Linear Transform:

It consists of scaling & shearing. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$

Rotation is one example of three shearings.

Rotation Martrix in 2D:

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Affine Transform:

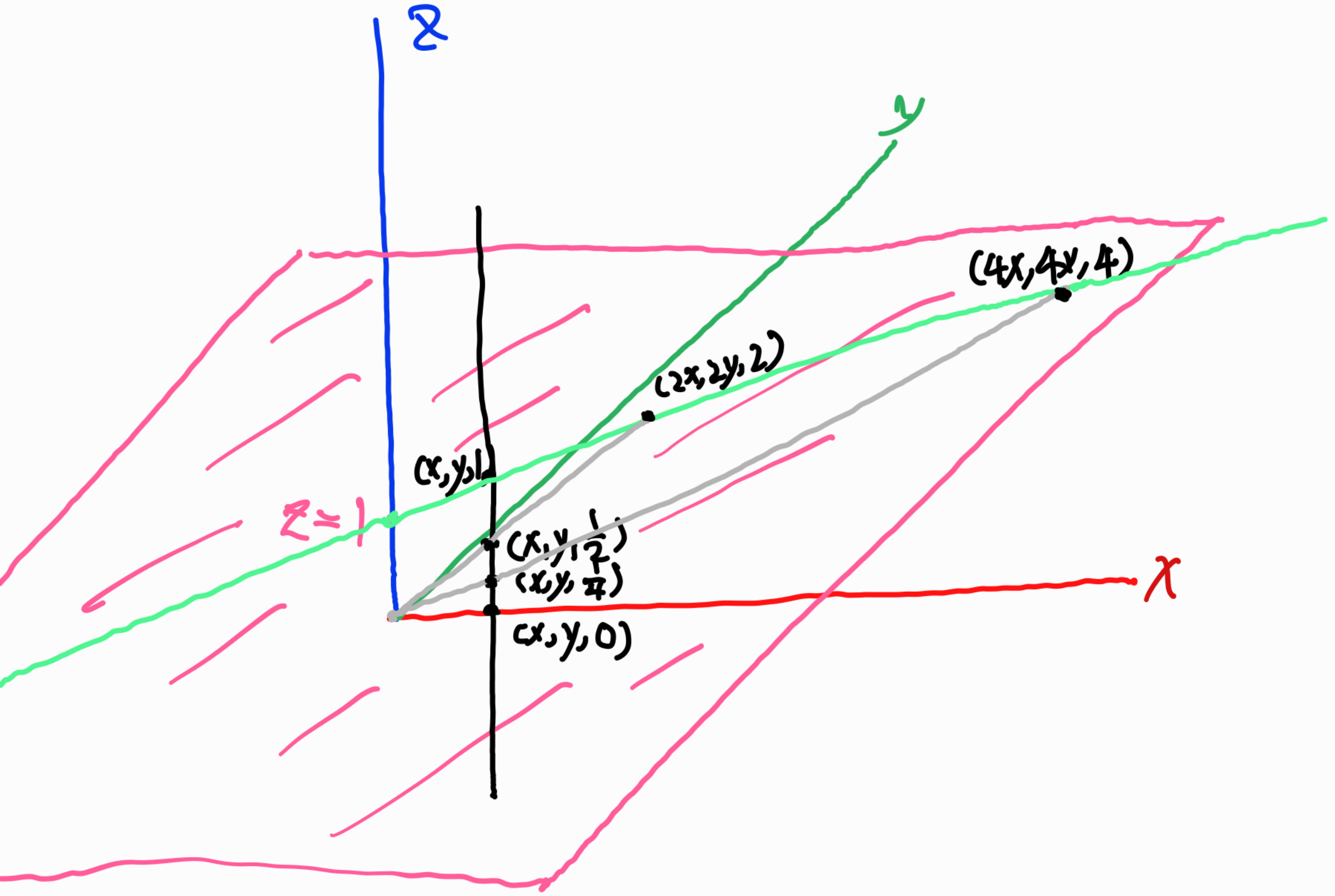
It's linear transformation plus translation.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ax+by+e \\ cx+dy+f \end{bmatrix}$$

$$\equiv \begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} ax+by+e \\ cx+dy+f \\ 1 \end{bmatrix}$$

Homogeneous Coordinates come handy in affine transformation.

Division by Zero in homogeneous coordinates:



$(x,y,0) \longmapsto (\lambda x, \lambda y, \lambda)$ where $\lambda = \infty$

in direction (x,y)

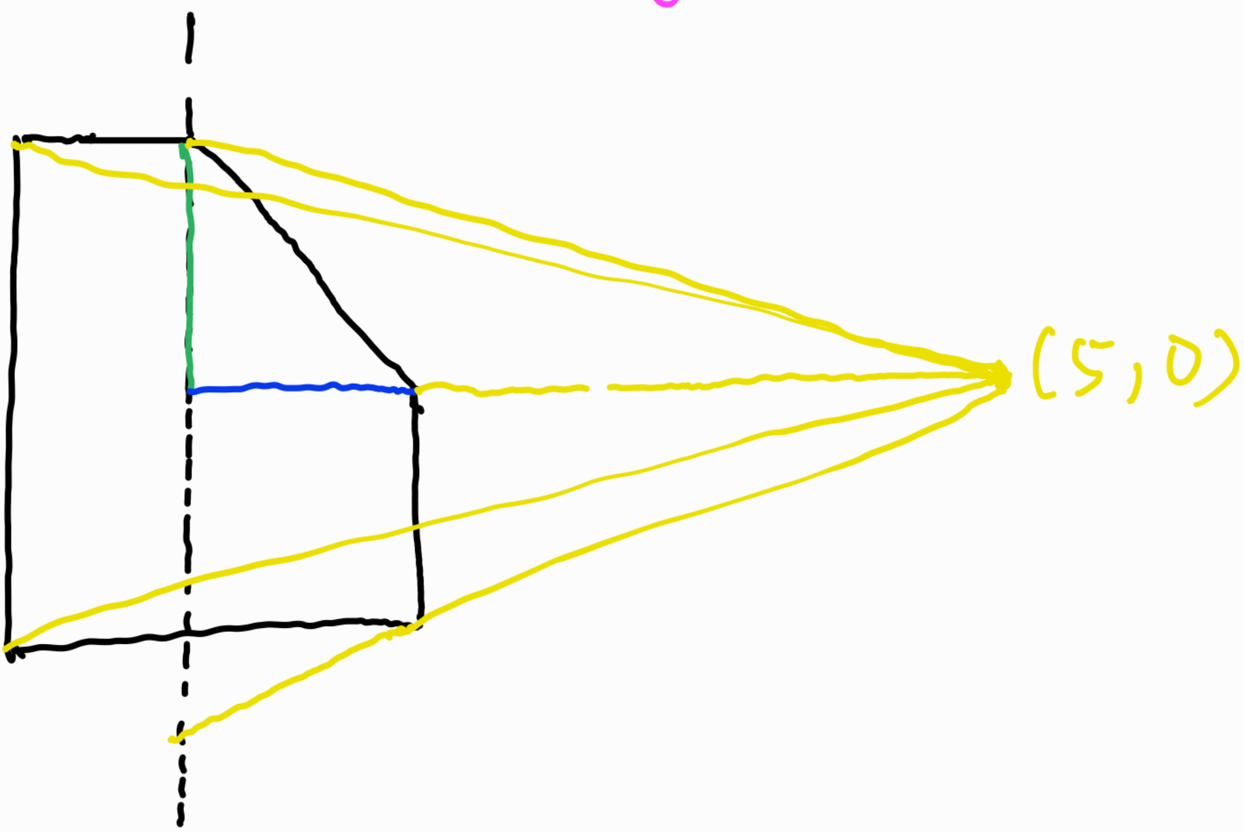
Homogeneous coordinates with the last

coordinate being zero represents a vector instead of point.

point $\neq$ vector

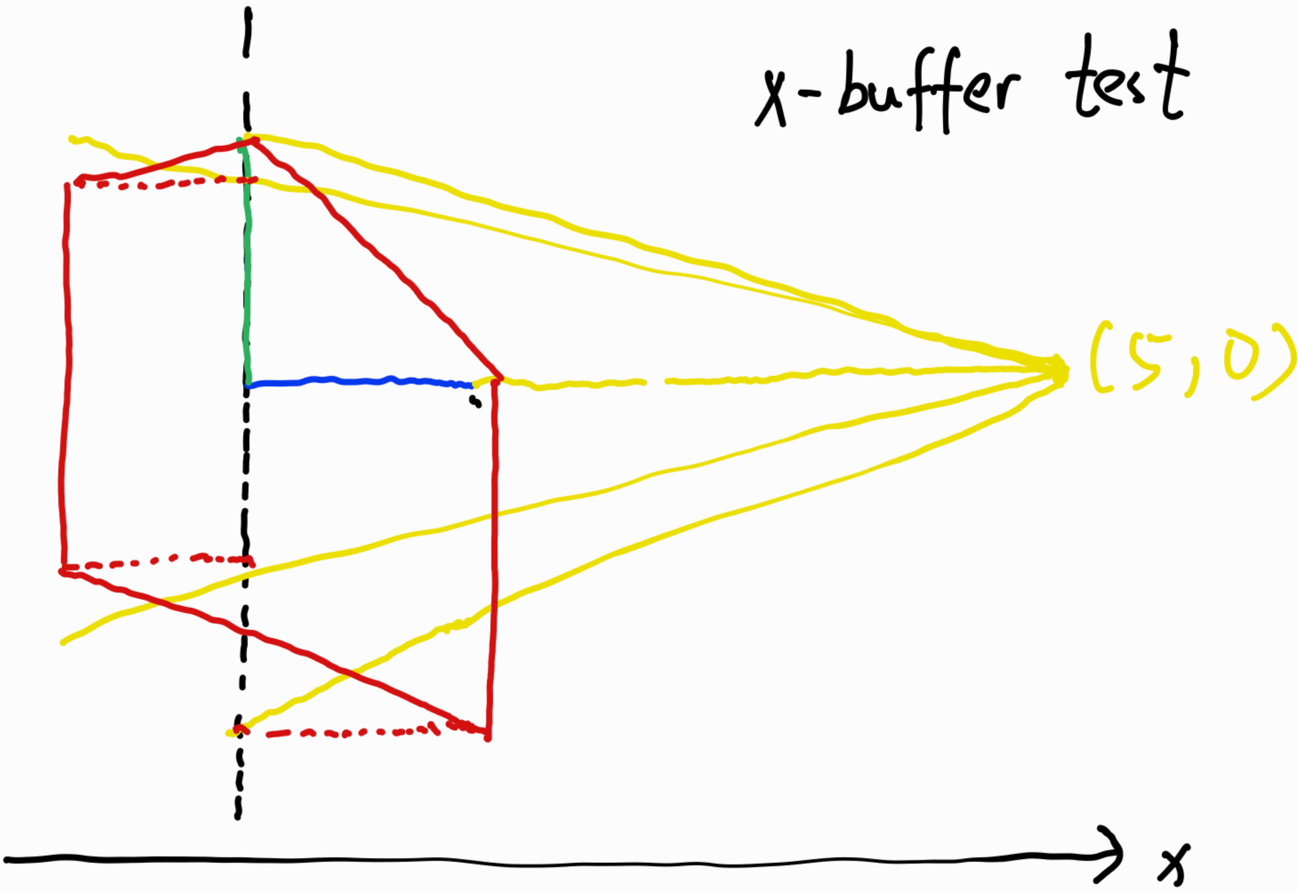
We can compose a number of matrix transformation into one matrix.

Last row of Homogeneous Matrix



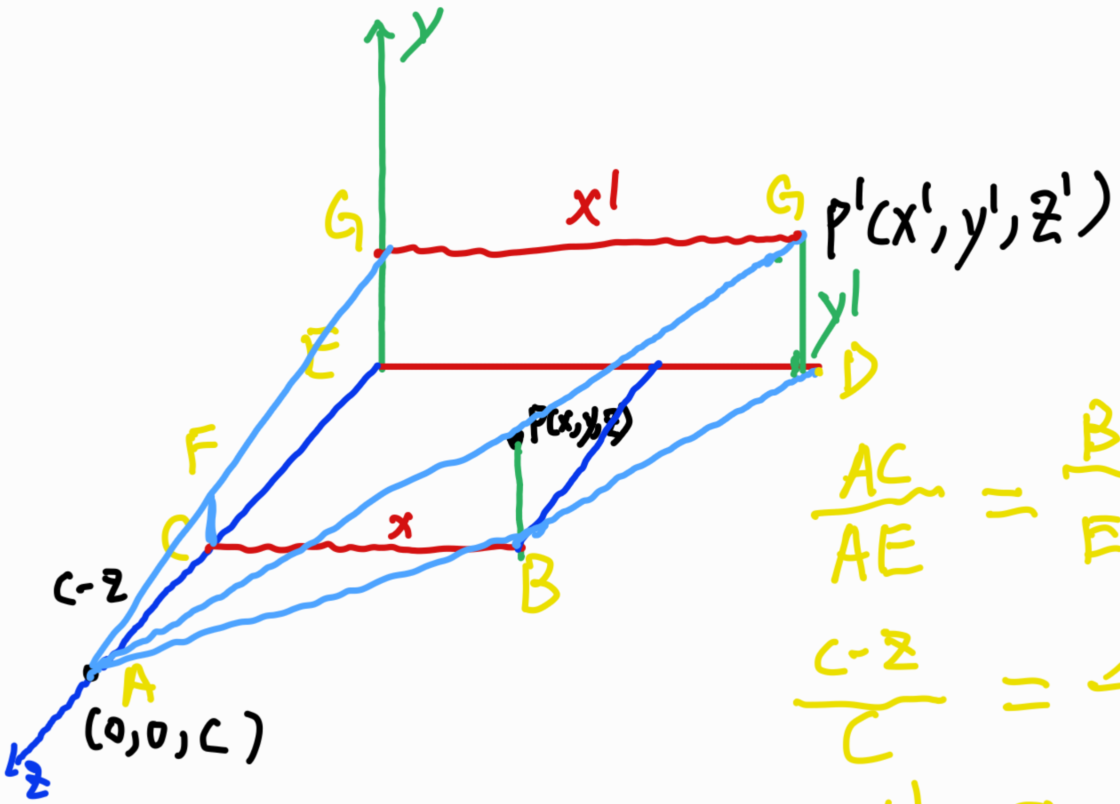
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{5} & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -\frac{1}{5}x+1 \end{bmatrix}$$

x-buffer test



Vertical line mapped to vertical lines
Close to camera are stretched
far from camera are shrunk

3D case:



$$\frac{AC}{AE} = \frac{BC}{ED}$$

$$\frac{c-z}{c} = \frac{x}{x'}$$

$$x' = x \cdot \frac{1}{1-\frac{z}{c}}$$

$\triangle ACF \sim \triangle AEG$

$$\frac{CF}{EG} = \frac{AC}{AE}$$

$$\frac{y}{y'} = \frac{c-z}{c}$$

$$y' = y \cdot \frac{1}{1-\frac{z}{c}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/c & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1-z/c \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \frac{x}{1-z/c} \\ \frac{y}{1-z/c} \\ \frac{z}{1-z/c} \end{bmatrix}$$

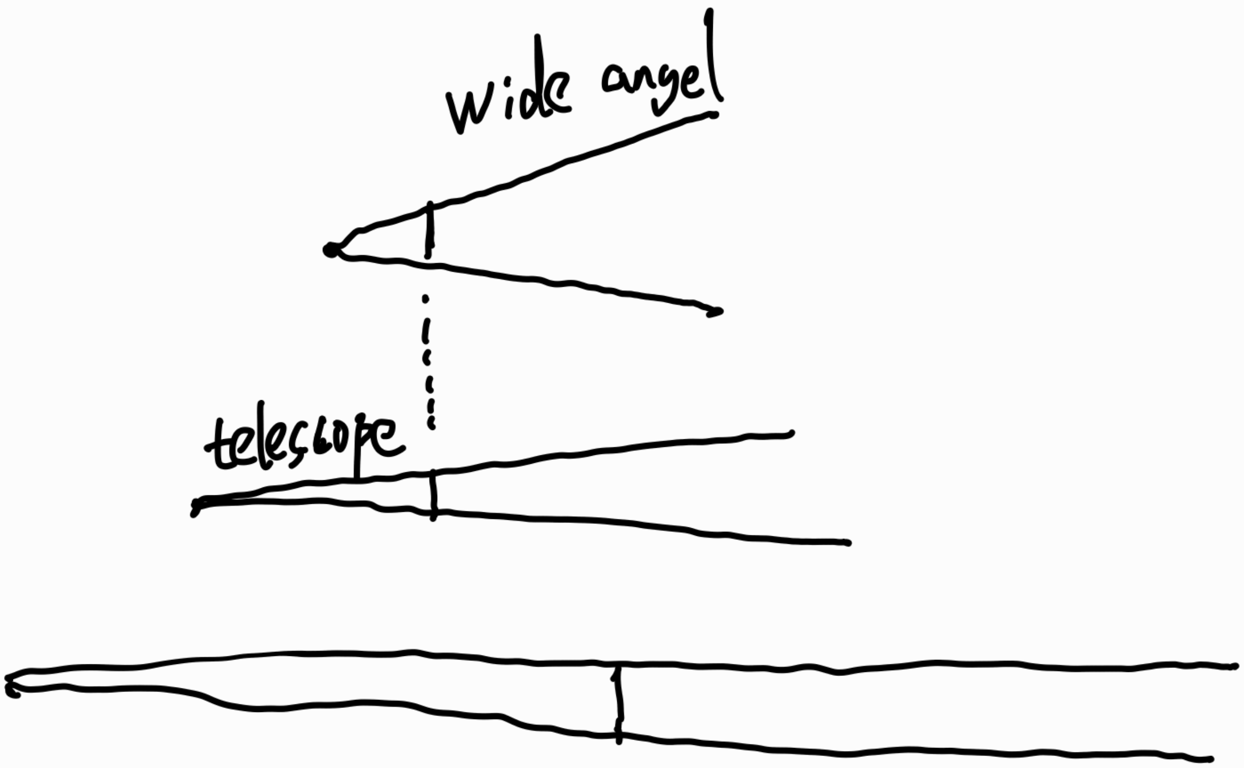
The closer z-coord to c, the larger the z val (z-fighting?)

$$
\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}
\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}
$$

for $i \in [1, 4]$

  for $j \in [1, 4]$

  $$c[i][j] = \sum_{k=1}^{4} a[i][k]\, b[k][j]$$



Viewport

$$\begin{bmatrix} \frac{w}{2} & 0 & 0 & a+\frac{w}{2} \\ 0 & \frac{h}{2} & 0 & b+\frac{h}{2} \\ 0 & 0 & \frac{d}{2} & \frac{d}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{w}{2} \cdot x + a + \frac{w}{2} \\ \frac{h}{2} \cdot y + b + \frac{h}{2} \\ \frac{d}{2} z + \frac{d}{2} \\ 1 \end{bmatrix} \qquad \begin{bmatrix} \frac{x}{1-z/c} \\ \frac{y}{1-z/c} \\ \frac{z}{1-z/c} \end{bmatrix}$$

$$x \in [-1, 1] \longmapsto x' \in [a, w+a]$$

$$y \in [-1, 1] \longmapsto y' \in [b, h+b]$$

$$z \in [-1, 1] \longmapsto z' \in [0, d]$$

Issue #97: $\frac{W}{2} \cdot \frac{x}{1-z/c} + \frac{W}{2}$ if $a=0$

$$\left[\frac{W}{2} \cdot \frac{-1}{1-z/c} + \frac{W}{2}, \quad \frac{W}{2} \cdot \frac{1}{1-z/c} + \frac{W}{2} \right]$$

$$\vec{v} = \frac{W}{2} \left[\frac{-1}{1-z/c} + 1, \ \frac{1}{1-z/c} + 1 \right]$$

if z approaches $-\infty$,

$\vec{v}$ approaches $\frac{W}{2}[1, 1]$

which is the center of the image.