

Referred from: A Practical Guide To Quantitative Finance Interviews

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Screwy Pirates

Quant trading interview question





Five pirates looted a chest full of 100 gold coins. Being a bunch of democratic pirates, they agree on the following method to divide the loot.



Most senior Pirate will propose a distribution of coins. All pirates, including the most senior pirate will then vote.

If $>50\%$ accept, the gold is divided.

If $<50\%$ accept, senior pirate killed and new senior pirate chosen

Every pirate wants maximum money.



Base Case

Let's think about a base case and then we move ahead with other cases.

No. of pirates = 2

Pirate P2 (senior) and P1.

What will happen?



Base Case

If most senior pirate P2 chooses to give himself all the gold, the only remaining pirate P1 will not be happy.

But since, the most senior pirate P2 votes for this, we have 50% votes, hence, most senior pirate gets all the gold and he lives.

Conclusion => Pirate P2 = 100 Gold Pirate P1 = 0 Gold



3 Pirates

Let's extend this to three pirates,

What happens?



3 Pirates

Senior pirates can decide to divide so majority of them are happy with his division.

So, now 2 pirates should agree.



3 pirates

Let Pirate P3 be the new senior pirate.

Let's say he decides to keep 100 Gold for himself, what happens?



3 pirates

He will be killed.

Pirate P2 becomes new senior pirate,

What happens next?



3 pirates

This becomes exactly the base case,

Pirate P2 gets all the gold

Pirate P1 gets nothing



3 pirates

Is there a way for most senior pirate P3 to survive?

Or

Is there a way to make P1 happy?



3 pirates

What if I give P1 one coin?

It's better than no coins right?

He's happy now.



3 pirates

Hence, P3 survives and P1 is happy.

Conclusion => P3 gets 99 gold, P2 gets nothing, P1 gets 1 coin



Extension

For 4 pirates, P4 will be the most senior pirate and by similar logic,
He can keep 99 coins, and give Pirate P2 1 coin.

For 5 pirates, P5 will need at least 3 votes,
So, he gives a coin each to pirate 1 and 3.



Final answer

Gold will be divided in the following manner,

$P_5 = 98$ coins, $P_4 = 0$ coins, $P_3 = 1$ coin, $P_2 = 0$ coins, $P_1 = 1$ coin.



Generalization

We can generalize the solution for any no. of pirates,

For, P_{2n+1} give coins to pirate $2n-1, 2n-3, \dots, 3, 1$ th pirate.



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