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1. $\int \frac{dx}{x^2-x-6}$

$= \frac{1}{5}\ln\frac{x-3}{x+2} + C$

2. $\tan^{10} x \cdot \sec^2 x\,\mathrm{d}x$

$= \frac{1}{11}\tan{}^{11}x + C$

3. $\int \sin^5 x\,\mathrm{d}x$

$-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

4. $\int \frac{dx}{(\arcsin x)^2\sqrt{1-x^2}}$

$= -\frac{1}{\arcsin x} + C$

5. $\int x \cdot \sqrt[4]{x+9}dx$

$= \frac{4}{9}\sqrt[4]{(x+9)^9} - \frac{36}{5}\sqrt[4]{(x+9)^5} + C$

6. $\int \frac{dx}{\sqrt{x^2+2x+2}}$

$= \ln\left|x+1+\sqrt{x^2+2x+2}\right| + C$

7. $\int \sqrt{x^2-a^2}dx$

$= \frac{1}{2}x\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

1.

$a = 12t^2 - 3\sin t.$

$\upsilon(0) = 5s(0) = -3$

(1) $\upsilon t(2)st.$

(1) $\upsilon = \int adt = \int \left(12t^2 - 3\sin t\right)dt = 4t^3 + 3\cos t + C_1.$

$\upsilon(0) = 55 = 3 + C_1C_1 = 2\upsilon = 4t^3 + 3\cos t + 2.$

(2) $s = \int \upsilon dt = \int \left(4t^3 + 3\cos t + 2\right)dt = t^4 + 3\sin t + 2t + C_2.$

$s(0) = -3C_2 = -3s = t^4 + 3\sin t + 2t - 3.$

1. $\left(e^2, 3\right).$

$\frac{dy}{dx} = \frac{1}{x}y = \int \frac{1}{x}dx = \ln|x| + C,$

$\left(e^2, 3\right)3 = \ln e^2 + CC = 1$

$y = \ln|x| + 1.$

1. $I_n = \int tan^n xdxn.$

$$
\begin{aligned}
I_n &= \int \tan^n x dx = \int \tan^{n-2} x \left(\sec^2 x - 1\right) dx \\
&= \int \tan^{n-2} x \sec^2 x dx - \int \tan^{n-2} x dx \\
&= \int \tan^{n-2} x d \tan x - \int \tan^{n-2} x dx \\
&= \frac{\tan^{n-1} x}{n-1} - I_{n-2} (n \geq 2)
\end{aligned}
$$

$$
I_1 = \int \tan x dx = -\ln|\cos x| + C, \quad I_0 = \int dx = x + C
$$

4. $f(x) \frac{\ln x}{x} f(x) \frac{\ln x}{x} \int x f'(x) dx.$

$f(x) = \left(\frac{\ln x}{x}\right)' = \frac{1 - \ln x}{x^2}$

$\int f(x) dx = \frac{\ln x}{x} + C$

$\int x f'(x) dx = \int x df(x) = x f(x) - \int f(x) dx = \frac{1 - 2\ln x}{x} + C.$

5. $y = y(x) y^2 (x - y) = x^2 \int \frac{dx}{y^2}.$

$y = t \cdot x t^2 x (1 - t) = 1 x = \frac{1}{t^2 (1-t)}$

$dx = \frac{3t - 2}{t^3 (1-t)^2} dt y = \frac{1}{t(1-t)}$

$\int \frac{dx}{y^2} = \int t^2 (1-t)^2 \cdot \frac{3t-2}{t^3(1-t)^2} dt = \int \left(3 - \frac{2}{t}\right) dt = 3t - 2\ln|t| + C = \frac{3y}{x} - 2\ln\left|\frac{y}{x}\right| + C.$

6. $f\left(\sin^2 x\right) = \frac{x}{\sin x} \int \frac{\sqrt{x}}{\sqrt{1-x}} f(x) dx.$

$\sin^2 x = t \sin x = \sqrt{t} x = \arcsin \sqrt{t} f(t) = \frac{\arcsin \sqrt{t}}{\sqrt{t}}$

$\int \frac{\sqrt{x}}{\sqrt{1-x}} f(x) dx = \int \frac{\sqrt{x}}{\sqrt{1-x}} \cdot \frac{\arcsin \sqrt{x}}{\sqrt{x}} dx = -2 \int \arcsin \sqrt{x} d\sqrt{1-x}$

$= -2\sqrt{1-x} \arcsin \sqrt{x} + 2 \int \sqrt{1-x} \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x}} dx$

$= -2\sqrt{1-x} \arcsin \sqrt{x} + 2\sqrt{x} + C.$

7. $f(\ln x) = \frac{\ln(1+x)}{x} \int f(x) dx.$

$\ln x = t x = e^t f(t) = \frac{\ln\left(1+e^t\right)}{e^t}$

$$\begin{aligned}\int f(x)dx &= \int \frac{\ln(1+e^x)}{e^x}dx = -\int \ln(1+e^x)\,de^{-x} \\ &= -e^{-x}\ln(1+e^x) + \int \frac{1}{1+e^x}dx \\ &= -e^{-x}\ln(1+e^x) + \int \left(1 - \frac{e^x}{1+e^x}\right)dx \\ &= x - \left(1+e^{-x}\right)\ln(1+e^x) + C\end{aligned}$$

8. $f\left(x^2-1\right) = \ln \frac{x^2}{x^2-2} f\left[\varphi(x)\right] = \ln x \int \varphi(x)dx.$

$\because f\left(x^2-1\right) = \ln \frac{\left(x^2-1\right)+1}{\left(x^2-1\right)-1} \therefore f(x) = \ln \frac{x+1}{x-1}.$

$f\left[\varphi(x)\right] = \ln \frac{\varphi(x)+1}{\varphi(x)-1} = \ln x$

$\Rightarrow \quad \frac{\varphi(x)+1}{\varphi(x)-1} = x \quad \Rightarrow \quad \varphi(x) = \frac{x+1}{x-1}$

$\int \varphi(x)\,\mathrm{d}x = x + 2\ln|x-1| + C.$

9. $f(x) = \begin{cases} x^2, x \le 0 \\ \sin x, x > 0 \end{cases} f(x).$

: $\int f(x)\,dx = \begin{cases} \frac{x^3}{3} + C_1, & x \le 0 \\ -\cos x + C_2, x > 0 \end{cases}$

$\lim_{x\to 0^-}\left(\frac{x^3}{3} + C_1\right) = \lim_{x\to 0^+}\left(-\cos x + C_2\right)$

$C_1 = C_2 - 1 C_1 = C_2 - 1 = C$

$\int f(x)\,dx = \begin{cases} \frac{x^3}{3} + C \quad x \le 0 \\ 1 - \cos x + C x > 0 \end{cases}.$

10. $\int \frac{ax^2+bx+c}{x^3(x-1)^2}dx$

: $\frac{ax^2+bx+c}{x^3(x-1)^2} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x^3} + \frac{B_1}{x-1} + \frac{B_2}{(x-1)^2}$

$A_1 \ne 0 B_1 \ne 0 \frac{A_1}{x} \frac{B_1}{x-1}$

$A_1 = B_1 = 0$

$\frac{ax^2+bx+c}{x^3(x-1)^2} = \frac{A_2}{x^2} + \frac{A_3}{x^3} + \frac{B_2}{(x-1)^2}$

$ax^2 + bx + c \equiv A_2 x\,(x-1)^2 + A_3\,(x-1)^2 + B_2 x^3.$

$x = 0 A_3 = c \; x = 1 B_2 = a + b + c$

$x^3 x^2$

$x^3 : A_2 + B_2 = 0 \; x^2 : A_3 - 2A_2 = a$

$a + 2b + 3c = 0.$

11. $f(x) f^{-1}(x)$

$\int f(x)dx = F(x) + C.$

$\int f^{-1}(x)dx.$

$: \because \quad x = f\left(f^{-1}(x)\right)$

$$\therefore \int f^{-1}(x)dx = xf^{-1}(x) - \int xdf^{-1}(x)$$
$$= xf^{-1}(x) - \int f\left(f^{-1}(x)\right)df^{-1}(x)$$
$$= xf^{-1}(x) - F\left(f^{-1}(x)\right) + \mathrm{C}$$

12. $f'\left(x\tan\frac{x}{2}\right) = (x + \sin x)\tan\frac{x}{2} + \cos x f(x)$ ..

$: f'\left(x\tan\frac{x}{2}\right) = x\tan\frac{x}{2} + \sin x\tan\frac{x}{2} + \cos x = x\tan\frac{x}{2} + 1.$

$f'(u) = u + 1 f(u) = \int (u+1)\,du = \frac{u^2}{2} + u + C$

$\therefore \quad f(x) = \frac{x^2}{2} + x + C.$

1. $F(x)$ $f(x)$ $f(x)$ $f^{-1}(x)$

$\int f^{-1}(x)\,dx = xf^{-1}(x) - F\left[f^{-1}(x)\right] + C.$

$\int f^{-1}(x)\,dx = xf^{-1}(x) - \int xd\left[f^{-1}(x)\right],$

$t = f^{-1}(x), \quad x = f(t),$

$\int xd\left[f^{-1}(x)\right] = \int f(t)\,dt$

$= F(t) + C = F\left[f^{-1}(x)\right] + C$

$\int f^{-1}(x)dx = xf^{-1}(x) - F\left[f^{-1}(x)\right] + C.$

2. $\frac{1}{2}e^{2x}$ $e^x sh\ x$ $e^x chx$ $\frac{e^x}{chx - shx}$.

$\because \left(\frac{1}{2}e^{2x}\right)' = e^{2x} = \frac{e^x}{\frac{e^x + e^{-x}}{2} - \frac{e^x - e^{-x}}{2}} = \frac{e^x}{chx - shx}$

$\therefore \quad \frac{1}{2}e^{2x}$ $\frac{e^x}{chx - shx}$.

$\because (e^x shx)' = e^x shx + e^x chx = e^x(chx + shx)$

$= \frac{e^x \cdot \left(ch^2x - sh^2x\right)}{chx - shx} = \frac{e^x}{chx - shx}$

$\therefore \quad e^x shx$ $\frac{e^x}{chx - shx}$.

$e^x chx$ $\frac{e^x}{chx - shx}$.

3. $f(x) = sgnx = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$ $y = \frac{x^2}{2}sgnx$ $y = |x|$.

$$y = \frac{x^2}{2} sgnx = \begin{cases} 0.5x^2 & x > 0 \\ 0 & x = 0 \\ -0.5x^2 & x < 0 \end{cases}, \; y' = \begin{cases} x & x > 0 \\ & \\ -x & x < 0 \end{cases}$$

$$y'|_{x=0} = \lim_{x \to 0} \frac{0.5x^2 \cdot sgnx - 0}{x - 0} = \lim_{x \to 0} 0.5x \cdot sgnx = 0$$

$y' = |x|$ $y = \frac{x^2}{2} sgnx$ $y = |x|$.