Random Access on Narrow Decision Diagrams in External Memory

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SPIN 2024

Adiar

I/O-efficient Decision Diagrams

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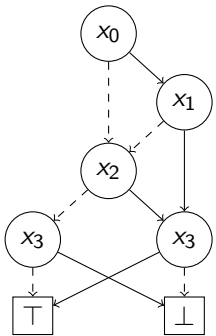
github.com/ssoelvsten/adiar

Features

Optimisations

Features

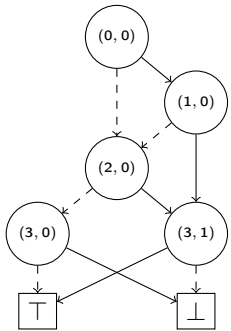
Optimisations



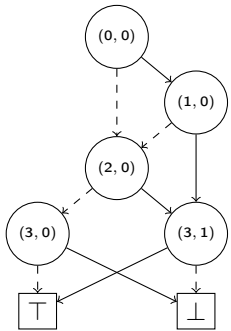
$$[\quad ((0,0),(2,0),(1,0)) \quad ,$$
$$((1,0),(2,0),(3,1)) \quad ,$$
$$((2,0),(3,0),(3,1)) \quad ,$$
$$((3,0),\top,\bot) \quad ,$$
$$((3,1),\bot,\top) \quad]$$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Serialized Representation of a Binary Decision Diagram.



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

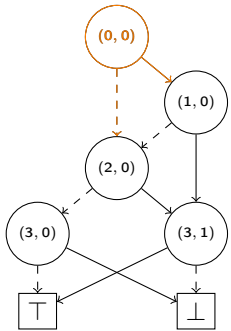


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]

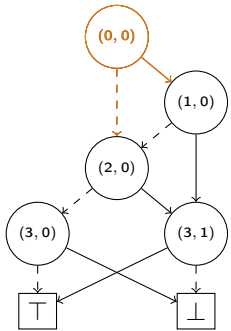


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]

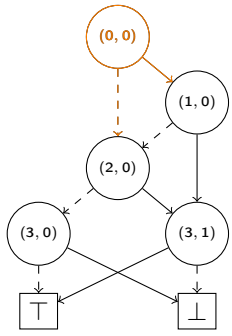


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

$$\left[\begin{array}{lll} ((0,0) \xrightarrow{\top} (1,0), & 1) & , \\ ((0,0) \xrightarrow{\perp} (2,0), & 1) & , \end{array} \right.$$

]



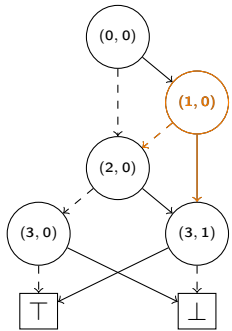
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0),\quad 1) \quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0),\quad 1) \quad ,$$

$]$

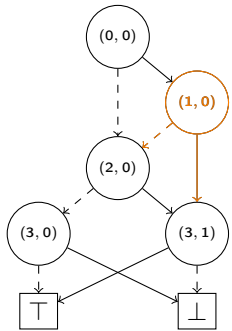


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1,0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0),\quad 1)\quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0),\quad 1)\quad ,$$
$$]$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

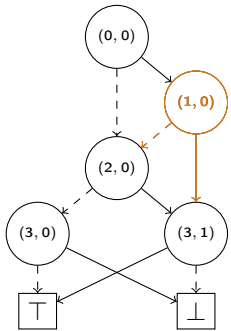
Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

[

$((0, 0) \xrightarrow{\perp} (2, 0), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1,0)$	1	0

Priority Queue: Q_{count}:

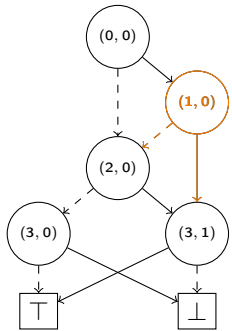
[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	0	0

Priority Queue: Q_{count}:

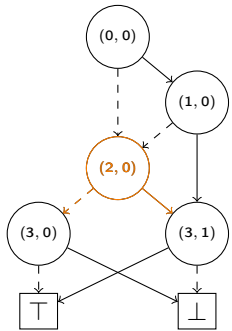
[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]

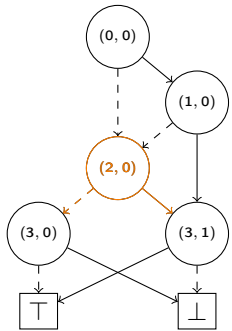


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	0	0

Priority Queue: Q_{count}:

$$
\begin{bmatrix}
((0,0) \xrightarrow{\perp} (2,0), & 1) & , \\
((1,0) \xrightarrow{\perp} (2,0), & 1) & , \\
 & & \\
((1,0) \xrightarrow{\top} (3,1), & 1) & , \\
 & &]
\end{bmatrix}
$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	1	0

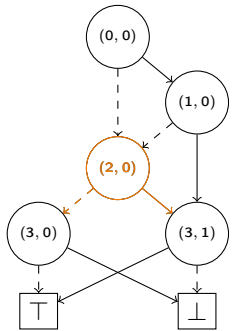
Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

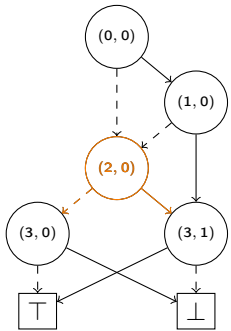
Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

]



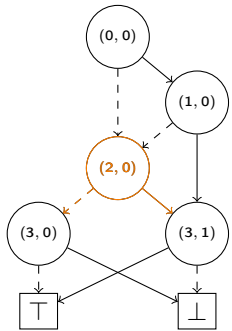
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



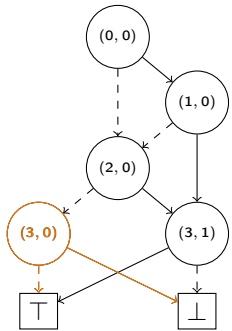
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



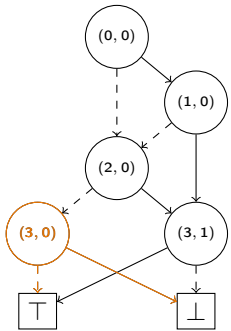
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,0)$	0	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

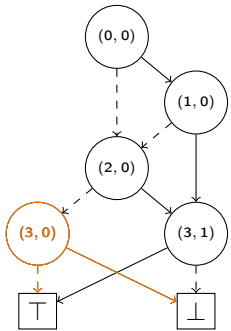
Seek	Sum	Result
$(3,0)$	2	0

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

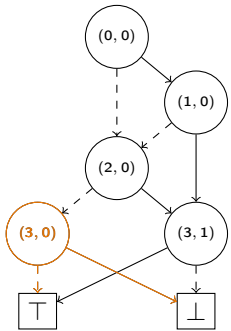
Seek	Sum	Result
$(3,0)$	2	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

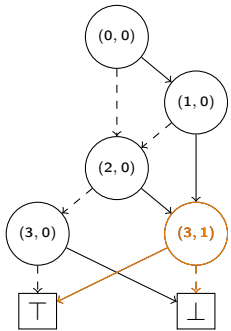
Seek	Sum	Result
$(3,1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

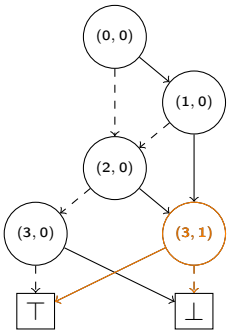
Seek	Sum	Result
$(3,1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



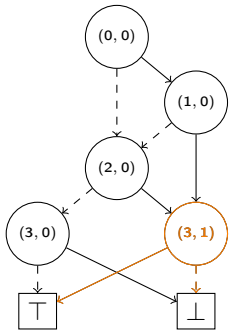
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,1)$	1	2

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



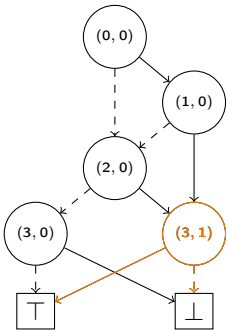
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]



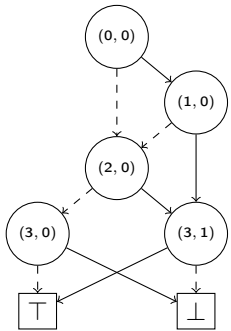
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue: Q_{count}:

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result

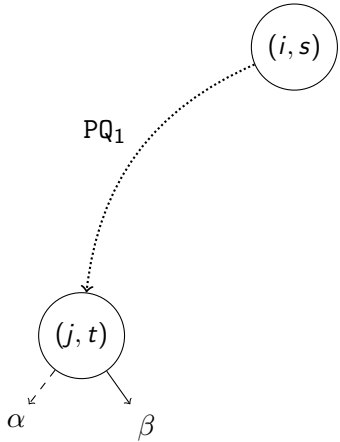
5

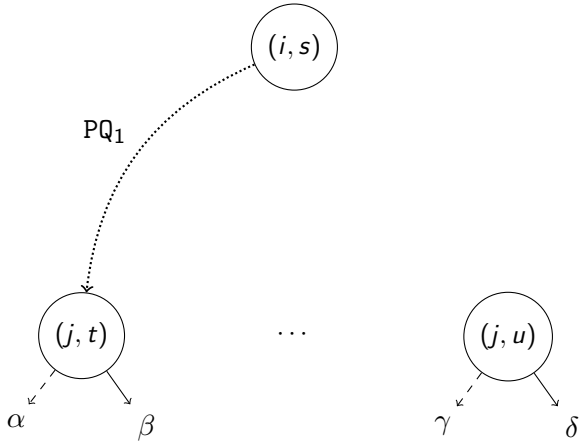
Priority Queue: Q_{count}:

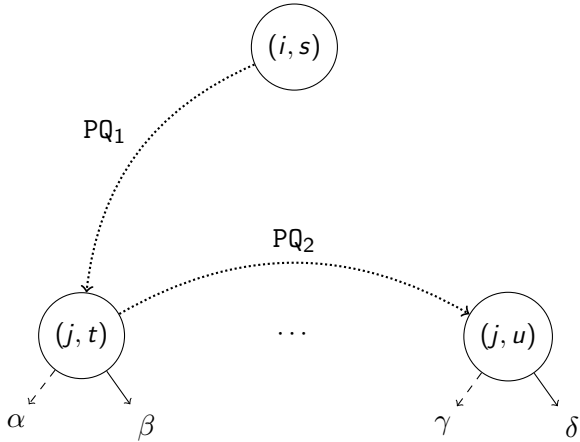
[

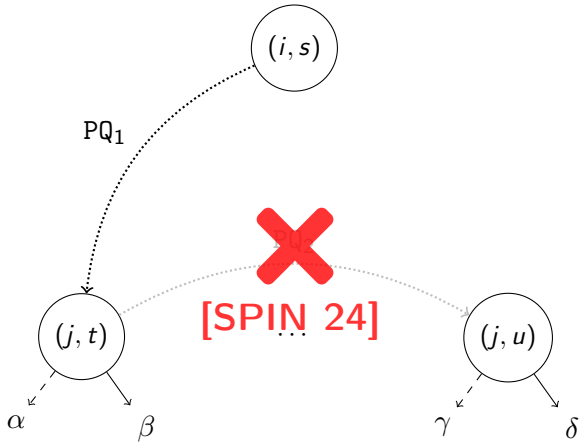
]

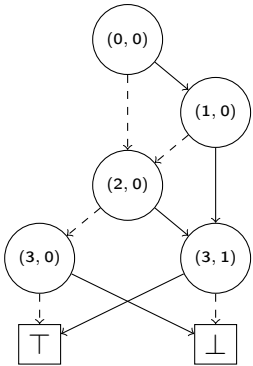

(i, s)

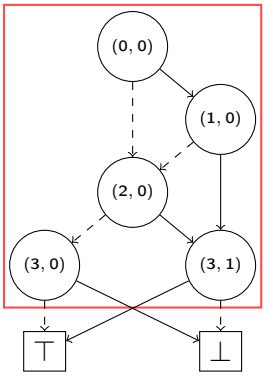

(i, s)
$\mathtt{PQ}_1$
(j, t)
α
β

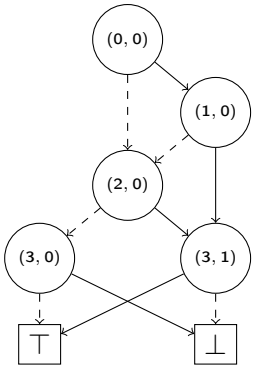

(i, s)
$\mathtt{PQ}_1$
(j, t)
$\cdots$
(j, u)
α
β
γ
δ

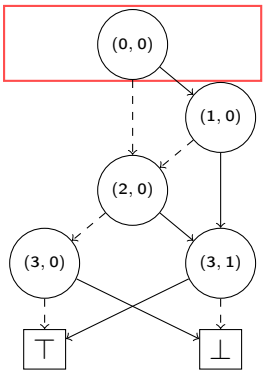

(i, s)
$\texttt{PQ}_1$
$\texttt{PQ}_2$
(j, t)
$\cdots$
(j, u)
α
β
γ
δ

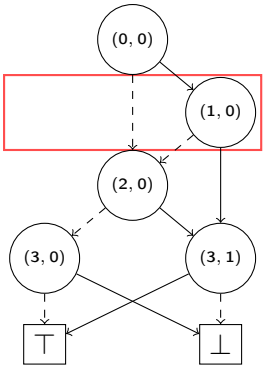

(i, s)
$\mathtt{PQ}_1$
(j, t)
(j, u)
[SPIN 24]
α
β
γ
δ

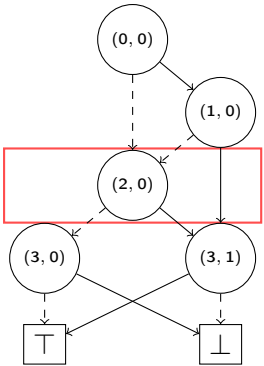

(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥

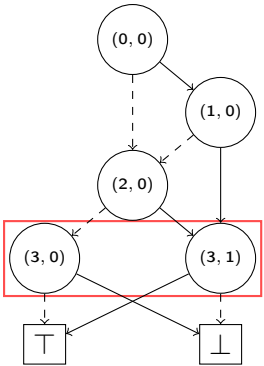

(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥

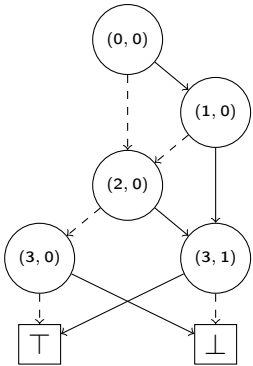

(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥

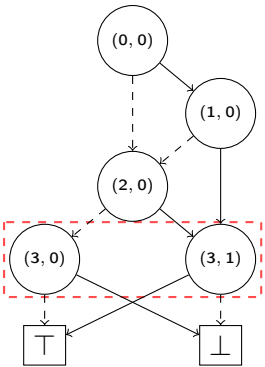

(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥


(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥


(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥

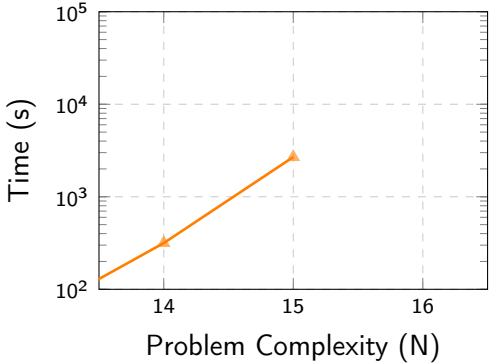

(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥


(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥



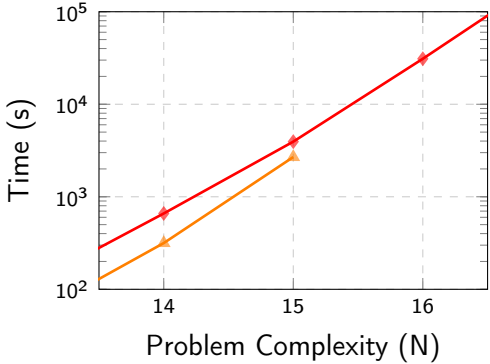
Width

2



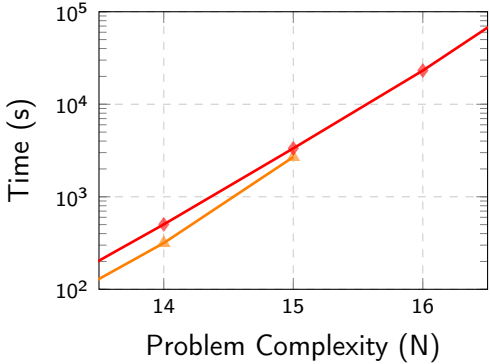
			⏱ $N = 15$
△	CUDD v3.0	:	44.8 min

Queens | 300 GiB of RAM



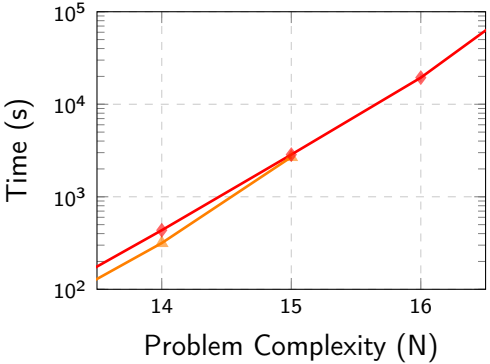
				⏱ $N = 15$
△	CUDD	v3.0	:	44.8 min
◇	Adiar	v1.0	:	66.7 min

Queens | 300 GiB of RAM



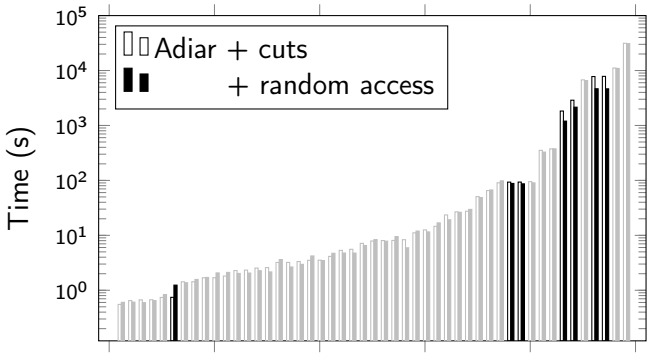
				⏱ $N = 15$
△	CUDD	v3.0	:	44.8 min
◇	Adiar	v1.0	:	66.7 min
	+ cuts		:	56.8 min

Queens | 300 GiB of RAM



				⏱ $N = 15$
△	CUDD	v3.0	:	44.8 min
◇	Adiar	v1.0	:	66.7 min
	+ cuts		:	56.8 min
	+ random access		:	47.2 min

Queens | 300 GiB of RAM



Speed Ups

6 (+26.9%)

Slowdowns

1 (-66.8%)

EPFL Circuit Verification | 300 GiB of RAM

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ssoelvsten.github.io

Adiar

github.com/ssoelvsten/adiar

ssoelvsten.github.io/adiar

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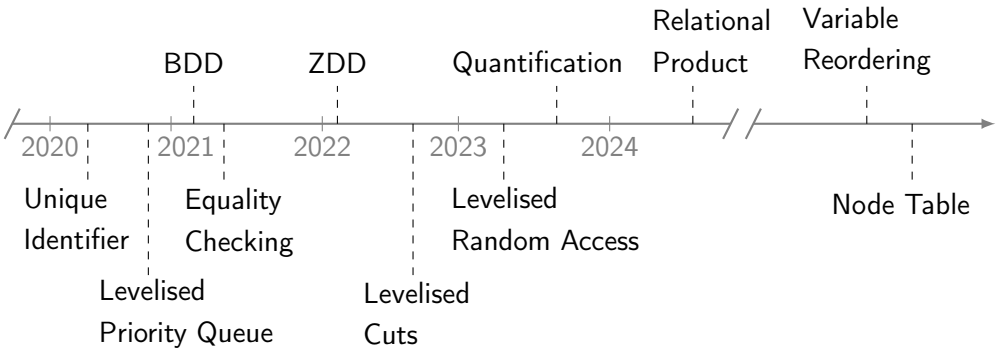
Relational
Product
Variable
Reordering
BDD
ZDD
Quantification
2020
2021
2022
2023
2024
Unique
Identifier
Equality
Checking
Levelised
Random Access
Node Table
Levelised
Priority Queue
Levelised
Cuts