Mathematical Model IN Image Proces.

① Array verses Matrix operation.

(array product)

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{22} & a_{22} \end{bmatrix} * \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{22} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

* Matrix product

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

② Linear verses Non-linear:-

Linear :- $H[a_i f_i(x,y) + a_j f_j(x,y)] = a_i H[f_i(x,y)] + a_j H[$

$$= \boxed{a_i g_i(x,y) + a_j g_j(x,y}$$

eg:- $f_1 = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$ $f_2 = \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}$

$$\max \left\{ (1) \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + (-1) \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix} \right\} = \max \left\{ \begin{bmatrix} -6 & -3 \\ -2 & -4 \end{bmatrix} \right\}$$

③ Arithmetic Operation:-

Sum

① $s(x,y) = f(x,y) + g(x,y)$.

Affrence

② $d(x,y) = f(x,y) - g(x,y)$.

③ $p(x,y) = f(x,y) * g(x,y)$.

④ $v(x,y) = f(x,y) \div g(x,y)$.

④ Set operations:-

$$A - B = \{w / w \in A, w \notin B\} = A \cap B^c$$

* gray scales sets A & B =

$$A \cup B = \{\max(a,b) \mid a \in A, b \in B\}$$

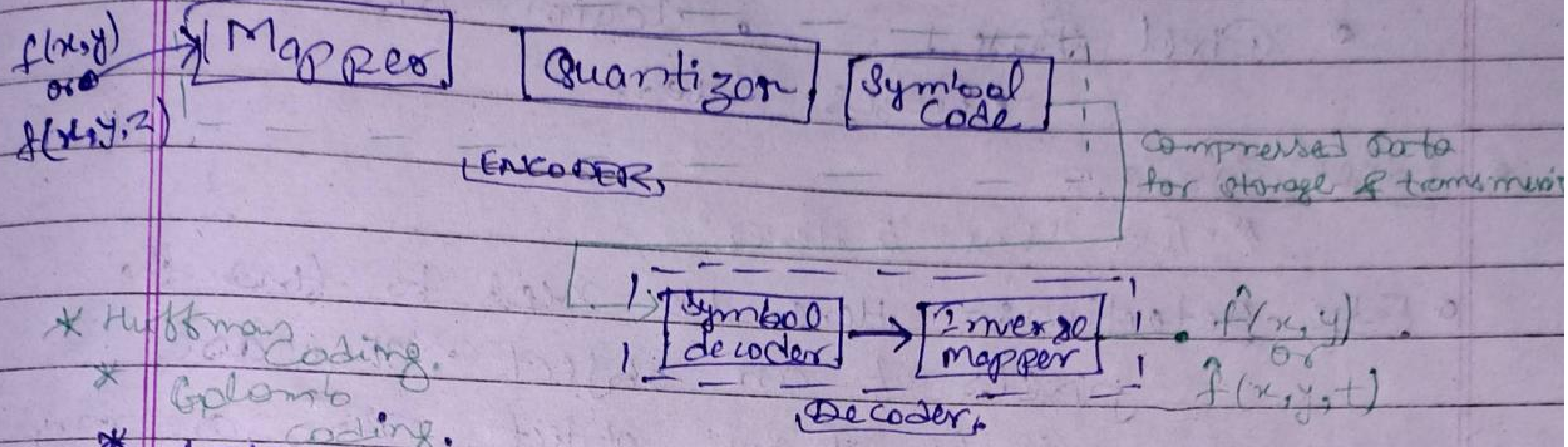
⑤ Logical operations:- [fuzzy set].

⑥ Spartial operation = single pixel, Neighbour pixe

⑦ Vector & Matrix operation =

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

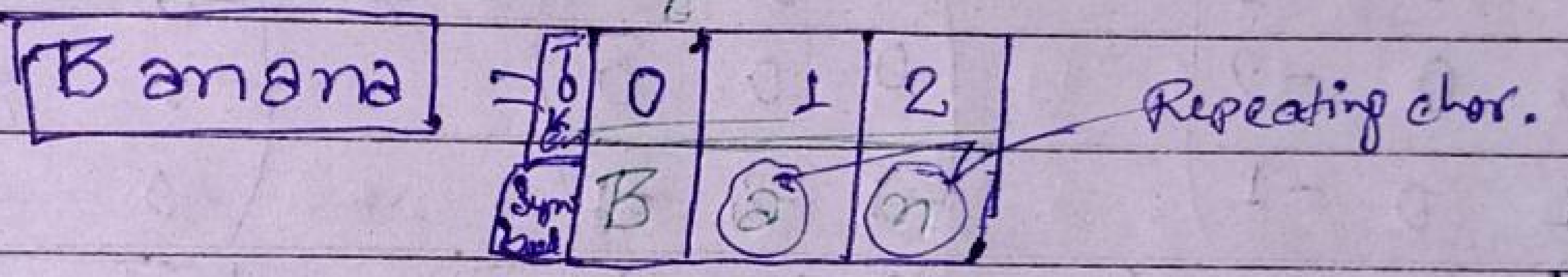
* Image compression Model.



- * Huffman Coding.
- * Golomb coding.
- * LZW coding

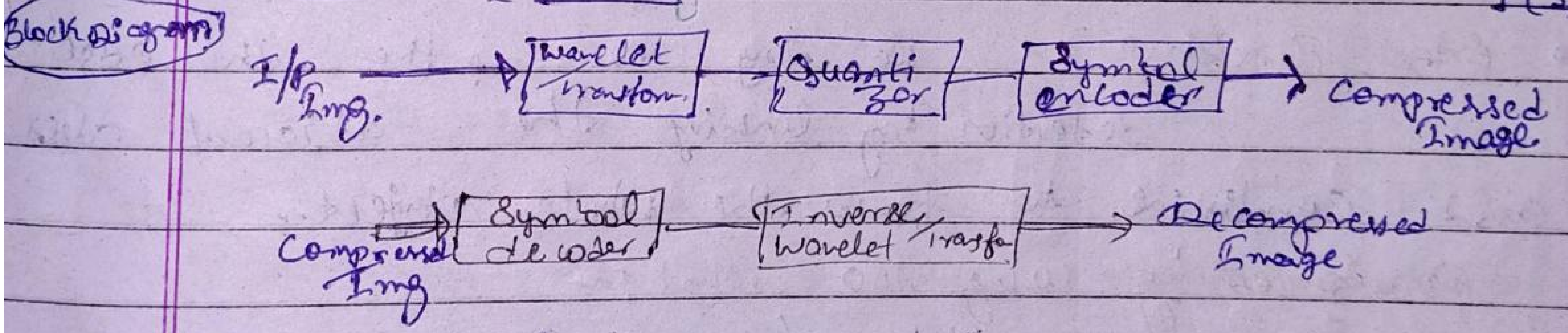
▷ Example of ~~BT~~ Compression:-

* Symbol based coding

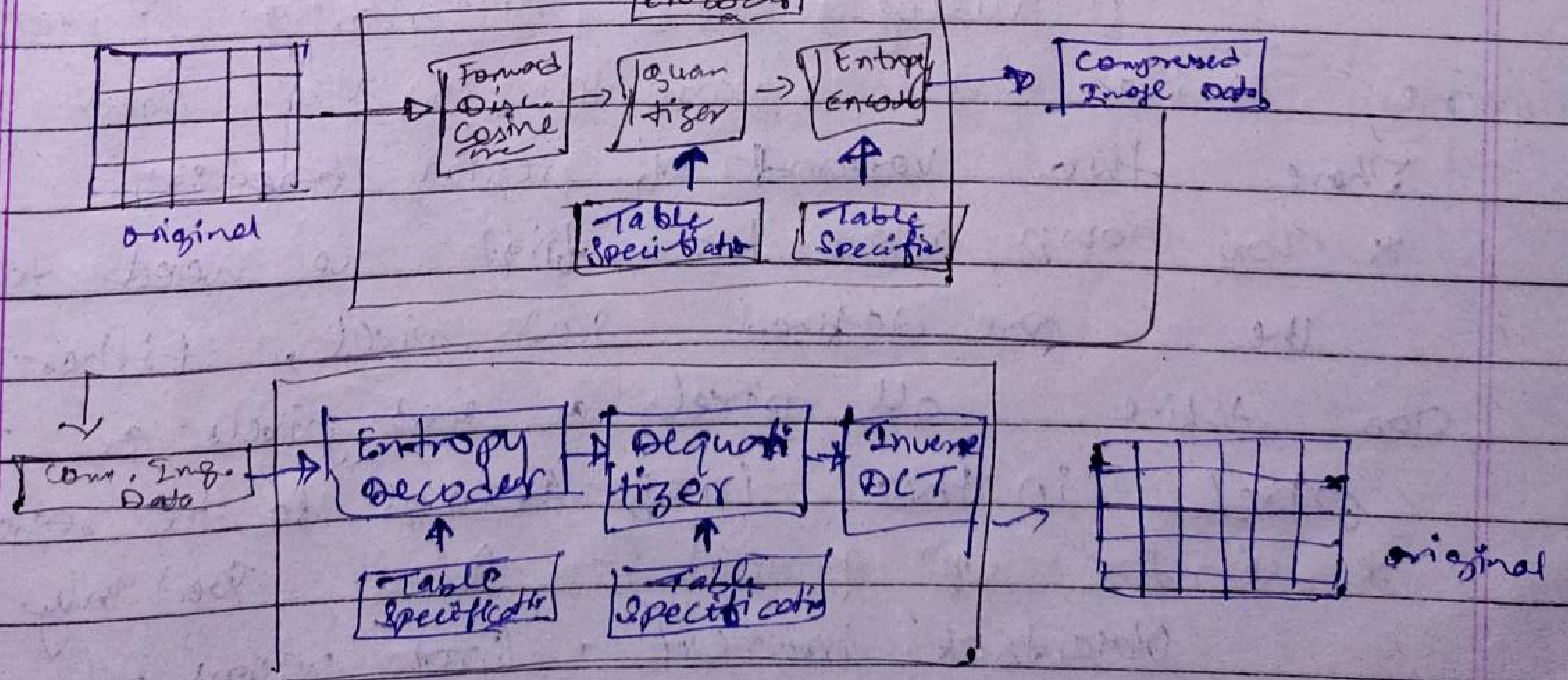


Triplet
(0,2,0)
(3,10,1)
(3,18,2)
(3,26,1)
(3,34)2
(3,42,1)

* Wavelet coding:-



* JPEG compression Block Diagram:-



* Region Based Segmentation *

PAGE NO.
DATE : / /20

* Three Methods of performing segmentation.
- Pixel Based
- Region Based
- Edges Based.

- Edges Based:-

  * Two steps is Heres.

  - Edge Detection:- Here, we need to find the pixels of an object. There are ~~many~~ edge pixels of an object. There are many object Detection methods such as
    - Sobel operator, Laplace operators, canny, etc.

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

Sobel vertical — Horizontal sobel — negative Laplace

  - Edge linking:- we try to refine the edge detection by linking the adjacent edges and combine to form the whole object. It performed by using two method:-
    * Local * Global processing.

- Region Based:- We grow regions by recursively including the neighboring pixels that are similar and connected to the seed pixel. There two variant of region based:-
  - * Top Down approach:- first we need to define the pre defined seed pixel, Either we can define all pixels as seed pixels or randomly pixel in the image belongs to the region.
  - * Bottom-UP approach:- select seed only from objects of interest. Grow regions only if the similarity criterion is fulfilled.

Basic Image

$$\boxed{A^{*}_{k,l} = a^{*}k \cdot a^{*}l}$$

$$\boxed{V = A U A^{T}}$$

$$\begin{bmatrix} 3 & 6 \\ 6 & 4 \end{bmatrix}$$

* Mopho Logical * Operators.

*→ Mophological image processing (or morphology) describes a range of image processing technique that deals with the "shape' of features" in an image.

△ This operations are applied to remove imperfections, introduced during segmentation. * Reflection & Translation.

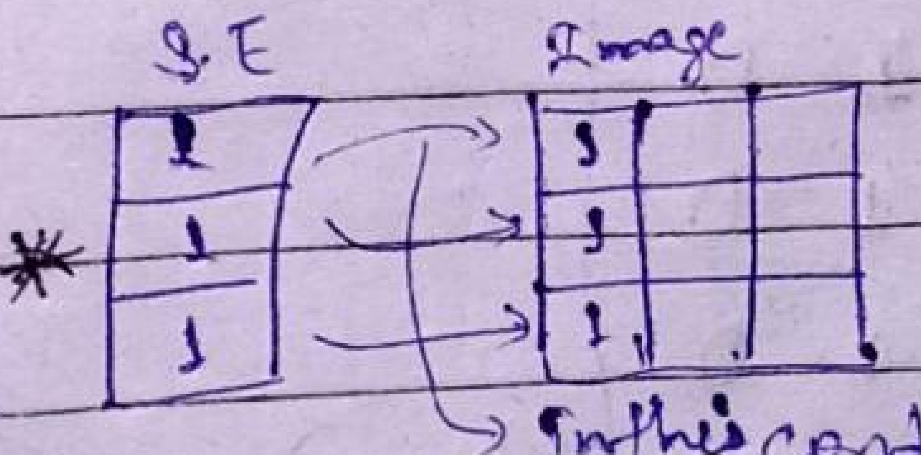
* Structuring element:-

△ It is a Small set to probe the Img Under Study

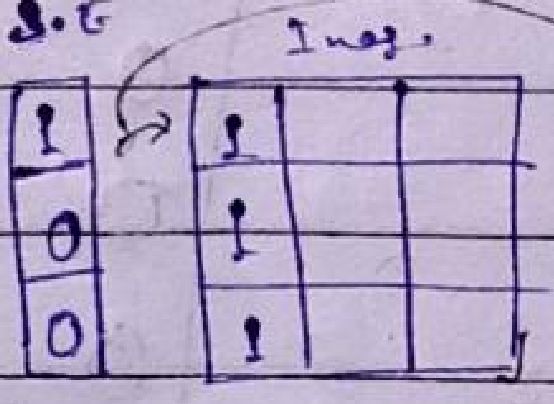
△ The shape & size must be adapted. to geometric properties for the object.

△ we check for three Condition While applying the SE.

↳ Fit ↳ Hit ↳ Miss.

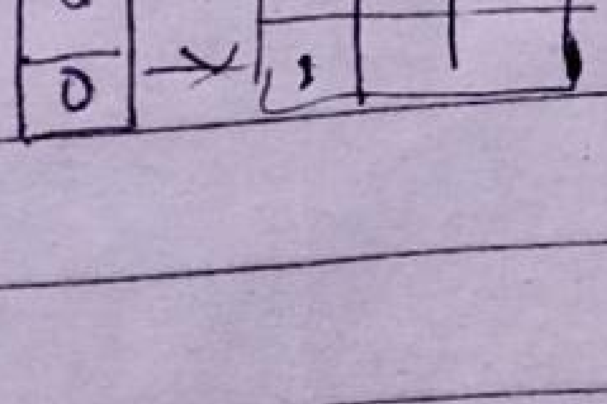


In this condition FIT



only one match

* Its HIT (when single Mat)



* Miss

No one match.

Dilation

Image

Numerical

Q) Compute A⊕B for the following input image

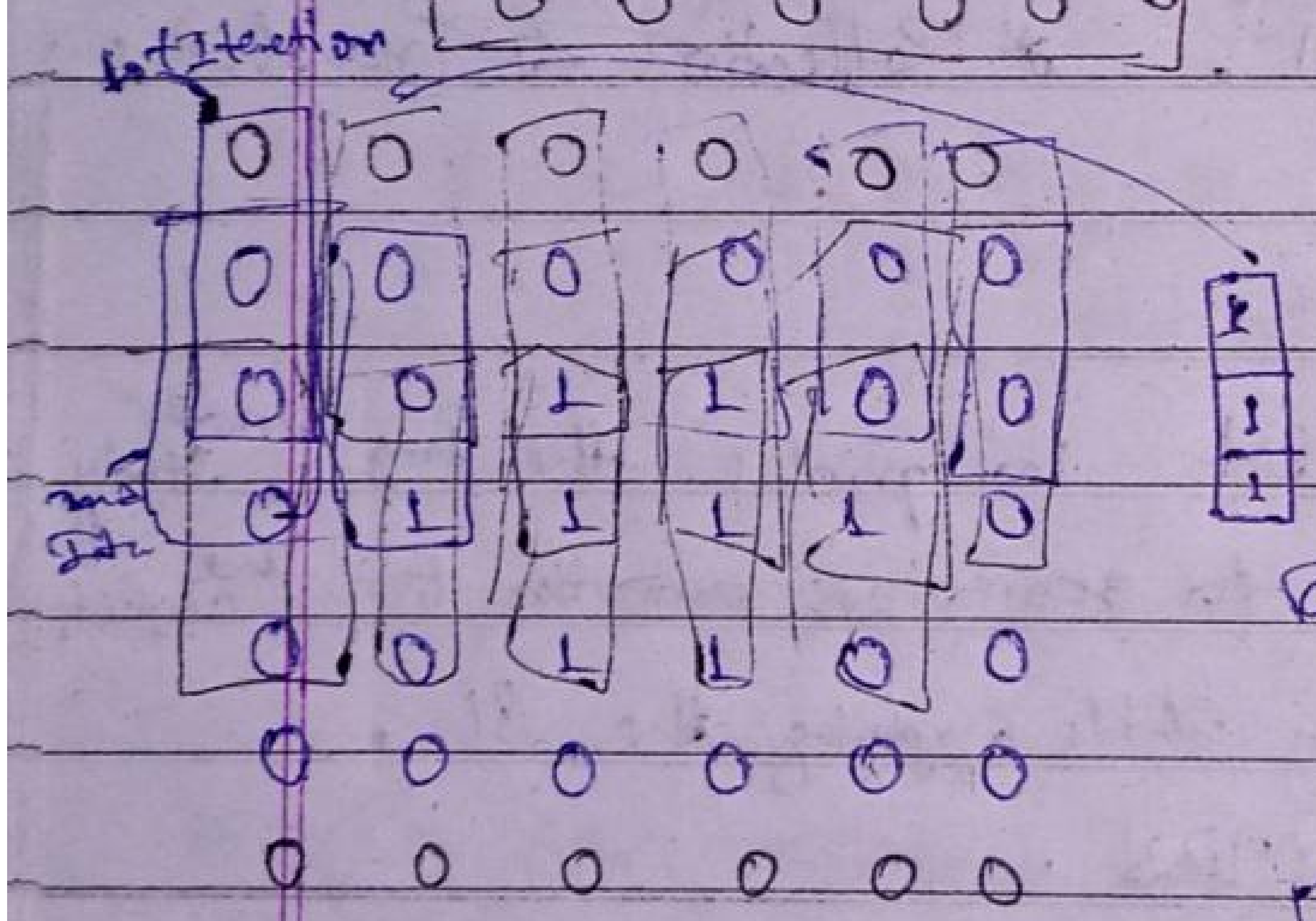
$$\underset{(A)}{\overset{\text{I/P Image}}{\begin{bmatrix} 0&0&0&0&0&0\\ 0&0&1&1&0&0\\ 0&1&1&1&1&0\\ 0&0&1&1&0&0\\ 0&0&0&0&0&0 \end{bmatrix}}} \qquad \underset{B}{\overset{SE}{\begin{bmatrix} 1\\ \textcircled{1}\\ 1 \end{bmatrix}}}$$

Sol^n^ First we will pad the Image A at top and bottom.

$$\begin{bmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&1&1&0&0\\ 0&1&1&1&1&0\\ 0&0&1&1&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{bmatrix} \text{ Zero padded}$$

* Then we will place B over A & check for following condition for dilation.

- ▷ Full Match = $\boxed{1}$
- ▷ Some Match or atleast one Match = $\boxed{1}$
- * No match = $\boxed{0}$



Checking one by one.

Output Img

$$\begin{bmatrix} 0&0&1&1&0&0\\ 0&1&1&1&1&0\\ 0&1&1&1&1&0\\ 0&1&1&1&1&0\\ 0&0&1&1&0&0 \end{bmatrix}$$

Answer

After dilation

Region-Based Segmentation

A edges and thresholds sometimes donot give good result ④ segmentation.

① Region Growing:— steps:—

① Find all Connected components in $S(x,y)$ and reduce each connected components to one pixel label all such pixels found as 1. All other pixels in 'S' are labelled 0.(zero).

Image.

② Form an image f_Q such that, at each point (x,y), $f_Q(x,y) = 1$ if the input image satisfies a given predicate, Q, at these Cordinate and $f_Q,(x,y) = 0$ otherwise

* Merging:

* Procedure for Region Splitting & Merging:—

① If a region R is inhomogeneous ($P(R) = \text{False}$) then R is split into four sub-region.

② If 2 adjacent region R_i, R_j are homogeneous ($P(R_i \cup R_j) = \text{True}$), then they are Merged.

③ The algorithm stops when no further Splitting or merging is possible.

Note condition for region growing:—

~~bb~~ [abs (seed value − pixel value) ≤ Threshold

Q) ② Apply region growing on the following image with seed point as 6 and threshold value as 3.

Solⁿ

seed

5^a	6^a	6^a	7^a	6^a	7^a	6^a	6^a
6	7^a	6^a	7^a	5^a	5^a	4^a	7^a
6^a	6^a	4^a	4^a	3	2	5^a	6^a
5^a	4^a	5^a	4	2	3	4^a	6^a
0	3	2	3	3	2	4^a	7^a
0	0	0	0	2	2	5^a	6^a
1	1	0	1	0	3	4^a	4^a
1	0	1	0	2	3	5	4

* seed point ⇒ $\boxed{6}$
* T = $\boxed{3}$

a

Condition → absolute difference <= ~~0~~ $\boxed{3}$ $\boxed{6}$

* 8-way connectivity

Now

a = 1

else 0;

Segmented Image

O/P

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	0	0	1	1
1	1	1	0	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0

Q). ① Apply region growing on the following image with initial point at (2,2) and threshold value as use 4-connectivity

Solⁿ

Conditions-

<= 2

	0	1	2	3
0	0	1	2	0
1	2	5^a	6^a	1
2	1	4^a	7	*3
3	0	2	5^a	1

next

(2,2) gi

$\boxed{T = 2}$

next

Frequency filters

① Spartial Domain :-

filter or mask

$$f(x,y) \rightarrow \boxed{h(x,y)} \rightarrow g(x,y).$$

Image

$$g(x,y) = f(x,y) * h(x,y)$$

$h(x,y)$: Input responce.

② frequency Domain

$$F(u,v) \rightarrow \boxed{H(u,v)} \rightarrow G(u,v)$$

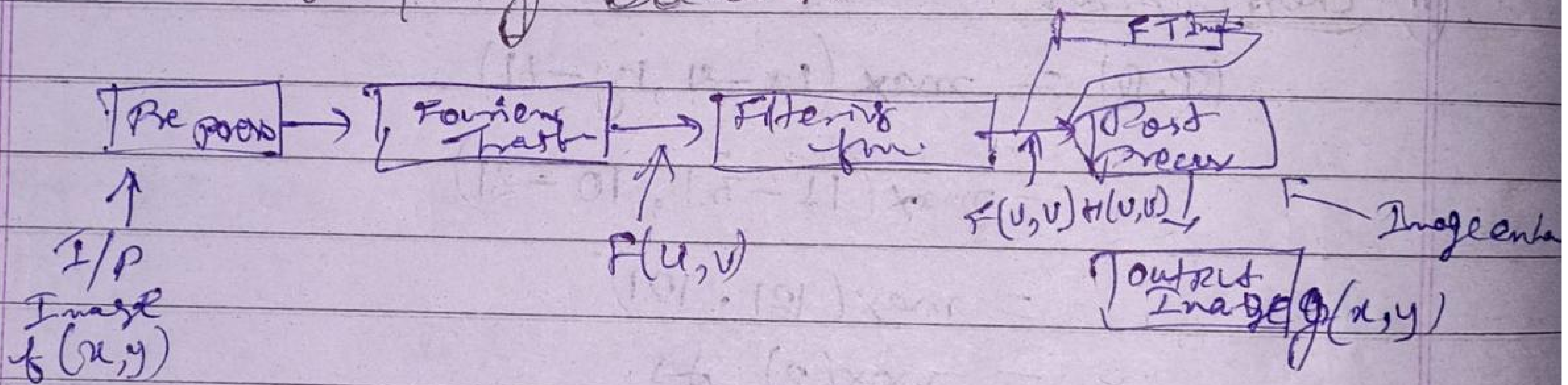
~~F(x,y)~~

$$G(u,v) = F(u,v) * H(u,v)$$

$$\left[g(x,y) = F^{-1}\left(F(u,v)\,H(u,v)\right)\right]$$

$H(u,v)$: Transfer function

** * Steps for filtering in frequency Domain :-



① Multiply the Ip image by $(-1)^{x+y}$ to center the transform.

② Compute F(u,v), the DFT of the image from (1)

③ Multiply F(u,v) by a filter function H(u,v)

④ compute the inverse DFT of the result in $\boxed{3}$ step

⑤ obtain the real part of the result in $\boxed{4}$

⑥ Multiply the result in ⑤ by $(-1)^{x+y}$,

used for origi

Q7. Convert the given spartial domain image using fourier transform & perform ideal low pass filter to ~~th~~ smoothen the image. Choose D_0 as $\boxed{D_0} = \boxed{\sqrt{0.5}}$ Show the step by step procedure for doing the same.

Sol^n Note: $= D(u,v) = \sqrt{(u-\frac{m}{2})^2+(v+\frac{N}{2})^2} = \sqrt{(u-2)^2+(v-2)^2}$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$ given

1st step: multiply the I/p Img. by $(-1)^{x+y}$ to center transform

$(-1)^{0+0}$ $(-1)^{0+1}$ $(-1)^{0+2}$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 \end{bmatrix}$$ Ans of step ①

2nd step: Compute the DFT of the given image from ①

$$\left[F(u,v) = \text{Kernel} \times f(x,y) \times \text{Kernel}^{T}\right]$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$