1. In the following quiz, you will practice using the chain rule, which allows us to differentiate functions of functions. We saw in the previous videos that for two functions $g(h)$ and $h(x)$, the derivative of g with respect to x is given by $\frac{dg}{dx} = \frac{dg}{dh}\frac{dh}{dx}$, where the two derivatives on the right hand side are in some way 'chained together'.

If $f(x) = g(h(x))$, which of the following is the equivalent to the chain rule in the $f'(x)$ notation?

- ○ $f'(x) = g'(h'(x))h'(x)$
- ○ $f'(x) = g'(h(x))$
- ◉ $f'(x) = g'(h(x))h'(x)$
- ○ $f'(x) = g'(h'(x))$

Correct
This is the chain rule, although it may be a little less clear that the derivatives are 'chained together'.

2. Much like the product rule, the art of the chain rule lies in identifying the components of the function that allow you to apply the rule.

Consider the function $f(x) = e^{x^2-3}$. We can break up $f(x)$ by writing $g(h) = e^h$ and $h(x) = x^2 - 3$. Now $f(x) = g(h(x))$

Use the chain rule to calculate $f'(x) = \frac{dg}{dx}$.

- ○ $f'(x) = (x^2 - 3)e^{x^2-3}$
- ○ $f'(x) = 2e^{x^2-3}$
- ◉ $f'(x) = 2xe^{x^2-3}$
- ○ $f'(x) = e^{x^2-3}$

Correct
$f'(x) = \frac{dg}{dh}\frac{dh}{dx}$.

3. Use this same process to identify the functions that make up $f(x) = \sin^3(x)$, and calculate $f'(x)$.

- ○ $f'(x) = 3\cos^2(x)$
- ○ $f'(x) = \cos^3(x)$
- ○ $f'(x) = 3\sin^2(x)$
- ◉ $f'(x) = 3\sin^2(x)\cos(x)$

Correct
You correctly identified that the functions are $g(h) = h^3$ and $h(x) = \sin(x)$, then applied the chain rule successfully. You can check that this gives the same answer as applying the product rule to $f(x) = \sin(x)\sin(x)\sin(x)$.

4.

Now you will tame a beast of your own by calculating the derivative of $\tan(x)$ with respect to x. You will need to use both the product rule and the chain rule.

If you want to try it on your own you can ignore the following hints, but they might be useful if you'd like some help starting.

Hint 1: The first step is to use the trigonometric relation $\tan(x) = \frac{\sin(x)}{\cos(x)}$.

Hint 2: The next step is to remember that $\frac{1}{\cos(x)} = [\cos(x)]^{-1}$. Now try to identify the functions that will allow you to calculate $\frac{d}{dx}\left(\frac{1}{\cos(x)}\right)$.

- ○ $\frac{d}{dx}\tan(x) = 1 - \tan^2(x)$
- ○ $\frac{d}{dx}\tan(x) = \tan^2(x)$
- ○ $\frac{d}{dx}\tan(x) = 0$
- ◉ $\frac{d}{dx}\tan(x) = 1 + \tan^2(x)$
- ○ $\frac{d}{dx}\tan(x) = 1 - \frac{\sin(x)}{\cos^2(x)}$

Correct
This is a nice example of how the chain rule and product rule can be very useful. It turns out that $1 + \tan^2(x) = \frac{1}{\cos^2(x)}$ through trig relations.

5.

The chain rule can also be applied to functions of functions of functions. Consider a function $f(g(h(x)))$. By applying the chain rule twice, it's possible to show that $\frac{df}{dx} = \frac{df}{dg}\frac{dg}{dh}\frac{dh}{dx}$. Notice how the derivatives chain together! If you'd like you can try to show this result yourself.

Use this information to find the derivative of $f(x) = e^{\sin(x^2)}$ with respect to x. Remember to write down the appropriate functions $f(g), g(h)$ and $h(x)$ to get started.

- ( ) $f'(x) = 2e^{\sin(x^2)}\sin(x)\cos(x)$
- ( ) $f'(x) = 2xe^{\sin(x^2)}$
- (•) $f'(x) = 2xe^{\sin(x^2)}\cos(x^2)$
- ( ) $f'(x) = e^{\sin(x^2)}\cos(x^2)$

Correct

You identified that $f(g) = e^g, g(h) = \sin(h)$ and $h(x) = x^2$, and applied the chain rule correctly. You might see how the chain rule can be applied to functions which are even more deeply nested by applying the chain rule over and over.