Data Structures

SY BTech(CSE)

Unit – 1

Introduction

PPT-8

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Objective:

Study of properties of asymptotic notations.

Properties:

Let f(n) and g(n) be such that

$$\lim_{n \to \infty} \frac{f(n)}{g(n)}$$

exists, then

1. Function $f(n) \in O(g(n))$ if

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = c < \infty,$$ also including the case in which limit is 0

2. Function $f(n) \in \Omega(g(n))$ if

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} > 0,$$ also including the case in which limit is ∞

3. Function $f(n) \in \Theta(g(n))$ if

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = c$$ for some constant c such that $0 < c < \infty$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = c < \infty$$

Prove that $10n^2 + 9 \neq O(n)$

Solution:

$f(n) = 10n^2 + 9$

$g(n) = n$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{10n^2 + 9}{n}$$

$$= 10n + 9/n$$

$$= \infty$$

$\therefore 10n^2 + 9 \neq O(n)$

Prove that $\sum i^2 = O(n^3)$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = c < \infty$$

$f(n) = \sum i^2$

$= \frac{1}{2} n^3 - \frac{1}{2} n$

$g(n) = n^3$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{\frac{1}{2} n^3 - \frac{1}{2} n}{n^3}$$

$= \frac{1}{2} - 1/(2n^2)$

$= \frac{1}{2} < \infty$

$f(n) = \sum i^2 = O(n^3)$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = c, \quad 0 < c < \infty$$

Prove that $5n^2 - 6n = \Theta(n^2)$

Solution:

$f(n) = 5n^2 - 6n$

$g(n) = n^2$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{5n^2 - 6n}{n^2}$$

$$= 5 - 6/n$$

$$= 5$$

AS $0 < 5 < \infty$

$\therefore 5n^2 - 6n = \Theta(n^2)$

Prove that $2n^2 2^n - n\log n = \Theta(n^2 2^n)$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = c, \quad 0 < c < \infty$$

Solution:

$f(n) = 2n^2 2^n - n\log n$

$g(n) = n^2 2^n$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{2n^2 2^n - n\log n}{n^2 2^n}$$

$$= 2 - \log n/n\, 2^n$$

$$= 2$$

AS $0 < 2 < \infty$

$\therefore 2n^2 2^n - n\log n = \Theta(n^2 2^n)$

Prove that $33n^3 + 4n^2 = \Omega(n^2)$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} > 0$$

Solution:

$f(n) = 33n^3 + 4n^2$

$g(n) = n^2$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{33n^3 + 4n^2}{n^2}$$

$$= 33n + 4$$

$$= \infty$$

AS $\infty > 0$

$\therefore 33n^3 + 4n^2 = \Omega(n^2)$

Prove that $33n^3 + 4n^2 = \Omega(n^3)$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} > 0$$

Solution:

$f(n) = 33n^3 + 4n^2$

$g(n) = n^3$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{33n^3 + 4n^2}{n^3}$$

$$= 33 + 4/n$$

AS $33 > 0$

$\therefore 33n^3 + 4n^2 = \Omega(n^3)$

Prove that $n! = O(n^n)$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = c < \infty$$

Solution:

$$n! = n(n-1)(n-2)\text{---}2 * 1$$

$$= n^n [1 - 1/n][1 - 2/n] \text{---} 2/n * 1/n$$

$f(n) = n!$

$g(n) = n^n$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \frac{\cancel{n^n}[1 - 1/n][1 - 2/n] \text{---} 2/n * 1/n}{\cancel{n^n}}$$

As $1/\infty = 0$

$= 0$

As $0 < \infty$

$\therefore n! = O(n^n)$

Practical Complexities:

$\log n$	n	$n \log n$	n^2	n^3	2^n
0	1	0	1	1	2
1	2	2	4	8	4
2	4	8	16	64	16
3	8	24	64	512	256
4	16	64	256	4096	65536
5	32	160	1024	32768	4294967296

2^n Grows very rapidly with n.

Performance Measurement:

- It is concerned with obtaining the space and time requirements of a particular algorithm.
- It depends on 1. Compiler

  2. Options used

  3. Computer specifications
- To compute time of a program we need a **clocking procedure**.

Gettime(): Returns the current time in milliseconds.

Ex: **Algo Main**

```
{
    h1 = gettime();
    binsrch(a,n,x);
    h2 = gettime();
    t = h2 – h1;
    Write "Time: ",t;
}
```

[Assignment 1.pdf]