

1. In this quiz, you will practice doing partial differentiation, and calculating the total derivative. As you've seen in the videos, partial differentiation involves treating every parameter and variable that you aren't differentiating by as if it were a constant.

Keep in mind that it might be faster to eliminate multiple choice options that can't be correct, rather than performing every calculation.

Given  $f(x, y) = \pi x^3 + xy^2 + my^4$ , with  $m$  some parameter, what are the partial derivatives of  $f(x, y)$  with respect to  $x$  and  $y$ ?

☐  $\frac{\partial f}{\partial x} = 3\pi x^3 + y^2,$

$\frac{\partial f}{\partial y} = 2xy^2 + 4my^4$

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2. Given  $f(x, y, z) = x^2y + y^2z + z^2x$ , what are  $\frac{\partial f}{\partial x}$ ,  $\frac{\partial f}{\partial y}$  and  $\frac{\partial f}{\partial z}$ ?

☐  $\frac{\partial f}{\partial x} = 3xyz,$

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☐  $\frac{\partial f}{\partial x} = 2xy + y^2z + z^2x,$

$$\frac{\partial f}{\partial y} = x^2 + 2yz + z^2x$$

$$\frac{\partial f}{\partial z} = x^2y + y^2 + 2zx$$

☐  $\frac{\partial f}{\partial x} = xy + z^2,$

$$\frac{\partial f}{\partial y} = x^2 + yz$$

$$\frac{\partial f}{\partial z} = y^2 + zx$$

☐  $\frac{\partial f}{\partial x} = 2xy + z^2,$

$$\frac{\partial f}{\partial y} = x^2 + 2yz$$

$$\frac{\partial f}{\partial z} = y^2 + 2zx$$

3. Given  $f(x, y, z) = e^{2x} \sin(y)z^2 + \cos(z)e^x e^y$ , what are  $\frac{\partial f}{\partial x}$ ,  $\frac{\partial f}{\partial y}$  and  $\frac{\partial f}{\partial z}$ ?

☐  $\frac{\partial f}{\partial x} = 4e^{2x} \cos(y)z - \sin(z)e^x e^y,$

$\frac{\partial f}{\partial y} = 4e^{2x} \cos(y)z - \sin(z)e^x e^y$

$\frac{\partial f}{\partial z} = 4e^{2x} \cos(y)z - \sin(z)e^x e^y$

☐  $\frac{\partial f}{\partial x} = 2e^{2x} \sin(y)z^2 + \cos(z)e^x e^y,$

$\frac{\partial f}{\partial y} = e^{2x} \cos(y)z^2 + \cos(z)e^x e^y$

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$\frac{\partial f}{\partial z} = 2e^{2x} \sin(y)z - \sin(z)e^x e^y$

4. Recall the formula for the total derivative, that is, for  $f(x, y)$ ,  $x = x(t)$  and  $y = y(t)$ , one can calculate  $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$ .

Given that  $f(x, y) = \frac{\sqrt{x}}{y}$ ,  $x(t) = t$ , and  $y(t) = \sin(t)$ , calculate the total derivative  $\frac{df}{dt}$ .

☐  $\frac{df}{dt} = \frac{1}{2\sqrt{t} \sin(t)} + \frac{\sqrt{t} \cos(t)}{\sin(t)}$

☐  $\frac{df}{dt} = -\frac{1}{\sqrt{t} \sin(t)} - \frac{\sqrt{t} \cos(t)}{\sin^2(t)}$

☐  $\frac{df}{dt} = \frac{1}{2\sqrt{t} \sin(t)} - \frac{\sqrt{t} \cos(t)}{\sin^2(t)}$

☐  $\frac{df}{dt} = \frac{1}{2\sqrt{t} \sin(t)} - \frac{\sqrt{t}}{\sin^2(t)}$

5. Recall the formula for the total derivative, that is, for  $f(x, y, z)$ ,  $x = x(t)$ ,  $y = y(t)$  and  $z = z(t)$ , one can calculate  $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$ .

Given that  $f(x, y, z) = \cos(x) \sin(y) e^{2z}$ ,  $x(t) = t + 1$ ,  $y(t) = t - 1$ ,  $z(t) = t^2$ , calculate the total derivative  $\frac{df}{dt}$ .

☐

$$\frac{df}{dt} = [-(t+1) \sin(t+1) \sin(t-1) + (t-1) \cos(t+1) \cos(t-1) + 4t \cos(t+1) \sin(t-1)] e^{2t^2}$$

☐

$$\frac{df}{dt} = [-\sin(t+1) \sin(t-1) + \cos(t+1) \cos(t-1) + 4t \cos(t+1) \sin(t-1)] e^{2t^2}$$

☐

$$\frac{df}{dt} = [-\sin(t+1) \sin(t-1) + \cos(t+1) \cos(t-1) + 2 \cos(t+1) \sin(t-1)] e^{2t^2}$$

☐

$$\frac{df}{dt} = [\cos(t+1) \sin(t-1) + \cos(t+1) \cos(t-1) + 4t \cos(t+1) \sin(t-1)] e^{2t^2}$$