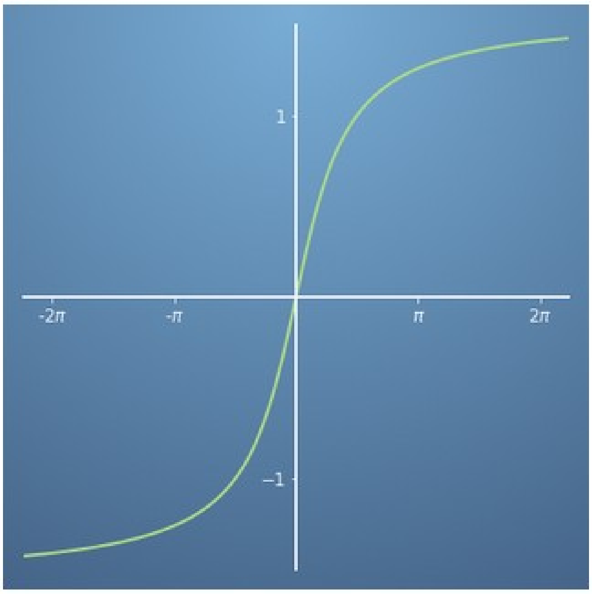
Congratulations! You passed!

Grade received 100% **To pass** 80% or higher

Go to next item

1. The graph below shows the function $f(x) = \tan^{-1}(x)$



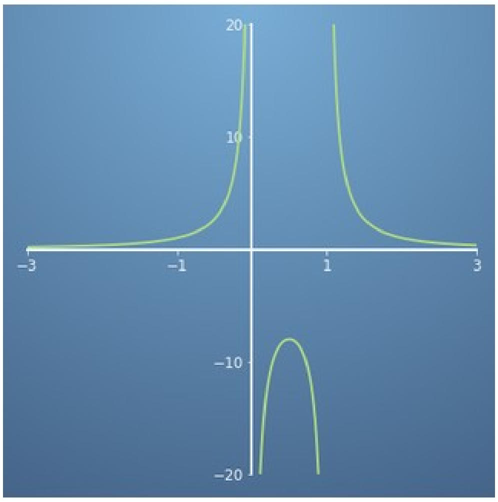
By using the Maclaurin series or otherwise, determine whether the function shown above is even, odd or neither.

- (●) Odd
- ( ) Even
- ( ) Neither odd nor even

Correct
For an odd function, $-f(x) = f(-x)$. We can also determine if a function is odd by looking at its symmetry. If it has rotational symmetry with respect to the origin, it is an odd function.

2. The graph below shows the discontinuous function $f(x) = \frac{2}{(x^2 - x)}$. For this function, select the starting points that will allow a Taylor approximation to be made.

1 / 1 point



- [x] $x = 0.5$

Correct
A Taylor approximation centered at $x = 0.5$ will allow us to approximate $f(x)$ for $0 < x < 1$ only.

- [x] $x = 2$

Correct
A Taylor approximation centered at $x = 2$ will allow us to approximate $f(x)$ for $x > 1$only.

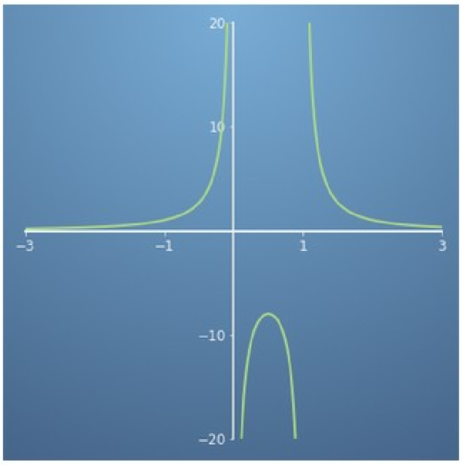
- [x] $x = -3$

Correct
A Taylor approximation centered at $x = -3$ will allow us to approximate $f(x)$ for $x < 0$ only.

- [ ] $x = 1$

3. For the same function as previously discussed, $f(x) = \frac{2}{(x^2 - x)}$, select all of the statements that are true about the resulting Taylor approximation.

1 / 1 point



- [ ] Approximation accurately captures the asymptotes
- [x] Approximation ignores segments of the function

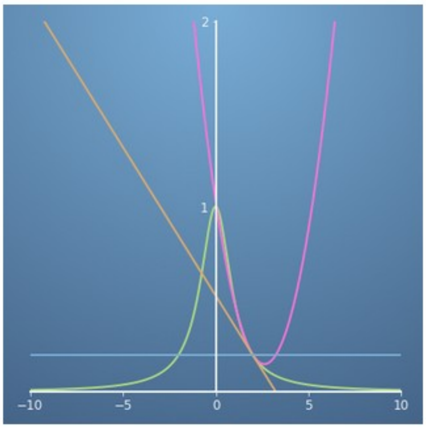
Correct
Due to the discontinues function and the range of x values in which it remains well behaved, the starting point of the Taylor series dictates the domain of the function we are trying to approximate.

- [ ] The approximation converges quickly
- [ ] This is a well behaved function
- [x] Approximation ignores the asymptotes

Correct
Taylor series approximations often find it difficult to capture asymptotes correctly. For example, the zeroth and first order terms cut directly through an asymptote in most cases.

4. The graph below highlights the function $f(x) = \frac{1}{(1+x^2)}$ (green line), with the Taylor expansions for the first 3 terms also shown about the point $x = 2$. The Taylor expansion is $f(x) = \frac{1}{5} - \frac{4(x-2)}{25} + \frac{11(x-2)^2}{125} + \ldots$. Although the function looks rather normal, we find that the Taylor series does a bad approximation further from its starting point, not capturing the turning point. What could be the reason why this approximation is poor for the function described.

0.8 / 1 point



- [ ] The function has no real roots
- [x] It is a discontinuous function in the complex plane

Correct
Although this function is well behaved in the real plane, in the imaginary plane, the asymptotes limit its convergence and the behaviour of the Taylor expansion, which is shown to behave badly for functions that are discontinuous.

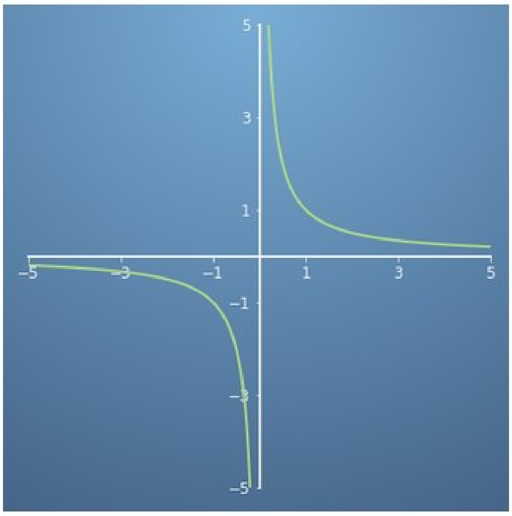
- [x] Asymptotes are in the complex plane

Correct
Although this function is well behaved in the real plane, in the imaginary plane, the asymptotes limit its convergence and the behaviour of the Taylor expansion, which is shown to behave badly for functions that are discontinuous.

- [ ] Function does not differentiate well
- [ ] None of these options

5. For the function $f(x) = \frac{1}{x}$, provide the linear approximation about the point $x = 4$, ensuring it is second order accurate.

1 / 1 point



- ○ $f(x) = 1/4 - x/16 + O(\boldsymbol{\Delta} x^2)$
- ○ $f(x) = 1/4 - (x-4)/16 + O(\boldsymbol{\Delta} x)$
- ○ $f(x) = 1/4 + x/16 - O(\boldsymbol{\Delta} x^2)$
- ◉ $f(x) = 1/4 - (x-4)/16 + O(\boldsymbol{\Delta} x^2)$

Correct
Second order accurate means we have a first order Taylor series. All the terms above are sufficiently small, assuming $\boldsymbol{\Delta} x$ is small.