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Unfolding-synthesis technique for digital pulse processing. Part 1: Unfolding

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ABSTRACT

The unfolding-synthesis technique is used in the development of digital pulse processing systems used in radiation measurements. This technique is applied to digital signals obtained by digitization of analog signals that represent the combined response of the radiation detectors and the associated signal conditioning electronics. The salient features of the unfolding-synthesis technique are first the unfolding of the digital signals into unit impulses, followed by the synthesis of digital signal processing systems with unit impulse responses equivalent to the desired pulse shapes. Part 1 of this paper covers the unfolding part of this technique.

1. Introduction

For more than two decades advancements in Digital Pulse Processing (DPP) have made it one of the most utilized techniques of pulse processing in radiation measurements today [1,2]. Early development of DPP was concentrated on direct synthesis of pulse shapes using sampled analog signals [3–5]. Other mathematically elaborated methods have also been considered and published [6,7]. In this paper we describe a technique that allows the synthesis of virtually any pulse shape, either exactly or as a close approximation. This method has been used extensively in creating algorithms suitable for real-time implementation. It has been also taught as part of university courses and lecturers [8]. However, no detailed and comprehensive description has been published prior to this paper. The unfolding-synthesis technique is applicable to linear signal processing systems that are either time-invariant or time-variant. In this paper, however, we will limit the discussions and the analysis to Linear Time-Invariant (LTI) systems. Unless explicitly noted otherwise, discrete-time systems will be considered.

2. Basics

Real-time digital pulse processing is accomplished using LTI systems (Fig. 1c) that are characterized and fully defined by their discrete-time impulse response $h(n)$ [9]. The impulse response $h(n)$ is the response of the system to the most fundamental digital signal – the unit impulse $\delta(n)$. The unit impulse is defined as:

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

The weighted unit impulse $\delta_w(n)$ is the unit impulse multiplied by a constant, $w\delta(n)$. Thus, $\delta_w(0)=w$. The unit impulse and the weighted unit impulse are depicted in Fig. 1a and b respectively.

The multiplication of the unit impulse by a constant, as in the case of the weighted unit impulse, is one of the basic digital signal processing operations. Other basic operations include addition (subtraction), signal multiplication, and delaying or shifting of the digital signals. Fig. 2 shows the basic operations with their graphical representations and the corresponding mathematical expressions. Digital signal processing algorithms incorporate these operations in order to achieve more complex system responses.

The transformation of an input signal $x(n)$ into an output signal $y(n)$ by a LTI system is mathematically expressed as the output signal as a convolution of the input signal and the impulse response of the system. In the discrete-time domain the convolution is given by the following sum:

$$y(n) = \sum_{i=-\infty}^{n} x(i)h(n-i), \tag{2}$$

where $x(n)$ is the input signal being transformed into the output signal $y(n)$ by a causal LTI system with a unit impulse response $h(n)$. The convolution is commonly written using the star ($*$) symbol:

$$y(n) = x(n) * h(n) \tag{3}$$

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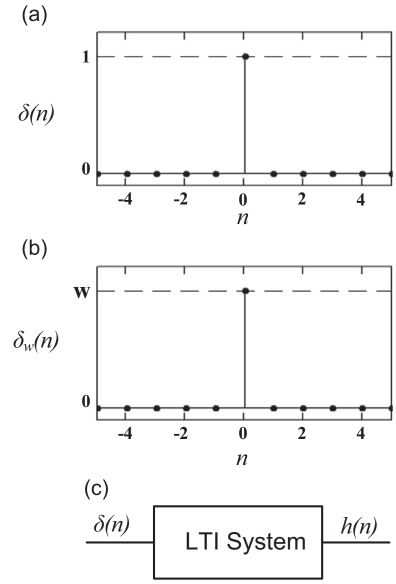

Fig. 1. (a) Unit impulse, (b) weighted unit impulse and (c) LTI system symbol and its unit impulse response definition.

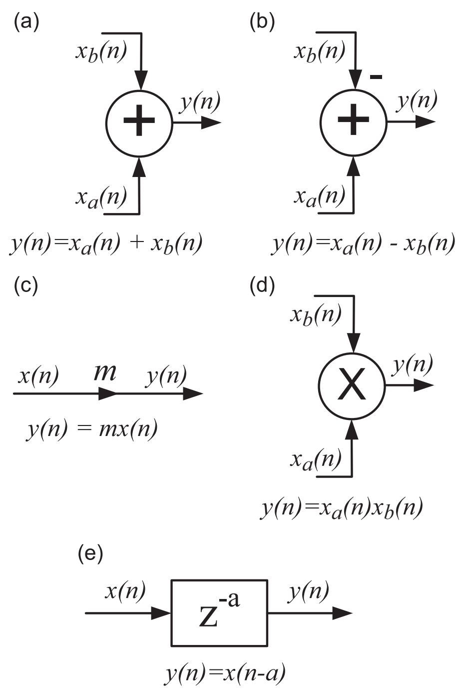

Fig. 2. Basic digital signal processing operations: (a) addition, (b) subtraction, (c) multiplication by a constant, (d) signal multiplication and (e) delaying or shifting.

The most important algebraic properties of the convolution are commutativity, associativity, and distributivity. We will use these convolution properties to explain and analyze various aspects of the unfolding-synthesis technique. These convolution properties are illustrated in Fig. 3.

3. Unfolding-synthesis technique

A system that implements the unfolding-synthesis technique is depicted in Fig. 4. Analog signals from radiation detectors and associated electronics are converted into discrete-time digital signals by a fast analog-to-digital converter (ADC). The ADC digitizes the analog signals by performing two operations: sampling and quantization. In the discussion that follows we consider that the quantization granularity is extremely fine and has little or no effect on the digital pulse processing algorithms. The quantization effects, however, should be taken into consideration when low-resolution ADCs are utilized by the DPP systems [10].

The digital signal that is produced by the ADC inherits the properties of the analog signal applied to its input. The analog signal is a convolution of the detector signal and the signal conditioning electronics. The detector signal can be approximated by a Dirac delta function $\delta(t)$ when it is very short compared to the DPP pulse shape. This approximation, for example, can be applied to signals from silicon drift detectors, small semiconductor detectors, and ultrafast scintillators. In such cases the analog signal applied to the fast ADC is characterized by the impulse response of the signal conditioning electronics.

In some cases, such as for large germanium detectors, the detector signal may vary in duration (charge collection time variation). In these cases, the pulse shaping in the discrete-time domain can be designed to unfold the impulse response of the signal conditioning electronics alone. The variability of the detector signal is then mitigated by features of the synthesized pulse shape, e.g., a flat top.

In other cases, the detector response may be combined with the response of the signal conditioning electronics. A typical example is a scintillation detector light pulse converted by a photo-multiplier tube (PMT) into an electric current. If the anode of the PMT is loaded by a C‖R network, the resulting voltage signal at the PMT anode will be defined as a convolution of the scintillation light signal and the exponential signal impulse response of the C‖R network. If the scintillator light emission pulse is a single time-constant exponential signal, then the anode signal will be a result of the convolution of the two exponential impulse responses. Digitizing this continuous-time signal will generate a digital signal that can be represented by the discrete-time convolution of two digital exponential signals.

For efficient implementation of the unfolding-synthesis technique, it is important to know the characteristics of the analog signals digitized by the ADC and to identify the impulse responses of the analog systems that will be unfolded. It is clear that these characteristics depend on both the response of the detector and the response of the signal conditioning electronics. The signal conditioning electronics are normally comprised of signal processing modules and networks with well-defined responses and transfer functions. These include detector preamplifiers, C-R and/or R-C networks, amplification stages, offset generators, base-line restorers, and other circuits that optimize the signal being digitized by the fast ADC.

The salient feature of the unfolding-synthesis technique is the transformation of the digitized analog impulse response into a unit impulse in the discrete-time domain. We call this process unfolding, or deconvolution. The unfolding transformation of a digital signal is performed by the unfolding system depicted in Fig. 4. The unfolding system has a unit impulse response $h_U(n)$ whose convolution with the digital signal from the ADC produces a unit impulse $h_N(n) * h_U(n)=\delta(n)$. Thus, the unfolding result is a convolution result and the unfolding is essentially a convolution. To avoid convolution–deconvolution tautology and confusion, the term “unfolding“ is used in this paper rather than “convolution” [11].

The synthesis of the desired shape is accomplished by a synthesizing system with impulse response $h_S(n)$ identical to the system pulse-shaped signal, which we will reference to as an optimal pulse shape. The optimal pulse shape is determined by various factors such as noise suppression, counting rate requirements and other constraints. There is extensive material published on this topic [12–14]. This paper does not focus on selecting the

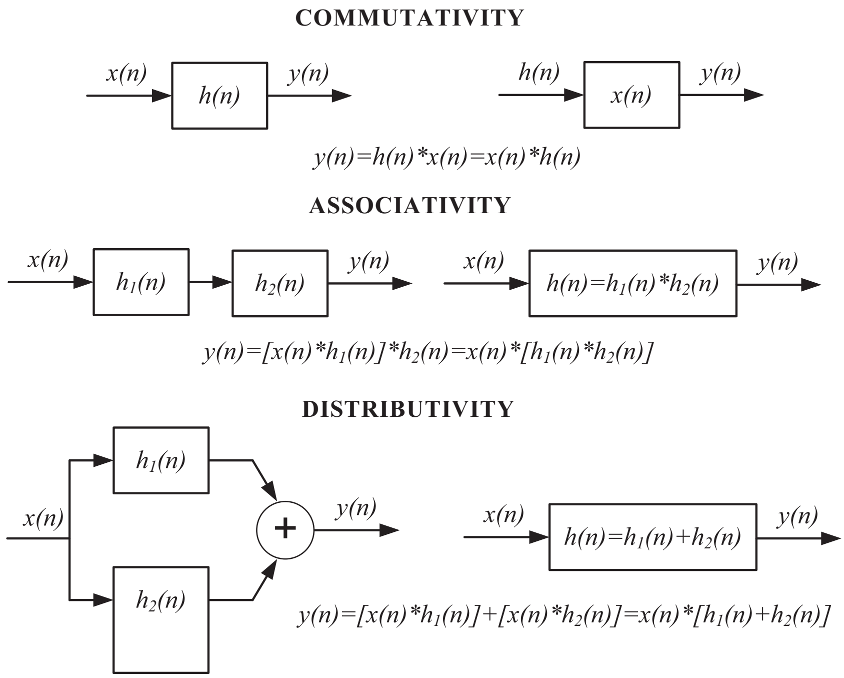

Fig. 3. Algebraic properties of convolution.

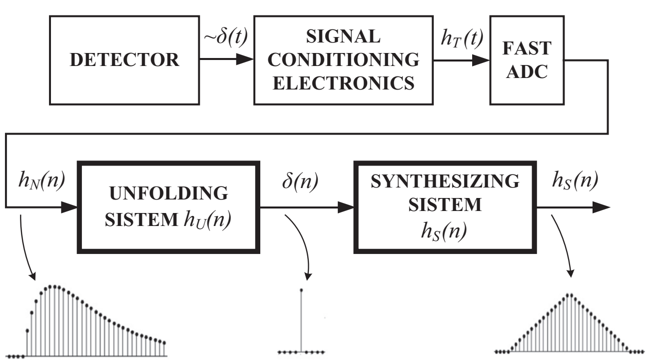

Fig. 4. Functional block diagram of a system using the unfolding-synthesis technique.

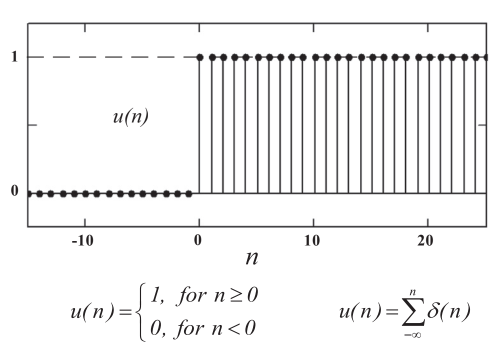

Fig. 5. Unit step signal.

most suitable pulse shape for a given detector or radiation measurement arrangement; instead, in this paper we describe the unfolding-synthesis technique as a design tool that allows the synthesis of a wide variety of pulse shapes or their approximations.

The first part of this technique is the unfolding part, which can be straightforwardly applied to the most common analog signals from detectors and associated signal conditioning networks.

4. Unfolding

As stated earlier, the purpose of the unfolding is to transform the digital signal from the ADC into a unit impulse. Some basic digital signals can be obtained by digitizing signals commonly found in radiation-measurement systems. The unit step digital signal is, perhaps, one of the easiest signals to unfold.

4.1. Unit step

The unit step can be obtained by digitizing an analog step signal. An example signal is the impulse response of a reset type charge-sensitive preamplifier. Fig. 5 shows the unit step signal that is described mathematically by a unit step function $u(n)$:

Note that the unit step signal is defined using only two values, zero and one. There is only one transition between these values–from zero at $n=-1$ to one at $n=0$. Intuitively, a backward finite difference $u(n)-d(n-m)$ operation performed on the unit step signal $u(n)$ will transform it into a unit impulse when $m=1$. In this paper we use the term “digital differentiation“ as an equivalent to the backward finite difference operation with $m=1$.

Fig. 6a shows the block diagram of a digital differentiation system (digital differentiator). The graphic symbol that is used in this paper for the digital differentiator response $y(n)$ to an arbitrary signal $x(n)$ is depicted in Fig. 6b.

The digital differentiator unit impulse response is given by:

$$h_{DIF}(n)=\delta(n)-\delta(n-1) \tag{4}$$

This unit impulse response has only two non-zero consecutive samples with the same magnitude but opposite signs. The unit impulse response of the digital differentiator is a unit doublet discrete function shown in Fig. 6c.

An important aspect associated with the unit step signal $u(n)$ is that it is the unit impulse response of an accumulator shown in Fig. 7c. The accumulator is a digital signal processing system, which adds together all samples presented at its input, known as accumulation. The accumulator block diagram and its graphical symbol are shown in Fig. 7a and b respectively.

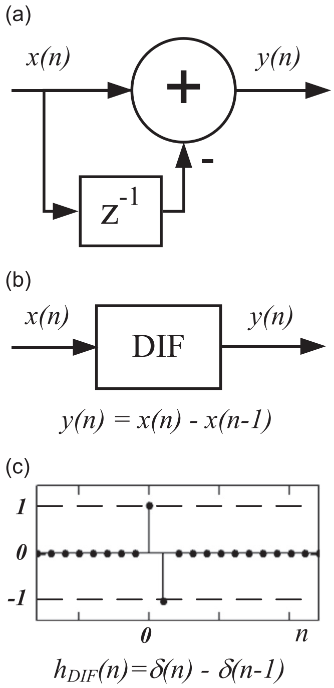

Fig. 6. Digital differentiator (a) block diagram, (b) graphical symbol and (c) unit impulse response.

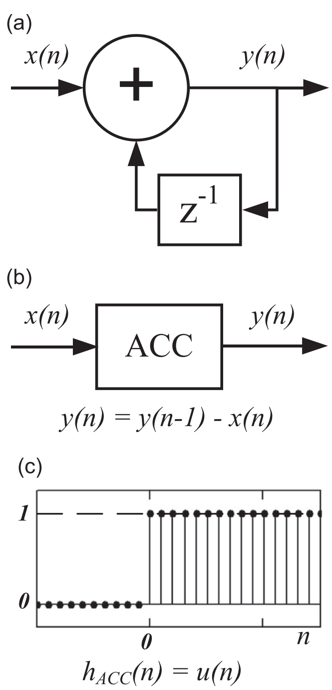

Fig. 7. Accumulator (a) block diagram, (b) graphical symbol and operational algorithm and (c) unit impulse response.

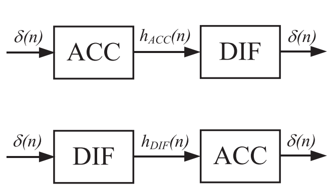

Fig. 8. Accumulator–digital differentiator unfolding system.

The unit impulse response of the accumulator $h_{ACC}(n)$ is a unit step:

$$h_{ACC}(n) = u(n) = \sum_{-\infty}^{n} \delta(n) \tag{5}$$

It is important to note that the accumulator is a system that needs to be relaxed before any non-zero signals are applied to its input. That is, prior to any non-zero values of the input signal, the accumulator output must be forced to zero by external means such as a logic reset signal.

A relaxed accumulator will transform a unit impulse into a unit step. Therefore, it is clear that a digital differentiator unfolds the unit impulse response of the accumulator. From the commutative property of the convolution it also follows that the accumulator acts as an unfolding system for the digital differentiator. This is also true for any pair of systems of which one unfolds the other. The accumulator-digital differentiator unfolding is paramount to the synthesis of pulse signals described by polynomial functions, which will be discussed later in Part 2 of this paper. Fig. 8 illustrates the unfolding of the unit impulse responses of the accumulator and the digital differentiator.

4.2. Exponential signal

Exponential signals are common in radiation measurement pulse-processing systems. The impulse response of a resistive feedback preamplifier, the impulse response of a R-C low pass network, and the step response of C-R differentiation network are only a few examples of systems with exponential responses in the continuous-time domain. The unfolding of the exponential pulses has been already discussed in our previous work [15]. Here we will give a brief summary and expand the unfolding process to systems representing summation and convolution of exponential pulses.

In the discrete-time domain an exponential unit impulse response is defined as:

$$h_N(n) = \begin{cases} a^n & \text{for } n \geq 0 \\ 0 & \text{elsewhere} \end{cases} \tag{6}$$

where a is called an exponential base [16]. In this paper we limit our considerations to an exponential base that is bounded between zero and one: $0 < a < 1$. For these values of the exponential base, $h_N(n)$ is a decaying exponential signal and it represents the most common signal found in radiation measurement systems.

In DPP the digital exponential signals are, in most cases, signals that result from the digitization of an analog exponential signal by a fast ADC. Fig. 9 shows an analog exponential signal and the corresponding digital exponential signal obtained by a fast ADC with a sampling period ΔT. The exponential base of the digital exponential signal can be related to the decay time-constant τ of

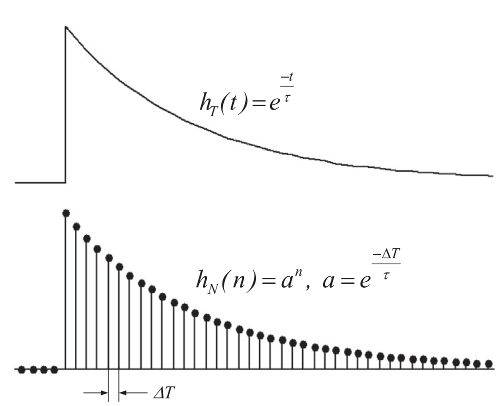

Fig. 9. Relationship between analog exponential signal and its digital counterpart.

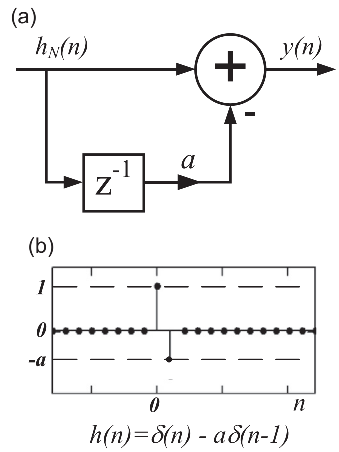

Fig. 10. Digital exponential signal unfolding system (a) block diagram and (b) unit impulse response.

the analog exponential signal by the following equation:

$$a = e^{-\Delta T/\tau} \quad (7)$$

Eq. (6) can be rewritten in the following recursive form:

$$h_N(n) = a h_N(n-1) \mathit{for}\ \ n > 0,\ h_N(0) = 1,\ 0\ \mathit{elsewhere} \quad (8)$$

Eq. (8) suggests that with appropriate scaling and shifting the exponential unit impulse response can be unfolded. The operational algorithm of a system that unfolds the exponential unit impulse response can be described by the following equation:

$$y(n) = h_N(n) - a h_N(n-1) \quad (9)$$

The functional block diagram and the unit impulse response of the system defined by Eq. (9) are depicted in Fig. 10. The unit impulse response is similar to the impulse response of the digital differentiator except that the magnitudes of the two non-zero samples are different. In this paper the function describing this impulse response will be referred to as the “asymmetric doublet“.

It is straightforward to verify that the system shown in Fig. 10 that operates according to Eq. (9) transforms an exponential unit impulse response into a unit impulse signal. From the definition of the exponential unit impulse response signal (Eq. (6)) all values of the samples preceding the sample at index $n=0$ are equal to zero. Therefore, $y(n)=0$ for $n<0$. For $n>0$, taking into account the definition of the digital exponential pulse (Eq. (6)), Eq. (9) can be rewritten as

$$y(n) = h_N(n) - a h_N(n-1) = a^n - a \cdot a^{n-1} = a^n - a^n = 0,\ \mathit{for}\ n \neq 0 \quad (10)$$

Finally, for $n=0$ Eq. (9) becomes

$$y(0) = x(0) - a x(-1) = a^0 - a \cdot 0 = 1 \quad (11)$$

Therefore, using Eqs. (10) and (11), the output signal of the unfolding system is defined by the following equation:

$$y(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{elsewhere} \end{cases} \quad (12)$$

Eq. (12) is identical to the definition of the unit impulse signal $\delta(n)$ given by Eq. (1). This proves that a system defined by Eq. (9) unfolds a system with an exponential unit impulse. In other words, such an unfolding system will transform a digital exponential unit impulse response into a unit impulse. As the unfolding system is linear, a weighted digital exponential signal $wh_N(n)$ will be transformed into a weighted unit impulse $w\delta(n)$.

The implementation of the system that operates according to Eq. (9) may be challenging because the exponential base a is a small, floating point number. Floating point multiplication is complex and often leads to rounding or truncations of the result. Integer multipliers are readily available in hardware such as field-programmable gate arrays (FPGA). Thus it is important, from an implementation point of view, to express Eq. (9) in a form that is suitable for integer multiplication. Let's rewrite Eq. (9) in the following form:

$$\frac{y(n)}{a} = \frac{h_N(n)}{a} - h_N(n-1) + h_N(n) - h_N(n) = h_N(n) - h_N(n-1) + \frac{1-a}{a} h_N(n) \quad (13)$$

Let us define a multiplication factor M as

$$M = \frac{a}{1-a} \quad (14)$$

By multiplying both sides of Eq. (13) by M, Eq. (13) transforms to:

$$y_w(n) = M\left[h_N(n) - h_N(n-1)\right] + h_N(n) \quad (15)$$

where $y_w(n)$ is the weighted response of the system. When a is small $y_w(n) \approx My(n)$. This result is similar to the algorithms presented in our previous work [15].

If the multiplication factor M is defined as

$$M = \frac{1}{1-a} \quad (16)$$

then Eq. (13) takes the form

$$y_w(n) = M\left[h_N(n) - h_N(n-1)\right] + h_N(n-1) \quad (17)$$

For this definition of M, $y_w(n)$ is exactly equal to $My(n)$. That is $y_w(n)=My(n)$. When the digital exponential unit impulse response is a result of digitization of an analog exponential impulse response, the multiplication factor M can be expressed in terms of the sampling period ΔT and the decay time-constant τ of the analog exponential signal. Using Eqs. (7) and (16) the multiplication factor M can be expressed as:

$$M = \frac{1}{1-e^{-\Delta T/\tau}} \approx \frac{\tau}{\Delta T} + 0.5 \quad (18)$$

4.3. Sum of exponential signals

The unfolding of an exponential unit impulse response can be extended to unit impulse responses that are a weighted sum of two or more exponential signals. Next we will define a system and the associated equations for unfolding an exponential unit impulse response that is a sum of two exponential signals. Such a signal can be expressed as

$$h_N(n) = Aa^n + Bb^n = x_A(n) + x_B(n) \quad (19)$$

where a is the exponential base of the first summed exponential signal $x_A(n)$, b is the exponential base of the second signal $x_B(n)$ and A and B are the respective amplitude weighting coefficients of these signals. The amplitude weighting coefficients are constants that define the sum amplitude weights of the exponential signals participating in the sum. We also define two unfolding systems of digital exponential signals. The first unfolds the digital exponential signal $x_A(n)$ with exponential base a, while the second unfolds the digital exponential signal $x_B(n)$ with the exponential base b. The unit impulse response of the first system is defined as:

$$h_A(n) = \begin{cases} 1 & \text{for } n=0 \\ -a & \text{for } n=1 \\ 0 & \text{elsewhere} \end{cases} \quad (20)$$

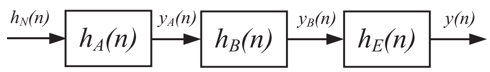

Fig. 11. Unfolding system of the sum of two exponential signals.

The unit impulse response of the second unfolding system is

$$h_B(n) = \begin{cases} 1 & \text{for } n = 0 \\ -b & \text{for } n = 1 \\ 0 & \text{elsewhere} \end{cases} \tag{21}$$

We can define an unfolding system shown in Fig. 11.

In this system the input signal is processed consecutively by the two exponential unfolding systems $h_A(n)$ and $h_B(n)$, producing a result $y_B(n)$ that is, in general, not a unit impulse. The result $y_B(n)$ is then unfolded by a system with a unit impulse response $h_E(n)$. Our goal is to find the unit impulse response $h_E(n)$. The first step is to find a mathematical expression for describing the signal $y_B(n)$.

The signal $y_A(n)$ is the result of the convolution of $h_A(n)$ and $x(n)$. Using the distributive property of the convolution the signal $y_A(n)$ can be described as:

$$y_A(n) = [x_A(n) + x_B(n)] * h_A(n) = A\delta(n) + x_B(n) * h_A(n) \tag{22}$$

where $a^n * h_A(n) = \delta(n)$.

Similarly, using the distributive and associative properties of the convolution the signal $y_B(n)$ can be expressed as:

$$y_B(n) = \left[A\delta(n) + x_B(n) * h_A(n)\right] * h_B(n) = Ah_B(n) + Bh_A(n) \tag{23}$$

Note that $\delta(n) * h_A(n) = h_A(n)$ and $\delta(n) * h_B(n) = h_B(n)$.

Substituting $h_A(n)$ and $h_B(n)$ with their definitions from Eqs. (20) and (21) respectively, the signal $y_B(n)$ can be rewritten explicitly as:

$$y_B(n) = \begin{cases} A + B & \text{for } n = 0 \\ -Ab - Ba & \text{for } n = 1 \\ 0 & \text{elsewhere} \end{cases} \tag{24}$$

Eq. (24) can be normalized by dividing both sides by $(A+B)$ resulting in the following normalized expression:

$$\dot{\delta}_K(n) = \frac{y_B(n)}{A+B} = \begin{cases} 1 & \text{for } n = 0 \\ -K & \text{for } n = 1 \\ 0 & \text{elsewhere} \end{cases} \tag{25}$$

where K is a weighting factor defined as:

$$K = \frac{Ab + Ba}{A + B} \tag{26}$$

The normalized signal described by Eq. (25) is an asymmetric doublet and is similar to the unit impulse response of an unfolding system of a digital exponential pulse as shown in Fig. 10.

Finally, we will define the system with a unit impulse response $h_E(n)$ that unfolds the asymmetric doublet given by Eq. (25). In other words, the system $h_E(n)$ will transform an asymmetric doublet into a unit impulse. This transformation can be expressed by the following equation:

$$\delta(n) = \dot{\delta}_K * h_E(n) = h_E(n) * \dot{\delta}_K \tag{27}$$

From the definition of the asymmetric doublet (Fig. 10b) Eq. (25) can be expressed in terms of unit impulses

$$\dot{\delta}_K(n) = \delta(n) - K\delta(n-1) \tag{28}$$

Eq. (28) is similar to Eq. (9) and, as stated earlier, it describes a system which unfolds a digital exponential signal with exponential base K. Therefore, in order to unfold the asymmetric doublet the unit impulse response $h_E(n)$ must be a digital exponential signal with exponential base K. The system that responds to a unit impulse by generating a digital exponential signal has been described in our previous work [16]. The block diagram of the

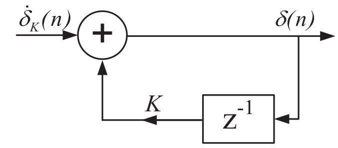

Fig. 12. Block diagram of a system that unfolds an asymmetric doublet.

system that unfolds an asymmetric doublet as defined by Eq. (28) is shown in Fig. 12

The system in Fig. 12 must be relaxed as in the case of an accumulator. In fact, when $K=1$ this system acts as an accumulator. The impulse response of the relaxed system $h_E(n)$ is, therefore, defined as:

$$h_E(n) = \delta(n) + Kh_E(n-1) \tag{29}$$

It is important to note that the weighting factor K depends on the amplitude weighting coefficients A and B, and the exponential factors a and b of the signals being added together. This dependence must be considered when unfolding signals with amplitude weighting coefficients that may change with time, temperature, or other factors. A typical example of such variability is the signals from scintillation detectors with multiple light-emitting components due to the temperature dependency of their light output and decay time-constants.

The unfolding algorithm is independent of the sign of the amplitude weighting coefficients except in the case when $A = -B$. In some cases the weighting factor K may become zero ($K=0$) making the system in Fig. 12a pass-through system ($h_E(n) = \delta(n)$). When $A = B$ the weighting factor K becomes independent of the amplitude weighting coefficients A and B.

If $A = -B$ the weighting factor K becomes undefined because the denominator in Eq. (26) becomes zero. An important signal in the radiation measurements is the signal that is a sum of two exponential signals with different exponential bases ($a \neq b$) and equal magnitude-opposite sign weighting factors $A = -B$. We will show below that such a signal is actually the result of a convolution of the two exponential signals forming the sum.

4.4. Convolution of exponential signals

The convolution of two exponential impulse responses is commonly found in analog pulse-processing systems. A resistive feedback preamplifier followed by a R-C low pass filter, cascaded low pass filters, and a C-R or pole-zero differentiated preamplifier signal followed by R-C low pass filter are all examples of system impulse responses that are a result of a convolution of analog exponential impulse responses. Fig. 13 illustrates some of these systems.

The convolution $h_T(t)$ of two signals $x_a(t)$ and $x_b(t)$ in the continuous-time domain is defined as:

$$h_T(t) = x_a(t) * x_b(t) = \int_{-\infty}^{t} x_a(\theta) * x_b(t-\theta) d\theta \tag{29}$$

Let's define $x_a(t)$ and $x_b(t)$ as exponential signals:

$$x_a(t) = \begin{cases} e^{-t/\tau_a} & \text{for } t \geq 0 \\ 0 & \text{elsewhere} \end{cases} \tag{30}$$

and

$$x_b(t) = \begin{cases} e^{-t/\tau_b} & \text{for } t \geq 0 \\ 0 & \text{elsewhere} \end{cases} \tag{31}$$

where τ_b and τ_a are the decay time constants of the exponential signals.

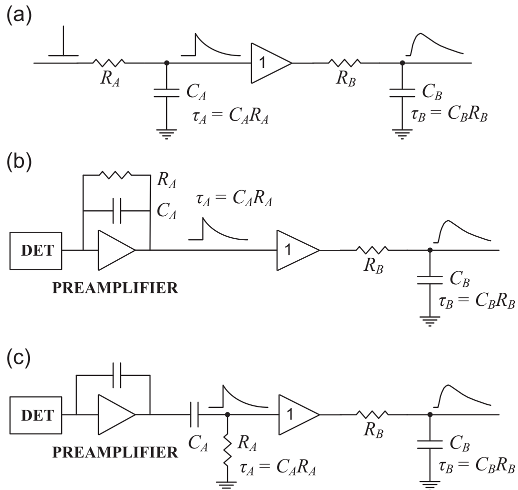

Fig. 13. Convolution of analog exponential impulse responses (a) cascaded R-C networks, (b) resistive feedback preamplifier followed by R-C network and (c) reset type preamplifier followed by R-C network.

By substituting Eqs. (30) and (31) in Eq. (29) the signal $h_T(t)$ can be expressed explicitly. It is zero for $t<0$. For $t\geq 0$ the signal $h_T(t)$ can be expressed as:

$$h_T(t) = \int_0^t e^{-\theta/\tau_a} * e^{-t-\theta/\tau_b} d\theta \tag{32}$$

The integral in Eq. (32) has two different evaluations depending on the relationship between τ_b and τ_a:

$$h_T(t) = \frac{ab}{a-b}(e^{-t/\tau_a} - e^{-t/\tau_b})\text{for} t_b \neq t_a \tag{33}$$

and

$$h_T(t) = te^{-t/\tau_a}\text{for } t_b = t_a \tag{34}$$

Eq. (33) indicates that the convolution of two exponential signals can be expressed as a difference of the two exponential signals with equal amplitude weighting coefficient. Fig. 14a shows the amplitude normalized signal $h_T(t)$ for $\tau_b \neq \tau_a$. Note that the digital signal (Fig. 14b) corresponding to the analog signal is also the difference (sum) of two digital exponential signals as shown in Fig. 14c.

One important observation about the digital signal depicted in Fig. 14b is that $h_N(0)=0$. As the real-time exponential unfolding systems are causal, the unfolding result cannot be a unit impulse $\delta(n)$ as defined by Eq. (1). Instead the unfolding can only result in a unit impulse at $n=1$ or greater. To unfold the signal defined by either Eq. (33) or Eq. (34) we use the unfolding systems as defined by Eqs. (20) and (21). Fig. 15 shows the unfolding system of a digital signal obtained by digitizing an analog signal that is a result of a convolution of two exponential signals in the continuous-time domain.

First we will unfold the signal $h_N(n)=a^n-b^n=x_A(n)-x_B(n)$. The system with the unit impulse response $h_A(n)$ unfolds the digital exponential signal $x_A(n)$. The convolution of $h_N(n)$ and $h_A(n)$ can be expressed as:

$$y_A(n) = [x_A(n) + x_B(n)] * h_A(n) = \delta(n) - x_B(n) * h_A(n) \tag{35}$$

The convolution with $h_B(n)$ will result in:

$$y(n) = \left[\delta(n) - x_B(n) * h_A(n)\right] * h_B(n) = h_B(n) - h_A(n) \tag{36}$$

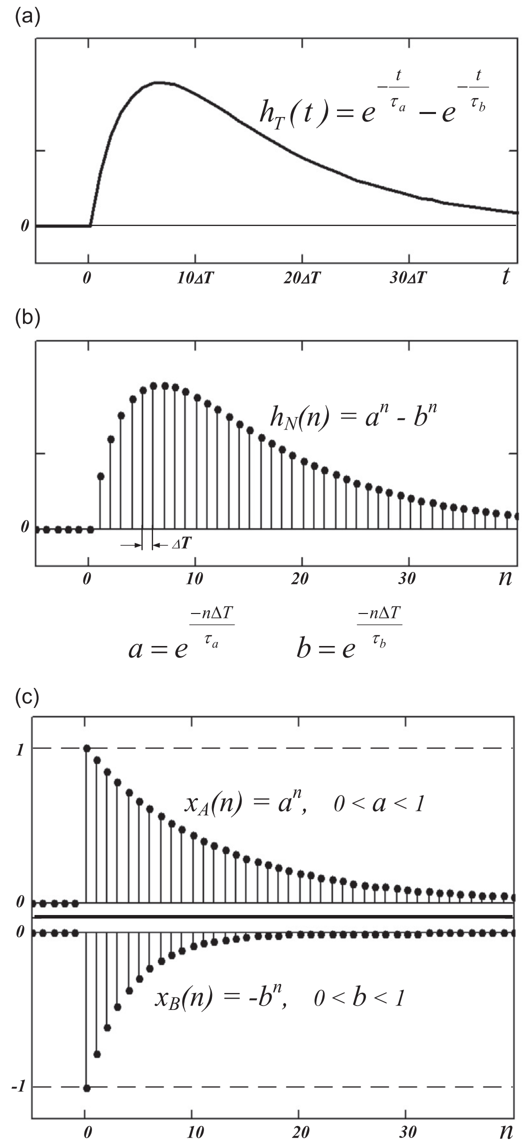

Fig. 14. Convolution of two exponential impulse responses (a) amplitude normalized signal in the continuous-time domain, (b) digital signal obtained by digitizing of the signal in the continuous time domain and (c) digital signal as the sum of two exponents.

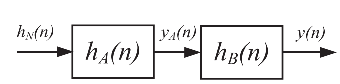

Fig. 15. Unfolding system of the digitized signal obtained by a convolution of two analog exponential signals.

Using Eqs. (20),(21) and (36) the result of the unfolding $y(n)$ can be found as:

$$y(n) = h_B(n) - h_A(n) = \begin{cases} a-b & \text{for } n=1 \\ 0 & \text{elsewhere} \end{cases} \tag{37}$$

This result clearly shows that $y(n)=\delta_w(n-1)$, a result that is a direct consequence of the fact that the unfolding system is a causal LTI system. The signal $y(n)$ can be normalized by a factor $(a-b)$ to obtain an unfolding result that is a unit impulse shifted by one sample. As the origin of the digital signal processing systems is relative, the shift may be disregarded when $y(n)$ is the only signal applied to the signal processing systems that follow the unfolding system.

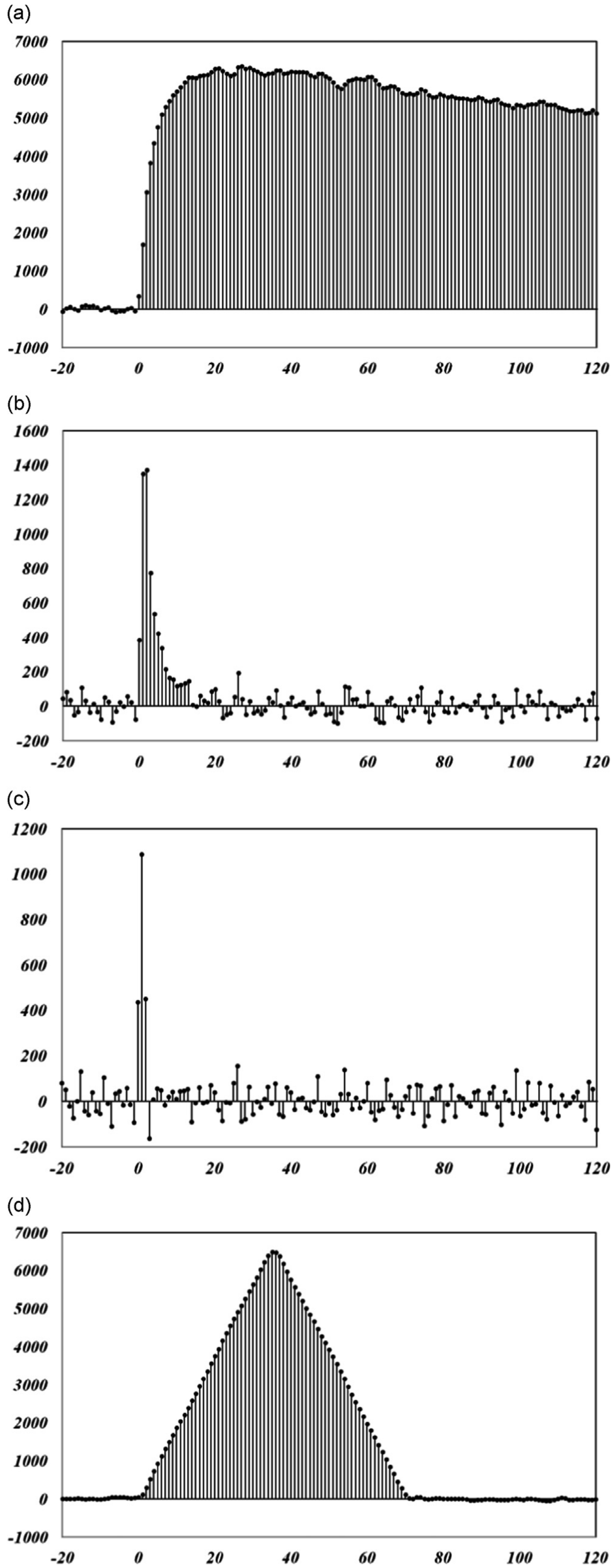

Fig. 16. Digital pulse shaping of experimental signal from a SDD: (a) ADC signal, (b) after unfolding the long time constant (Eq. (35)) (c) after unfolding the short time constant (Eq. (37)) and (d) triangular pulse synthesized from the unfolded signal.

Next we will unfold the signal $x_A(n)=n\Delta Ta^n$. As ΔT is a constant, our interest is to unfold the normalized signal $h_N(n)=na^n$. The unfolding procedure will be carried out by applying the unit impulse response $h_A(n)$ twice in sequence. Using the definition of the exponential unfolding system given by Eq. (9) and the properties of the convolution we obtain:

$$y(n)=\left[na^n * h_A(n)\right] * h_A(n)=\left[na^n-a(n-1)a^{n-1}\right] * h_A(n)=a^n * h_A(n)=\delta(n) \tag{38}$$

Eqs. (37) and (38) indicate that the system in Fig. 5 can be used to unfold an exponential unit impulse response that is a result of a convolution of two continuous-time exponential signals regardless of the relationship between their decay time constants ($\tau_b \neq \tau_a$ *or* $\tau_b=\tau_a$).

Fig. 16 shows an example of applying the unfolding to a signal from a silicon drift detector (SDD) embedded in labZY X-Ray spectrometer nanoXRS [17]. The SDD is connected to a reset type preamplifier followed by a C-R differentiation network with time constant τ_a=6200 ns (a long time constant). Next, the C-R differentiated signal passes through a R-C low-pass filter with time constant τ_b=32 ns (a short time constant). The signal from the low-pass filter is digitized by a fast ADC with sampling period ΔT=12.5 ns. The signal from the ADC is shown in Fig. 16a. The captured signal is from an interaction of 5.9 keV $K\alpha$ manganese X-ray quantum. Fig. 16b shows the result of using Eq. (35) to unfold the response of the C-R differentiation network to a step signal, which is an exponential pulse. The exponential base a in Eq. (35) is $a \approx 0.998$. The signal shown in Fig. 16c is the result of unfolding the exponential impulse response of the R-C low pass filter according to Eq. (37). The exponential base b in Eq. (35) is $b \approx 0.677$.

The result from the unfolding is not a unit impulse as there are unaccounted and unfolded responses such as the finite preamplifier rise time, bandwidth limiters of the ADC input stage, and finite charge collection time of the SDD. As expected from Eq. (37) the sum of the 3 samples representing the unfolded signal is about a–b times less than the amplitude of the ADC signal.

The signal in Fig. 16d is a triangular signal synthesized using the unfolded signal in Fig. 16c. The amplitude of the triangular signal is normalized to the amplitude of the ADC signal. It is important to emphasize that the digital signal processing is noiseless. Therefore, if a digital signal is processed by a chain of signal processing systems the outcome is the same regardless of the application order of the systems in the chain. The triangular signal in Fig. 16c can be synthesized exactly the same if other algorithms are used to process the ADC digital signal preserving the same signal-to-noise ratio.

5. Conclusions

In Part 1 of this paper we have introduced the unfolding-synthesis technique and have described the unfolding algorithms applicable to most common signals in radiation measurements. The unfolding technique can be extended to other digital signals obtained by digitization of analog signals with known characteristics. Our intent was to demonstrate and to provide a basic procedure for unfolding commonly used signals in radiation measurements. The unfolding technique may be applied to more complex signals as long as they are well defined and described by mathematical equations. It is important that not all responses can be unfolded completely, especially ones on very short time scales. Still, the unfolding of the major, long time scale, impulse responses is key for achieving high throughput while utilizing pulse shapes with optimal characteristics for a given measurement setup.

Acknowledgments

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References

[1] Glenn. F. Knoll, Radiation Detection and Measurement, 4th edition, Wiley, 2010.
[2] Instrumentation for Digital Nuclear Spectroscopy, IAEA-TECDOC-1706, 2010.
[3] V.T. Jordanov, et al., Nuclear Instruments and Methods in Physics Research A 353 (1994) 261.
[4] G. Ripamonti, A. Castoldi, E. Gatti, Nuclear Instruments and Methods in Physics Research A A340 (1994) 583.
[5] P.C.P.S Simoes, J.C. Martins, C.M.B.A Correia, IEEE Transactions on Nuclear Science NS43 (3) (1996) 1804.
[6] N. Petrick, A.O. Hero, N.H. Clinthorne, W.L. Rogers, IEEE Transactions on Nuclear Science NS41 (4) (1994) 758.
[7] J.M Monzo, et al., Transactions on Nuclear Science 55 (1) (2008) 48.
[8] Valentin T. Jordanov, Digital Signal Processing in Radiation Measurements, Lecture as part of NE204 course, University of California, Berkeley, 2011.
[9] J.G. Proakis, D. K Manolakis, Digital Signal Processing: Principles, Algorithms and Applications, 3rd Edition, Prentice Hall, 1995.
[10] V.T Jordanov, K.V Jordanova, Transactions on Nuclear Science 59 (4) (2012) 1282.
[11] Glenn F. Knoll, Personal Communication (1995).
[12] F. Goulding, Nuclear Instruments and Methods in Physics Research A 100 (3) (1972) 493.
[13] V. Radeka, IEEE Transactions on Nuclear Science NS15 (3) (1968) 455–470.
[14] E Gatti, M Sampietro, P.F Manfredi, Nuclear Instruments and Methods in Physics Research A 287 (1990) 513.
[15] V. T Jordanov, Nuclear Instruments and Methods in Physics Research A 351 (1994) 592–594.
[16] V.T. Jordanov, Nuclear Instruments and Methods in Physics Research A 670 (2012) 18–24.
[17] www.labzy.com.