

Nearly Incompressible APIC Timestep

1. P2G

- $p_p = -K(J_p^{-\gamma} - 1)$
- $\mathbf{f}_i = \sum_p -v_p^0 J_p p_p \nabla N_{ip}^2$
- $m_i = \sum_p N_{ip}^2 m_p$
- $\mathbf{D}_p = \sum_i N_{ip}^2 (\mathbf{x}_i - \mathbf{x}_p)(\mathbf{x}_i - \mathbf{x}_p)^T = \frac{1}{4} \Delta x^2 \mathbf{I}$
- $\mathbf{m}\mathbf{u}_i = \sum_p N_{ip}^2 m_p (\mathbf{u}_p + \frac{4}{\Delta x^2} \mathbf{B}_p (\mathbf{x}_i - \mathbf{x}_p))$

2. Field Solve

- $\mathbf{u}_i = (\mathbf{m}\mathbf{u}_i + \Delta t \mathbf{f}_i) / m_i + \Delta t \mathbf{g}$
- $\mathbf{u}_i \leftarrow BC(\mathbf{u}_i)$

3. G2P

- $\mathbf{u}_p = \sum_i N_{ip}^2 \mathbf{u}_i$
- $\mathbf{B}_p = \sum_i N_{ip}^2 \mathbf{u}_i (\mathbf{x}_i - \mathbf{x}_p)^T$
- $(\nabla \cdot \mathbf{u})_p = \sum_i \nabla N_{ip}^2 \cdot \mathbf{u}_i$
- $J_p \leftarrow J_p e^{\Delta t (\nabla \cdot \mathbf{u})_p}$
- $\mathbf{x}_p \leftarrow \mathbf{x}_p + \Delta t \mathbf{u}_p$
- $\rho_c = \sum_p N_{cp}^{1'} \frac{m_p}{\Delta x^3}$
- $k_c = \sum_p N_{cp}^{0'}$

4. Position Correction

- if cell in boundary $\rightarrow k_c = 1$
- $\rho_c \leftarrow (k_c > 0) ? \rho_c : \max(\rho_c, \rho_0)$
- $\delta \mathbf{x}_i = - \sum_c \nabla N_{ic}^1 \frac{\Delta t^2}{\rho_0} \kappa (1 - \frac{\rho_c}{\rho_0})$
- $\delta \mathbf{x}_i \leftarrow BC(\delta \mathbf{x}_i)$
- $\mathbf{x}_p \leftarrow \mathbf{x}_p + \sum_i N_{ip}^{2'} \delta \mathbf{x}_i$