**Slide**: The Black-Scholes Equation

* For our problem we used the Black-Scholes PDE for financial modeling – specifically for the European Call Price option
* This is a PDE based on Geometric Brownian Motion (stochastic process) solved with stochastic integration (Ito calculus)
* This is a very basic model based on stocks instead of both stocks and bonds
  + Interest: theoretical no risk of loss interest rate
  + Stock volatility: variation of price over time
  + Stock drift: average value of asset over time

**Slide**: Martingale Representation

* Based in probability
* Stochastic process that uses the normal distribution

**Slide**: Replication Strategy

* Self-financing: once the initial value is placed it doesn’t have outside value added
* Using theory and references, this theoretical example has a constant bond to simplify model for those with minimal stochastic knowledge
* Values
  + =martingale process, = bond val, = stock val, = # bonds, = # stock, = portfolio value, = GBM

**Slide**: Non-zero interest rates

* is forward contract (exchange of assets at certain time) which leads us to find non-zero interest rates
* This leads to the stochastic proves for discounted stock, which we have not elected
* Using to create a more dynamic and accurate theoretical model

**Slide**: Black-Scholes Formula for European Call Option

* If you use a constant , GBM is not used. Making this less accurate for a continuous model but good for specified times
* But if substitute in the equation, you get a model over time

**Impacts**: blue = base, red = higher, green = lower

**Slide**: Stock Volatility Impact

* Higher = less profitable over time due variation over time
* Lower = more profitable as price remains more constant

**Slide:** Riskless Interest Rate Impact

* Higher = more profitable, lower = less profitable
* Minimal impact

**Slide:** Stock Drift Impact

* Higher = dramatic impact, higher value
* Lower = not as impactful, lower value

**Impacts**: blue = base, red = both lower, green = both higher, yellow, black = mixed

**Slide**: Mixed Interest and Volatility Impact

* Red = significantly higher
* Green = higher at beginning, but negligible over time
* Black ( high, low) = lowest return
* Yellow ( low, high) = highest return, but similar to red

**Slide**: Mixed Interest and Drift Impact

* Red = dramatically lower (lowest)
* Green = much higher
* Black ( low, high) = lower, but more profitable than red
* Yellow ( high, low) = highest, comparable to green

**Slide**: Mixed Volatility and Drift Impact

* Red = lower
* Green = relatively higher
* Black ( low, high) = lowest
* Yellow ( high, low) = highest, dramatic

**Slide**: Interpretation

* Behaviors:
  + have a positive correlation/relationship
  + has an inverse relationship
* Impact:
  + is only in GBM and has most impact
  + is in GBM and Black-Scholes with significant impact
  + is only in Black-Scholes has least impact

**Slide**: Linear Stability of the Black-Scholes

* Popular linear b.c. where vanishes when asset price gets large
* Growth has no effect on convergence
* Stability holds for all

**Slide**: Weaknesses of the Black-Scholes

* Assumptions
  + GBM, is smooth/continuous, fractional stocks, no transaction costs, all assets can move freely
* Normal distribution, constant volatility, riskless interest rate

**Slide**: How We Can Combat These Weaknesses

* Assumptions violated means invalid
* GBM: not easy to combat and no consensus of how to
* Normal distribution to leptokurtic (outliers common)
* Constant volatility to volatility clusters
* Give risk to

**Slide**: The Black-Scholes Partial Differential Equation

* Still commonly used
* Our model
  + Theoretical
  + Simplified for ease
  + Possible steps:
    - Stochastic process for
    - Run two models: one for and one for both

**References**