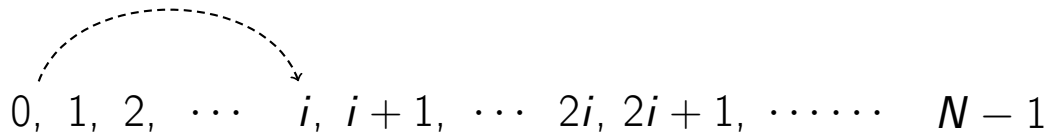# I/O-Efficient Algorithms and Data Structures

Steffan Christ Sølvsten
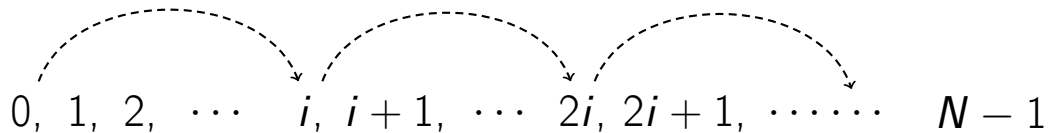
8th of September, 2023
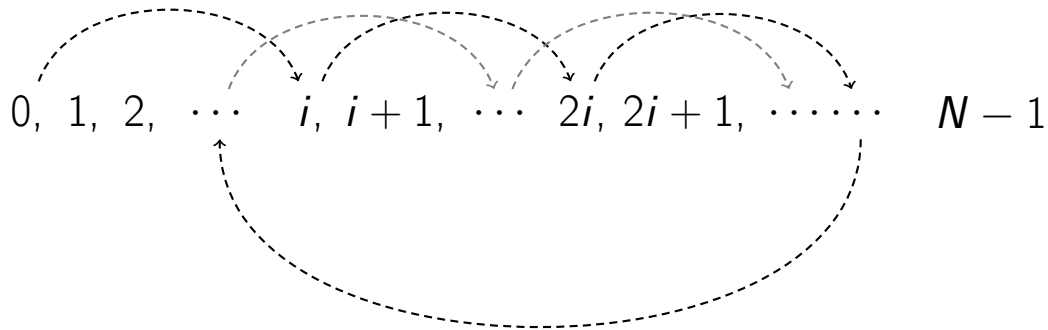
AARHUS
UNIVERSITY

$$0, \; 1, \; 2, \; \cdots \quad i, \; i+1, \; \cdots \quad 2i, \; 2i+1, \; \cdots\cdots \quad N-1$$

$$0,\ 1,\ 2,\ \cdots\quad i,\ i+1,\ \cdots\ 2i,\ 2i+1,\ \cdots\cdots\quad N-1$$

$$0, \; 1, \; 2, \; \cdots \quad i, \; i+1, \; \cdots \; 2i, \; 2i+1, \; \cdots\cdots \quad N-1$$

$$0, \ 1, \ 2, \ \cdots \quad i, \ i+1, \ \cdots \ 2i, \ 2i+1, \ \cdots\cdots \quad N-1$$

$0, \ 1, \ 2, \ \cdots \quad i, \ i+1, \ \cdots \ 2i, \ 2i+1, \ \cdots\cdots \quad N-1$
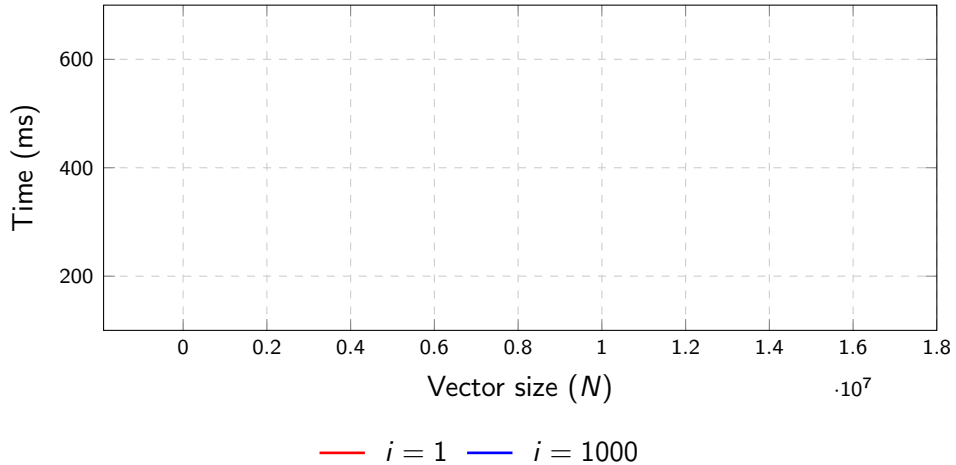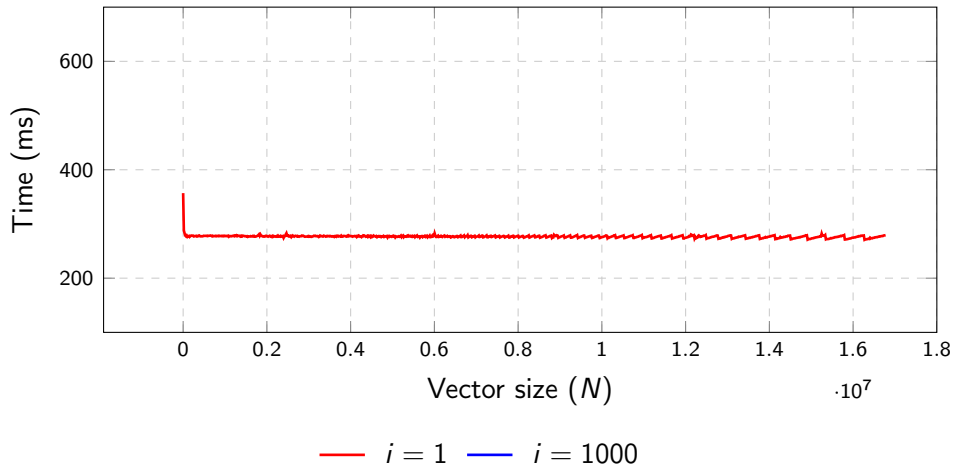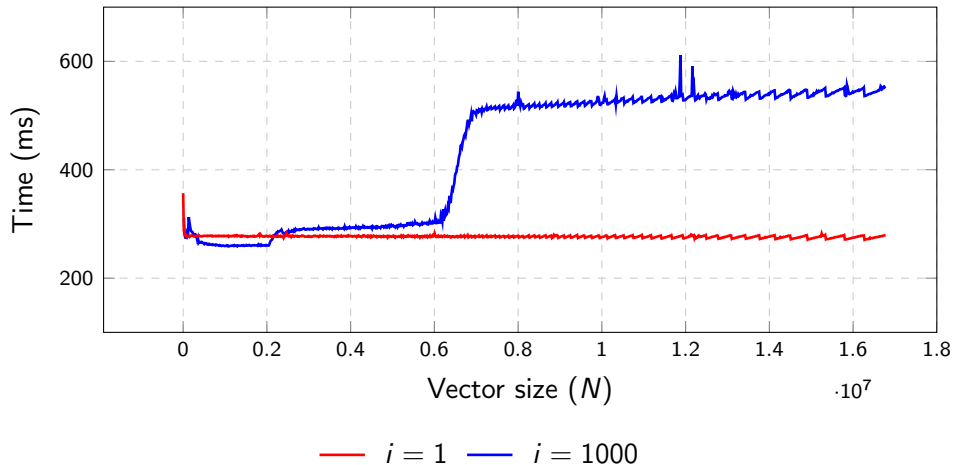
I/O Model
Aggarwal and Vitter '87

I/O Model : Sequential Access
Aggarwal and Vitter '87

a, b, c	d, e, f	$\cdots$	x, y, z

I/O Model : Sequential Access
Aggarwal and Vitter '87

I/O Model : Sequential Access
Aggarwal and Vitter '87

I/O Model : Sequential Access
Aggarwal and Vitter '87

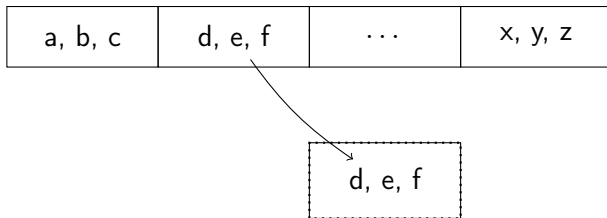| a, b, c	d, e, f	$\cdots$	x, y, z

I/O Model : Sequential Access
Aggarwal and Vitter '87

I/O Model : Sequential Access
Aggarwal and Vitter '87

a, b, c	d, e, f	$\cdots$	x, y, z

I/O Model : Sequential Access
Aggarwal and Vitter '87

a, b, c	d, e, f	$\cdots$	x, y, z

æ

I/O Model : Sequential Access
Aggarwal and Vitter '87

a, b, c	d, e, f	$\cdots$	x, y, z

æ, ø

I/O Model : Sequential Access
Aggarwal and Vitter '87

a, b, c	d, e, f	$\cdots$	x, y, z

æ, ø, å

I/O Model : Sequential Access
Aggarwal and Vitter '87

I/O Model : Sequential Access
Aggarwal and Vitter '87

a, b, c	d, e, f	$\cdots$	x, y, z	æ, ø, å

Time	:	N
I/O	:	N/B
Memory	:	B

I/O Model : Stack

I/O Model : Stack

I/O Model : Stack

a, b

a, b, c

I/O Model : Stack

a, b, c	d

I/O Model : Stack

a, b, c	d, e

I/O Model : Stack

a, b, c	d, e, f

I/O Model : Stack

I/O Model : Stack

a, b, c

g | d, e, f

I/O Model : Stack

a, b, c

| g, h | d, e, f |

I/O Model : Stack

a, b, c

g, h, i | d, e, f

I/O Model : Stack

I/O Model : Stack

a, b, c	d, e, f

g, h, i	j

I/O Model : Stack

a, b, c	d, e, f

g, h, i	j, k

I/O Model : Stack

a, b, c	d, e, f

g, h, i	j

I/O Model : Stack

a, b, c	d, e, f

g, h, i	

a, b, c	d, e, f

g, h	

I/O Model : Stack

a, b, c	d, e, f

g	

I/O Model : Stack

a, b, c	d, e, f

I/O Model : Stack

I/O Model : Stack

a, b, c	d, e, f

	d, e

I/O Model : Stack

a, b, c	d, e, f

	d, e, f'

I/O Model : Stack

a, b, c	d, e, f

g'	d, e, f'
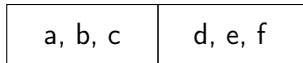
I/O Model : Stack

a, b, c	d, e, f

g', h'	d, e, f'

I/O Model : Stack

a, b, c	d, e, f

g', h', i'	d, e, f'

I/O Model : Stack

I/O Model : Stack

a, b, c	d, e, f'

g', h', i'	j'

I/O Model : Stack

a, b, c	d, e, f'

g', h', i'	j'

$$\begin{aligned} \text{Time} &: \quad O(N) \\ \text{I/O} &: \quad O(N/B) \\ \text{Memory} &: \quad 2B \end{aligned}$$

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

$M/B - 1$

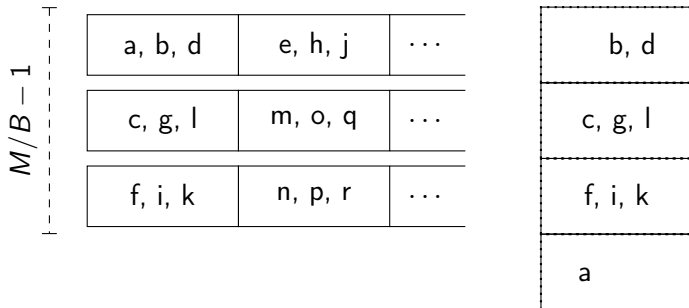| a, b, d	e, h, j	$\cdots$
c, g, l	m, o, q	$\cdots$
f, i, k	n, p, r	$\cdots$

d
c, g, l
f, i, k
a, b

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

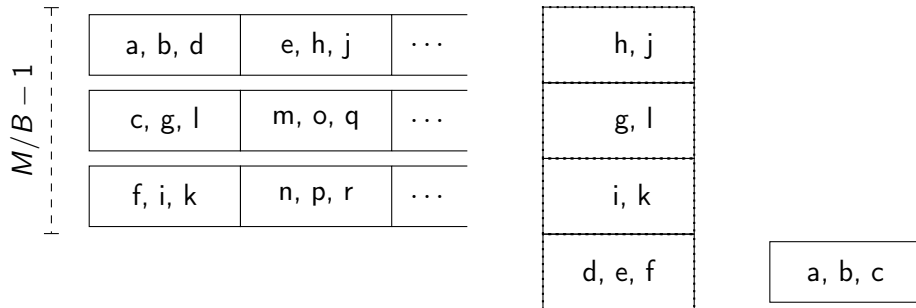# I/O Model : M/B-way Mergesort
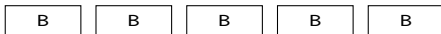## Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

| B |

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

B		B

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

| B | | B | | B |

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

B		B		B		B

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

| B | | B | | B | | B | | B |

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

| B | | B | | B | | B | | B | | B |

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

| B | | B | | B | | B | | B | | B | | $\cdots$ | | B |

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

| B | | B | | B | | B | | B | | B | $\cdots$ | B | | B |

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

N/B

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

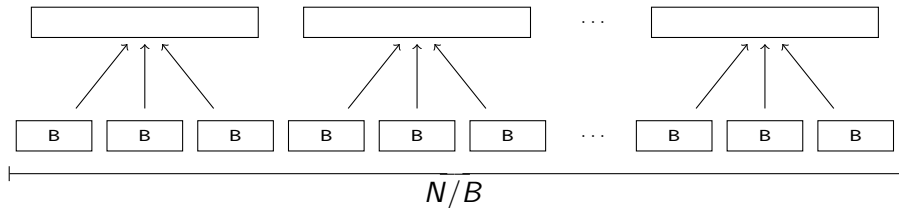# I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87

I/O Model : M/B-way Mergesort
Aggarwal and Vitter '87
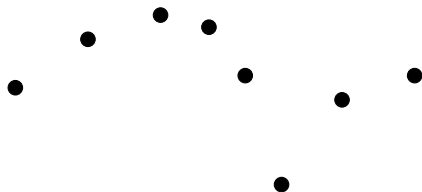
Theorem
N elements can be sorted in $\Theta(N/B \cdot \log_{M/B}(N/B))$ I/Os.

Convex Hull : Graham Scan
Graham '72

Convex Hull

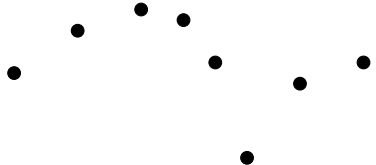Compute the *convex hull* for N points in the plane.

Theorem
Convex Hull can be computed in $O(N/B \cdot \log_{M/B}(N/B))$ I/Os.

Convex Hull : Graham Scan
Graham '72

Upper Hull:

Convex Hull : Graham Scan
Graham '72

Upper Hull:

- Sort input points by x-axis

Convex Hull : Graham Scan
Graham '72

Upper Hull:

- Sort input points by x-axis
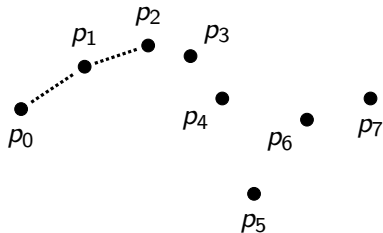- Initialize stack $S = [p_0, p_1]$

Convex Hull : Graham Scan
Graham '72

Upper Hull:

- Sort input points by x-axis

- Initialize stack $S = [p_0, p_1]$

- For remaining points $p_i \in p_2, p_3, \ldots, p_{N-1}$:

  1. Let p_s, p_t be the two top-most points of S
  2. While $p_s - p_t - p_i$ is a "left-turn":
     - Pop p_t and go-to 1
  3. Push p_i onto S

Convex Hull : Graham Scan
Graham '72

Upper Hull:

- Sort input points by x-axis

- Initialize stack $S = [p_0, p_1]$

- For remaining points $p_i \in p_2, p_3, \ldots, p_{N-1}$:

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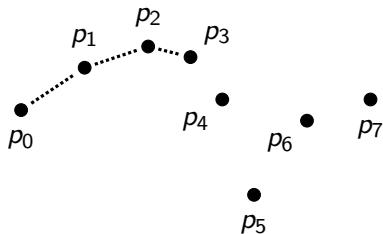 3. Push p_i onto S

Convex Hull : Graham Scan
Graham '72

Upper Hull:

- Sort input points by x-axis

- Initialize stack $S = [p_0, p_1]$

- For remaining points $p_i \in p_2, p_3, \ldots, p_{N-1}$:

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Convex Hull : Graham Scan
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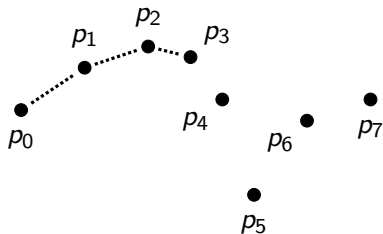 3. Push p_i onto S

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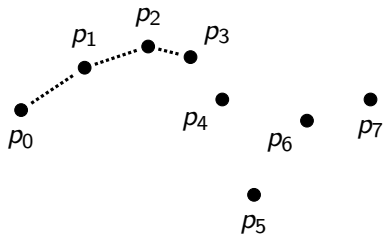   3. Push p_i onto S

Convex Hull : Graham Scan
Graham '72

Upper Hull:

- Sort input points by x-axis

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Convex Hull : Graham Scan
Graham '72

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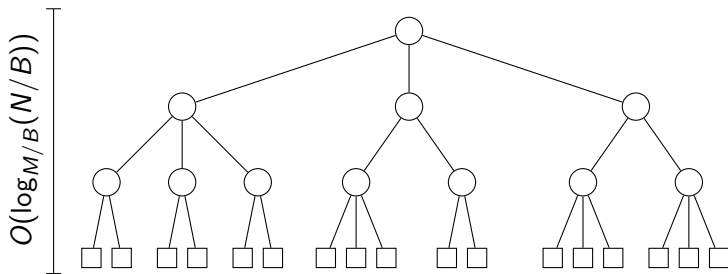 3. Push p_i onto S

Lower Hull:

- Symmetric...

a–b Tree
Huddleston and Mehlhorn '82
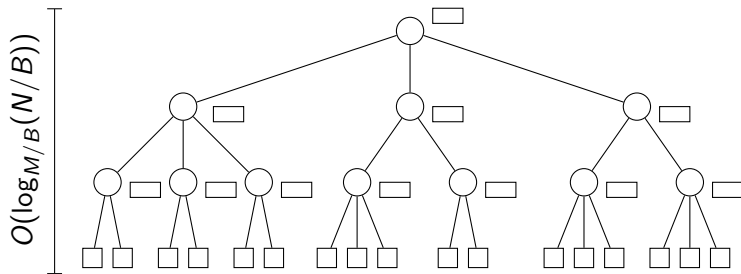
Buffer Tree
Arge '95

$$a = \frac{1}{4}M/B, \qquad b = M/B, \qquad \text{Leaf Size} = B$$

Buffer Tree
Arge '95

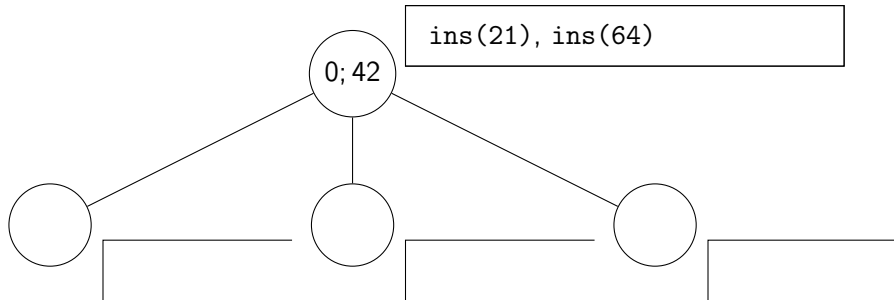$$a = \frac{1}{4}M/B, \qquad b = M/B, \qquad \text{Leaf Size} = B$$
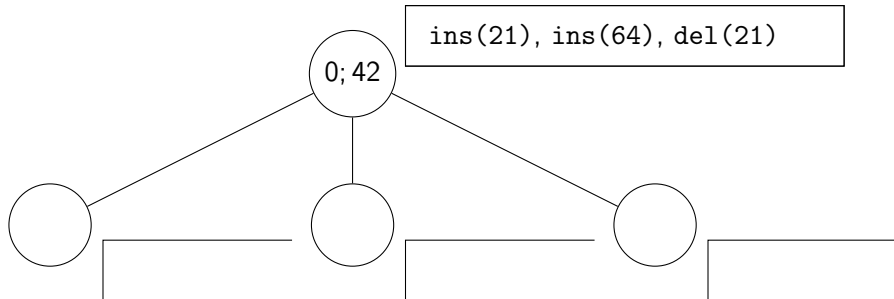
0; 42

Buffer Tree
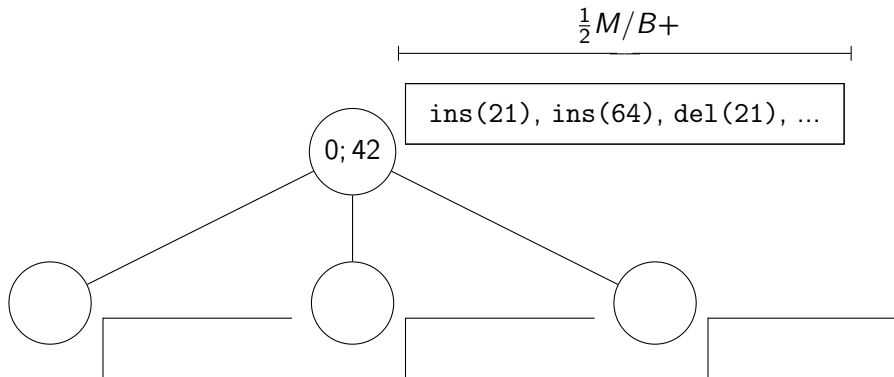## Arge '95
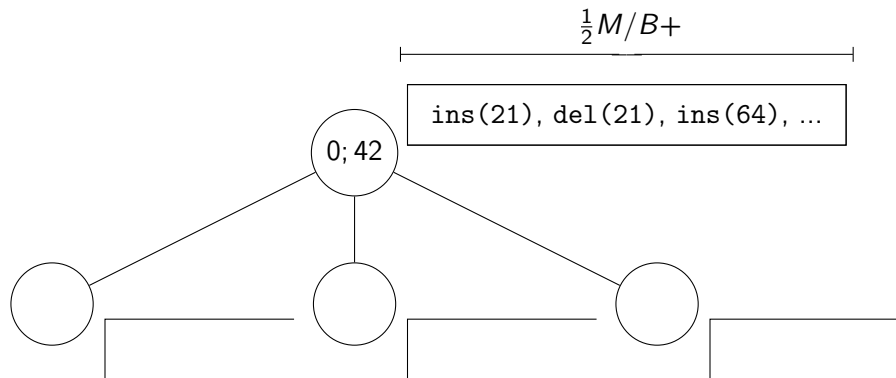
Buffer Tree
Arge '95
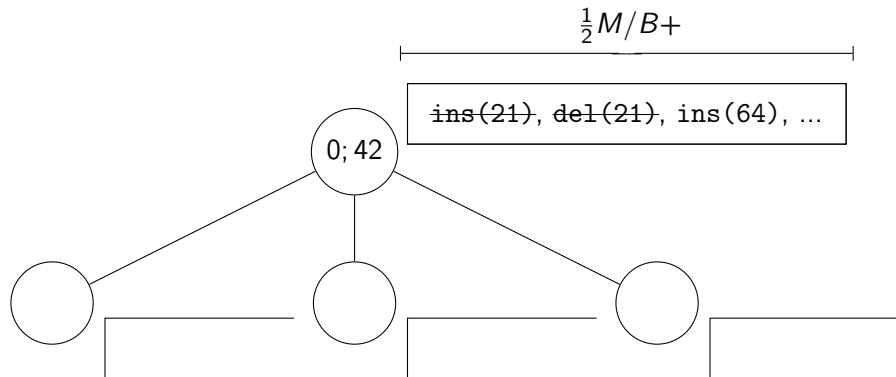
Buffer Tree
Arge '95
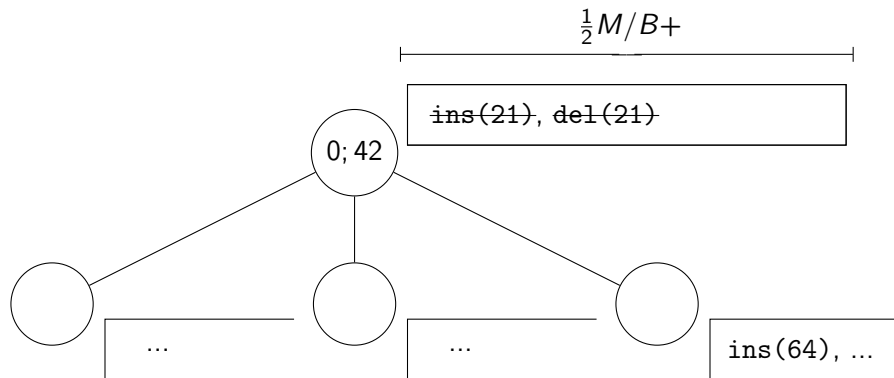
Buffer Tree
Arge '95

Buffer Tree
Arge '95

Theorem
A Buffer Tree can resolve N `inserts` and `deletes` in $\Theta(N/B \cdot \log_{M/B}(N/B))$ I/Os.

Buffer Tree
Arge '95

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A Buffer Tree can resolve N `inserts` and `deletes` in $\Theta(N/B \cdot \log_{M/B}(N/B))$ I/Os.

Theorem
A Buffer Tree with N requests can empty all its buffers, and output all remaining sorted elements, in $\Theta(N/B)$ I/Os.

Buffer Tree
Arge '95

Theorem
A Buffer Tree can resolve N `inserts` and `deletes` in $\Theta(N/B \cdot \log_{M/B}(N/B))$ I/Os.

Theorem
A Buffer Tree with N requests can empty all its buffers, and output all remaining sorted elements, in $\Theta(N/B)$ I/Os.

Corollary
An I/O-efficient Priority Queue can resolve N `push` and `deletemin` operations in $\Theta(N/B \cdot \log_{M/B}(N/B))$ I/Os.

Proof.
Use an $M/2$ sized internal memory priority queue, pq. If pq overflows, move $M/4$ the largest elements to a Buffer Tree, t. If pq underflows, obtain the $M/4$ smallest elements from t. $\quad\square$

Binary Decision Diagrams
Arge '96, Sølvsten '22

#Paths

Given a Binary Decision Diagram of N nodes, compute the number of paths from the root to the $\top$ terminal.

Theorem
#Paths can be computed in
$O(N/B \cdot \log_{M/B}(N/B))$ I/Os.

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Binary Decision Diagrams
Arge '96, Sølvsten '22

$$[\quad ((0,0),(2,0),(1,0))\quad,$$
$$((1,0),(2,0),(3,1))\quad,$$
$$((2,0),(3,0),(3,1))\quad,$$
$$((3,0),\top,\bot)\quad,$$
$$((3,1),\bot,\top)\quad]$$

Decision Diagram   *On-Disk Format*

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$
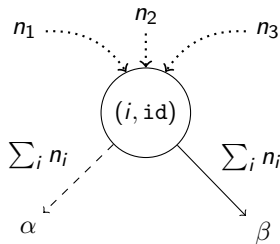
Idea

Count the number of in-going paths to each node.
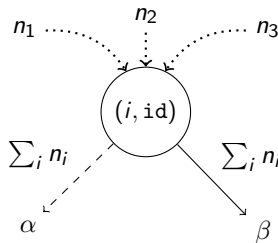
Time-Forward Processing

Defer work with Q_{count} : PriorityQueue$\langle(s \to t, \quad \mathbb{N})\rangle$ sorted on t in ascending order.

$$((i, \mathtt{id}) \xrightarrow{\bot} \alpha, \quad \textstyle\sum_i n_i), \qquad ((i, \mathtt{id}) \xrightarrow{\top} \beta, \quad \textstyle\sum_i n_i)$$

Binary Decision Diagrams
Arge '96, Sølvsten '22

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Binary Decision Diagrams
Arge '96, Sølvsten '22

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]

Binary Decision Diagrams
Arge '96, Sølvsten '22

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]

Binary Decision Diagrams
Arge '96, Sølvsten '22

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:
$$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$$

$$]$$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:
$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$
$\quad ((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$

$]$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:
$$\left[\begin{array}{llll} ((0,0) \xrightarrow{\top} (1,0), & 1 & , \\ ((0,0) \xrightarrow{\bot} (2,0), & 1 & , \end{array} \right.$$

$$\left. \vphantom{\begin{array}{l} a \\ b \end{array}} \right]$$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

[

$((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

]

Binary Decision Diagrams
Arge '96, Sølvsten '22

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

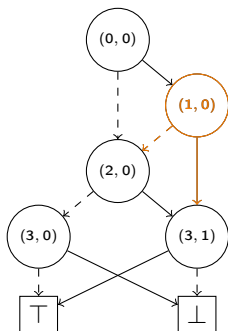| Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

$[$

$\quad ((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$
$\quad ((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$\quad ((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$]$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

$[$

$\quad ((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$
$\quad ((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$\quad ((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$]$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

[

$\quad ((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$
$\quad ((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$\quad ((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad]$

Binary Decision Diagrams
Arge '96, Sølvsten '22

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	1	0

Priority Queue: Q_{count}:

$[$

$((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$

$]$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

]

Binary Decision Diagrams
Arge '96, Sølvsten '22

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

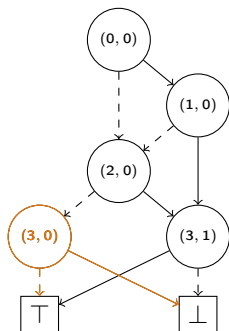| Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\perp} (3, 0),\quad 2)$,
$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)$,
$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)$]

Binary Decision Diagrams
Arge '96, Sølvsten '22

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\perp} (3, 0),\quad 2)\quad ,$

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)\quad ,$

$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)\quad]$

Binary Decision Diagrams
Arge '96, Sølvsten '22

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

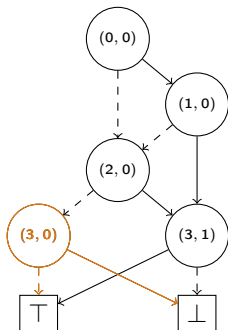| Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\perp} (3, 0), \quad 2) \quad ,$
$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)\qquad,$
$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)\qquad]$

Binary Decision Diagrams
Arge '96, Sølvsten '22

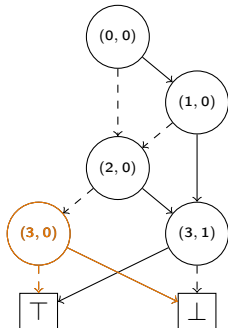**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	2	2

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

$[$

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$

Binary Decision Diagrams
Arge '96, Sølvsten '22

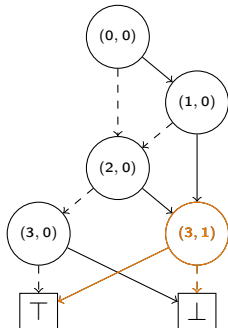(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:
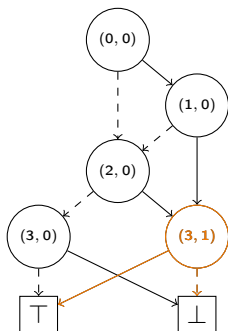
$[$

$$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$$
$$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$$

Binary Decision Diagrams
Arge '96, Sølvsten '22

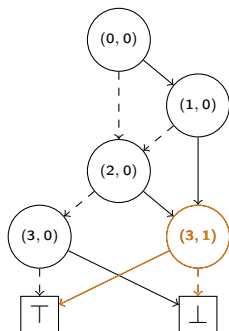(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	1	2

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2 \quad)$]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

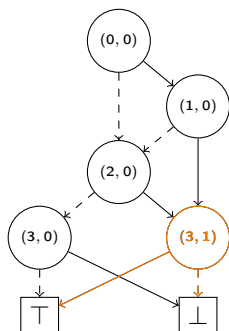| Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue: Q_{count}:

[

]

Binary Decision Diagrams
Arge '96, Sølvsten '22

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result
5

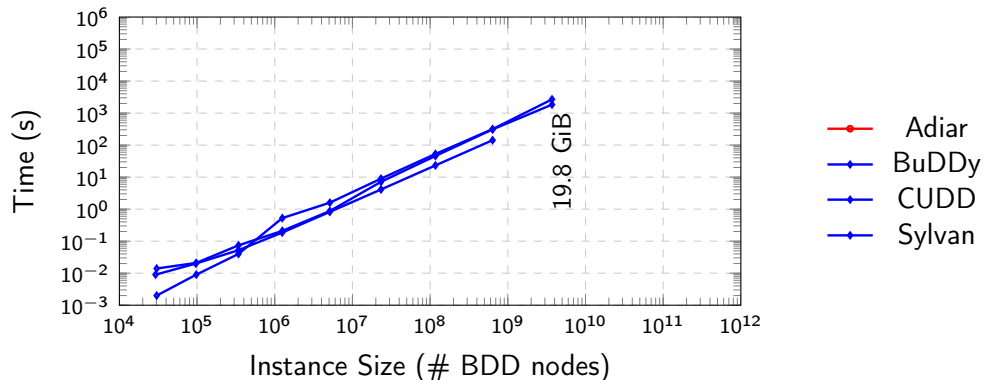Priority Queue: Q_{count}:
[

]

Adiar

I/O-efficient Decision Diagrams

github.com/ssoelvsten/adiar
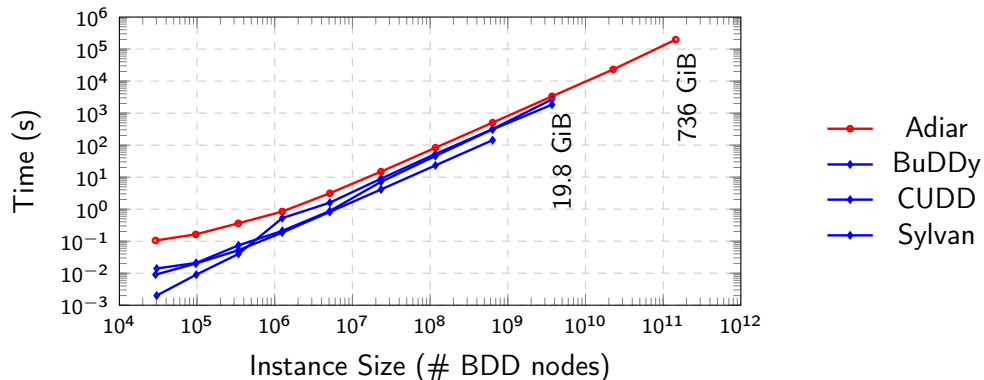
Binary Decision Diagrams
Arge '96, Sølvsten '22

Running time for the *N-Queens* problems.

Binary Decision Diagrams
Arge '96, Sølvsten '22

Running time for the *N-Queens* problems.

Further Reading : Foundations

- **Aggarwal and Vitter (1987)**
  "The Input/Output Complexity of Sorting and Related Problems"
  The I/O-model, Sorting, Permutation, FFT, and Matrix transposition.

- **Arge, Goodrich, Nelson, and Sitchinava (2008)**
  "Fundamental Parallel Algorithms for Private-cache Chip Multiprocessors."
  The I/O-model for Multi-Threading.

Further Reading : Data Structures

- **Arge (1995)**
  "The Buffer Tree: A new technique for Optimal I/O-algorithms"
  An I/O-efficient Tree, Priority Queue, and Range Tree.

- **Sanders (2002)**
  "Fast Priority Queues for Cached Memory"
  A much faster I/O-efficient Priority Queue.

- **Agarwal, Arge and Yi (2006)**
  "I/O-Efficient Batched Union-Find and Its Applications to Terrain Analysis"
  An I/O-efficient (Lazy) Union-Find.

Further Reading : Algorithms

- **Goodrich, Tsay, Vengroff, and Vitter (1993)**
  "External-Memory Computational Geometry"
  Distribution Sweeping and other algorithms.

- **Chiang, Goodrich, Grove, Tamassia, Vengroff, and Vitter (1995)**
  "External-memory Graph Algorithms"
  Time-forward Processing and other algorithms.

- **Arge, Toma, Vitter (2001)**
  "I/O-Efficient Algorithms for Problems on Grid-Based Terrains"
  The TERRAFLOW algorithm.

Further Reading : Libraries (C++)

- **TPIE : Templated Portable I/O Environment**
  github.com/thomasmoelhave/tpie
  Duke University and Aarhus University

- **STXXL : Standard Template library for XXL data sets**
  github.com/stxxl/stxxl
  University of Karlsruhe

Steffan Christ Sølvsten

✉ soelvsten@cs.au.dk

🌐 ssoelvsten.github.io

Adiar

‹/› github.com/ssoelvsten/adiar

📕 ssoelvsten.github.io/adiar

AARHUS
UNIVERSITY

Distribution Sweeping
Goodrich, Tsay, Vengroff, and Vitter '93

Batched Range Searching

Given N axis-parallel rectangles and N points in the plane, compute for each point p all rectangles containing p.

Theorem
Batched Range Searching can be solved in $O(sort(N) + scan(T))$ I/Os.

Distribution Sweeping
Goodrich, Tsay, Vengroff, and Vitter '93

Preprocessing:

Algorithm:

Distribution Sweeping
Goodrich, Tsay, Vengroff, and Vitter '93

Preprocessing:

- Split each rectangle into two vertical lines.

Algorithm:

Distribution Sweeping
Goodrich, Tsay, Vengroff, and Vitter '93

Preprocessing:

- Split each rectangle into two vertical lines.

- Sort all lines and points by their x-value.

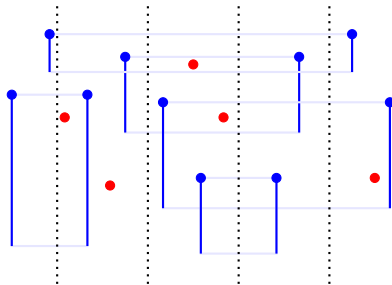Algorithm:

Distribution Sweeping
Goodrich, Tsay, Vengroff, and Vitter '93

Preprocessing:

- Split each rectangle into two vertical lines.

- Sort all lines and points by their x-value.

Algorithm:

- Split all data into $\Theta(\sqrt{M/B})$ *slabs*. Solve these recursively; output is given sorted by y-value.

Distribution Sweeping
Goodrich, Tsay, Vengroff, and Vitter '93

Preprocessing:

- Split each rectangle into two vertical lines.
- Sort all lines and points by their *x*-value.

Algorithm:

- Split all data into $\Theta(\sqrt{M/B})$ *slabs*. Solve these recursively; output is given sorted by *y*-value.
- Merge slabs together, report points between line segments outside its slab.
  - Use $\Theta(\sqrt{M/B}^2) = \Theta(\sqrt{M/B})$ multi-slabs to maintain each *active* rectangle.
  - Output points and un-matched line segments.