$$(a+b)^n = C_n^0 a^n b^0 + C_n^1 a^{n-1} b^1 + \cdots + C_n^n a^0 b^n$$

$$C_n^m = \frac{n\cdot(n-1)\cdots(n-m+1)}{m\cdot(m-1)\cdots 1}$$

$$\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$t = 1-x$

$x = 1-t$

$\tan(\frac{\pi}{2}-\alpha) = \cot\alpha$

$$= \lim_{t\to 0} t\tan(\frac{\pi}{2}-\frac{\pi}{2}t)$$

$$= \lim_{t\to 0} t\cot\frac{\pi}{2}t$$

$$= \lim_{t\to 0} t\,\frac{\cos\frac{\pi}{2}t}{\sin\frac{\pi}{2}t}$$

$$= \lim_{t\to 0}\frac{\frac{\pi}{2}t}{\sin\frac{\pi}{2}t}\cdot\frac{2}{\pi}\cdot\cos\frac{\pi}{2}t$$

$$= \frac{2}{\pi}$$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$\sin(\alpha+\beta) - \sin(\alpha-\beta)$

$= 2\sin\beta\cos\alpha$

$x = \alpha+\beta$

$a = \alpha-\beta$

$\alpha = \frac{\alpha+\beta+(\alpha-\beta)}{2} = \frac{x+a}{2}$

$\beta = \frac{\alpha+\beta-(\alpha-\beta)}{2} = \frac{x-a}{2}$

2.5 T1.7

$$\lim_{x\to a}\frac{\sin x-\sin a}{x-a}$$

$$= \lim_{x\to a}\frac{2\sin\frac{x-a}{2}\cdot\cos\frac{x+a}{2}}{x-a}$$

$$= 1\cdot\cos a$$

$$= \cos a$$

T2.8

$$\lim_{x\to\frac{\pi}{3}}\frac{\sin(x-\frac{\pi}{3})}{1-2\cos x}$$

$x-\frac{\pi}{3} = t$

$x = t+\frac{\pi}{3}$

$$= \lim_{t\to 0}\frac{\sin t}{1-2\cos(\frac{\pi}{3}+t)}$$

$$\lim_{t\to 0}\frac{1-\cos t}{t} = \frac{\frac{1}{2}t^2}{t}$$

$$= \lim_{t\to 0}\frac{\sin t}{1-2[\frac{1}{2}\cos t-\frac{\sqrt{3}}{2}\sin t]}$$

$$= \lim_{t\to 0}\frac{\sin t}{1-\cos t+\sqrt{3}\sin t} = \lim_{t\to 0}\frac{\frac{\sin t}{t}\to 1}{\underbrace{\frac{1-\cos t}{t}}_{=0}+\underbrace{\sqrt{3}\frac{\sin t}{t}}_{\sqrt{3}}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

2-5.T3.2 $A = \max\{a_1 \cdots a_m\}$ $(a_i > 0)$

$$\lim_{n\to\infty} \sqrt[n]{a_1^n + \cdots + a_m^n} = A$$

$$A^n \le a_1^n + \cdots + a_m^n \le A^n + \cdots + A^n = mA^n$$

$$A \le \sqrt[n]{a_1^n + \cdots + a_m^n} \le \sqrt[n]{m}\, A$$

$$\lim_{n\to\infty} \sqrt[n]{m} = 1 \quad \Rightarrow \quad \lim_{n\to\infty} \sqrt[n]{a_1^n + \cdots + a_m^n} = A$$

(3) $x_1 = \sqrt{2}$, $x_2 = \sqrt{2+\sqrt{2}}$ $\cdots$ $x_n = \sqrt{2+\cdots\sqrt{2+\sqrt{2}}}$

$x_n \uparrow$ 显然 $\qquad x_n = \sqrt{2 + x_{n-1}}$

$x_1 < 2$, $\underbrace{\qquad}_{\text{假设 } x_n < 2}$ $x_{n+1} = \sqrt{x_n + 2} < \sqrt{2+2} = 2$

由数学归纳法得 $x_n < 2$ 对任意 n 成立

由单调收敛准则 $\lim\limits_{n\to\infty} x_n = A$ 存在

$$x_{n+1} = \sqrt{x_n + 2}$$

$$\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} \sqrt{x_n + 2}$$

$$A = \sqrt{A+2} \quad \Rightarrow \quad A = 2$$

3.4 $x_{n+1} = \frac{1}{2}\left(x_n + \frac{1}{x_n}\right) \ge \frac{1}{2}\cdot 2\sqrt{x_n \cdot \frac{1}{x_n}} = 1$

$$x_{n+1} - x_n = \frac{1}{2}x_n + \frac{1}{2x_n} - x_n = \frac{1}{2x_n} - \frac{1}{2}x_n$$

$$= \frac{1 - x_n^2}{2x_n} \le 0$$

$x_n \downarrow$ 有下界 $\Rightarrow$ $\lim\limits_{n\to\infty} x_n$ 存在，记为 A

$A = \frac{1}{2}\left(A + \frac{1}{A}\right) \Rightarrow A = 1$ $(-1$ 舍去$)$

2-7 T6.4

$$\lim_{x\to 0}(\cos x)^{\frac{4}{x^2}}$$

$$1^{\infty} = (1+u(x))^{\frac{1}{u(x)}}$$

$$= \lim_{x\to 0}(1+\underbrace{\cos x-1}_{\downarrow 0})^{\frac{4}{x^2}}$$

$$= \lim_{x\to 0}(1+\cos x-1)^{\frac{1}{\cos x-1}\cdot\frac{4(\cos x-1)}{x^2}}$$

$$= \lim_{x\to 0}\left[(1+\cos x-1)^{\frac{1}{\cos x-1}}\right]^{\frac{4(\cos x-1)}{x^2}}$$

$$f(x)^{g(x)} = A^B$$

$$= \left[\lim_{x\to 0}(1+\cos x-1)^{\frac{1}{\cos x-1}}\right]^{\lim_{x\to 0}\frac{4(\cos x-1)}{x^2}}$$

$$1-\cos x \sim \frac{1}{2}x^2$$

$$\cos x-1 = -\frac{1}{2}x^2$$

$$= e^{-2}$$

2-7. T7

讨论 $f(x) = \lim_{n\to\infty}\frac{x+x^2e^{\frac{n}{x}}}{1+e^{\frac{n}{x}}}$ $\quad e^{\frac{n}{x}} \to x>0$

① $x>0$ $\quad e^{\frac{n}{x}} \to +\infty$ $\quad f(x) = \lim_{n\to\infty}\frac{x+x^2\cdot e^{\frac{n}{x}}}{1+e^{\frac{n}{x}}}$

$$= \lim_{n\to\infty}\frac{e^{-\frac{n}{x}}\cdot x + x^2}{e^{-\frac{n}{x}}+1} = x^2$$

② $x<0$ $\quad e^{\frac{n}{x}} \to 0$ $\quad f(x) = \lim_{n\to\infty}\frac{x+x^2\cdot e^{\frac{n}{x}}}{1+e^{\frac{n}{x}}} = \frac{x}{1} = x$

$$f(x) = \begin{cases} x^2 & x>0 \\ x & x<0 \end{cases}$$

$$\lim_{x\to 0^-} f(x) = 0 \qquad \lim_{x\to 0^+} f(x) = 0$$

/

$x=0$ 处为可去间断点。

$$A_0 e^{6.5\%\cdot 10} = 1000$$

$$A_0 e^{rk}$$

$$A_0 = 1000\cdot e^{-0.65} = 522$$

2-8 .T 3 证: 令 $F(x) = f(x) - f(x+a)$, (a.)

$F(0) = f(0) - f(a)$

$F(a) = f(a) - f(2a) = f(a) - f(0)$

$F(0) \cdot F(a) = -(f(0) - f(a))^2$

≤ 0

若 $F(0) \cdot F(a) = 0 \Rightarrow f(0) = f(a)$

$F(0) = f(0+a) \Rightarrow \xi = 0$

若 $F(0) \cdot F(a) < 0 \Rightarrow$ 零点定理 存在 $\xi \in (0, a)$, 使 $F(\xi) = 0$

即 $f(\xi) - f(\xi + a) = 0$

$\Rightarrow f(\xi) = f(\xi + a)$

$f(\xi) = f(\xi + a)$

$F(x) = f(x) - f(x+a)$

~~f~~

$F(0) = f(0) - f(a)$

$F(a) = f(a) - f(2a)$

$= f(a) - f(0)$

2-3 T3. 5

$$\lim_{x \to 0} \frac{1}{x} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right)$$

$$= \lim_{x \to 0} \frac{1}{x} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{x \sin x} = \lim_{x \to 0} \frac{\frac{1}{2} x^2}{x \cdot x} = \frac{1}{2}$$