Information Theory. 7th Chapter Problems

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2. Prove convexity of function rate-distortion for stationary source without use of memoryless property.
3. For a stationary source with memory, prove more precise estimation of D_0 in this property: For arbitrary stationary source $H(D) = 0$ holds

$$D \geq D_0 = \min_y \int_X f(x)d(y,x)dx \qquad (1)$$

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hint. Prove, that transformation preserves the norm. Square of norm of vector $\boldsymbol{x}$ can be written as $\|\boldsymbol{x}\|^2 = \boldsymbol{x}\boldsymbol{x}^T$. Substitute into this statement instead of $\boldsymbol{x}$ vector $\boldsymbol{y} = \boldsymbol{x}A$, where A – orthonormal transformation matrix.

5 Find conditional distribution $\varphi(y|x)$, which reaches rate-distortion function of binary source.

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6 Find conditional distribution $\varphi(y|x)$, which reaches rate-distortion function of Gaussian source.

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7. As the set of codes for binary source, binary codes with parity check can be used. Even-weight sequences are approximating code words. Depending on the code length, different coding rates and distortions are obtained obtained. For Binary Source uniformly distributed characters draw a plot of rate-distortion function $R(D)$ for such a coding method and compare with theoretical function $H(D)$.

8 In analog sources coding, the quantisation quality is measured in *signal-to-noise*, expressed *in decibels*, calculated as

$$10 \log_{10} \frac{\sigma^2}{D} \quad \text{dB},$$

where σ^2 and D – dispersion and mean square error respectively.
Draw a plot of dependence between the maximum achievable signal-to-noise ratio and a Gaussian source rate. What will be the expected gain in dB of the speed increase by 1 bit per sample at different coding rates?

9 Use Bleyhut algorithm for finding a rate-distortion function for a Binary Source without memory. Compare this result with obtained analytically.

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10 Use Bleyhut algorithm for finding a rate-distortion function for a Gaussian Source without memory. Compare this result with obtained analytically.