



Knowledge Representation and Reasoning (KRR)

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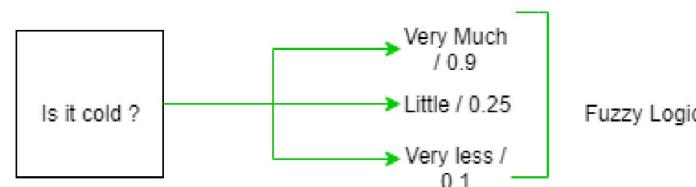
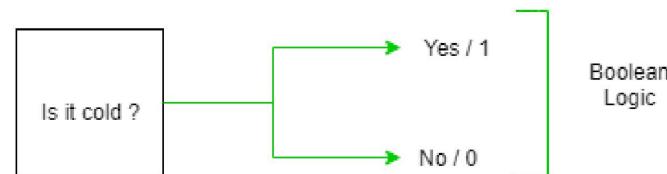
Knowledge Representation

- Propositional Logic
- Predicate Logic (chapter 8)
- Semantic Networks
- Frames
- **Fuzzy Logic (FL)**



Fuzzy Logic

- A logic based on the two truth values ***True*** and ***False*** is sometimes **inadequate** when describing **human reasoning**.
- Fuzzy logic uses the **whole interval** between 0 (False) and 1 (True) to describe human reasoning.
- A form of **many-valued logic** in which the truth values of variables may be any real number between 0 and 1 both inclusive.
- The concept was introduced by **Lotfi Zadeh** (the University of California) in 1965.



What is Fuzzy Logic?

- Fuzzy Logic (FL) is a **method of reasoning** that resembles **human reasoning**.
- The approach of FL **imitates** the way of **decision making** in humans that involves all intermediate possibilities between digital values YES (1) and NO (0).
- Fuzzy logic is determined as **a set** of mathematical principles for **knowledge representation** based on **degrees of membership** and **degrees of truth**.
- **Fuzzy logic** is a **method for reasoning with logical expressions** describing membership in fuzzy sets. For example, the complex sentence **Tall(Tom) \wedge Heavy(Tom)** has a fuzzy truth value that is a function of the truth values of its components. The standard rules for evaluating the fuzzy truth, T, of a complex sentence are:

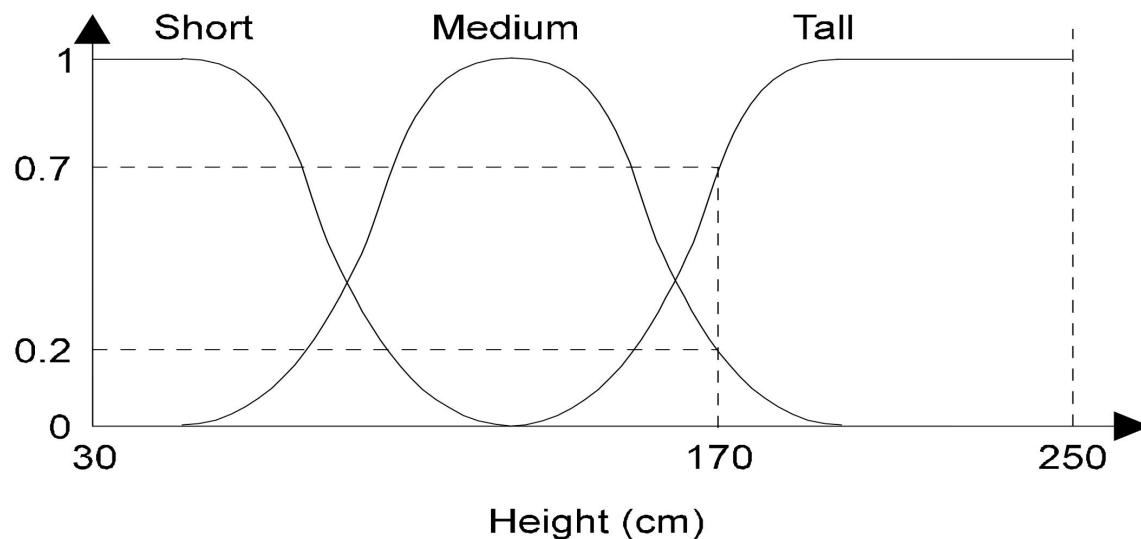
$$T(A \wedge B) = \min(T(A), T(B)), T(A \vee B) = \max(T(A), T(B)), \quad T(\neg A) = 1 - T(A).$$

Fuzzy Logic...

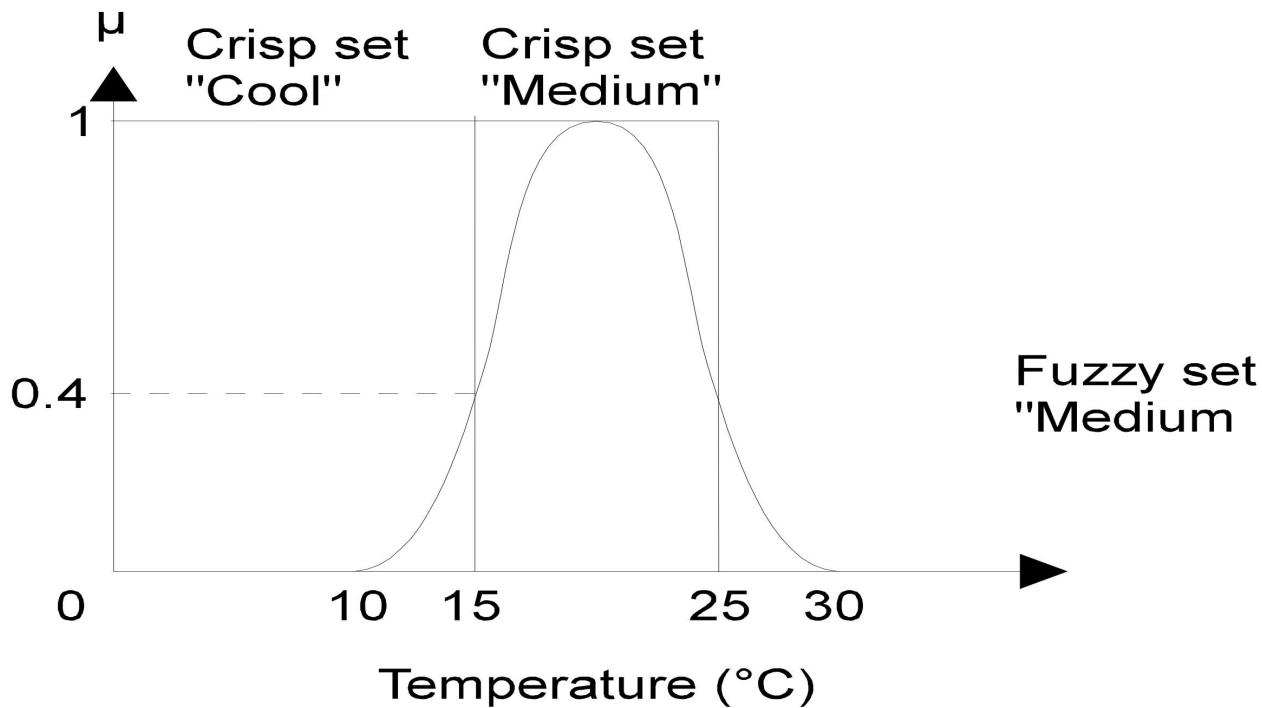
- *Fuzzy sets*
- *Fuzzy input and output variables*
- *Fuzzy rules*
- *Fuzzy inference mechanism*

Fuzzy Set

- A fuzzy set can be defined as a set with **fuzzy boundaries**.
- Example: For person's "height", the fuzzy set can be defined as: {'short', 'average', 'tall'}
- To represent a fuzzy set in a computer, we express it as **a function** and then **map** the elements of the set to their **degree of membership**.

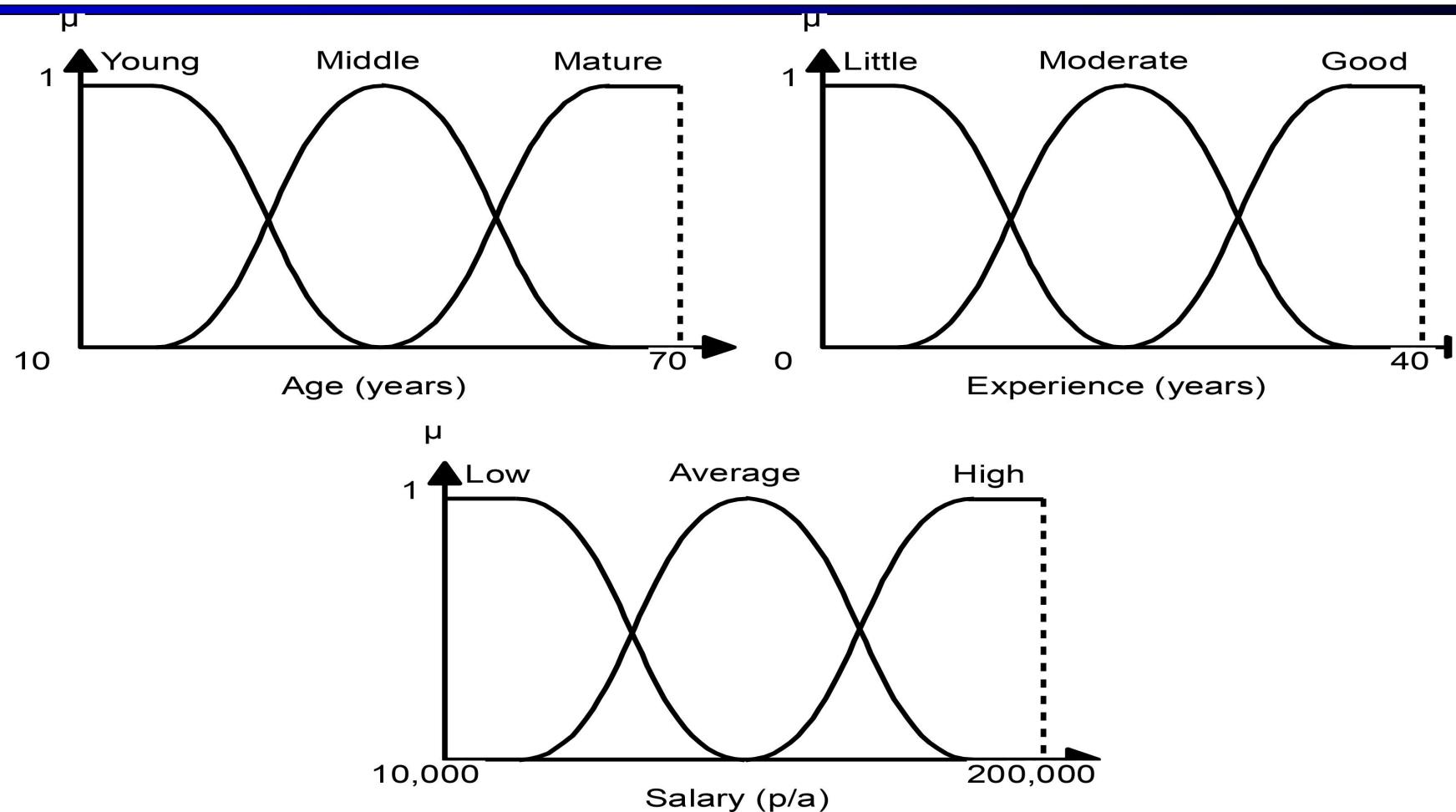


Fuzzy Systems

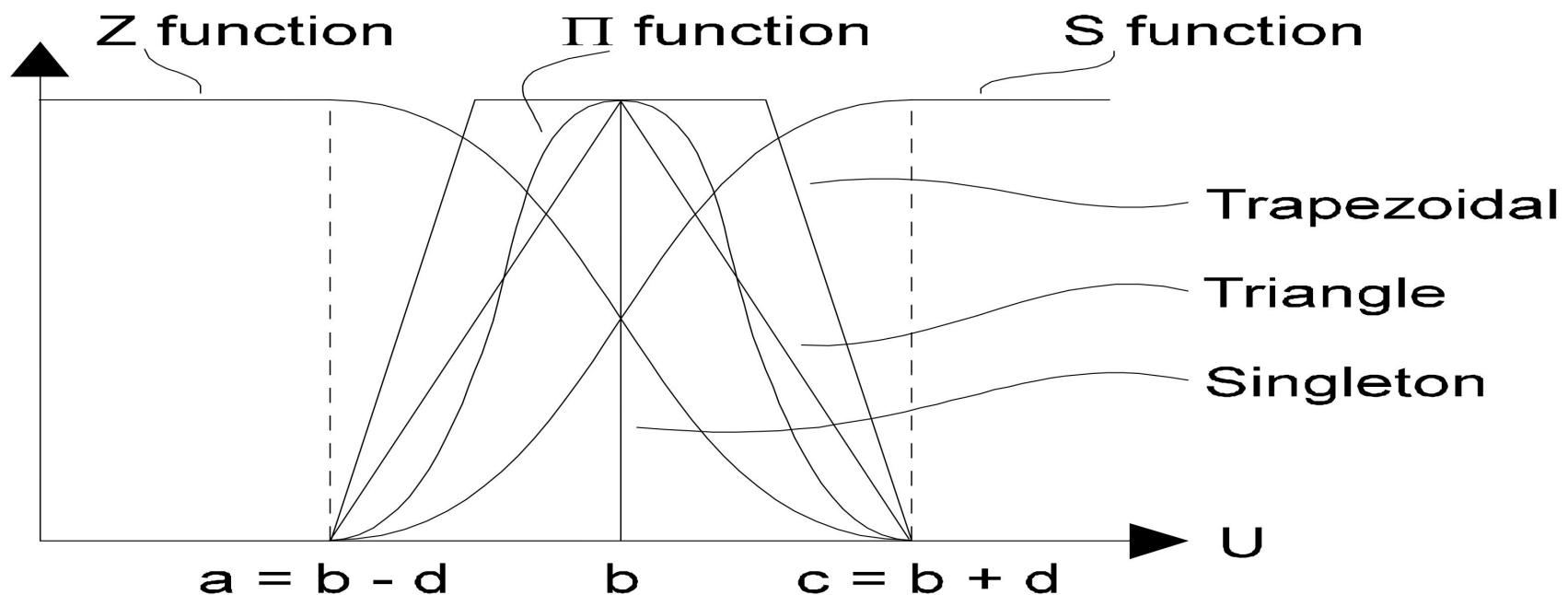


- *Representing crisp and fuzzy sets as subsets of a domain (universe of discourse) U .*

Membership Functions



Standard Membership Functions

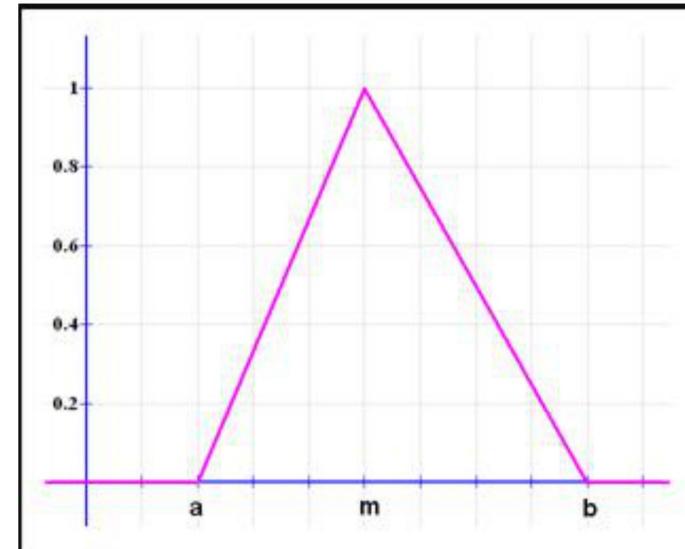


- *Most commonly used shapes of membership functions: triangular, trapezoidal, piecewise linear, and Gaussian.*

Standard Membership Functions

Triangular function: defined by a lower limit **a**, an upper limit **b**, and a value **m**, where **a < m < b**.

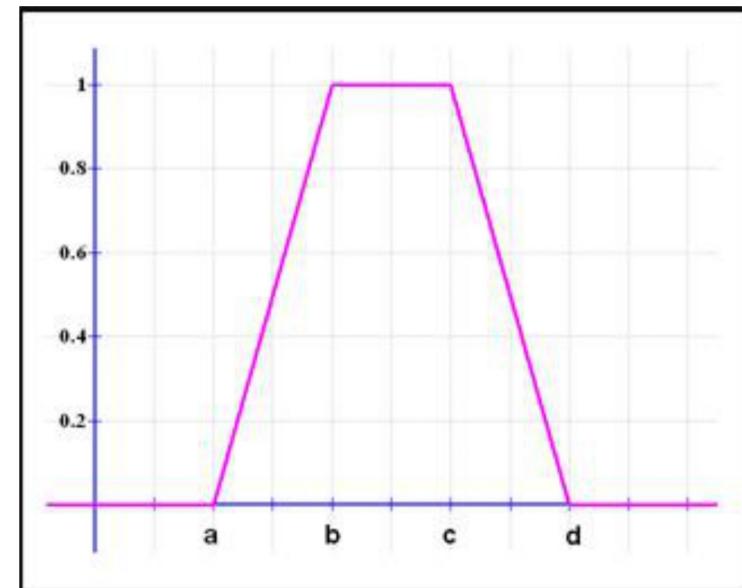
$$\mu_A(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{m-a}, & a < x \leq m \\ \frac{b-x}{b-m}, & m < x < b \\ 0, & x \geq b \end{cases}$$



Standard Membership Functions

Trapezoidal function: defined by a lower limit a , an upper limit d , a lower support limit b , and an upper support limit c , where $a < b < c < d$.

$$\mu_A(x) = \begin{cases} 0, & (x < a) \text{ or } (x > d) \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \end{cases}$$

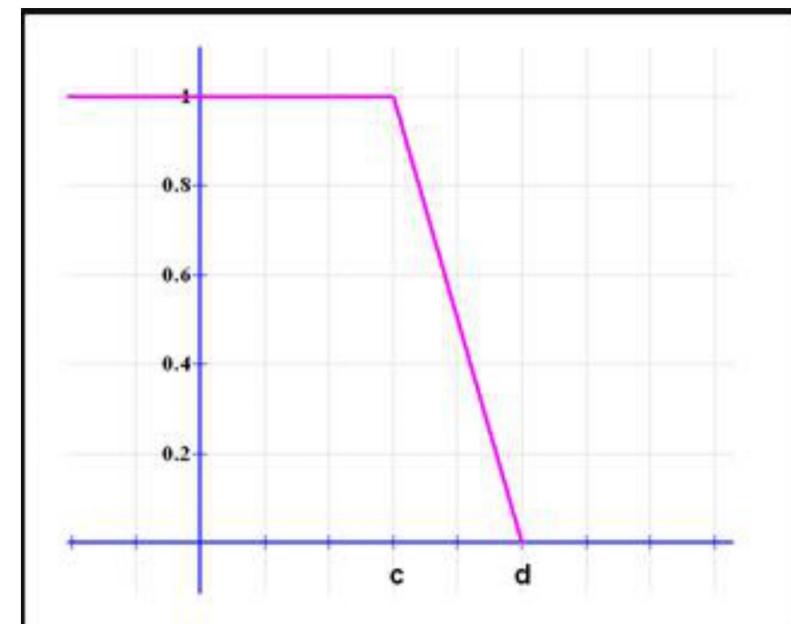


Standard Membership Functions

There are two special cases of a trapezoidal function, which are called R-functions and L-functions:

R-functions: with parameters $a = b = -\infty$

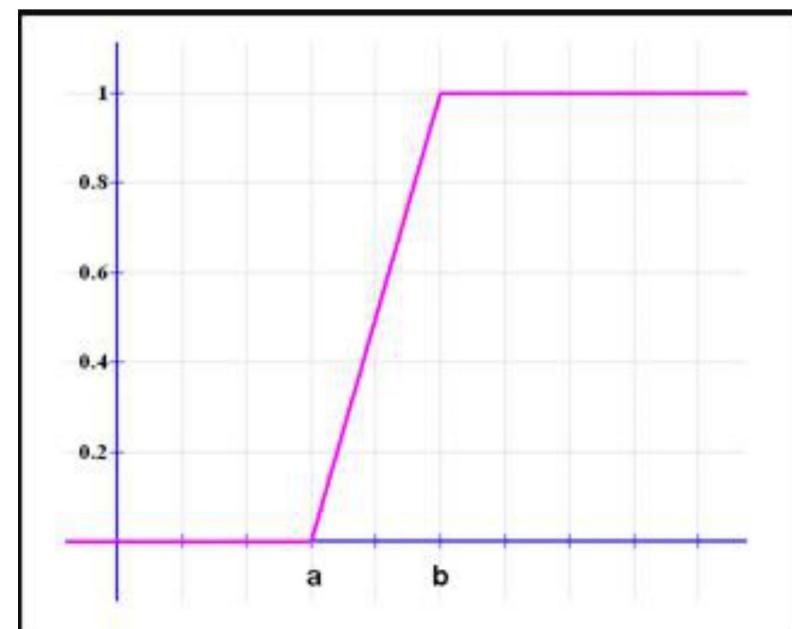
$$\mu_A(x) = \begin{cases} 0, & x > d \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 1, & x < c \end{cases}$$



Standard Membership Functions

L-Functions: with parameters $c = d = +\infty$

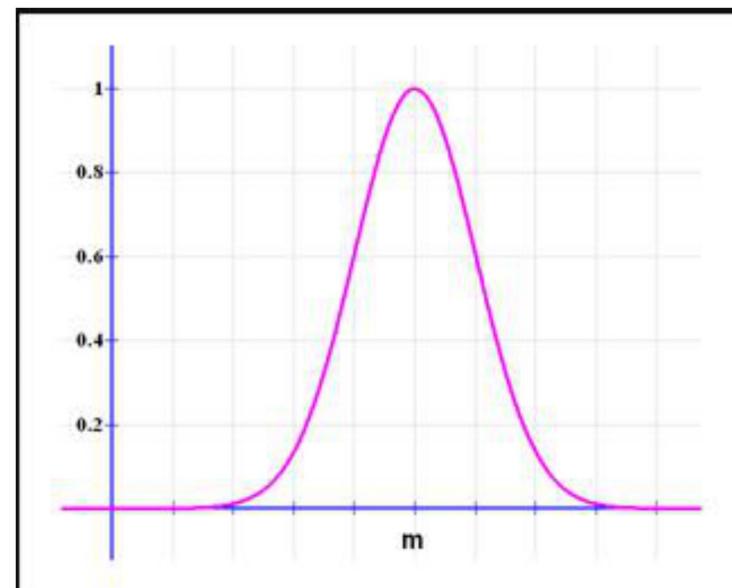
$$\mu_A(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b \end{cases}$$



Standard Membership Functions

Gaussian function: defined by a central value m and a standard deviation $k > 0$. The smaller k is, the narrower the “bell” is.

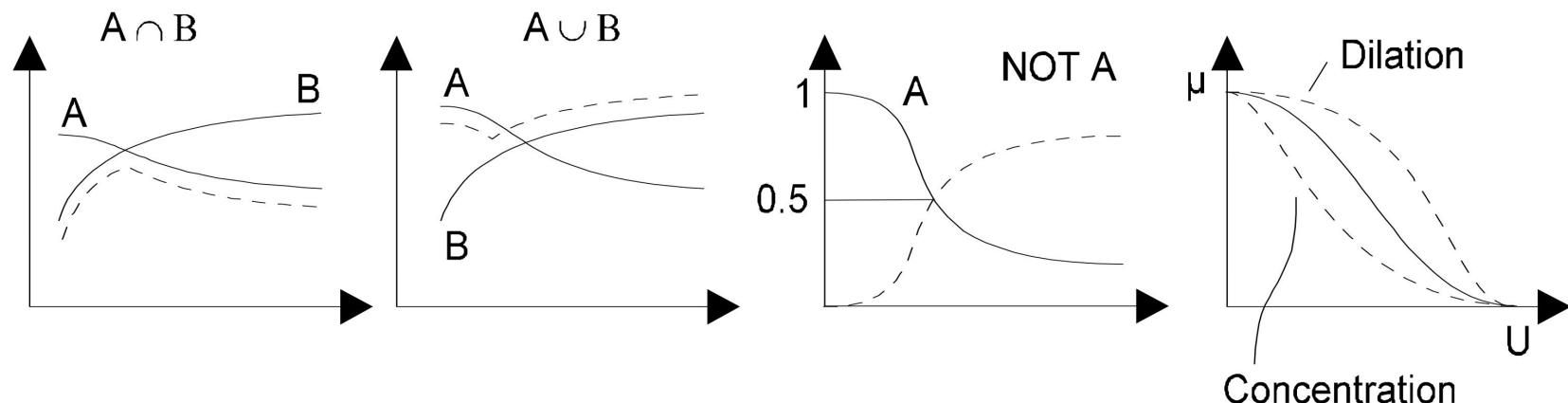
$$\mu_A(x) = e^{-\frac{(x-m)^2}{2k^2}}$$



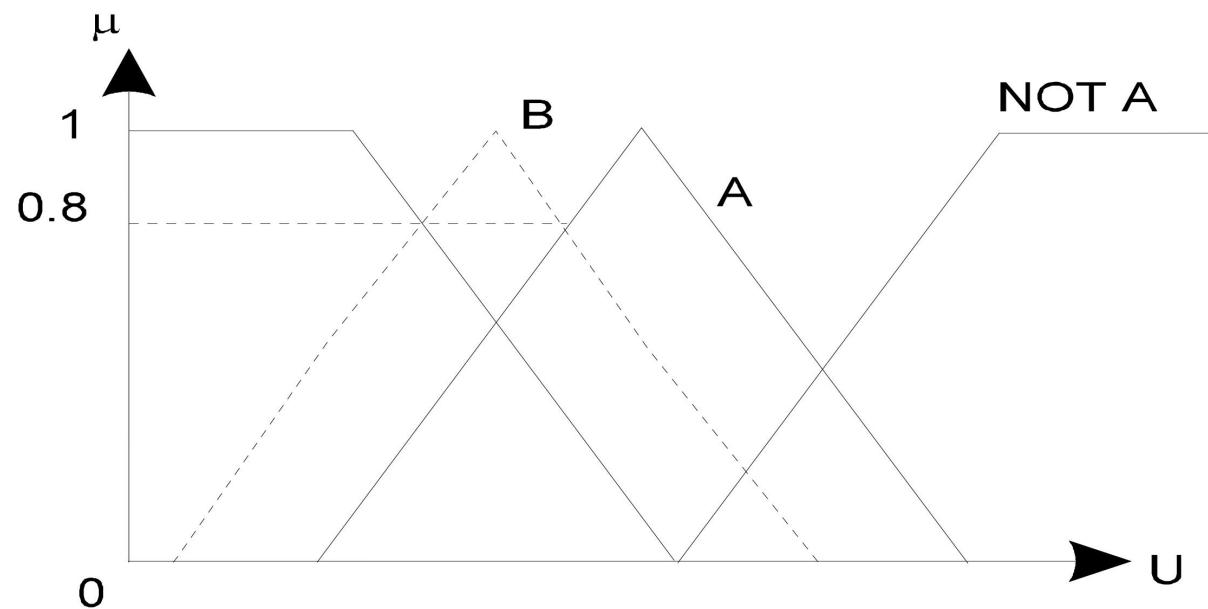
Operations on Fuzzy sets

- Operations on fuzzy sets is done by means of their membership functions. Let A and B be fuzzy sets on a mutual universe.

1. Intersection of A and B is:	$A \cap B =$	= $a \min b$, corresponding items in a and b .
2. Union of A and B is:	$A \cup B =$	= $a \max b$, corresponding items in a and b .
3. Complement of A is: $A =$		= $1 - a$
4. Dilation (increases the degree of membership of all members by spreading out the curve) of A is:		= DIL (A) = $(u_A(x)^{1/n})$ for all x in U
5. Concentration (decreases the degree of membership of all members) of A is:		= CON (A) = $(u_A(x)^n)$ for all x in U



Operations with Fuzzy Sets...



- A graphical representation of **similarity S**, between **two fuzzy sets B and A** based on possibility P and necessity N measures (see the formulas in the text).

Linguistic Variables and Hedges

- A *linguistic variable* is a *fuzzy variable*. A linguistic variable takes words or sentences as values.
- For example, the fuzzy variable ‘temperature’, might have values as ‘hot’, ‘medium’, and ‘cold’.
- **Example1:** Let x be a *linguistic variable* with the label *age*. Terms of this linguistic variables, which are fuzzy sets, could be: “old”, “young”, “very old” from the term set
- $T=\{Old, VeryOld, NotSoOld, MoreOrLessOld, QuiteOld, Young, VeryYoung, NotSoYoung, etc\}$
- If *wind* is strong Then sailing is *good*
- If *project duration* is long Then completion risk is *low*
- If *speed* is slow Then stopping distance is *short*

Linguistic Variables and Hedges

- A linguistic **hedge** or modifier is an operation that **modifies** the meaning of a term or fuzzy set.
 - Hedges are **some word** that **modify** the linguistic variable of a fuzzy set.
 - For example, if **weak pressure** is a **fuzzy set**, then:
 - {**very weak pressure**, **more-or-less weak pressure**, **extremely weak pressure**, and **not so-weak pressure**}
- are examples of hedges which are applied to this fuzzy set.

Fuzzy Modifier..

- A linguistic modifier is an operation that modifies the meaning of a term. For example, in the sentence, **very close to 0**, the word **very** modifies **Close to 0** which is a fuzzy set. Examples of other modifiers are a little, more or less, possibly, definitely.

$$\text{Very } a = a^2$$

$$\text{Extremely } a = a^3$$

$$\text{Very Very } a = a^4$$

$$\text{Somewhat } a = \text{Morl } a = a^{1/2}$$

$$\text{Little bit } a = a^{1.3} \quad \text{Slightly } a = a^{1/3}$$

- Young = [1 0.36 0.01 0 0]
- Very very young = young⁴ = [1 0.13 0 0 0]

Normalization:

- A fuzzy set is normalized if its **largest membership** value equals 1.
- A fuzzy set can be normalized by **dividing** each membership value by the **largest** membership value in the set, $a/\max(a)$.

Psychological Continuum

- *Very young, young, not so young, old, very old, not so old* ----->
derived from young and old.
- *Pressure* ----->*Strong, low, Okay, high (100-2000 Pascal)*
- *Bitter, sweet, sour, salt, hot,*

Fuzzy systems...

- A simple *fuzzy rule* for the smoker and the risk of cancer case example.

Rule1 IF a person is a "heavy_smoker"
 THEN the risk of cancer is "high",

where the two fuzzy concepts "heavy-smoker" and "high" can be represented by their membership functions, for example:

A fuzzy concept "heavy-smoker":

No.of cigarettes per day	0	2	4	6	8	10
Grade (membership)	0	0.1	0.6	0.8	0.9	1.0

A fuzzy concept: "High risk of cancer"

Level of risk	1	2	3	4	5
Grade (membership)	0.0	0.2	0.7	0.9	1.0

Fuzzy Inference and Rules

Fuzzy Inference: The process of reasoning based on Fuzzy logic.

Example:

If the power transformer is **slightly** overloaded,
Then keep this load for a while.

Fuzzy rule: A conditional statement in the form:

IF x is A THEN y is B, where x and y are linguistic variables, and A and B are linguistic values determined by fuzzy sets.

- **Rule 1:** *IF (GPA is high) and (Exam is satisfactory) and (Approach is Smart)
then (Decision is Select)*
- **Rule 2:** *IF (GPA is low) and (Exam is bad) or (Approach is Stupid)
then (Decision is Reject)*

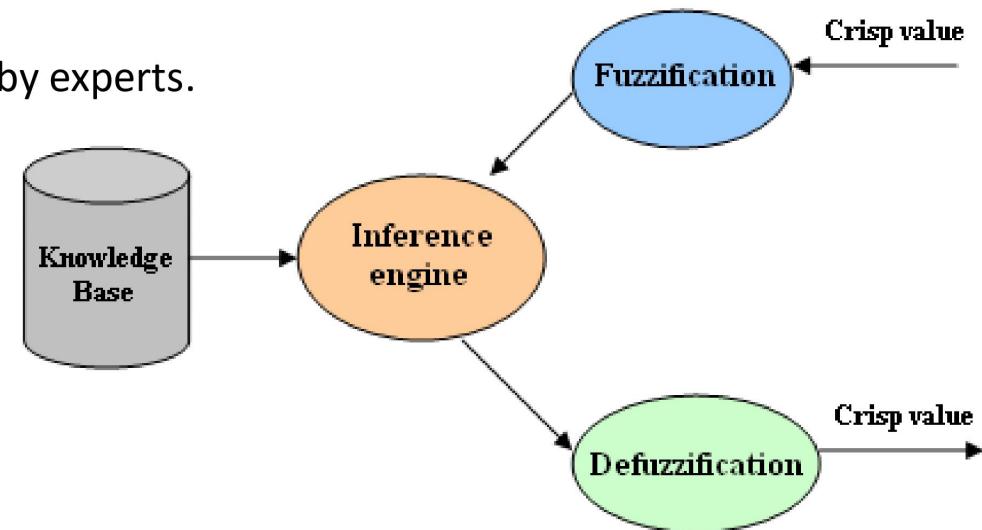
Structure of Fuzzy Inference System

In general, a fuzzy inference system consists of four modules:

Fuzzification Module: Transforms the system inputs, which are crisp numbers, into fuzzy sets.

This is done by applying a fuzzification function.

Knowledge Base: (KB) Stores IF-THEN rules provided by experts.



Inference Engine: Simulates the human reasoning process by making fuzzy inference on the inputs and IF-THEN rules.

Defuzzification Module: Transforms the fuzzy set obtained by the inference engine into a crisp value.

Fuzzy Rules and Fuzzy Inference

- *Inputs to a fuzzy system can be:*
 - fuzzy, e.g. (Score = Moderate), defined by membership functions
 - exact, e.g.: (Score = 190); (Theta = 35), defined by crisp values.
- *Outputs from a fuzzy system can be:*
 - fuzzy, i.e. a whole membership function, or
 - exact, i.e. a single value is produced on the output.

Mamdani-Style Inference

Four steps: fuzzification, rule evaluation, aggregation of the rules, defuzzification

Rule 1:

IF x is A3
OR y is B1
THEN z is C1

Rule 1:

IF project_funding is adequate
OR project_staffing is small
THEN risk is low

Rule 2:

IF x is A2
AND y is B2
THEN z is C2

Rule 2:

IF project_funding is marginal
AND project_staffing is large
THEN risk is normal

Rule 3:

IF x is A1
THEN z is C3

Rule 3:

IF project_funding is inadequate
THEN risk is high

Fuzzy Expert System

- An expert system that uses fuzzy logic instead of Boolean logic.
- A fuzzy expert system is a collection of fuzzy rules and membership functions that are used to reason about data.

Problem

- A 4-person family wants to buy a house. An indication of how **comfortable** they want to be is the **no. of bedrooms** in the house. But they also want a **large** house.

Let $u=\{1,2,3,4,5,6,7,8,9,10\}$ be the set of available houses described by their number of bedrooms. Then the fuzzy set C (for comfortable) may be described as

$$C=[0.2 \ 0.5 \ 0.8 \ 1 \ 0.7 \ 0.3 \ 0 \ 0 \ 0 \ 0]$$

Let, L be the fuzzy set large defined as:

$$L=[0 \ 0 \ 0.2 \ 0.4 \ 0.6 \ 0.3 \ 1 \ 1 \ 1 \ 1]$$

The intersection of Comfortable and Large is then

$$=[0 \ 0 \ 0.2 \ 0.4 \ 0.6 \ 0.3 \ 0 \ 0 \ 0 \ 0]$$

To interpret this, five bedrooms is optimal, but only satisfactory to the grade of 0.6. The second best solution is four bedrooms.

The union of Comfortable and Large is:

$$[0.2 \ 0.5 \ 0.8 \ 1 \ 0.7 \ 0.8 \ 1 \ 1 \ 1 \ 1]$$

Here, four bedrooms is fully satisfactory because it is comfortable, and 7-10 bedrooms also, because that would mean a large house.

These properties are important, because they help to predict the outcome of long sentences.

Applications of Fuzzy Logic

Control Applications: Aircraft control, Sendai subway operation (Hitachi), Cruise control (Nissan), Automatic Transmission (Nissan, Subaru), Self parking model car (Tokyo University), Space Shuttle docking (NASA)

Scheduling and Optimization:

Elevator Scheduling (Hitachi, Fujitsu, Mitsubishi)

Stock Market Analysis: Yamichi Securities

Signal Analysis for Tuning and Interpretation:

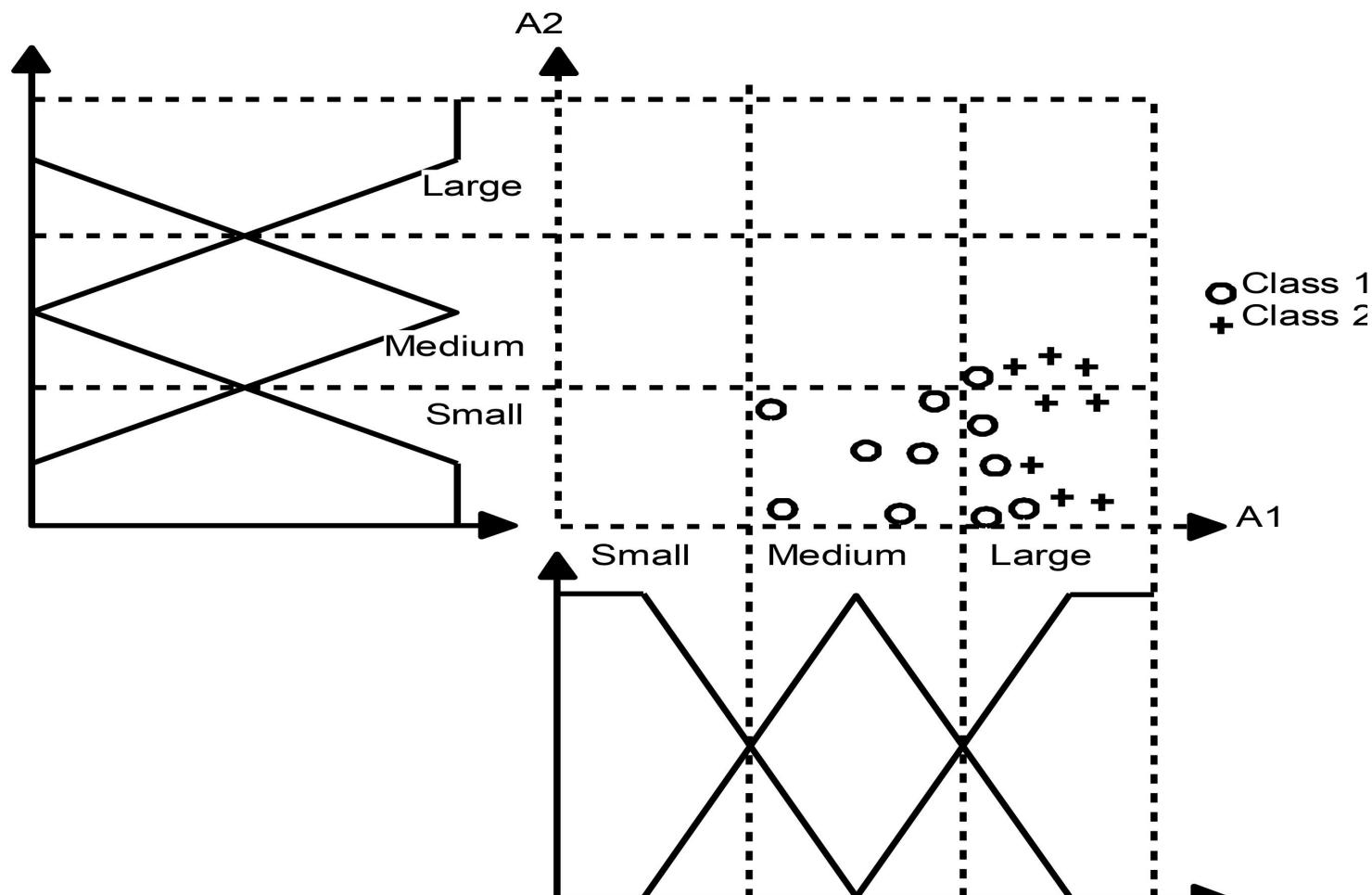
TV picture adjustment (Sony Corporation)

Handwriting Recognition: Sony Palmtop computer

Video Camera Autofocus: Sony and Canon

Video Image Stabilizer: Matsushita, Panasonic

Pattern Recognition and Classification



Fuzzy System Applications

- Pattern recognition and classification
- Fuzzy clustering
- Image and speech processing
- Fuzzy systems for prediction
- Fuzzy control
- Monitoring
- Diagnosis
- Optimization and decision making
- Group decision making

Acknowledgement

- AIMA = Artificial Intelligence: A Modern Approach by Stuart Russell and Peter Norving (3rd edition)
- U of toronto
- Other online resources

Thank You