3-1.T8 设 $f(x)$ 可导，且 $f'(3)=2$，求 $\lim\limits_{x\to 0}\frac{f(3-x)-f(3)}{2x}$.

$$f'(3)=\lim_{\Delta x\to 0}\frac{f(3+\Delta x)-f(3)}{\Delta x}=2$$

$$\lim_{x\to 0}\frac{f(3-x)-f(3)}{2x}=-\frac{1}{2}\underbrace{\lim_{x\to 0}\frac{f(3-x)-f(3)}{-x}}$$

$$=-\frac{1}{2}\cdot 2=-1$$

14. 已知 $f(x)$ 在 $x=1$ 处连续，且 $\lim\limits_{x\to 1}\frac{f(x)}{x-1}=2$，求 $f'(1)$.

$$\cancel{f'(x)}\quad f'(1)=\lim_{\Delta x\to 0}\frac{f(1+\Delta x)-f(1)}{\Delta x}=\lim_{\Delta x\to 0}\underbrace{\frac{f(1+\Delta x)}{\Delta x}}=2$$

$$f(1)=\underbrace{\lim_{x\to 1}f(x)}=\lim_{x\to 1}\frac{f(x)}{x-1}\cdot(x-1)=2\cdot 0=0$$

$$\lim_{x\to 1}\frac{f(x)}{x-1}\overset{x-1=t}{=\!=}\lim_{t\to 0}\underbrace{\frac{f(t+1)}{t}}=2$$

9. $f(x)$ 为偶函数，且 $f'(0)$ 存在，证明 $f'(0)=0$

$$f(-x)=f(x)$$

$$f'(0)=\lim_{\Delta x\to 0}\frac{f(\Delta x)-f(0)}{\Delta x}$$

$$\lim_{\Delta x\to 0^+}\frac{f(\Delta x)-f(0)}{\Delta x}=\lim_{\Delta x\to 0^-}\frac{f(\Delta x)-f(0)}{\Delta x}$$

A

$$\overset{\Delta x=-t}{=\!=}\lim_{t\to 0^+}\frac{f(-t)-f(0)}{-t}$$

$$=-\lim_{t\to 0^+}\frac{f(t)-f(0)}{t}$$

$$A=-A\Rightarrow A=0\Rightarrow f'(0)=0$$

3-2 T4.6 求 $y=\arcsin\frac{1}{x}$ 的导数.

$$\frac{1}{x}=x^{-1}$$

$$y=\arcsin u$$
$$u=\frac{1}{x}$$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{1}{\sqrt{1-u^2}}\cdot(-1)x^{-2}=\frac{1}{\sqrt{1-(\frac{1}{x})^2}}\cdot(-x^{-2})$$

$$=\frac{-1}{\sqrt{1-(\frac{1}{x})^2}\cdot x^2}=\frac{-1}{\sqrt{1-\frac{1}{x^2}}\cdot|x|\cdot|x|}=\frac{-1}{\sqrt{x^2-1}\cdot|x|}$$

3-4. T7.4 求 $y=x\ln x$ 的 n 阶导数

$$y'=1\cdot\ln x+x\cdot\frac{1}{x}=\ln x+1$$
$$y''=\frac{1}{x}=x^{-1}$$
$$y'''=-1\cdot x^{-2}$$
$$y^{(4)}=(-1)\cdot(-2)\cdot x^{-3}$$
$$\vdots$$
$$y^{(n)}=(-1)\cdot(-2)\cdots[-(n-2)]\cdot x^{-(n-1)}$$
$$=(-1)^{n-2}\cdot(n-2)!\,x^{-(n-1)}\quad(n\geq 2)$$

3-2 T7.12 求 $y=\arcsin\frac{e^x-e^{-x}}{e^x+e^{-x}}$

$$y=\arcsin u$$
$$u=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{1}{\sqrt{1-u^2}}\left(\frac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})^2}{(e^x+e^{-x})^2}\right.$$

$$=\frac{1}{\sqrt{1-\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)^2}}\cdot\frac{(e^x+e^{-x})^2-(e^x-e^{-x})^2}{(e^x+e^{-x})^2}$$

$$= \frac{1}{\sqrt{\frac{(e^x+e^{-x})^2-(e^x-e^{-x})^2}{(e^x+e^{-x})^2}}} \cdot \frac{\cancel{e^{2x}}+2+\cancel{e^{-2x}}-\cancel{e^{2x}}+2-\cancel{e^{-2x}}}{(e^x+e^{-x})^2}$$

$$= \frac{e^x+e^{-x}}{\sqrt{4}} \cdot \frac{4}{(e^x+e^{-x})^2} = \frac{2}{e^x+e^{-x}}$$

3-2 T7.2

求 $y = \ln\cot\frac{x}{2}$ 的导数

$y = \ln u$

$u = \cancel{\cot\frac{x}{2}}\ \cot v$

$v = \frac{x}{2}$

$(\cot x)' = -\csc^2 x$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx}$$

$$= \frac{1}{u}\cdot(-\csc^2 v)\cdot\frac{1}{2}$$

$$= \cancel{\cot\frac{x}{2}}\ \frac{1}{\cot\frac{x}{2}}\cdot -\csc^2\frac{x}{2}\cdot\frac{1}{2}.$$

$$= -\frac{1}{2}\cdot\tan\frac{x}{2}\cdot\csc^2\frac{x}{2}$$

3-3. T3

试从 $\frac{dx}{dy} = \frac{1}{y'}$ 导出

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}\ y'$$

① $\frac{d^2x}{dy^2} = -\frac{y''}{(y')^3}$

$$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{y'}\right)$$

$$= \frac{d}{dx}\left(\frac{1}{y'}\right)\cdot\frac{dx}{dy}$$

$$= \frac{-y''}{(y')^2}\cdot\frac{1}{y'} = -\frac{y''}{(y')^3}$$

② $\frac{d^3x}{dy^3} = \frac{3(y'')^2 - y'y'''}{(y')^5}$

$$\frac{d^3x}{dy^3} = \frac{d}{dy}\left(\frac{d^2x}{dy^2}\right) = \frac{1}{dy}\left(-\frac{y''}{(y')^3}\right)$$

$$= \frac{d}{dx}\left(-\frac{y''}{(y')^3}\right) \cdot \frac{dx}{dy}$$

$$= \frac{-y'''(y')^3 + y'' 3(y')^2 y''}{(y')^6} \cdot \frac{1}{y'}$$

$$= \frac{-y'''(y')^3 + 3(y')^2(y'')^2}{(y')^7}$$

$$= \frac{-y'y''' + 3(y'')^2}{(y')^5}$$

3-1 T16 设 $f(x) = (x-x_0)g(x)$，$g(x)$ 在点 x_0 处连续，求 $f'(x_0)$

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x \cdot g(x_0+\Delta x) - 0}{\Delta x}$$

$$= \lim_{\Delta x \to 0} g(x_0+\Delta x) = g(x_0)$$

3-1 T17. 证明双曲线 $xy=a^2$ 上任一点处的切线与两坐标轴构成的三角形面积等于 $2a^2$

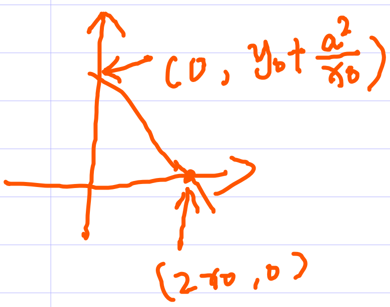
证：(x_0, y_0) 是双曲线上的一点 $y = \frac{a^2}{x}$

$$\left.\frac{dy}{dx}\right|_{x_0} = (a^2 \cdot x^{-1})'\Big|_{x=x_0} = -a^2x^{-2}\Big|_{x=x_0} = -a^2x_0^{-2}$$

$$y - y_0 = -\frac{a^2}{x_0^2}(x - x_0)$$

$x=0 \Rightarrow \quad y = y_0 + \frac{a^2}{x_0} \qquad (0, \ y_0+\frac{a^2}{x_0})$

$y=0 \Rightarrow \quad -y_0 = -\frac{a^2}{x_0^2}(x-x_0) \Rightarrow x = \frac{-y_0 - \frac{a^2}{x_0^2}\cdot x_0}{-\frac{a^2}{x_0^2}}$



$= \frac{y_0 x_0^2}{a^2} + x_0$

$= \frac{a^2 x_0}{a^2} + x_0 = 2x_0$

$(2x_0, 0)$

$x_0 y_0 = a^2$

$S = \frac{1}{2}\cdot 2|x_0| \cdot |y_0 + \frac{a^2}{x_0}|$

$= \frac{1}{2}\cdot 2|x_0| \cdot |y_0 + y_0|$

$= 2|x_0|\cdot|y_0| = 2a^2$

3-3 T8 设 $y = x^2 e^{2x}$，求 $y^{(20)}$

$y^{(20)} = C_{20}^0 \cdot (x^2)^{(0)}(e^{2x})^{(20)} + C_{20}^1 \cdot (x^2)^{(1)}(e^{2x})^{(19)}$

$+ C_{20}^2 (x^2)^{(2)}(e^{2x})^{(18)} + \cancel{C_{20}^3 (x^2)^{(3)} (e^{2x})^{(17)}}$

$= C_{20}^0 \cdot x^2 \cdot 2^{20} e^{2x}$

$+ C_{20}^1 \cdot 2x \cdot 2^{19} e^{2x}$

$+ C_{20}^2 \cdot 2 \cdot 2^{18} e^{2x}$

$(e^{2x})^n$