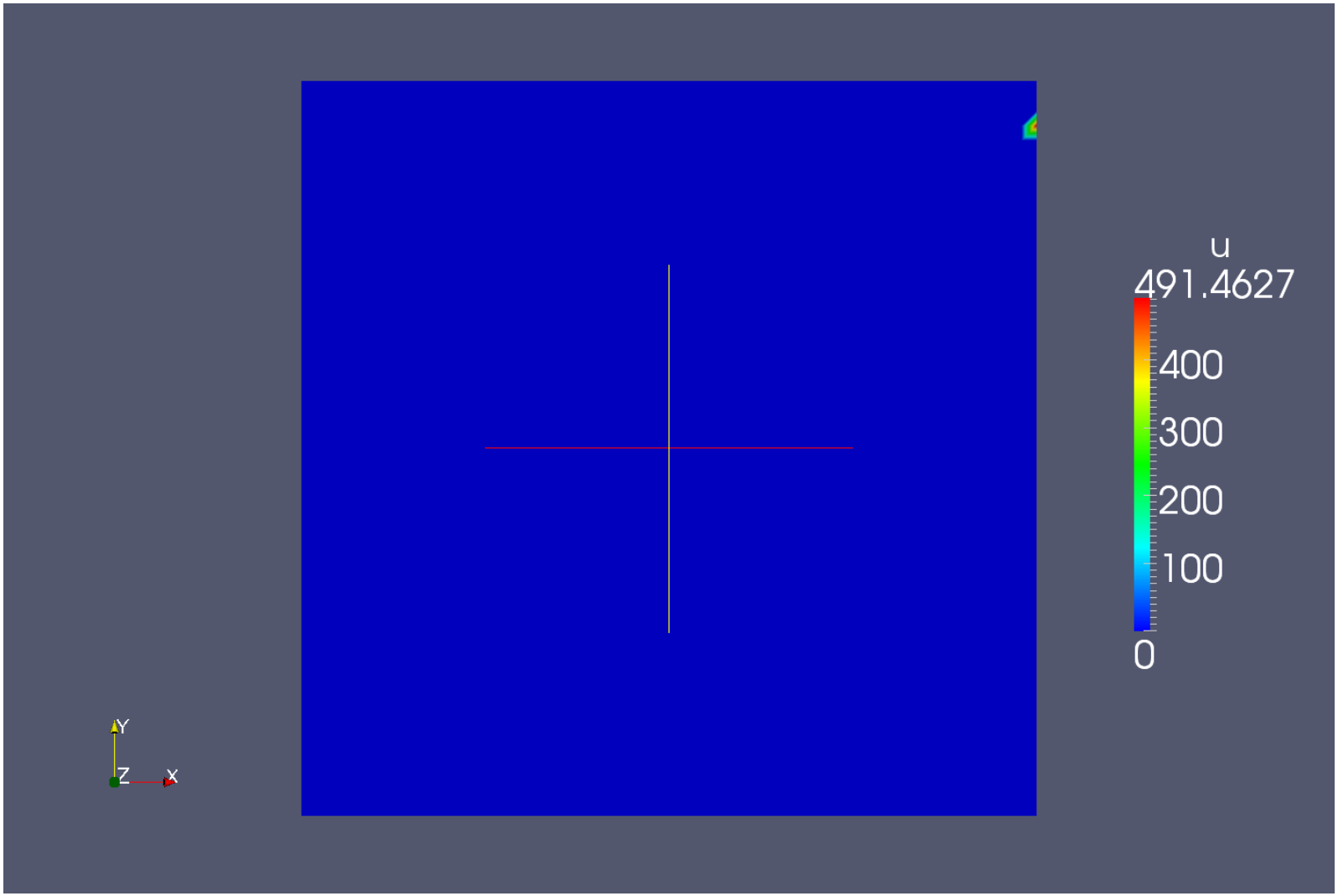

Figure: **Adjoint solution to Poisson Equation.** 1×10^0 *total histories, 0.286 seconds CPU time.*

Evolution of a Solution

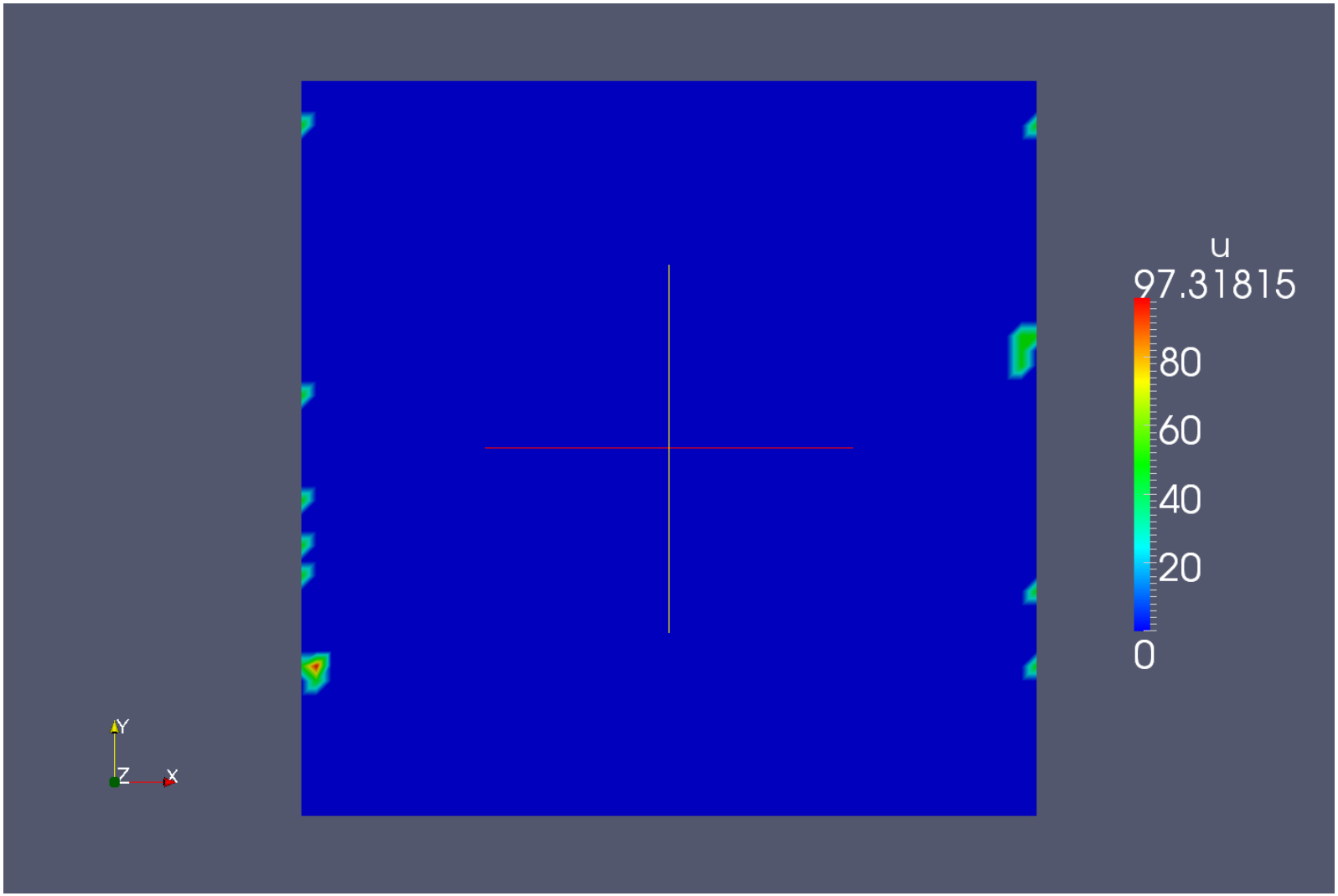

Figure: **Adjoint solution to Poisson Equation.** 1×10^1 *total histories, 0.278 seconds CPU time.*

Evolution of a Solution

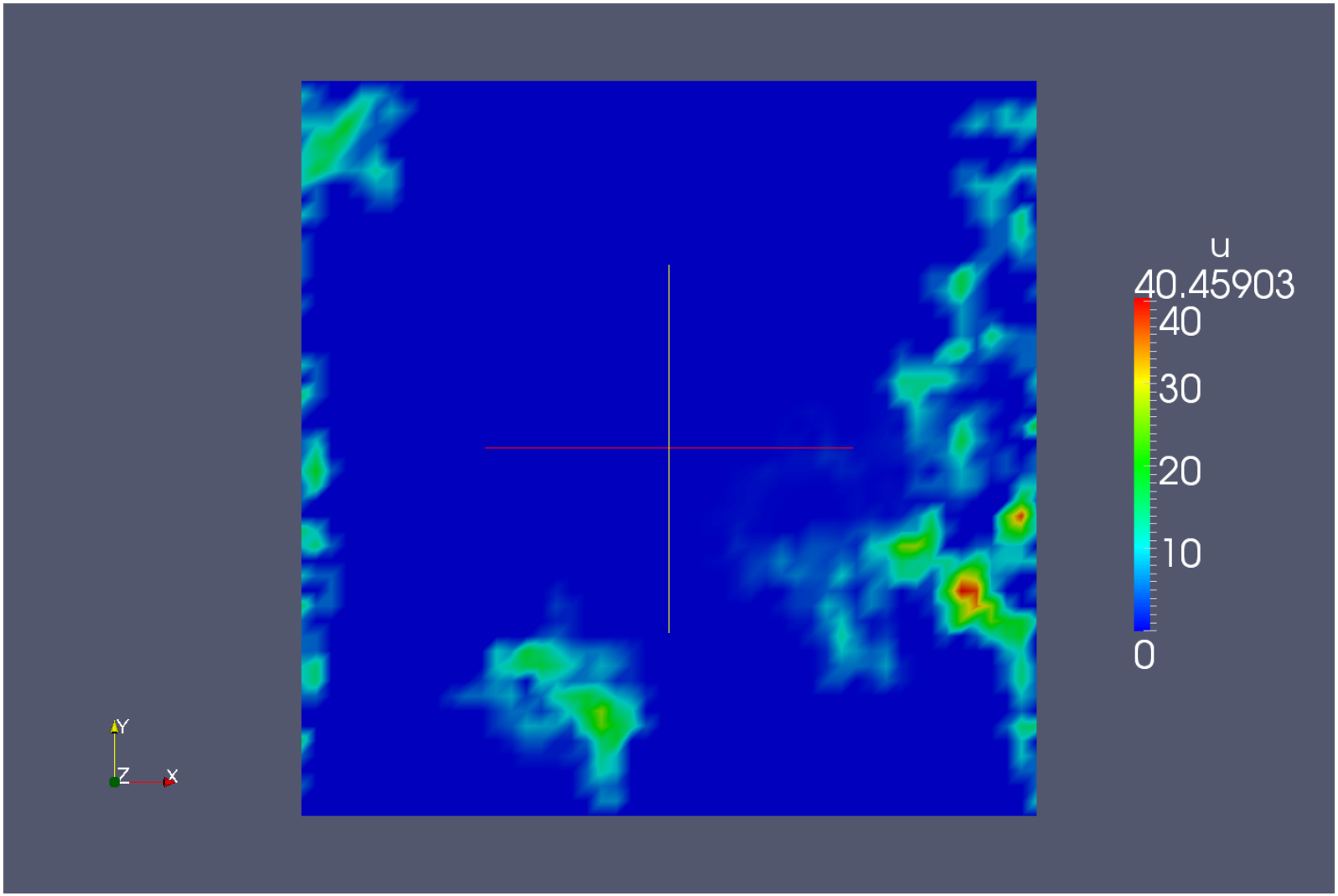

Figure: **Adjoint solution to Poisson Equation.** 1×10^2 *total histories, 0.275 seconds CPU time.*

Evolution of a Solution

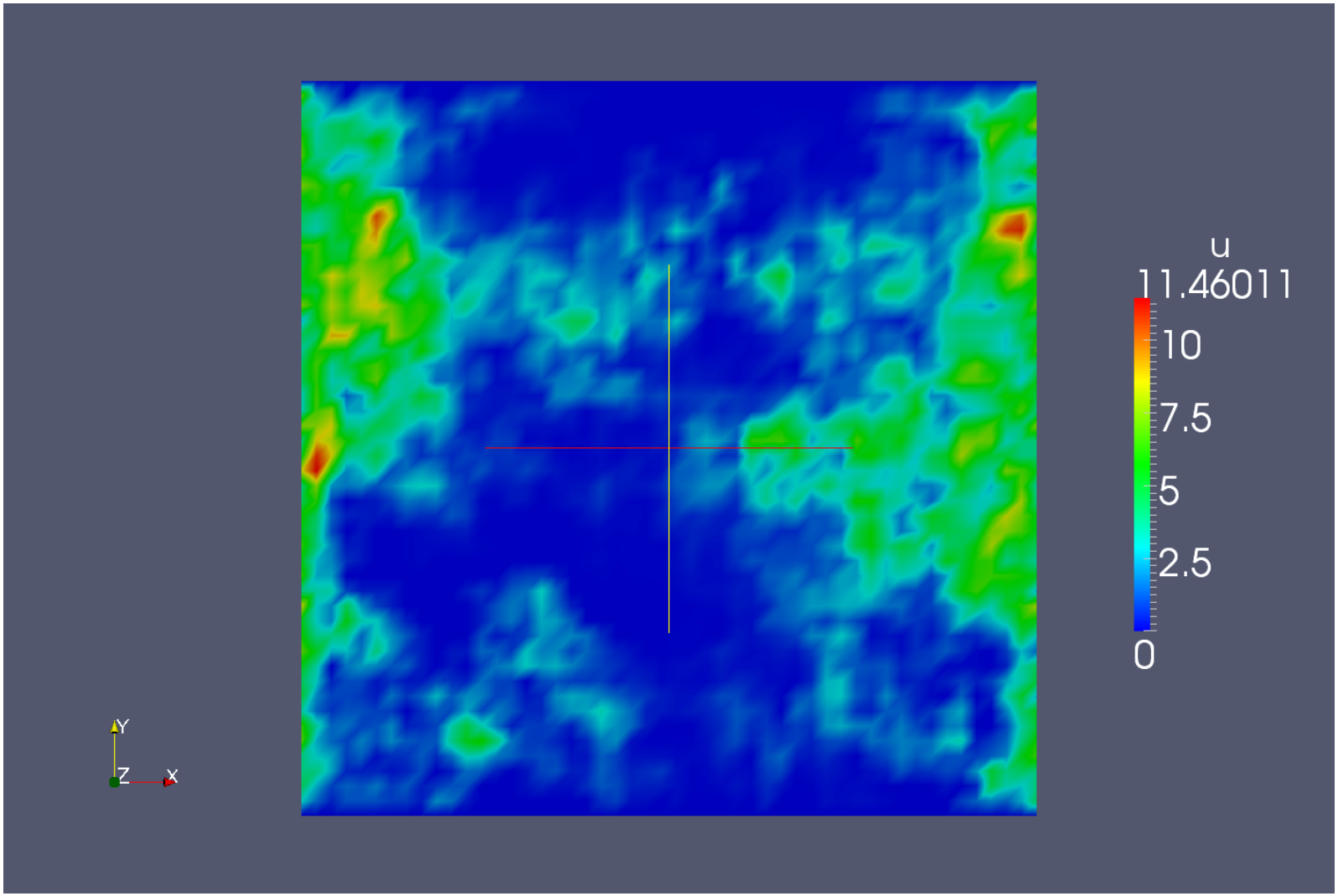

Figure: **Adjoint solution to Poisson Equation.** 1×10^3 *total histories, 0.291 seconds CPU time.*

Evolution of a Solution

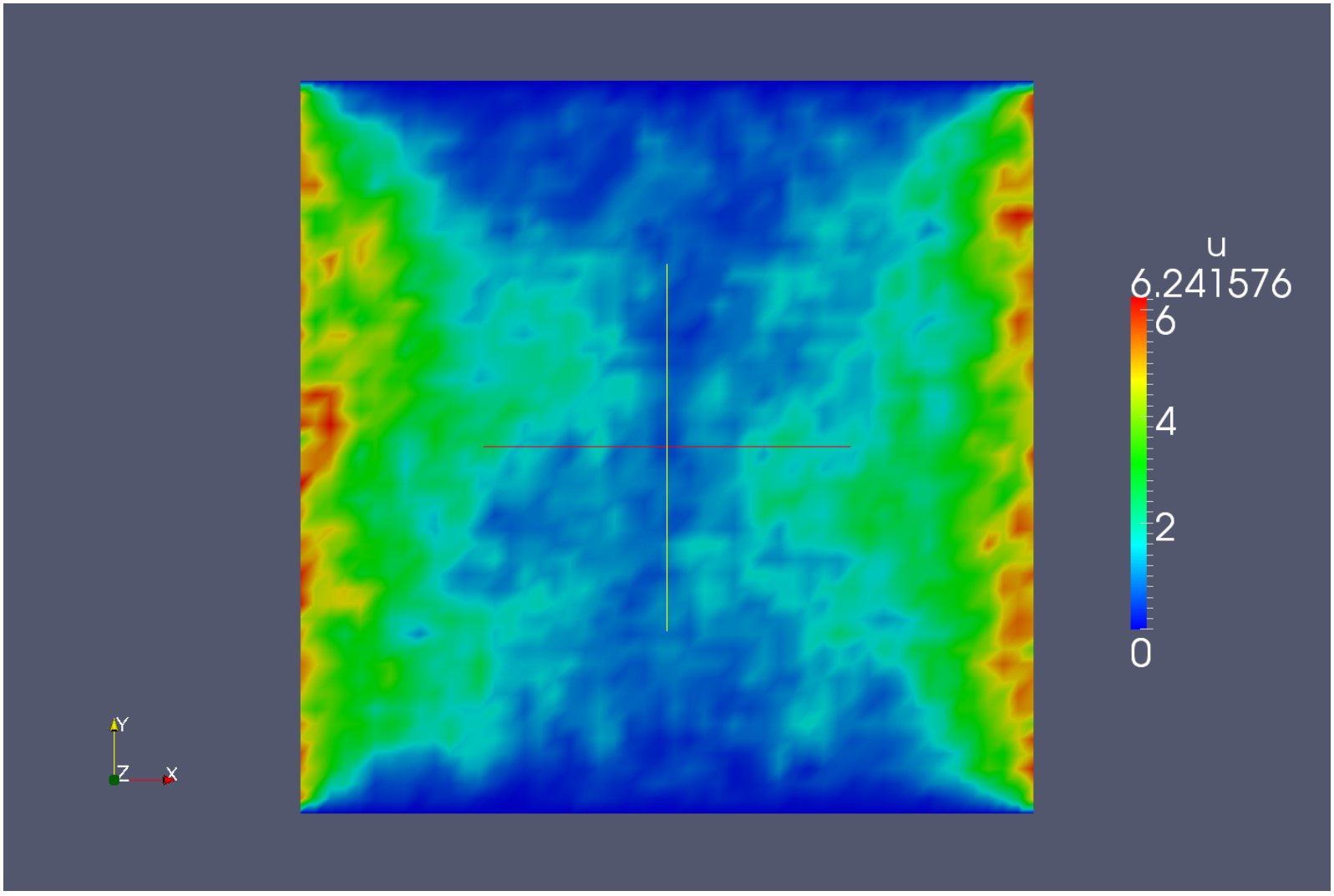

Figure: **Adjoint solution to Poisson Equation.** 1×10^4 *total histories, 0.428 seconds CPU time.*

Evolution of a Solution

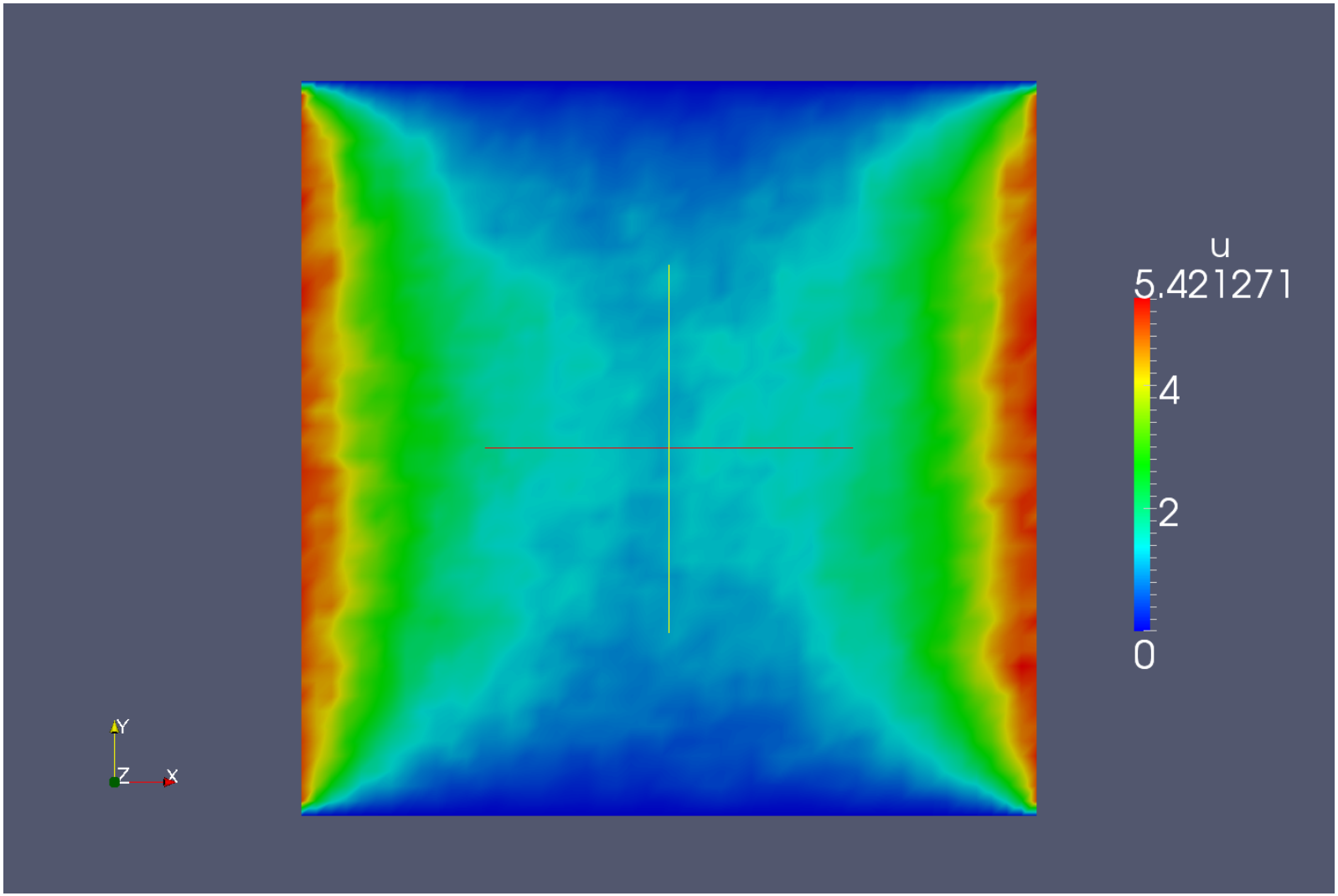

Figure: **Adjoint solution to Poisson Equation.** 1×10^5 *total histories, 1.76 seconds CPU time.*

Evolution of a Solution

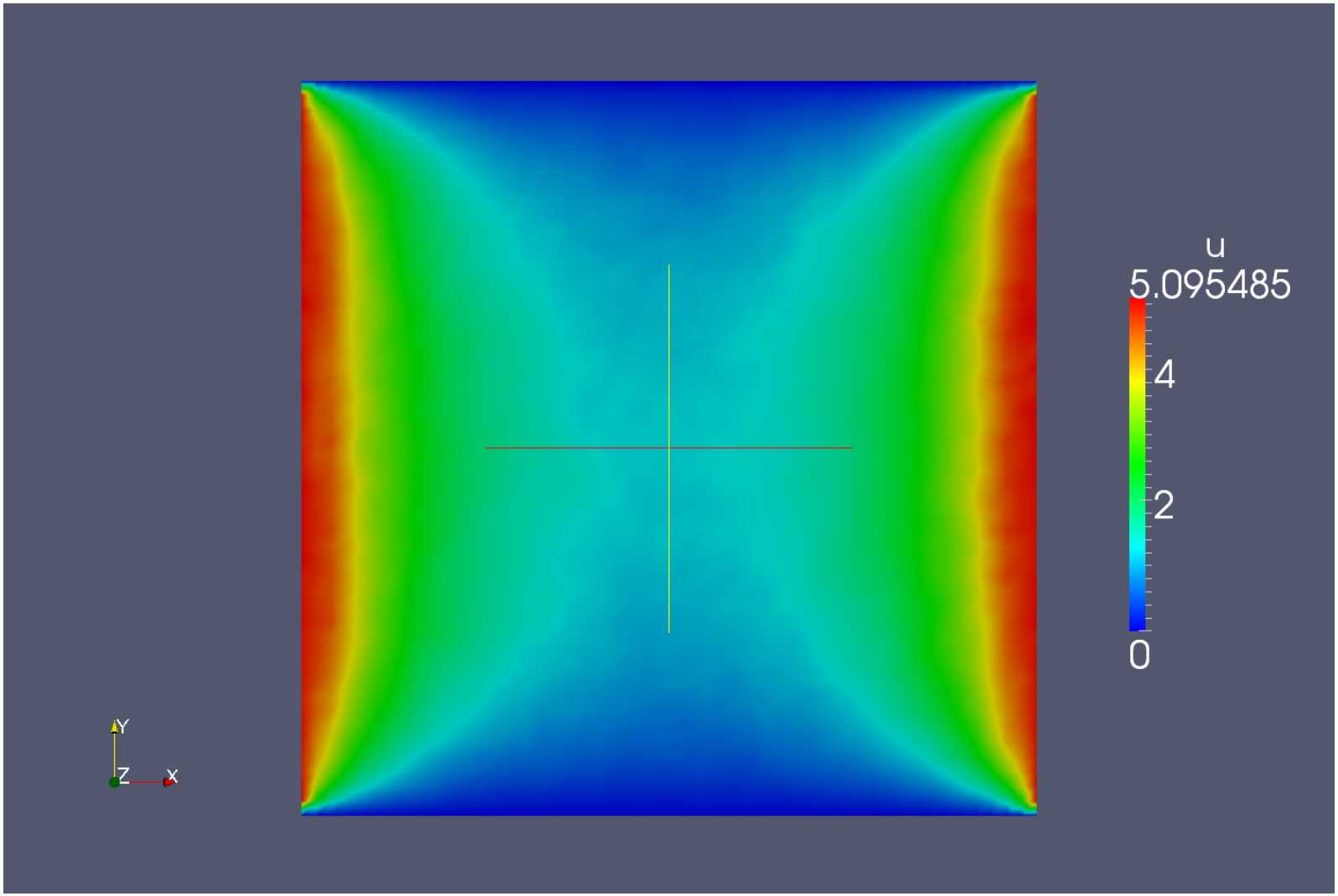

Figure: **Adjoint solution to Poisson Equation.** 1×10^6 *total histories, 15.1 seconds CPU time.*

Evolution of a Solution

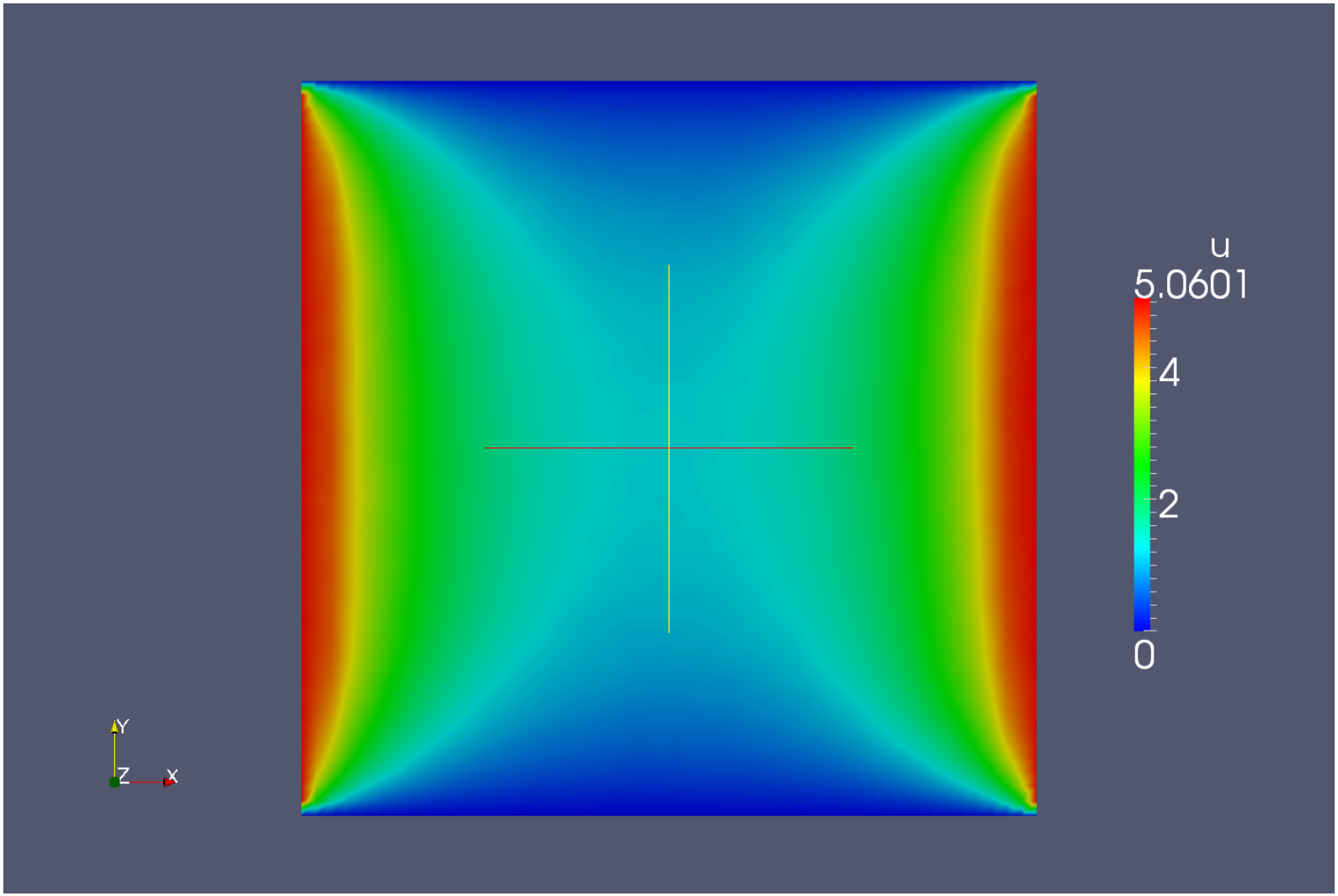

Figure: **Adjoint solution to Poisson Equation.** 1×10^7 *total histories, 149 seconds CPU time.*

Monte Carlo Synthetic-Acceleration

MCSA Iteration

$$\mathbf{r}^k = \mathbf{b} - \mathbf{A}\mathbf{x}^k$$

$$\mathbf{x}^{k+1/2} = \mathbf{x}^k + \mathbf{r}^k$$

$$\mathbf{r}^{k+1/2} = \mathbf{b} - \mathbf{A}\mathbf{x}^{k+1/2}$$

$$\hat{\mathbf{A}}\delta\mathbf{x}^{k+1/2} = \mathbf{r}^{k+1/2}$$

$$\mathbf{x}^{k+1} = \mathbf{x}^{k+1/2} + \delta\mathbf{x}^{k+1/2}$$

- Neumann-Ulam methods bound by the Central Limit Theorem
- Build on Halton's 1962 Sequential Monte Carlo method
- Neumann-Ulam Monte Carlo solver computes the correction
- Decouples MC error from solution error, exponential convergence

Monte Carlo Linear Solvers Library (MCLS)

- Preliminary development during January of 2013
- General asynchronous MSOD MCSA implementation
  - Forward and adjoint Monte Carlo with method of expected values
  - Parallel row matrix/vector interface
  - General fixed point iteration strategy
  - Explicit algebraic preconditioner suite
- Implemented in C++
- Heavy use of the Trilinos scientific computing libraries
- Open-source BSD 3-clause license
- https://github.com/sslattery/MCLS

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SP_N Equations

$$\hat{\Omega}\cdot\vec{\nabla}\psi(\vec{r},\hat{\Omega},E)+\sigma(\vec{r},E)\psi(\vec{r},\hat{\Omega},E)= \iint \sigma_s(\vec{r},E'\to E,\hat{\Omega}'\cdot\hat{\Omega})\psi(\vec{r},\hat{\Omega}',E')d\Omega' dE'+q(\vec{r},\hat{\Omega},E)\,, \quad (1)$$

$$-\nabla\cdot\left[\frac{n}{2n+1}\frac{1}{\Sigma_{n-1}}\nabla\left(\frac{n-1}{2n-1}\phi_{n-2}+\frac{n}{2n-1}\phi_n\right) + \frac{n+1}{2n+1}\frac{1}{\Sigma_{n+1}}\nabla\left(\frac{n+1}{2n+3}\phi_n+\frac{n+2}{2n+3}\phi_{n+2}\right)\right] + \Sigma_n\phi_n = q\delta_{n0} \qquad n=0,2,4,\cdots,N\,, \quad (2)$$

$$-\nabla\cdot\mathbb{D}_n\nabla\mathbb{U}_n+\sum_{m=1}^{4}\mathbb{A}_{nm}\mathbb{U}_m=\frac{1}{k}\sum_{m=1}^{4}\mathbb{F}_{nm}\mathbb{U}_n \qquad n=1,2,3,4\,. \quad (3)$$

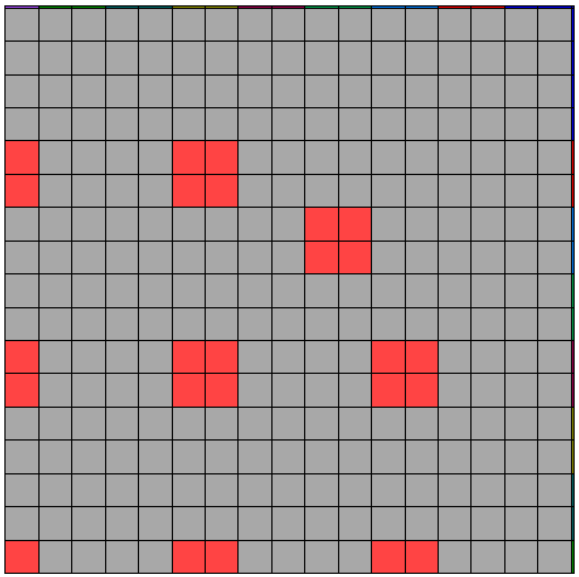
k-eigenvalue Solutions with MCSA

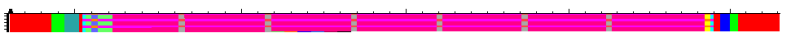
Algorithm 1 Power Iteration MCSA Scheme

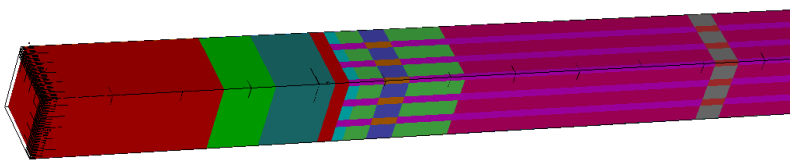
$k_0 =$ initial guess
$\mathbf{\Phi}_0 =$ initial guess
$n = 0$
while $|\frac{k^n - k^{n-1}}{k^n}| < \epsilon$ **do**
  $\mathbf{M\Phi}^{n+1} = \frac{1}{k^n}\mathbf{F\Phi}^n$ {Solve for the new flux state with MCSA}
  $k^{n+1} = k^n \frac{\int \mathbf{F\Phi}^{n+1} d\mathbf{r}}{\int \mathbf{F\Phi}^n d\mathbf{r}}$
  $n = n + 1$
end while

- Swap in MCSA as the solver at each eigenvalue operation
- Use Thyra/Stratimikos interfaces for plug and play - Thanks Ross!
- Current Exnihilo implementation gives the full operator

Fuel Assembly Criticality Calculations







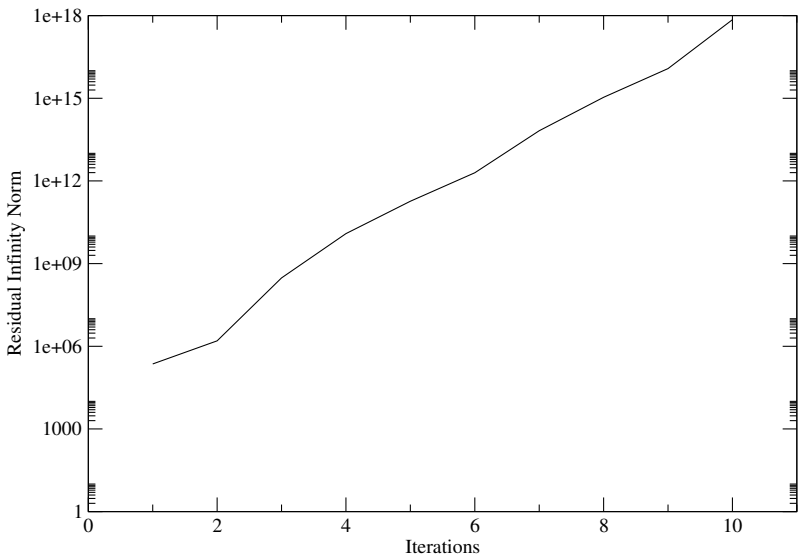
- CASL Problem 3: 17×17 quarter symmetry HZP LWR fuel assembly
- SP_N criticality calculation
- MCLS leveraged by the Exnihilo code base (ORNL)
- Comparison to Trilinos Aztec Krylov solvers with ILUT

Parameter	**Value**
Power Level	0 MW
Inlet Temperature	326.85C
Fuel Temperature	600C
Boron Concentration	1300 ppm
Moderator Density	0.743 g/cc
Helium Density	1.79×10^{-4} g/cc
Zirconium Density	6.56 g/cc
Stainless Steel Density	8.0 g/cc
Inconel Density	8.19 g/cc
UO2 Density	10.257 g/cc
Fuel Pin Radius (w/o clad)	0.4096 cm

Jacobi Preconditioned Calculations

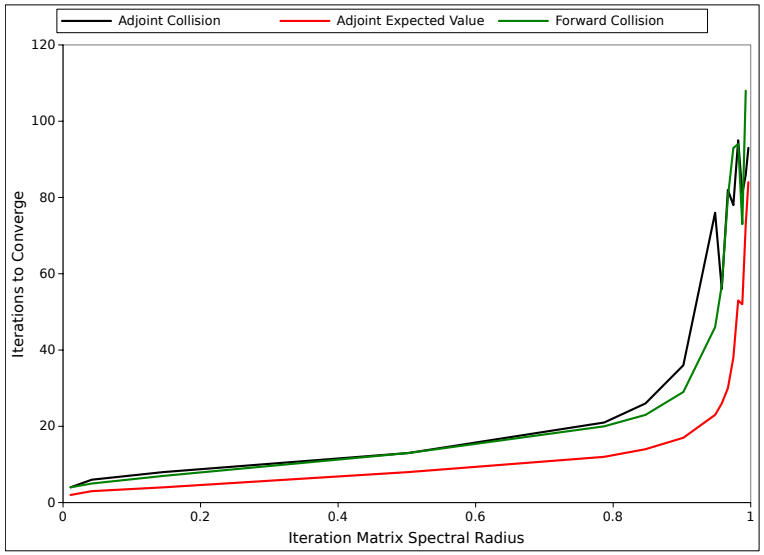
		SP_N Order			
		1	**3**	**5**	**7**
P_N Order	**0**	0.0647	0.1275	0.1449	0.1514
	1	0.0686	0.1338	0.1484	0.1547
	3	0.0687	0.1399	0.1582	0.1625
	5	0.0692	0.1399	0.1582	0.1657
	7	0.0678	0.1393	0.1624	0.166

Table: **Spectral radius results for the block Jacobi preconditioned iteration matrix with 10 energy groups and full downscatter for sample problem.**

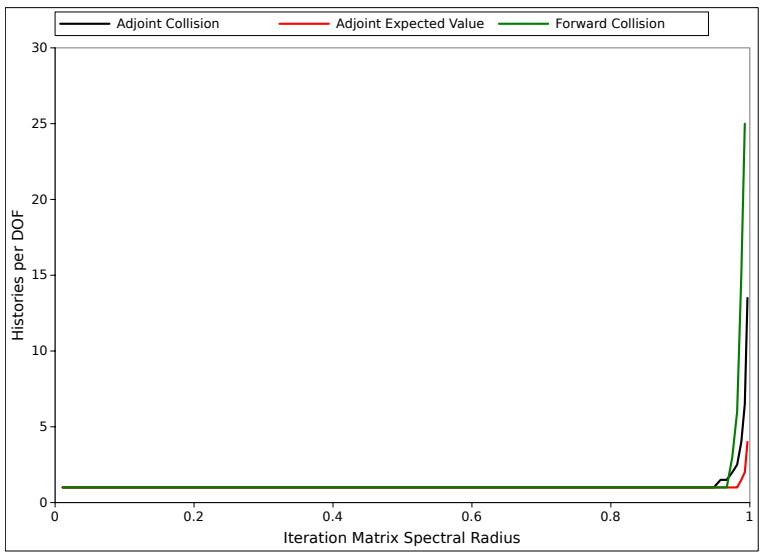


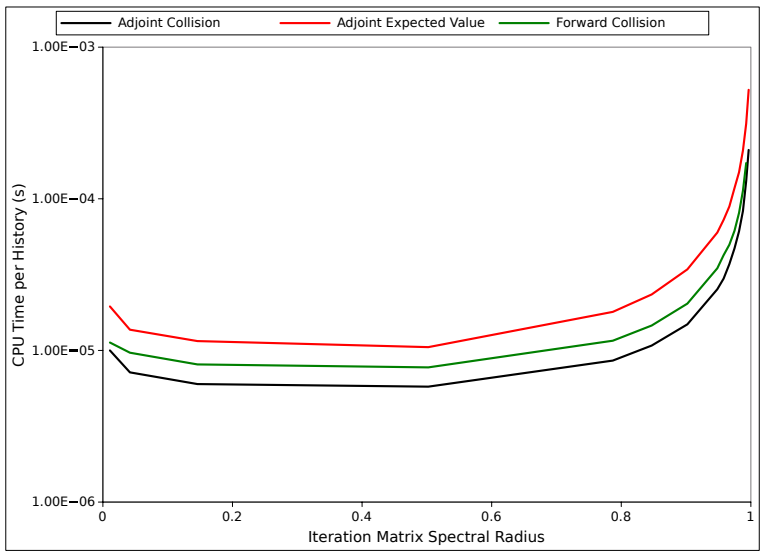
- Light water moderator creates a lot of scattering and $\rho(\mathbf{H}) \approx 1$
- Intractable number of histories required for MCSA convergence

MCSA Breakdown



- As $\rho(\mathbf{H}) \rightarrow 1$ terrible things happen...
- A more robust set of preconditioners is required for the SP_N equations





Explicit Preconditioning

$$\mathbf{M}_L^{-1}\mathbf{A}\mathbf{M}_R^{-1}\mathbf{M}_R\mathbf{x} = \mathbf{M}_L^{-1}\mathbf{b} \quad \rightarrow \quad \mathbf{M}_L^{-1}\mathbf{A}\mathbf{M}_R^{-1}\mathbf{u} = \mathbf{M}_L^{-1}\mathbf{b}$$

$$\mathbf{x} = \mathbf{M}_R^{-1}\mathbf{u}$$

Left/Right Preconditioned MCSA Iteration

$$\mathbf{r}^k = \mathbf{M}_L^{-1}(\mathbf{b} - \mathbf{A}\mathbf{M}_R^{-1}\mathbf{u}^k)$$

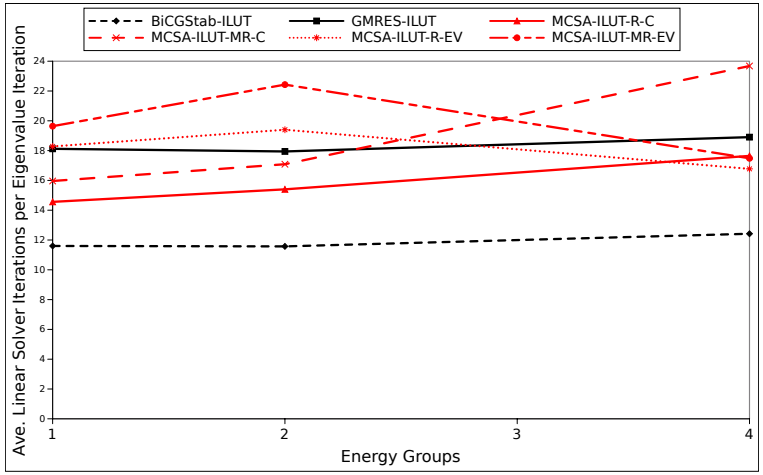
$$\mathbf{u}^{k+1/2} = \mathbf{u}^k + \mathbf{r}^k$$

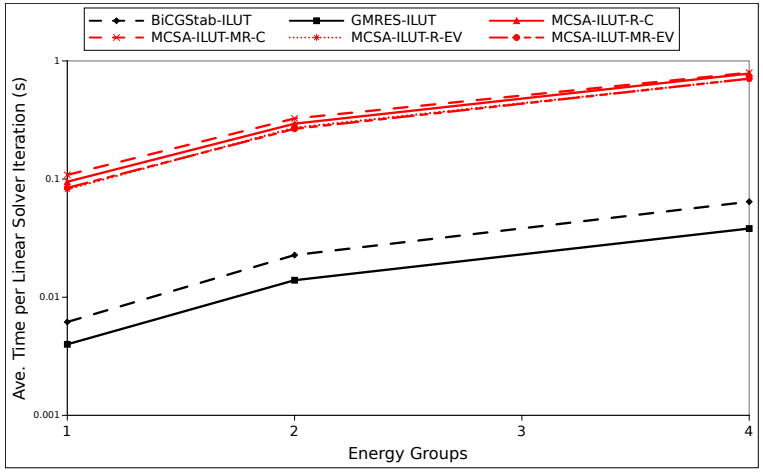
$$\mathbf{r}^{k+1/2} = \mathbf{M}_L^{-1}(\mathbf{b} - \mathbf{A}\mathbf{M}_R^{-1}\mathbf{u}^{k+1/2})$$

$$\mathbf{M}_L^{-1}\mathbf{A}\mathbf{M}_R^{-1}\delta\mathbf{u}^{k+1/2} = \mathbf{r}^{k+1/2}$$

$$\mathbf{u}^{k+1} = \mathbf{u}^{k+1/2} + \delta\mathbf{u}^{k+1/2}$$

Fuel Assembly Results





- MCLS generates the same k-eigenvalue and neutron flux in all groups and spatial locations
- MCSA converged in fewer iterations than GMRES, more iterations than BiCGStab
- Explicit preconditioning strategy destroys sparsity and elevates CPU times (Ifpack ILUT)
- Spectral radius and memory limitations combined to prevent solutions at finer discretizations

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- Monte Carlo Synthetic Acceleration Methods
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- **Application to Fluid Flow**
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Monte Carlo Synthetic Acceleration for Nonlinear Problems

- Many physics problems of interest are nonlinear
- Signficant research on Newton methods since the 1980's
- Newton methods often leverage Krylov solvers
  - Robust implementations
  - No operator required
- Monte Carlo methods need the full operator
- Automatic construction of the linear model is available
  - Operator overloading for nonlinear residual differentiation
  - Ideal for Monte Carlo
  - Potential scaling improvements
  - Resiliency benefits

Newton's Method

- Seek solutions of the general nonlinear problem

$$\mathbf{F}(\mathbf{u}) = \mathbf{0}$$

$$\mathbf{u} \in \mathbb{R}^n, \ \mathbf{F} : \mathbb{R}^N \to \mathbb{R}^N$$

- Interpret the exact solution $\mathbf{u}$ to be the roots of $\mathbf{F}(\mathbf{u})$

$$\mathbf{F}(\mathbf{u}^{k+1}) = \mathbf{F}(\mathbf{u}^k) + \mathbf{F}'(\mathbf{u}^k)(\mathbf{u}^{k+1} - \mathbf{u}^k) + \frac{\mathbf{F}''(\mathbf{u}^k)}{2}(\mathbf{u}^{k+1} - \mathbf{u}^k)^2 + \cdots$$

- Form Newton's method

$$\mathbf{J}(\mathbf{u})\delta\mathbf{u}^k = -\mathbf{F}(\mathbf{u}^k)$$

$$\mathbf{u}^{k+1} = \mathbf{u}^k + \delta\mathbf{u}^k$$

The FANM Method

Forward-Automated Newton-MCSA

Algorithm 2 FANM

1: $k := 0$
2: **while** $||\mathbf{F}(\mathbf{u}^k)|| > \epsilon||\mathbf{F}(\mathbf{u}^0)||$ **do**
3: $\quad \mathbf{J}(\mathbf{u}^k) \leftarrow AD(\mathbf{F}(\mathbf{u}^k))$ {Automatic differentiation}
4: $\quad \mathbf{J}(\mathbf{u}^k)\delta\mathbf{u}^k = -\mathbf{F}(\mathbf{u}^k)$ {Solve for the Newton correction with MCSA}
5: $\quad \mathbf{u}^{k+1} \leftarrow \mathbf{u}^k + \delta\mathbf{u}^k$
6: $\quad k \leftarrow k + 1$
7: **end while**

- Robustness of Newton's method (inexact)
- Accuracy and convenience of FAD
- Potential parallelism, and resiliency benefits of MCSA
- Requires only nonlinear function evaluations
- Can utilize globalization and forcing term selection methods

FANM Numerical Experiments

- Sequence of Navier-Stokes benchmarks for the nonlinear methods
  - Thermal convection cavity problem (De Vahl Davis, 1983)
  - Lid driven cavity problem (Ghia et al., 1982)
  - Backward-Facing step problem (Gartling, 1990)
- Tuning benchmark parameters varies the strength of nonlinearities
- Newton-Krylov method leveraging Aztec GMRES used for comparisons
- All problems preconditioned with algebraic multigrid (ML) and leveraged some kind of globalization (e.g. backtracking)

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} - \nabla \cdot \mathbf{T} - \rho \mathbf{g} = \mathbf{0}$$

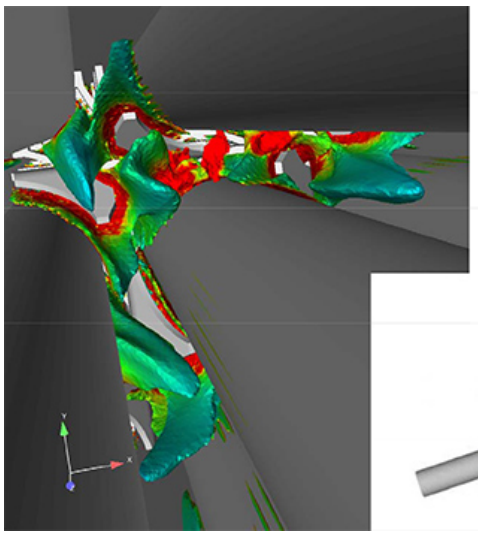
$$\nabla \cdot \mathbf{u} = 0$$

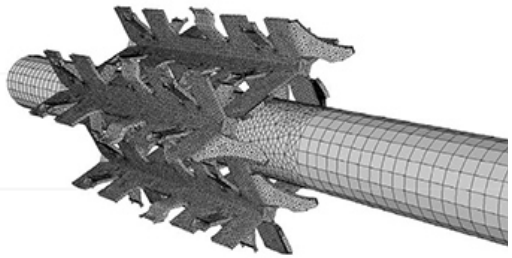
$$\rho C_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = 0$$

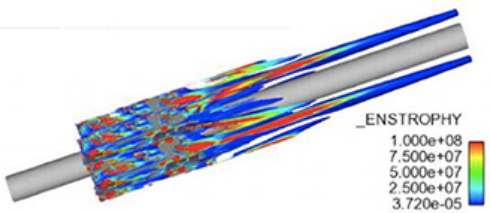
$$\mathbf{T} = -P\mathbf{I} + \mu[\nabla \mathbf{u} + \nabla \mathbf{u}^T]$$

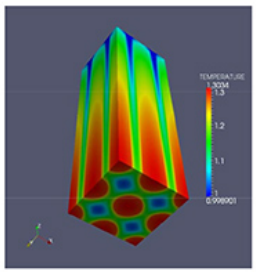
$$\mathbf{q} = -k \nabla T$$

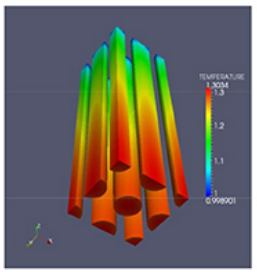
FANM Numerical Experiments

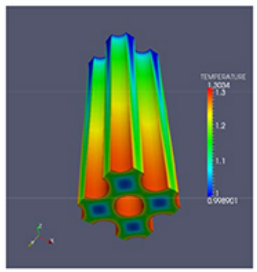


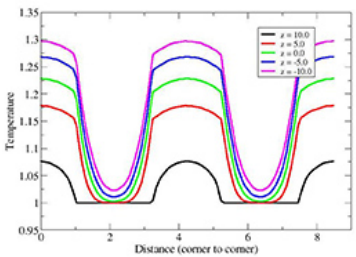


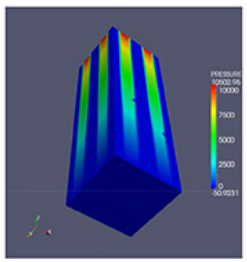


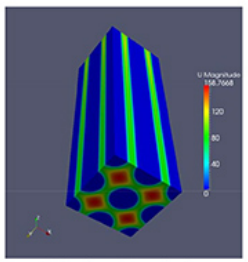

Temperature


Temperature (Pins)


Temperature (Fluid)




Pressure


Velocity Magnitude

- Drekar is a production physics suite that leverages Newton methods with FAD
- MCLS incorporated into the Drekar nonlinear solver scheme using Thyra/Stratimikos to implement FANM
- Thanks Roger!

Images source: www.casl.gov

FANM Nonlinear Iteration Results

Benchmark	Newton-Krylov	FANM
Convection, Ra=1×10^3	5	5
Convection, Ra=1×10^4	7	7
Convection, Ra=1×10^5	9	10
Convection, Ra=1×10^6	11	11
Lid Driven, Re=100	6	6
Lid Driven, Re=300	9	9
Lid Driven, Re=500	11	11
Lid Driven, Re=700	14	10
Backward Step, Re=200	10	9
Backward Step, Re=300	15	14
Backward Step, Re=400	10	10
Backward Step, Re=500	19	20

Table: **Navier-Stokes benchmark comparison for nonlinear iterations.** *Over all benchmarks, FANM performed better in terms of nonlinear iterations for 1 more case than the Newton-Krylov method.*

FANM Linear Solver Iteration Results

Benchmark	Newton-Krylov	FANM
Convection, Ra=1×10^3	32	18
Convection, Ra=1×10^4	23	17
Convection, Ra=1×10^5	25	20
Convection, Ra=1×10^6	39	25
Lid Driven, Re=100	27	42
Lid Driven, Re=300	35	52
Lid Driven, Re=500	41	56
Lid Driven, Re=700	21	14
Backward Step, Re=200	24	13
Backward Step, Re=300	23	17
Backward Step, Re=400	18	12
Backward Step, Re=500	30	52

Table: **Navier-Stokes benchmark comparison for total linear solver iterations.** *Over all benchmarks, FANM performed better in terms of linear solver iterations for twice as many cases as the Newton-Krylov method.*

FANM CPU Time Results

Benchmark	Newton-Krylov Speedup
Convection, Ra=1×10^3	338
Convection, Ra=1×10^4	336
Convection, Ra=1×10^5	346
Convection, Ra=1×10^6	465
Lid Driven, Re=100	299
Lid Driven, Re=300	322
Lid Driven, Re=500	288
Lid Driven, Re=700	488
Backward Step, Re=200	400
Backward Step, Re=300	593
Backward Step, Re=400	825
Backward Step, Re=500	1057

Table: **Newton-Krylov speedup over FANM.** *For all benchmarks the explicit MCSA preconditioning strategy caused significantly larger CPU times for FANM when compared to the Newton-Krylov solutions.*

Outline

- Monte Carlo Synthetic Acceleration Methods
- Application to Neutron Transport
- Application to Fluid Flow
- **Parallelization of MCSA**

Parallelization of Monte Carlo Methods

- No literature observed for parallel Neumann-Ulam solvers beyond history-level parallelism
- Numerous references for modern parallel Monte Carlo methods in reactor physics
- Build a strategy for applying modern methods to the Neumann-Ulam method
- MCSA iteration-level parallelism comes from parallel matrix/vector operations

Domain Decomposed Monte Carlo

- Each parallel process owns a piece of the domain (linear system)
- Random walks must be transported between adjacent domains through parallel communication
- Domain decomposition determined by the input system

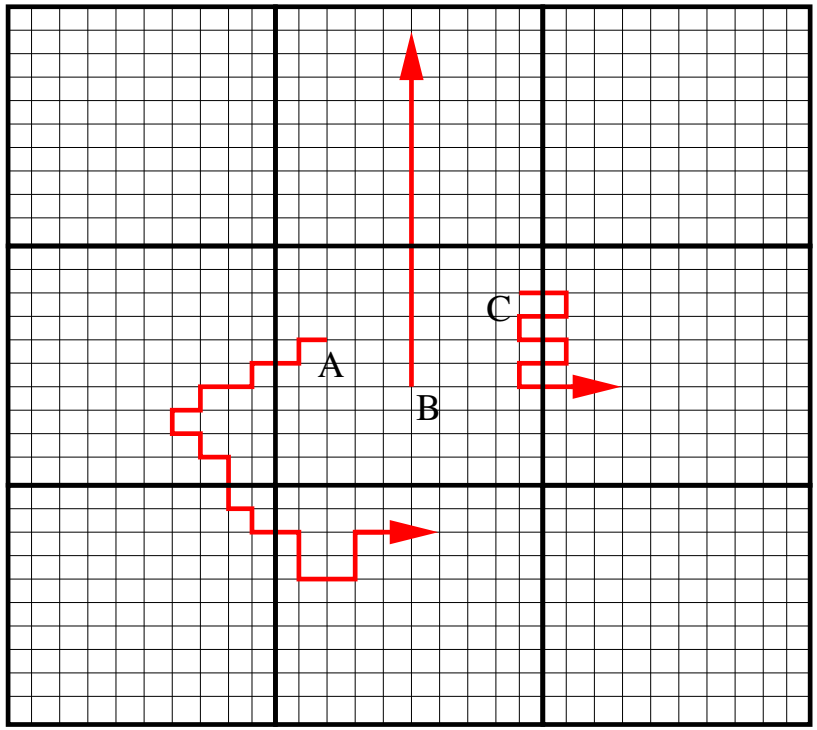
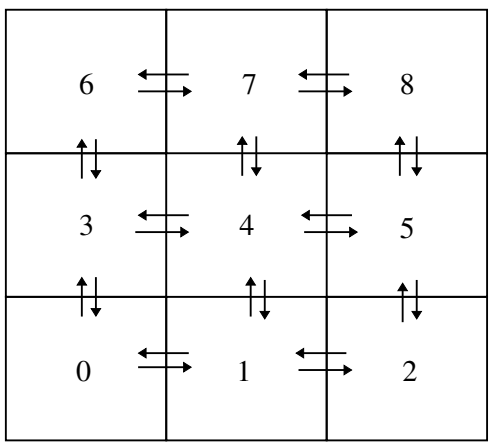


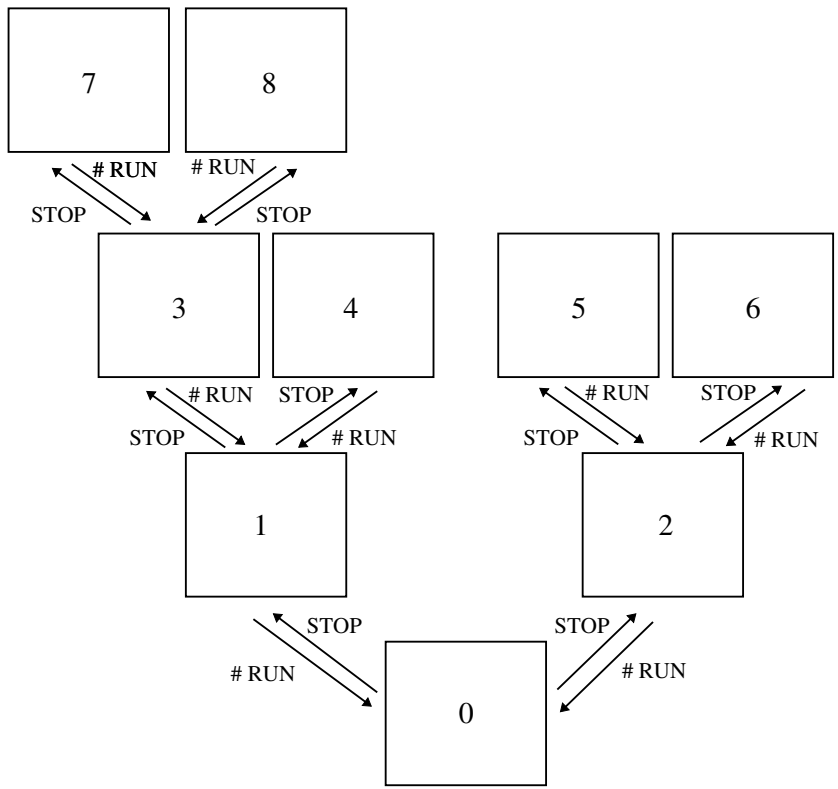
Figure: *Domain decomposition example illustrating how domain-to-domain transport creates communication costs.*

Asynchronous Monte Carlo Transport Kernel

- Developed by Brunner and Brantley in 2009
- Asynchronous nearest neighbor communication of histories
- Binary asynchronous communication tree for completing transport



- Extensible to problems where histories may be created (i.e. variance reduction)



Thomas A. Brunner and Patrick S. Brantley, "An efficient, robust, domain-decomposition algorithm for particle Monte Carlo", Journal of Computational Physics, vol. 228, pp.3882-3890, 2009.

Multiple-Set Overlapping-Domain Decomposition

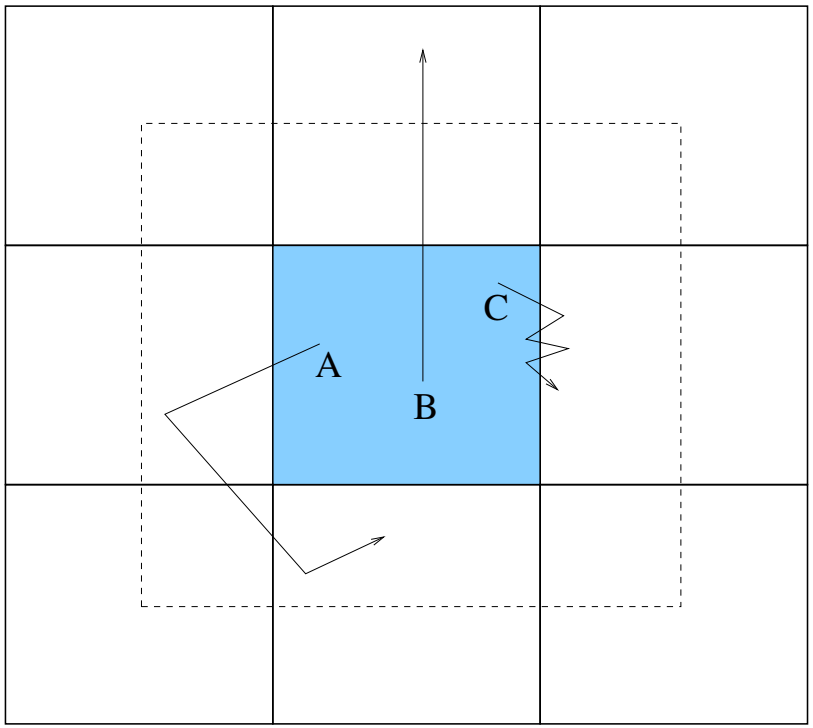


Figure: *Overlapping domain example illustrating how domain overlap can reduce communication costs.*

- Developed by Wagner and colleagues in 2010
- Each set contains the full domain
- Multiple sets replicate the domain
- Domains overlap within a set
- Redundancy for resiliency (and useful work)

Wagner et. al., "Hybrid and parallel domain-decomposition methods development to enable Monte Carlo for reactor analysis", Joint International Conference on Supercomputing in Nuclear Applications and Monte Carl (SNA+MC 2010), 2010.

Multiple-Set Overlapping-Domain Decomposition

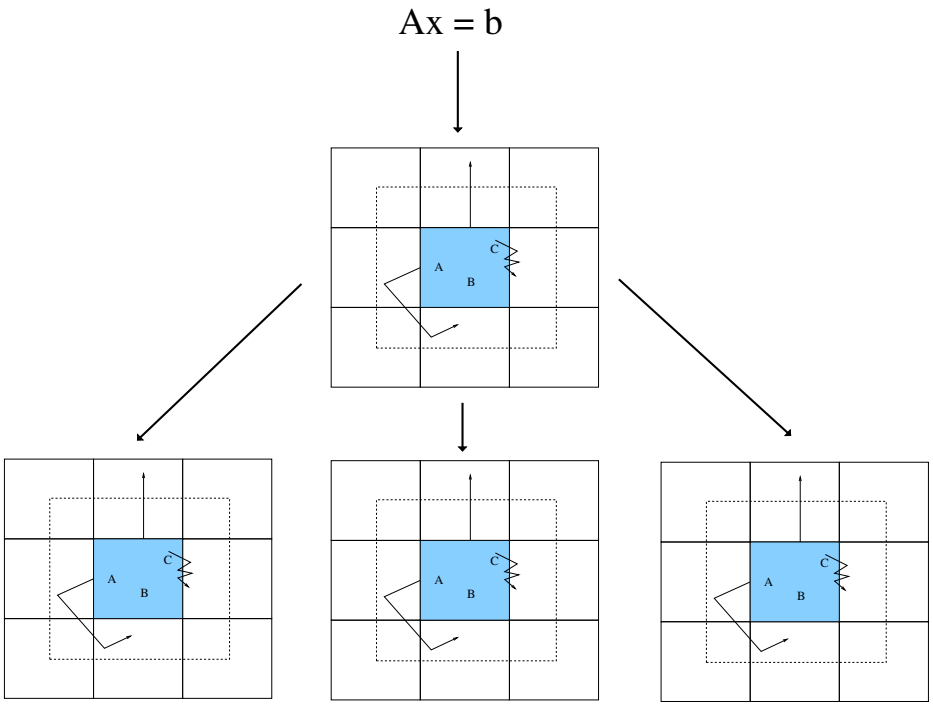


Figure: *MSOD construction.*

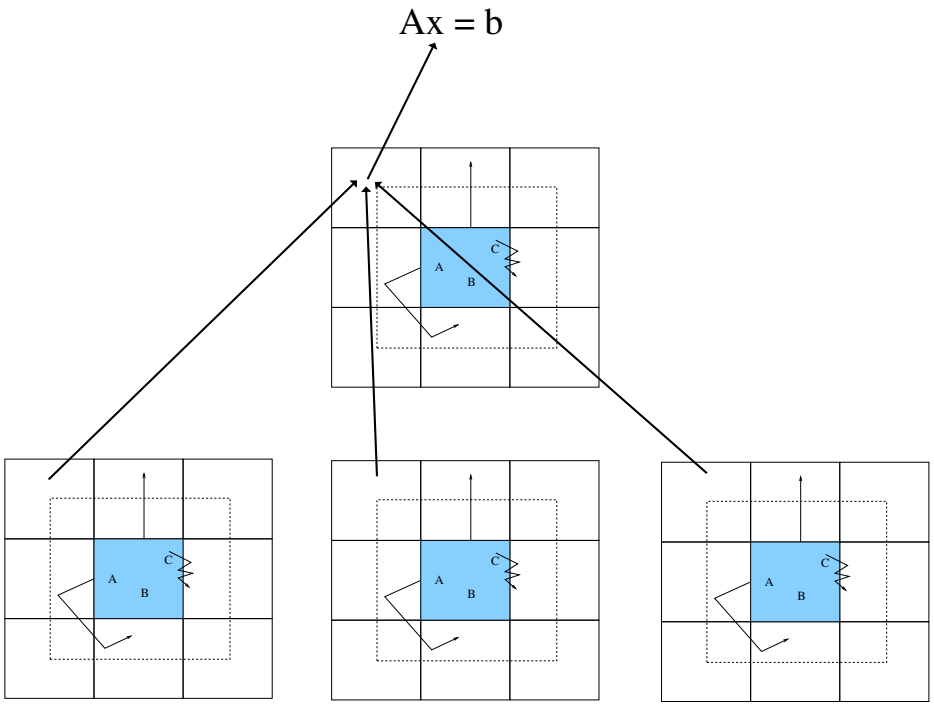


Figure: *MSOD tally reduction.*

Parallel Scaling

- Simple 2D neutron diffusion problem for control - spectral radius is maintained as global problem size grows
- Comparison to Trilinos Belos Krylov solvers with Jacobi preconditioning - conjugate gradient and GMRES
- Strong scaling - Global size fixed at 1.6E7 DOFs
- Weak scaling - Local size fixed at 4.0E4 DOFs
- Calculations performed on the Titan Cray XK7 machine at ORNL (MPI only)
- Limited MCLS arithmetic optimization artificially inflates efficiencies

Strong Scaling Results

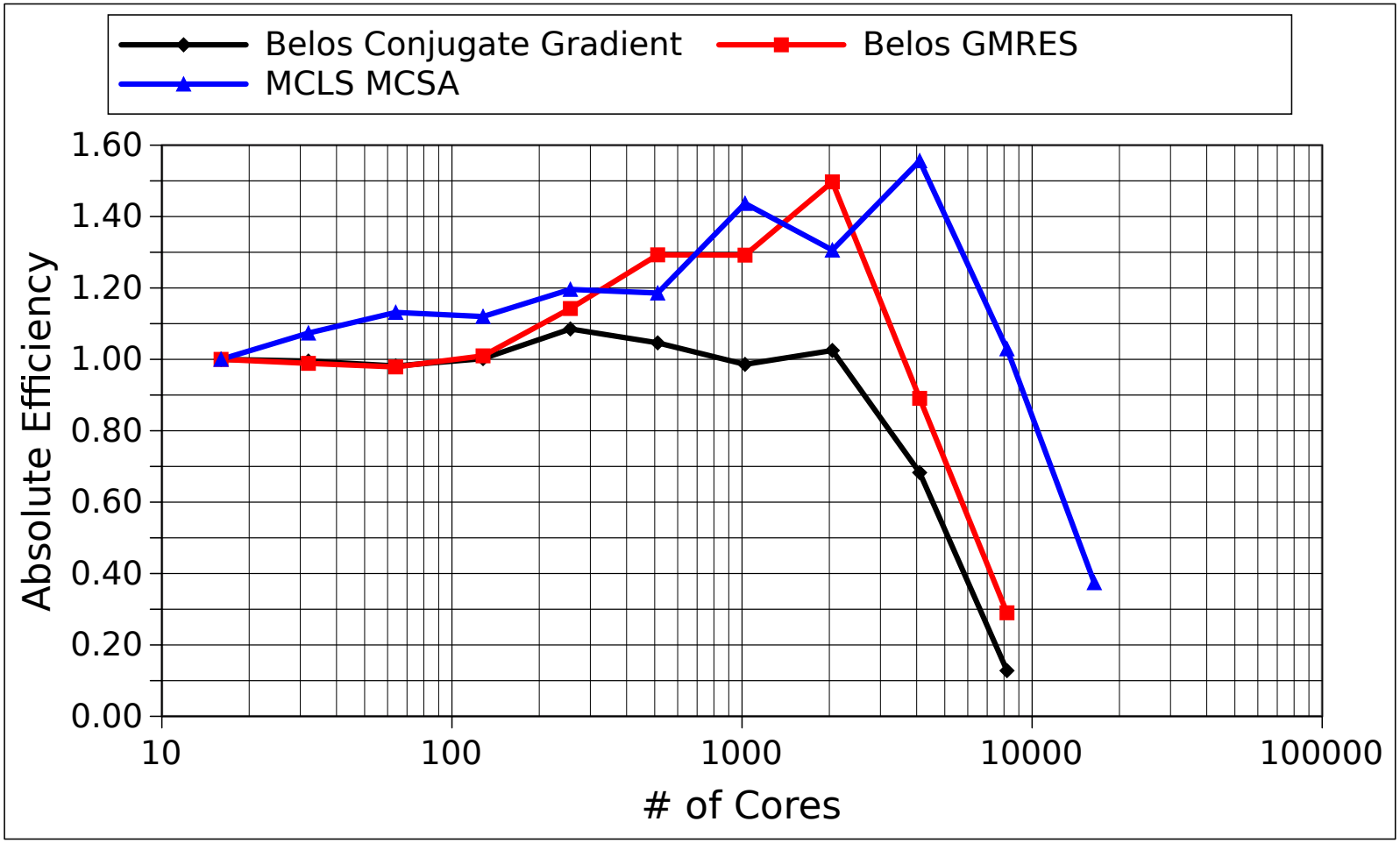
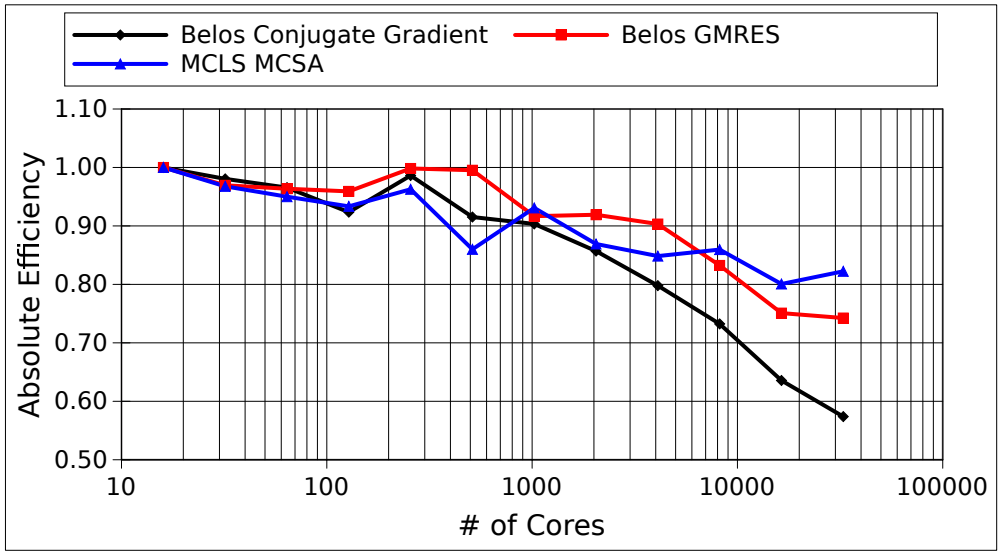
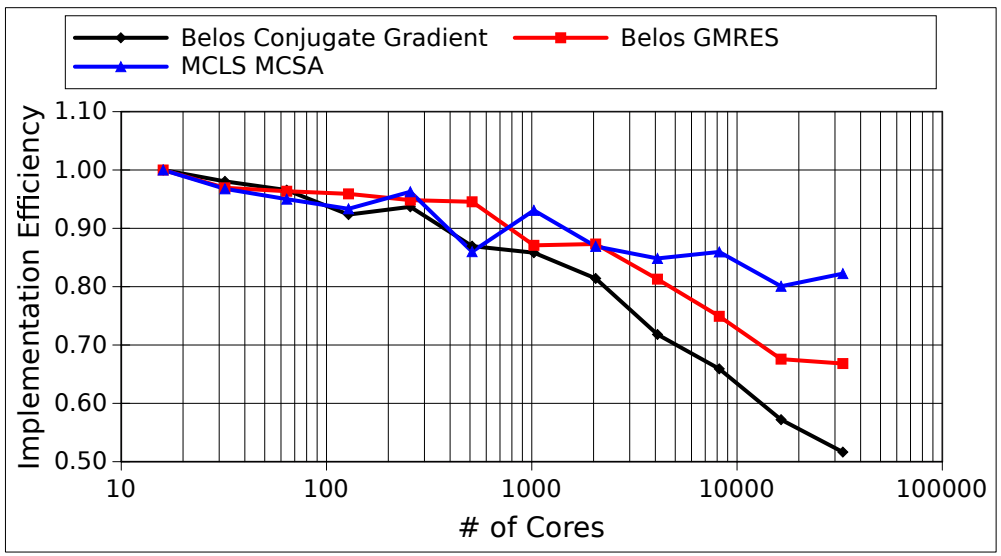


Figure: **Pure domain decomposition.** *Super-linear speed-up from memory thrashing in base case. MCLS is an order of magnitude slower arithmetically.*

Weak Scaling Results





- CG demonstrates poor scaling due to the cheaper iteration sequence
- Implementation efficiency is effectively the parallel efficiency of a single iteration
- Improved implementation efficiency means improved iterative performance will potentially give a better time to solution

Strong Scaling Results with Multiple Sets

Splitting: same global number of histories as the single set

Replicating: set-multiple of single set problem global histories

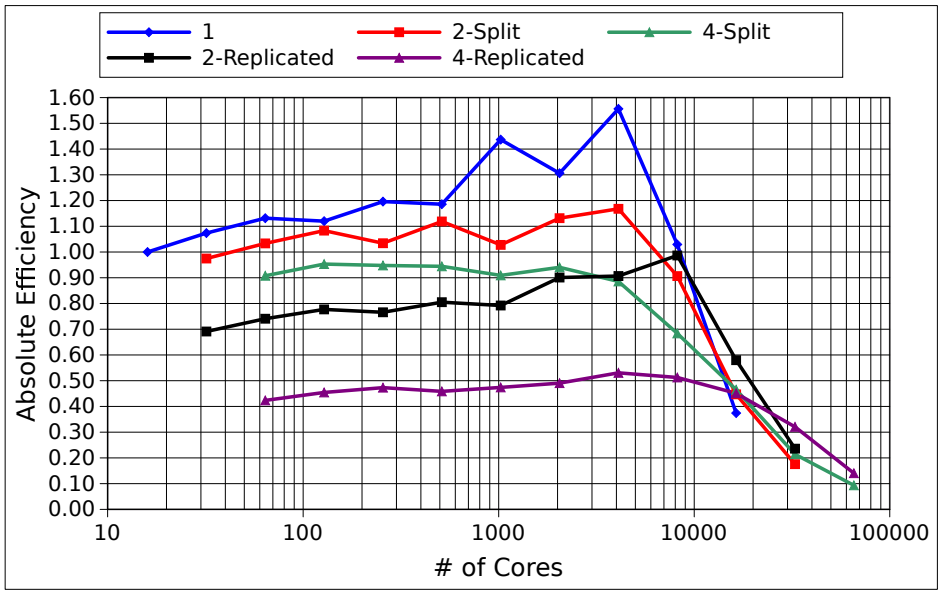


Figure: Absolute parallel efficiency relative to 16-core 1-set base case.

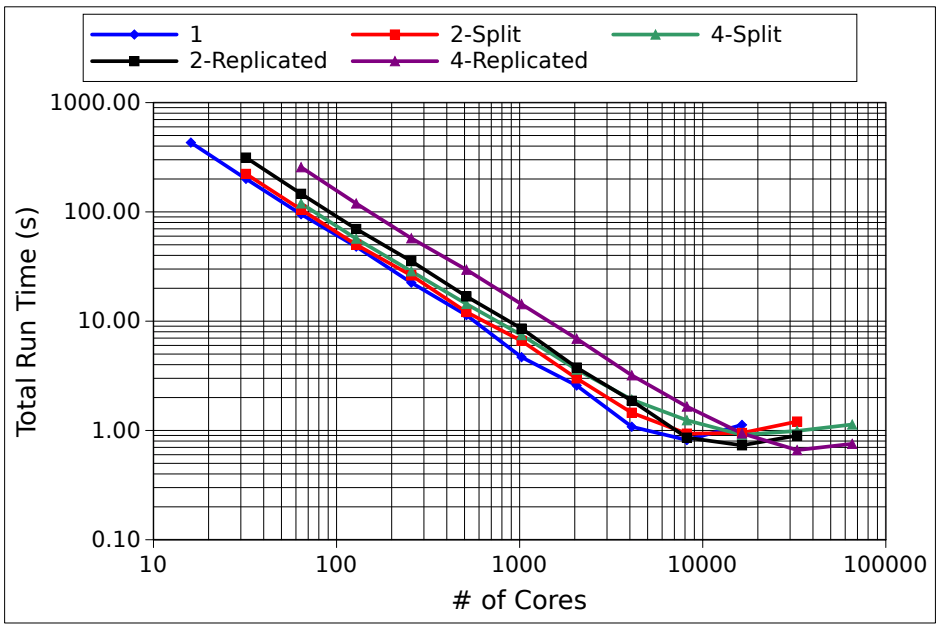


Figure: Wall time in seconds to solution for each case.

Weak Scaling Results with Multiple Sets

- Need to consider adding sets is a strong scaling exercise
- Modify the weak scaling efficiency computation to account for these extra resources
- Superposition of Monte Carlo results enhances time to solution!

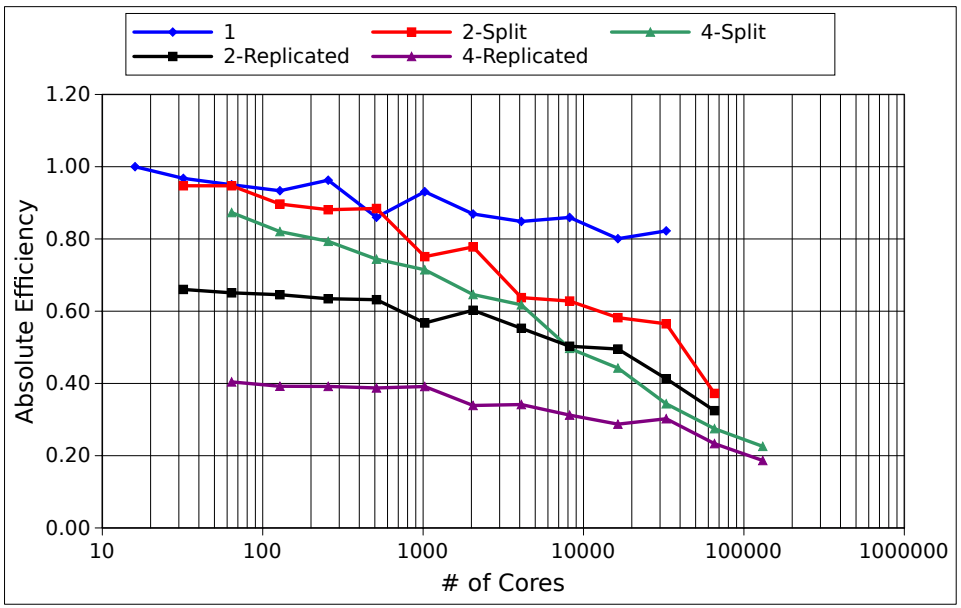


Figure: Absolute parallel efficiency relative to 16-core 1-set base case.

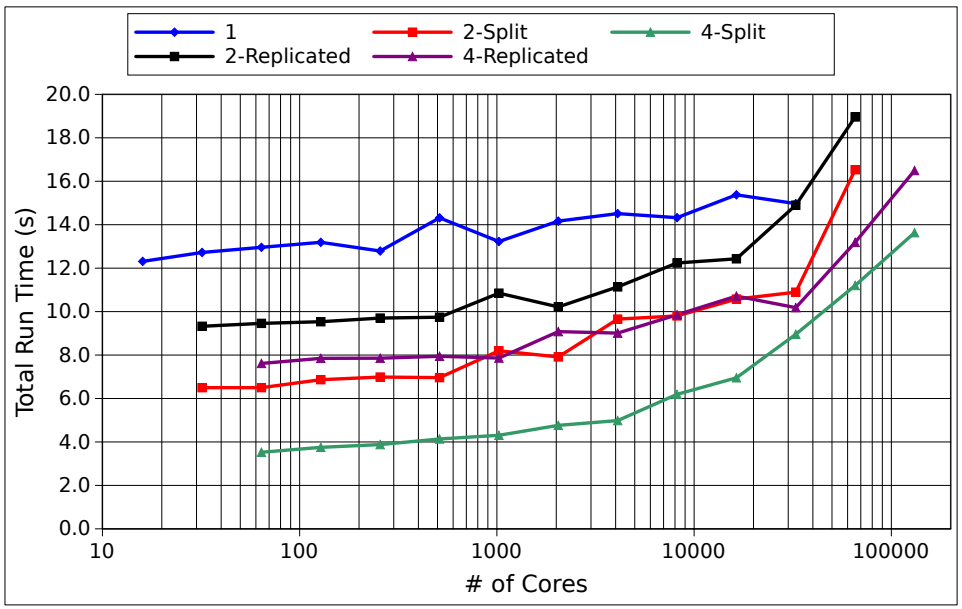


Figure: Wall time in seconds to solution for each case.

Strong Scaling Results with Overlap

- Overlap values selected based on average 'diffusion length' of a history in the system of 2.6 discrete states
- Overlap eliminates communication in the Monte Carlo sequence but simply differs it to an overlapping tally vector reduction

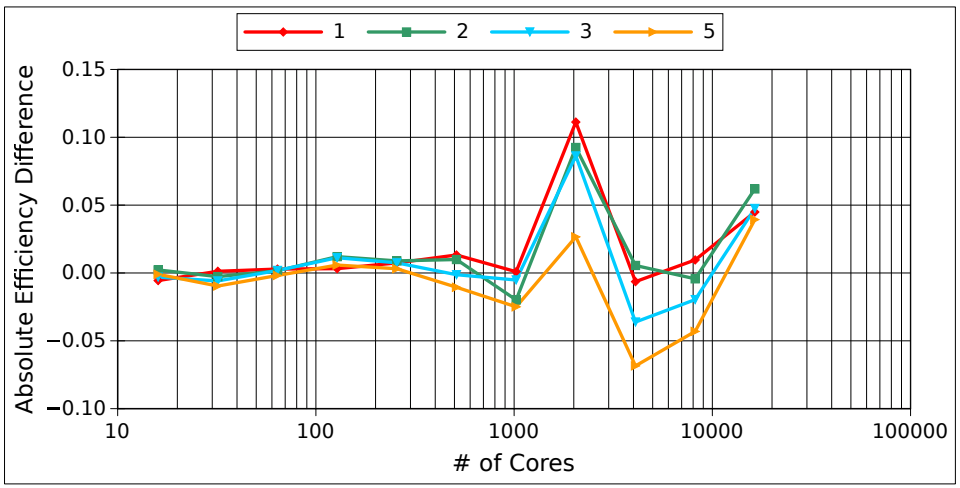


Figure: Strong scaling efficiency difference compared to the 0 overlap case.

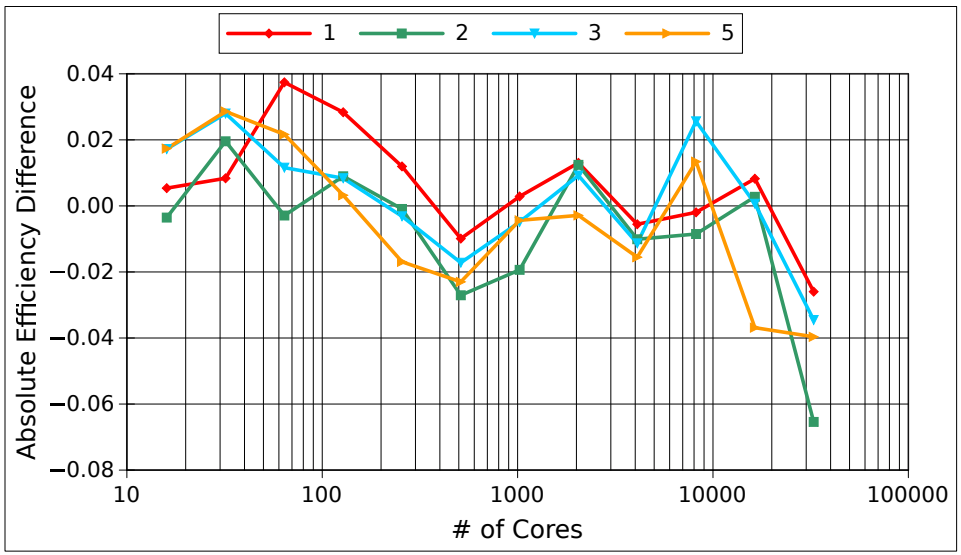


Figure: Weak scaling efficiency difference compared to the 0 overlap case.

MCSA as a Stochastic Additive Schwarz Method

- No domain-to-domain communication in Monte Carlo sequence
- Fixed point iteration acts as a smoother
- Observed to converge in the same number of iterations
- Can add overlap to preserve iterative performance for more ill-conditioned problems

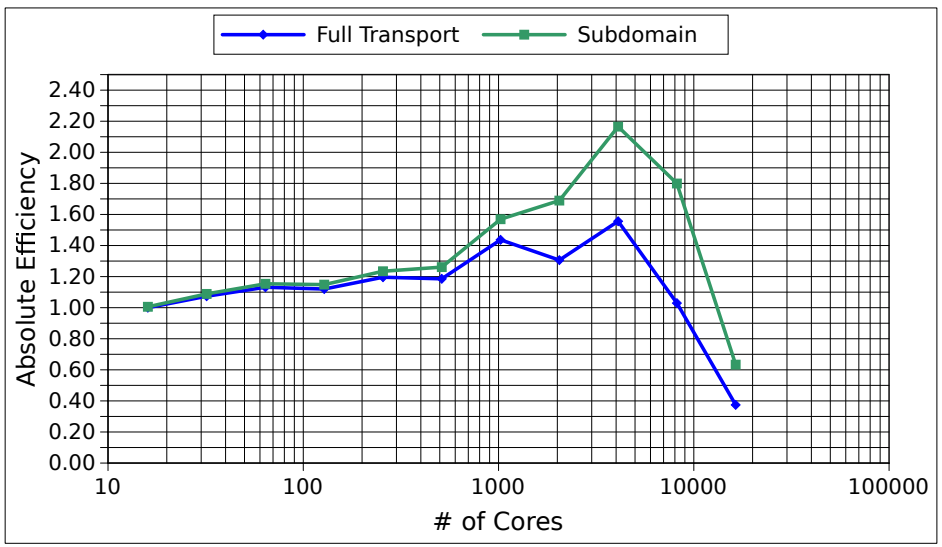


Figure: Strong scaling absolute efficiency for pure domain decomposition.

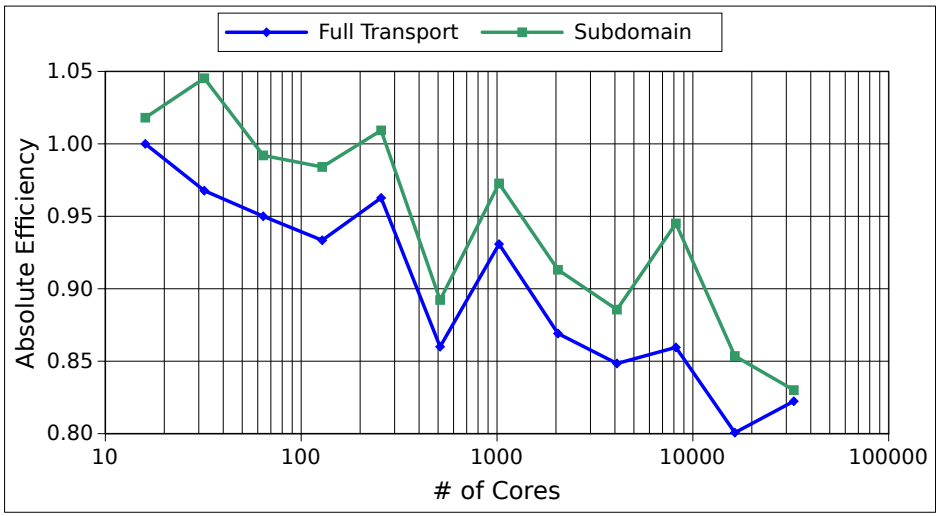


Figure: Weak scaling absolute efficiency for pure domain decomposition.

MCSA Application to Neutronics Summary

- MCSA can solve the asymmetric system generated by the SP_N equations
- Light water reactor problems are difficult to solve with MCSA as they have large spectral radii due to the neutron scattering in the moderator
- Advanced algebraic preconditioning strategies were applied to the SP_N equations to obtain convergence with ILUT chosen for subsequent investigations
- MCSA was verified to produce the same flux distribution and k-eigenvalue for the fuel assembly as production Krylov methods
- MCSA was observed to converge in fewer iterations per eigenvalue iteration than GMRES for the fuel assembly criticality problem and more than Bi-CGStab using the same preconditioning

MCSA Application to Fluid Flow Summary

- Forward-Automated Newton-MCSA (FANM) has been developed
- The FANM method has been verified to produce the same solutions as a production Newton-Krylov method for three difficult benchmark problems
- The FANM method has better iterative performance than the Newton-Krylov method for convection dominated problems, converging in fewer linear solver iterations with the same preconditioning for high and low Rayleigh numbers
- The spectral radius convergence restriction on MCSA was observed to be a significant hindrance by preventing solutions to forced flow problems at high Reynolds numbers
- More Monte Carlo histories at every FANM iteration can reduce the number of linear and nonlinear iterations required to converge the problem

Parallelization of MCSA Summary

- The multiple-set overlapping-domain (MSOD) parallel algorithm for particle transport has been adapted to parallelize MCSA
- MCSA scales favorably compared to production Krylov methods for both strong and weak scaling cases
- Overlap in small quantities can provide parallel efficiency boosts of a few percent in strong scaling cases but is ineffective in weak scaling cases
- Multiple sets offers a means to reduce time to solution by solving multiple copies of the original problem and combining the solutions using superposition
- MCSA is most efficiently used in parallel as a stochastic realization of an additive Schwarz method

Future Work

- Shortcomings observed on real problems
  - Significant optimization required to determine production feasibility and true scalability
  - Explicit algebraic preconditioning methods not sufficient
  - Spectral radius limitation is severe
- Performance improvements
  - Random walk optimizations
  - Alias sampling implementations
  - Multiple set reduction analysis
  - FANM forcing term and MCSA history relationships
- Preconditioning improvements
  - Alternative left/right sampling schemes
  - Variance reduction based strategy
  - Reduced order physics/PDE models for acceleration
- Breaking away from $\rho(\mathbf{H}) < 1$
  - Monte Carlo methods of the second degree
  - Stochastic projection methods

Discussion