HA 5.1

(a) $\max(X,Y) = X + Y - \min(X,Y)$

(b) $\cdot \min(X,Y) \sim Geo(\underbrace{1-(1-p)(1-q)}_{p+q-pq})$

Bew.

$$\Pr[\min(X,Y) > k] = \Pr[X > k \wedge Y > k]$$

X, Y unabh.

$$= \Pr[X>k]\Pr[Y>k]$$

$$= (1-p)^k (1-q)^k$$

$$= ((1-p)(1-q))^k$$

$$\leadsto \mathbb{E}[\max(X,Y)] = \underbrace{\mathbb{E}[X]}_{=\frac{1}{p}} + \underbrace{\mathbb{E}[Y]}_{=\frac{1}{q}} - \underbrace{\mathbb{E}[\min(X,Y)]}_{=\frac{1}{p+q-pq}}$$

$$= 5 + \frac{5}{4} - \frac{25}{21}$$

$$= \frac{425}{84}$$

Aufg. 2

$$\mathbb{E}\left[(\max(U,Z)+Y)^{W^3} (2WV + X^{2-2W}) \right]$$

$$= \mathbb{E}\left[(\max(U,Z)+Y)^{W^3} (2WV + X^{2-2W}) \,\middle|\, W=0 \right] \frac{1}{5}$$

$$+ \mathbb{E}\left[(\max(U,Z)+Y)^{W^3} (2WV + X^{2-2W}) \,\middle|\, W=1 \right] \frac{4}{5}$$

$$= \mathbb{E}\left[X^2 \mid W=0 \right] \frac{1}{5} + \mathbb{E}\left[(\max(U,Z)+Y)(2V+1) \mid W=1 \right] \frac{4}{5}$$

$$\mathbb{E}[X^2 \mid W=0] = \mathbb{E}[X^2] = \mathrm{Var}[X] + \mathbb{E}[X]^2$$

X, W unabhängig impliziert X^2, W unabh.

$$= 5 \cdot (21/25)/(4/25)^2 + \left(5 \cdot \frac{25}{4}\right)^2$$

$X \sim \mathrm{NegBin}\left(5, \frac{4}{25}\right)$

$\leadsto X$ ist die Summe von 5 unabh. ZV $\sim \mathrm{Geo}\left(\frac{4}{25}\right)$

$$= \frac{125 \cdot 21}{16} + \frac{125^2}{16} = \frac{9125}{8}$$

$$\mathbb{E}[(\max(U,Z)+Y)\ (2V+1) \mid W=1]$$

$$= \mathbb{E}[\max(U,Z)+Y] \cdot \mathbb{E}[2V+1]$$

$(2V+1)$ unabh von $(\max(U,Z)+Y)$,

da V, U,Z,Y unabh.

- $\mathbb{E}[\max(U,Z)+Y] \overset{\text{Linearität}}{=} \mathbb{E}[\max(U,Z)]+\mathbb{E}[Y]$

- $\mathbb{E}[Y] \underset{Y\sim Bin(15,\frac{4}{5})}{=} 15 \cdot \frac{4}{5} = 12$

- $\mathbb{E}[\max(U,Z)] \underset{\text{siehe HA 5.1}}{=} \mathbb{E}[U]+\mathbb{E}[Z]-\mathbb{E}[\min(U,Z)]$

- $\mathbb{E}[U] \underset{U\sim Geo(\frac{1}{5})}{=} 1/1/5 = 5$, $\mathbb{E}[Z] \underset{Z\sim Geo(4/5)}{=} 5/4$

- $\mathbb{E}[\min(U,Z)] \underset{\min(U,Z)\sim Geo(1-(1-\frac{1}{5})(1-\frac{4}{5}))}{=} \frac{25}{21}$

- $\mathbb{E}[\max(U,Z)+Y]$

$$= 12 + 5 + 5/4 - \frac{25}{21}$$

$$= \frac{1433}{84}$$

- $\mathbb{E}[2V+1] \overset{\text{Linearität}}{=} 2\,\mathbb{E}[V]+1$

$$= 2 \cdot 10 \cdot \frac{4}{5} + 1$$

$V \sim \mathrm{Bin}(10, \frac{4}{5})$

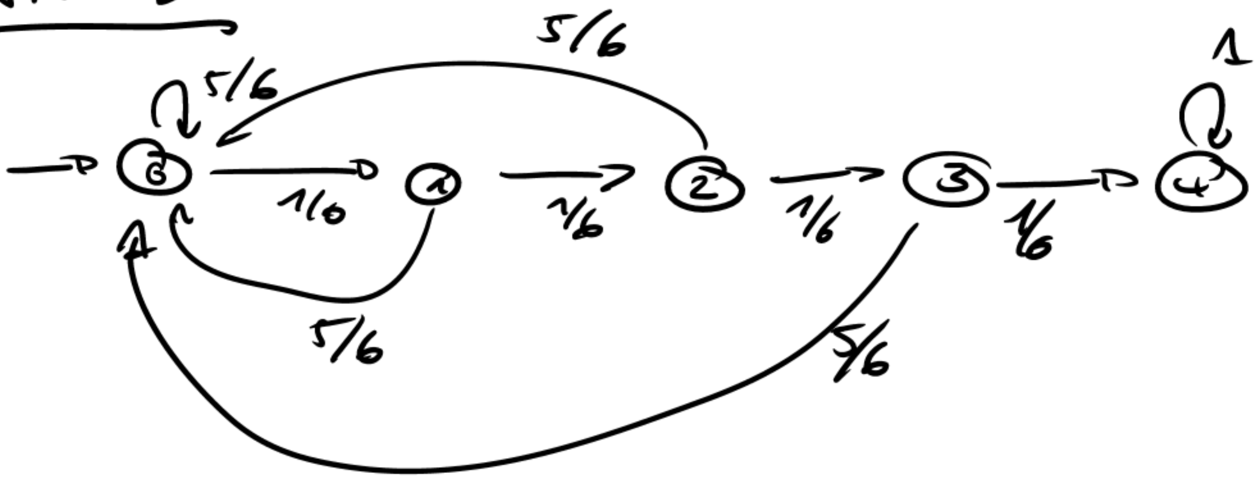
$$= 17$$

$$\Rightarrow \mathbb{E}[\ldots] = \frac{1}{5} \cdot \frac{9125}{8} + \frac{4}{5} \cdot 17 \cdot \frac{1433}{84}$$

$$= \frac{1825}{8} + \frac{24361}{105}$$

$$= \frac{386513}{840} \approx 460.135.$$

HA 5.3



(a) $\mathbb{E}_0[N] = 1 + \frac{1}{6}\mathbb{E}_1[N] + \frac{5}{6}\mathbb{E}_0[N]$

$\mathbb{E}_1[N] = 1 + \frac{1}{6}\mathbb{E}_2[N] + \frac{5}{6}\mathbb{E}_0[N]$

$\mathbb{E}_2[N] = 1 + \frac{1}{6}\mathbb{E}_3[N] + \frac{5}{6}\mathbb{E}_0[N]$

$\mathbb{E}_3[N] = 1 + \frac{1}{6}\underbrace{\mathbb{E}_4[N]}_{=0} + \frac{5}{6}\mathbb{E}_0[N]$

(Substituieren)

$$\rightsquigarrow \mathbb{E}_0[N] = 1 + \frac{5}{6}\mathbb{E}_0[N] + \frac{1}{6} + \frac{5}{6^2}\mathbb{E}_0[N] + \frac{1}{6^2} + \frac{5}{6^3}\mathbb{E}_0[N] + \frac{1}{6^3} + \frac{5}{6^4}\mathbb{E}_0[N]$$

$$\rightsquigarrow \mathbb{E}_0[N] = 1554, \quad \mathbb{E}_1[N] = 1548, \quad \mathbb{E}_2[N] = 1512, \quad \mathbb{E}_3[N] = 1296$$

(b) $\mathbb{E}_4[N^2] = 0$

$$\mathbb{E}_i[N^2] = \frac{5}{6}\mathbb{E}_0[(N+1)^2] + \frac{1}{6}\mathbb{E}_{i+1}[(N+1)^2]$$

$$= \frac{5}{6}\mathbb{E}_0[N^2] + \frac{5}{3}\mathbb{E}_0[N] + \frac{5}{6}$$

$$+ \frac{1}{6}\mathbb{E}_{i+1}[N^2] + \frac{1}{3}\mathbb{E}_{i+1}[N] + \frac{1}{6}$$

$$= 2591 + \frac{1}{3}\mathbb{E}_{i+1}[N] + \frac{1}{6}\mathbb{E}_{i+1}[N^2] + \frac{5}{6}\mathbb{E}_0[N^2]$$

$$\leadsto \mathbb{E}_0[N^2] = 3107 + \frac{1}{6}\mathbb{E}_1[N^2] + \frac{5}{6}\mathbb{E}_0[N^2]$$

$$\mathbb{E}_1[N^2] = 3095 + \frac{1}{6}\mathbb{E}_2[N^2] + \frac{5}{6}\mathbb{E}_0[N^2]$$

$$\mathbb{E}_2[N^2] = 3023 + \frac{1}{6}\mathbb{E}_3[N^2] + \frac{5}{6}\mathbb{E}_0[N^2]$$

$$\mathbb{E}_3[N^2] = 2591 + \frac{5}{6}\mathbb{E}_0[N^2]$$

Gleiche Matrix wie in (a)

$$\leadsto \mathbb{E}_0[N^2] = 4\,819\,566$$

$$\leadsto \mathrm{Var}[N] = \underline{2404650}$$

Kontrolle

$$G_i(z) := \mathbb{E}_i\left[z^N\right]$$

$$G_4(z) = 1$$

$$G_i(z) = \frac{1}{6}\,\mathbb{E}_{i+1}\left[z^{N+1}\right] + \frac{5}{6}\,\mathbb{E}_1\left[z^{N+1}\right]$$

$$= \frac{1}{6}\, z\, G_{i+1}(z) + \frac{5}{6}\, z\, G_1(z)$$

$$\leadsto G_0(z) = \frac{5}{6} z G_0(z) + \frac{5}{6^2} z^2 G_0(z) + \frac{5}{6^3} z^3 G_3(z) + \frac{5}{6^4} z^4 G_3(z) + \frac{1}{6^4} z^4$$

$$\leadsto G_0(z) = \frac{1/6^4\, z^4}{1 - 5\cdot\left(\frac{1}{6} z + \frac{1}{6^2} z^2 + \frac{1}{6^3} z^3 + \frac{1}{6^4} z^4\right)}$$

$$\leadsto \frac{d}{dz} G_0\Big|_{z=1} = 1554 \checkmark = \mathbb{E}_0[N]$$

$$\frac{d^2}{dz^2} G_0\Big|_{z=1} = 4\,818\,012 = \mathbb{E}_0[N(N-1)]$$

AA 5.4

(a) $\Omega = [5] \times [5]$

$Pr[\omega] = \frac{1}{|\Omega|}$

Ω

	1	2	3	4	5
1				B	
2				B	
3			A C	A C B	
4		A	A C	A C B	
5					

$\rightsquigarrow Pr[A] = \frac{5}{25}$

$Pr[B] = \frac{4}{25}$

$Pr[A \cap B] = \frac{2}{25}$

$\to$ A, B nicht unabh.

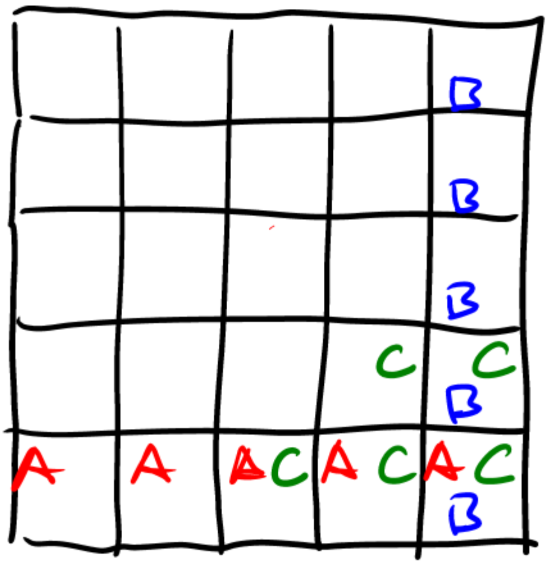
$Pr[A \mid C] = \frac{4/25}{4/25} = 1$

$Pr[B \mid C] = \frac{2/25}{4/25} = 1/2$

$Pr[A \cap B \mid C] = \frac{2/25}{4/25} = 1/2$

$\to$ A, B bzgl. C unabh.

b



$$\Pr[A] = 1/5$$

$$\Pr[B] = \frac{1}{5}$$

$$\Pr[A \cap B] = \frac{1}{25}$$

A, B unabh.

$$\Pr[A \mid C] = 3/5$$

$$\Pr[B \mid C] = 2/5$$

$$\Pr[A \cap B \mid C] = 1/5$$

A, B bzgl. C nicht bedingt unabh.