8.1

Soll gelten: $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$(\text{und } \forall x \in \mathbb{R}: f(x) \geq 0)$

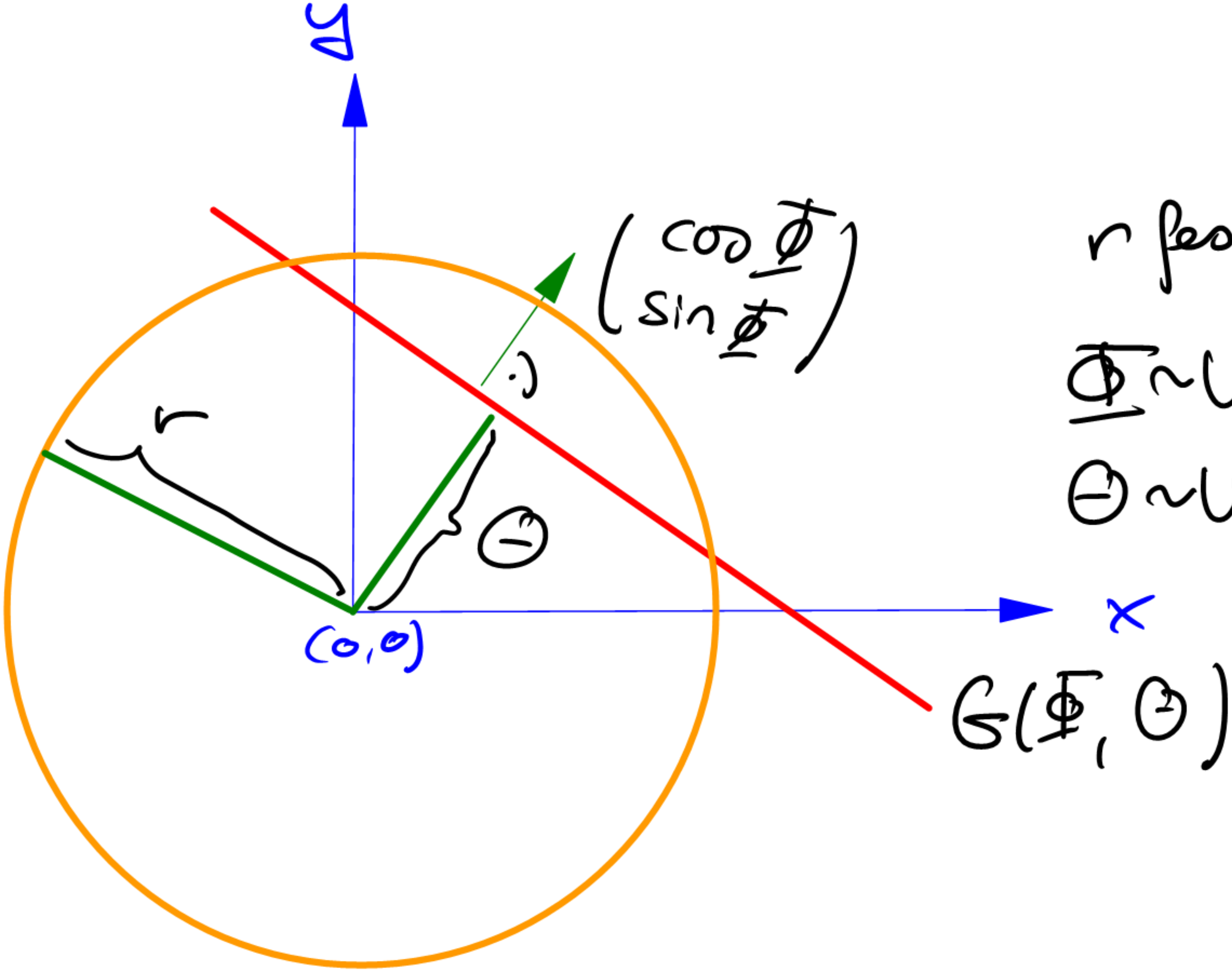
$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx \underset{x=\tan\varphi}{=} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+\tan^2\varphi}\,\frac{d\varphi}{\cos^2\varphi}$$

$$dx = \frac{d\varphi}{\cos^2\varphi}$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\cos^2\varphi+\sin^2\varphi}\,d\varphi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi = \pi$$

$\rightsquigarrow c = \frac{1}{\pi}.$

8.2



r fest aus $[0,\infty)$

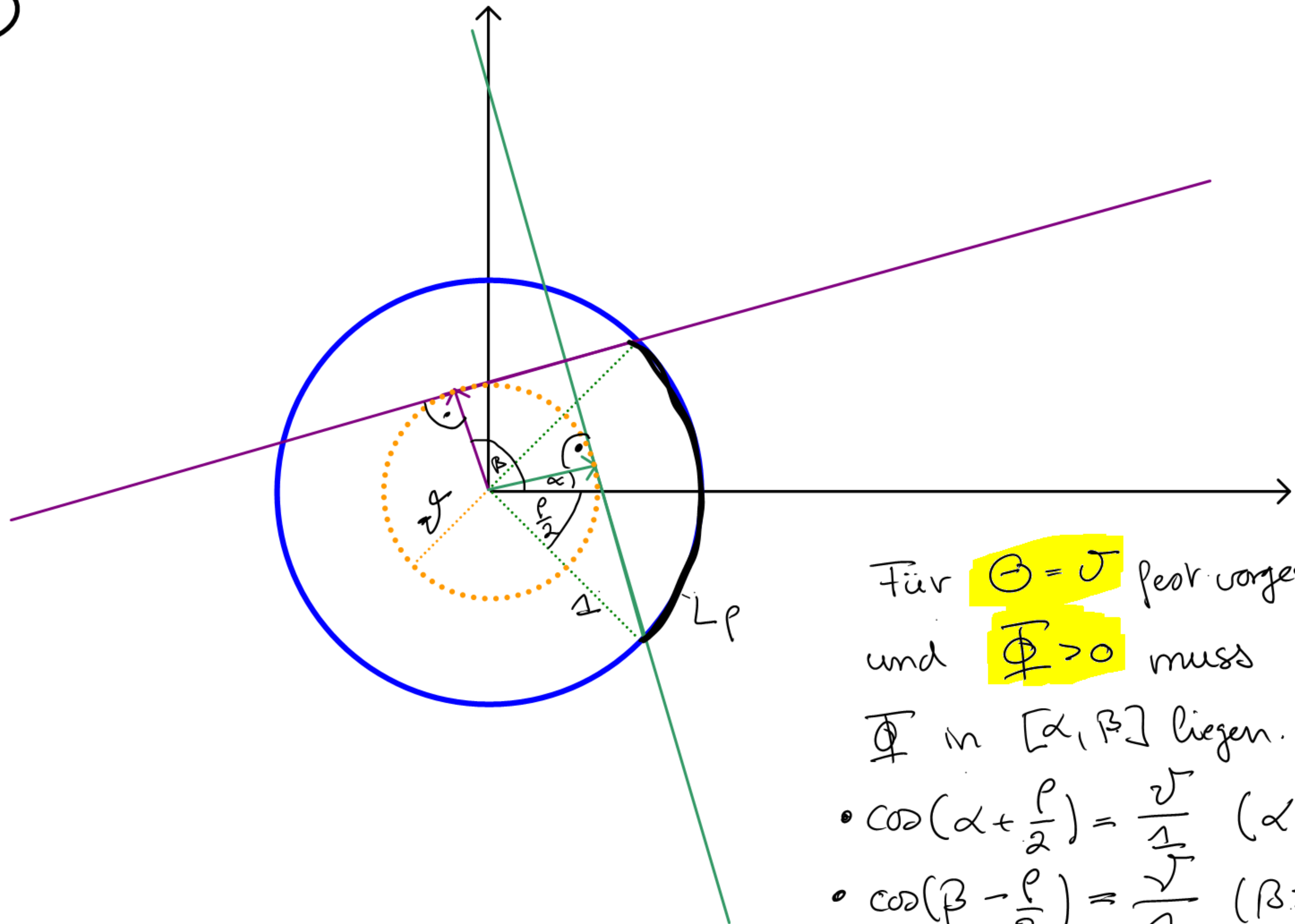
$\Phi \sim Uni\,[0, 2\pi)$

$\Theta \sim Uni\,[0, 1]$

ⓐ $Pr[G(\Phi, \Theta) \cap K_r \neq \emptyset] = Pr[\Theta \leq r]$

$= 1$, falls $r \geq 1$, sonst $= \int_0^r 1 \, dr = r$

(b)



Für $\Theta = \vartheta$ fest vorgegeben und $\Phi > 0$ muss Φ in $[\alpha, \beta]$ liegen mit:

- $\cos\left(\alpha + \frac{\rho}{2}\right) = \frac{\vartheta}{1}$ $(\alpha \geq 0)$
- $\cos\left(\beta - \frac{\rho}{2}\right) = \frac{\vartheta}{1}$ $(\beta \geq \alpha \geq 0)$

$$\rightsquigarrow \Phi \in \left[\max\left(0, \arccos(\vartheta) - \frac{\rho}{2}\right), \arccos(\vartheta) + \frac{\rho}{2}\right]$$

$$\rightsquigarrow \Pr\left[G(\Phi, \Theta) \cap L_\rho \neq \emptyset \;\wedge\; \Phi > 0 \right]$$

$$= \frac{1}{2\pi} \int_{\vartheta=0}^{1} \int_{\varphi=\max(0,\arccos(\vartheta)-\frac{\rho}{2})}^{\arccos(\vartheta)+\frac{\rho}{2}} d\varphi \; d\vartheta$$

$$= \frac{1}{2\pi} \left[\int_{\vartheta=0}^{\cos(\frac{\rho}{2})} \underbrace{\int_{\arccos(\vartheta)-\frac{\rho}{2}}^{\arccos(\vartheta)+\frac{\rho}{2}} d\varphi}_{\rho} \; d\vartheta + \int_{\vartheta=\cos(\frac{\rho}{2})}^{1} \underbrace{\int_{0}^{\arccos(\vartheta)+\frac{\rho}{2}} d\varphi}_{\frac{\rho}{2}+\arccos(\vartheta)} \; d\vartheta \right]$$

$$= \underbrace{\qquad\qquad\qquad}_{\rho\cos(\frac{\rho}{2})} \qquad \underbrace{\qquad\qquad\qquad\qquad}_{(1-\cos(\frac{\rho}{2}))\frac{\rho}{2} + \int_{\cos(\frac{\rho}{2})}^{1} \arccos(\vartheta)}$$

$$\int_{\cos(\frac{\rho}{2})}^{1} \arccos(\sigma)\, d\sigma = -\int_{\frac{\rho}{2}}^{0} x \sin x \, d = \int_{0}^{\rho/2} x \sin x \, dx$$

$\sigma = \cos x$

$d\sigma = -\sin x \, dx$

$u = x \quad v' = \sin x$

$u' = 1 \quad V = -\cos x$

$$= \left[-x \cos x\right]_0^{\frac{\rho}{2}} + \int_0^{\frac{\rho}{2}} \cos x \, d$$

$$= -\frac{\rho}{2} \cos\left(\frac{\rho}{2}\right) + \sin\left(\frac{\rho}{2}\right)$$

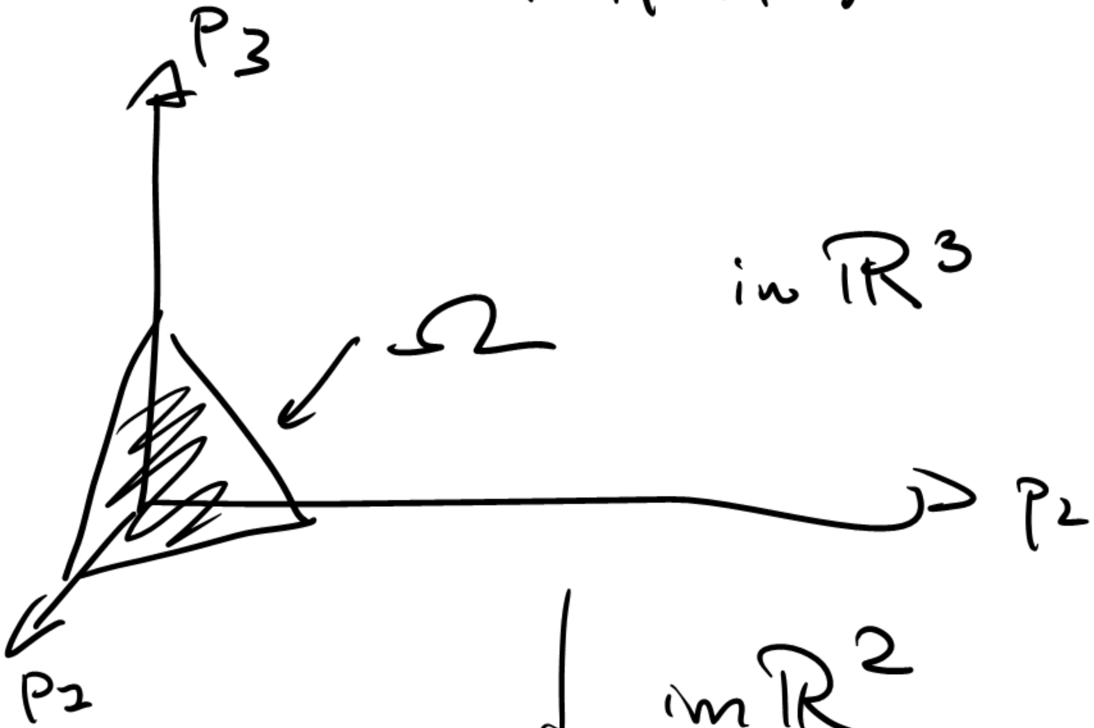
$$\rightsquigarrow \Pr\left[G(\Phi, \Theta) \cap L_\rho \neq \emptyset\right] = \frac{\frac{\rho}{2} + \sin\left(\frac{\rho}{2}\right)}{\pi}$$

- Für $\rho > \pi$ folgt analog $\dfrac{\frac{\rho}{2} + \sin\left(\pi - \frac{\rho}{2}\right)}{\pi}$

$$= \frac{\frac{\rho}{2} + \sin\left(\frac{\rho}{2}\right)}{\pi}$$

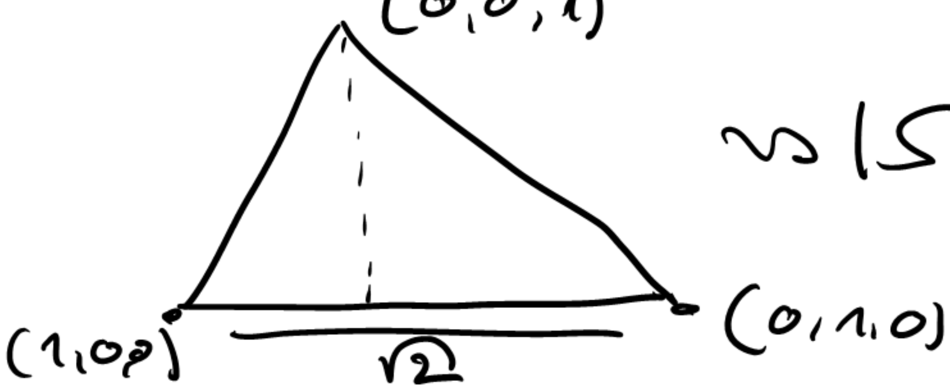
8.3

$$\Omega = \{ (p_1, p_2, p_3) \in [0,1]^3 \mid p_1 + p_2 + p_3 = 1 \}$$



$\downarrow$ im $\mathbb{R}^2$

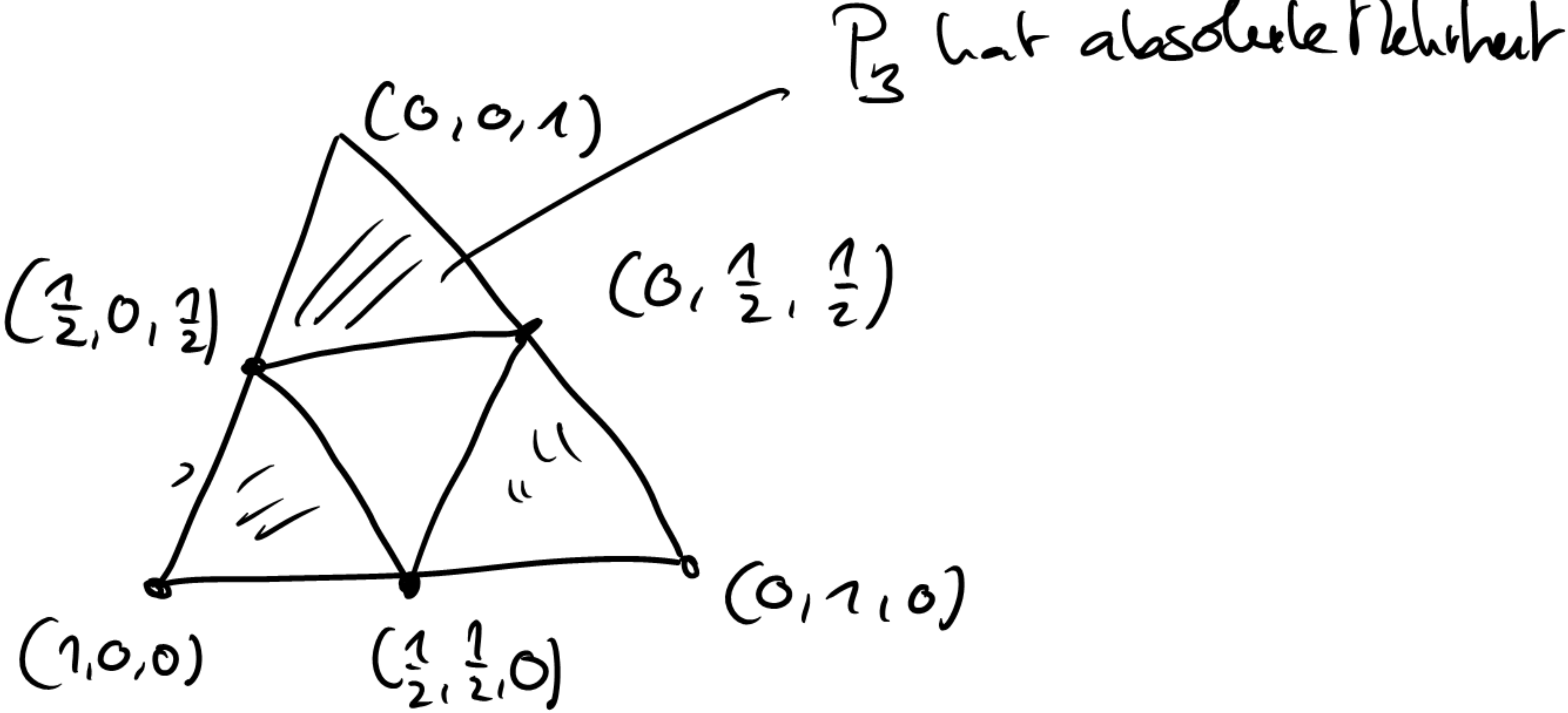
Gleichseitiges Dreieck mit Kantenlänge $\sqrt{2}$



$$\rightsquigarrow |\Omega| = \frac{1}{2}\sqrt{2} \cdot \sqrt{2 - \frac{1}{2}}$$

$$= \frac{1}{2}\sqrt{3}$$

W'keit für A:



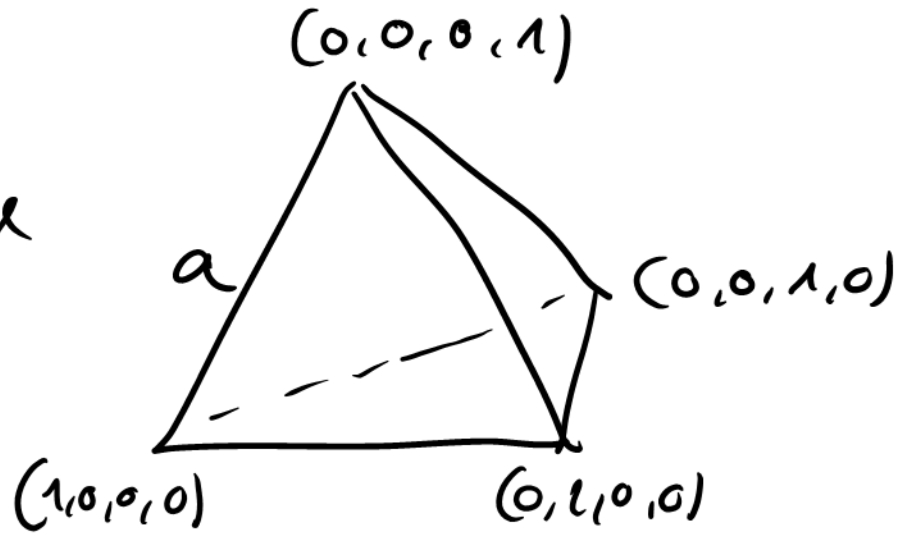
4 gleichseitige Dreiecke mit derselben Kantenlänge $\rightsquigarrow$ $Pr[A] = \frac{3}{4}$

(b)

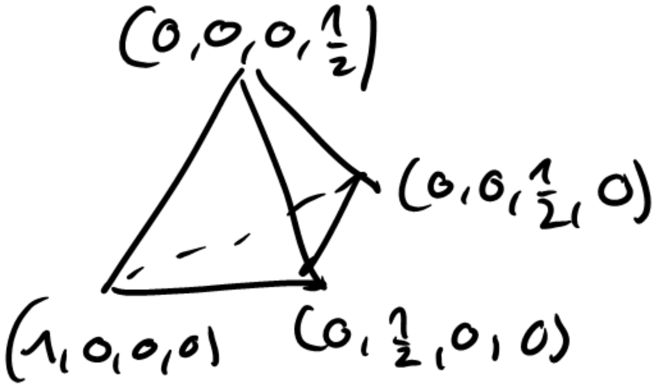
$$\Omega = \{(p_1, p_2, p_3, p_4) \in [0,1]^4 \mid \sum p_i = 1\}$$

$$A_i = \{(p_1, \dots, p_4) \in \Omega \mid p_i \geq 1/2\}$$

Ω: Tetraeder mit Kantenlänge a



A_1:



$\rightsquigarrow$ Tetraeder mit halber Kantenlänge $\frac{a}{2}$

Volumen eines Tetraeders mit Kantenlänge a:

$$\frac{\sqrt{2}}{12} a^3$$

$$\rightsquigarrow \Pr[A_i] = \frac{\frac{\sqrt{2}}{12}\left(\frac{a}{2}\right)^3}{\frac{\sqrt{2}}{12} a^3} = \frac{1}{8}$$

$$\rightsquigarrow \Pr[\underbrace{A_1 \cup A_2 \cup A_3 \cup A_4}_{\text{disjunkt}}] = \frac{4}{8} = \frac{1}{2}$$

8.4

(a) $\rightsquigarrow$ Negation : mindestens 4 fehlerhafte Übertragungen

$\rightsquigarrow$ $\mathbb{P} \setminus Q^* F Q^* F Q^* F Q^* F Q^\omega$

aus TA 7.3

(b) $\rightsquigarrow$ Eine Übertragung hat die Form:

$$S\ \Pi\ (VH)^*\ (\varepsilon + F)\ S$$

$\rightsquigarrow$ Wollen keine Übertragungen der Form

$\rightsquigarrow$ $S\ \Pi\ F S$

$\rightsquigarrow$ $\mathbb{P} \setminus Q^*\ S \Pi F S\ Q^\omega$