9.1

a) $K \sim \mathrm{Bin}(100, 7.5)$

Gesucht: $\Pr\left[a < \frac{K}{n} \leq b\right]$

$$= \Pr\left[\frac{K}{n} \leq b\right] - \Pr\left[\frac{K}{n} \leq a\right]$$

$$\Pr\left[\frac{K}{n} \leq b\right] = \Pr\left[K \leq b \cdot n\right]$$

$$= \Pr\left[\frac{K - \mathbb{E}[K]}{\sqrt{\mathrm{Var}[K]}} \leq \frac{b \cdot n - p \cdot n}{\sqrt{np(1-p)}}\right]$$

$$\approx \Phi\left(\frac{b-p}{\sqrt{p(1-p)}}\sqrt{n}\right)$$

$$b \in \left\{ \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10} \right\}$$

$$\leadsto \quad \frac{b-p}{\sqrt{p(1-p)}}\sqrt{n} \underset{\substack{n=100 \\ p=0.75}}{=} 4 \cdot \frac{10b - 7.5}{\sqrt{3}}$$

$$= \frac{40 \cdot b - 30}{\sqrt{3}}$$

$$\leadsto \Phi\left(\frac{-6}{\sqrt{3}}\right) = 1 - \Phi\left(\frac{6}{\sqrt{3}}\right) \approx 0$$

$$\Phi\left(\frac{-2}{\sqrt{3}}\right) = 1 - \Phi\left(\frac{2}{\sqrt{3}}\right) \approx 0.123$$

$$\Phi\left(\frac{2}{\sqrt{3}}\right) \approx \Phi(1.1547) \approx 0.87698$$

$$\Phi\left(\frac{6}{\sqrt{3}}\right) \approx \Phi(3.4641) \approx 0.99974$$

Notenverteilung:

5: ≈ 0

4: ≈ 0.123

3: ≈ 0.754

2: ≈ 0.123

1: ≈ 0

(ii) Nok 3 $\hat{=}$ $\quad \frac{K}{n} \in \left(\frac{7}{10}, \frac{8}{10}\right]$

$$\Pr\left[\frac{K}{n} \in \left(\frac{7}{10}, \frac{8}{10}\right]\right]$$

$$\approx \Phi\left(4 \cdot \frac{+0.05}{\sqrt{3}} \cdot \sqrt{n}\right) - \Phi\left(4 \cdot \frac{-0.05}{\sqrt{3}} \sqrt{n}\right)$$

$$= 2\,\Phi\left(\frac{0.2}{\sqrt{3}}\sqrt{n}\right) - 1 \overset{!}{\geq} 0.99$$

$$\rightsquigarrow \Phi\left(\frac{0.2}{\sqrt{3}}\sqrt{n}\right) \geq 0.995$$

$$\rightsquigarrow \frac{0.2}{\sqrt{3}}\sqrt{n} \geq 2.58$$

$$\rightsquigarrow n \geq 500$$

(b) $K \sim \mathrm{Bin}(n,p)$

$$\Pr[K > u] \geq 0.99$$

$$\| $$

$$\Pr\left[\frac{K - \mathbb{E}[K]}{\sqrt{\mathrm{Var}[K]}} > \frac{u - np}{\sqrt{np(1-p)}}\right] \approx \Phi\left(-\frac{u-np}{\sqrt{np(1-p)}}\right)$$

$$\rightsquigarrow \quad -\frac{u-np}{\sqrt{np(1-p)}} \geq 2.33$$

$$\rightsquigarrow \quad u \leq \left\lfloor np - 2.33\sqrt{np(1-p)} \right\rfloor$$

$$u(n,p) = np - 2.33\sqrt{np(1-p)}$$

$$u\left(100, \tfrac{6}{10}\right) \approx \lfloor 48.6 \rfloor = 48 =: u_4$$

$$u\left(100, \tfrac{7}{10}\right) \approx \lfloor 59.32 \rfloor = 59 =: u_3$$

$$u\left(100, \tfrac{8}{10}\right) \approx \lfloor 70.68 \rfloor = 70 =: u_2$$

$$u\left(100, \tfrac{9}{10}\right) \approx \lfloor 83.01 \rfloor = 83 =: u_1$$

Notenverteilung für $p = 0.75$, $n = 100$

$$\Pr[K \le u] = \Pr\left[K/n \le \frac{u}{n}\right]$$

wie in ⓐ $\approx \Phi\left(\frac{u/n - p}{\sqrt{p(1-p)}} \sqrt{n}\right)$

$u_4 = 48 \rightsquigarrow \Phi(-6.24) \approx 0.0$

$u_3 = 59 \rightsquigarrow \Phi(-3.70) \approx 0.00011 \approx 0.0$

$u_2 = 70 \rightsquigarrow \Phi(-1.15) \approx 0.13$

$u_1 = 83 \rightsquigarrow \Phi(1.85) \approx 0.97$

1: $1 - 0.97 \approx 0.03$

$\rightsquigarrow$ Notenverteilung:

5: $0.0 - 0 \approx 0.0$

4: $0.00011 - 0.0 \approx 0.0$

3: $0.13 - 0.00011 \approx 0.13$

2: $0.97 - 0.13 \approx 0.64$

9.2

$$X_1, X_2, \dots, X_{10} \sim \mathcal{N}(\underbrace{9.66}_{\mu}, \underbrace{0.011}_{\sigma^2})$$

ⓐ $$\Pr[X_{(1)} \le t] = 1 - (1 - \Phi_{\mu,\sigma}(t)) \overset{!}{\ge} 0.99$$

$$\boxed{\begin{aligned} &\Pr[X_i \le t] = \Phi_{\mu,\sigma}(t) \\ &\quad \shortparallel \\ &\Pr\left[\frac{X_i - \mu}{\sigma} \le \frac{t-\mu}{\sigma}\right] \\ &\qquad = \Phi\left(\frac{t-\mu}{\sigma}\right) \end{aligned}}$$

$$\rightsquigarrow 1 - \Phi_{\mu,\sigma}(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma}\right) = \Phi\left(-\frac{t-\mu}{\sigma}\right) \le 0.63$$

$$\rightsquigarrow -\frac{t-\mu}{\sigma} \le 0.33 \Rightarrow t \ge 9.65$$

ⓑ $$\Pr[X_{(3)} \le t] = \sum_{k=3}^{10} \binom{10}{3} \Phi_{\mu,\sigma}(t)^k \, (1 - \Phi_{\mu,\sigma}(t))^{10-k} \ge 0.99$$

$$\Leftrightarrow \Pr[X_{(3)} > t] = \sum_{k=0}^{2} \binom{10}{k} \Phi_{\mu,\sigma}(t)^k \, (1 - \Phi_{\mu,\sigma}(t))^{10-k}$$

$$\overset{!}{\le} 0.01$$

<u>Gesucht:</u> Nullstelle von

$$x^{10} + 10\ x^9(1-x) + 45\ x^8(1-x)^2 - 0.01$$

für $x \in [0, 1]$

$\rightsquigarrow x \approx 0.3883$

$\rightsquigarrow \Phi_{\mu,\sigma}(t) = \Phi\left(\frac{t-\mu}{\sigma}\right) \approx 0.3883$

$\rightsquigarrow \Phi\left(-\frac{t-\mu}{\sigma}\right) \approx 0.6117$

$\rightsquigarrow -\frac{t-\mu}{\sigma} \approx 0.3$

$\rightsquigarrow t = 9.65$

9.3

- $f_1(t) = I_{[0,1]}(t)$, $F_1(t) = t \cdot I_{[0,1]}(t) + I_{(1,\infty)}(t)$

$$f_2(t) = \int_{-\infty}^{\infty} f_1(s)\, f_1(t-s)\, ds$$

vorgegeben

$$= \int_{-\infty}^{\infty} I_{\boxed{[0,1]}}(s)\, \underbrace{I_{[0,1]}(t-s)}_{= I_{\boxed{[t-1,t]}}(s)}\, ds$$

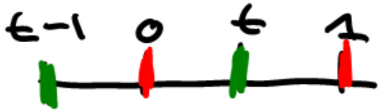
Fälle:

(a) 

$\rightsquigarrow t > 2: f_2(t) = 0$

(b) 

$\rightarrow t < 0: f_2(t) = 0$

(c) 

$\rightsquigarrow 0 < t \le 1: f_2(t) = \int_0^t 1\, ds = t$

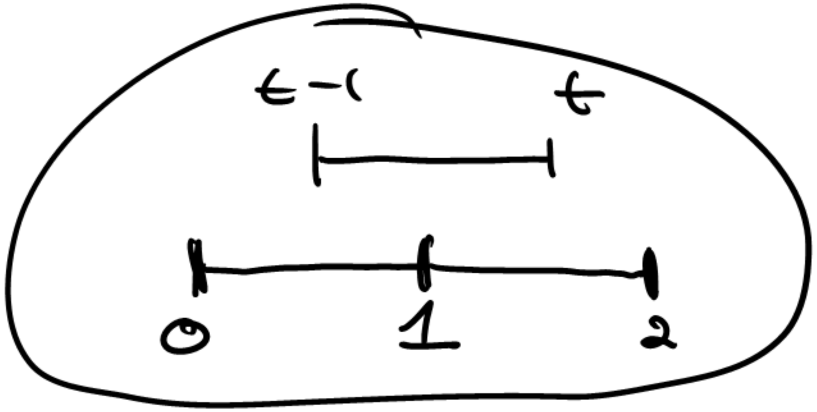
(d) 

$\rightsquigarrow 1 < t \le 2: f_2(t) = \int_{t-1}^{1} 1\, ds = 2 - t$

so $$f_2(t) = t\, I_{[0,1]}(t) + (2-t)\, I_{[1,2]}(t)$$

$$f_3(t) = \int_{-\infty}^{\infty} f_2(s)\; f_1(t-s)\; ds$$

$$= \int_{-\infty}^{\infty} \left(s\, I_{[0,1]}(s) + (2-s)\, I_{[1,2]}(s) \right) \overset{I_{[t-1,t]}(s)}{\overset{\parallel}{I_{[0,1]}(t-s)}}\; ds$$



$$= \begin{cases} t<0 \vee t>3 : & 0 \\ 0 \le t < 1 : & \int_0^t s\, ds = \frac{1}{2} t^2 \\ 1 \le t < 2 : & \int_{t-1}^{1} s\, ds + \int_1^t (2-s)\, ds = -\left(t - \frac{3}{2}\right)^2 + \frac{3}{4} \\ 2 \le t < 3 : & \int_{t-1}^{2} (2-s)\, ds = \frac{1}{2}(t-3)^2 \end{cases}$$

$$F_3(t) = \int_{-\infty}^{t} f_2(s)\,ds$$

$$= \frac{1}{6} t^3 I_{[0,1]}(t) +$$

$$+ \left(\frac{1}{6} + \left(\frac{3}{4}s - \frac{1}{3}\left(s - \frac{3}{2}\right)^3\right)\Big|_1^t\right) I_{(1,2]}(t)$$

$$+ \left(\frac{5}{6} + \left(\frac{1}{6}(s-3)^3\right)\Big|_2^t\right) I_{(2,3]}(t)$$

$$+ I_{(1,\infty)}(t)$$

$$= \begin{cases} \bullet\ t < 0 : 0 \\ \bullet\ 0 \le t \le 1 : \frac{1}{6} t^3 \\ \bullet\ 1 < t \le 2 : -\frac{5}{8} + \frac{3}{4}t - \frac{1}{3}\left(t - \frac{3}{2}\right)^3 \\ \bullet\ 2 < t \le 3 : 1 - \frac{1}{6}(t-3)^3 \end{cases}$$

$\bullet\ t > 3 : 1$