

# Fuzzy Logic

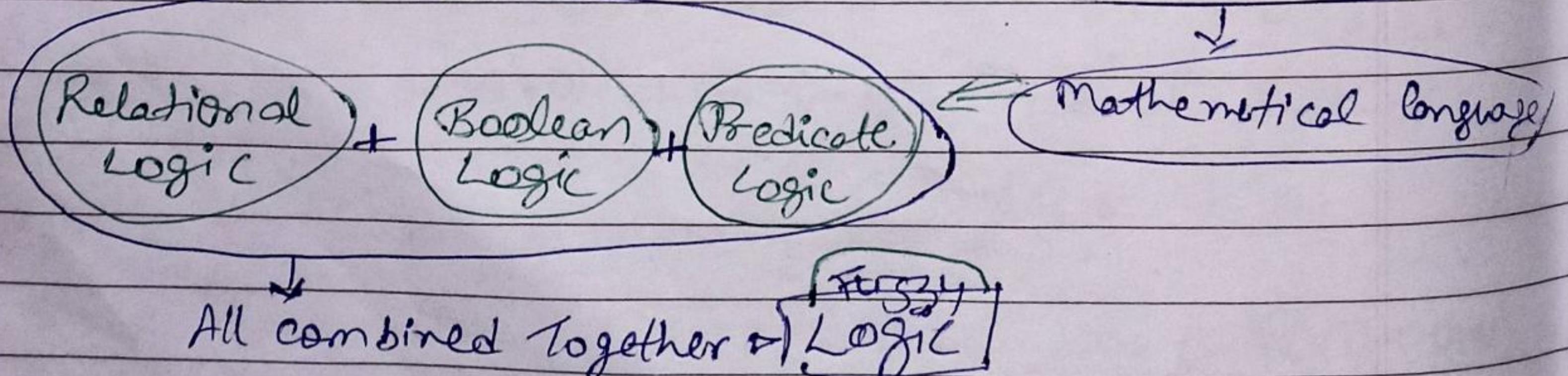
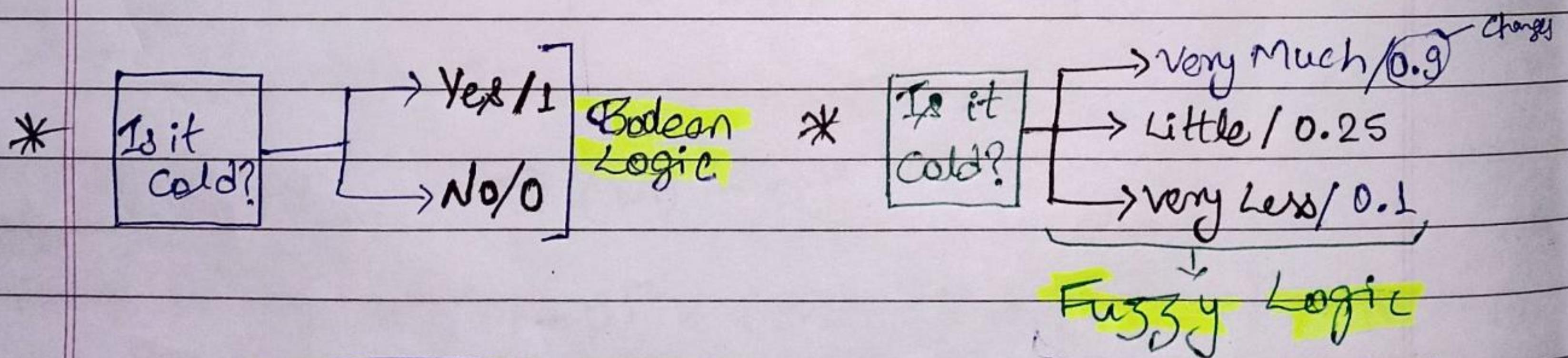
\* Fuzzy logic:- It is the term Fuzzy refers to things that are not clear or are vague. \* The condition which don't have desire Yes or No.

\* Many situation we can't determine whether the state is true or false, their fuzzy logic provides very valuable flexibility for reasoning.

► In the boolean system, value,  $1.0$  represents the absolute truth value and  $0.0$  represents the absolute 'false' value.

Note But, In fuzzy system, there is no logic for the absolute truth or false value.

► It contains only partially true and partially false



\* Fuzzy logic deals with Fuzzy set / Fuzzy Algebra.

# Crisp Logic vs Fuzzy logic

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## \* Crisp Logic

Crisp O/P

→ Yes or No

→ True or False

Fuzzy O/P

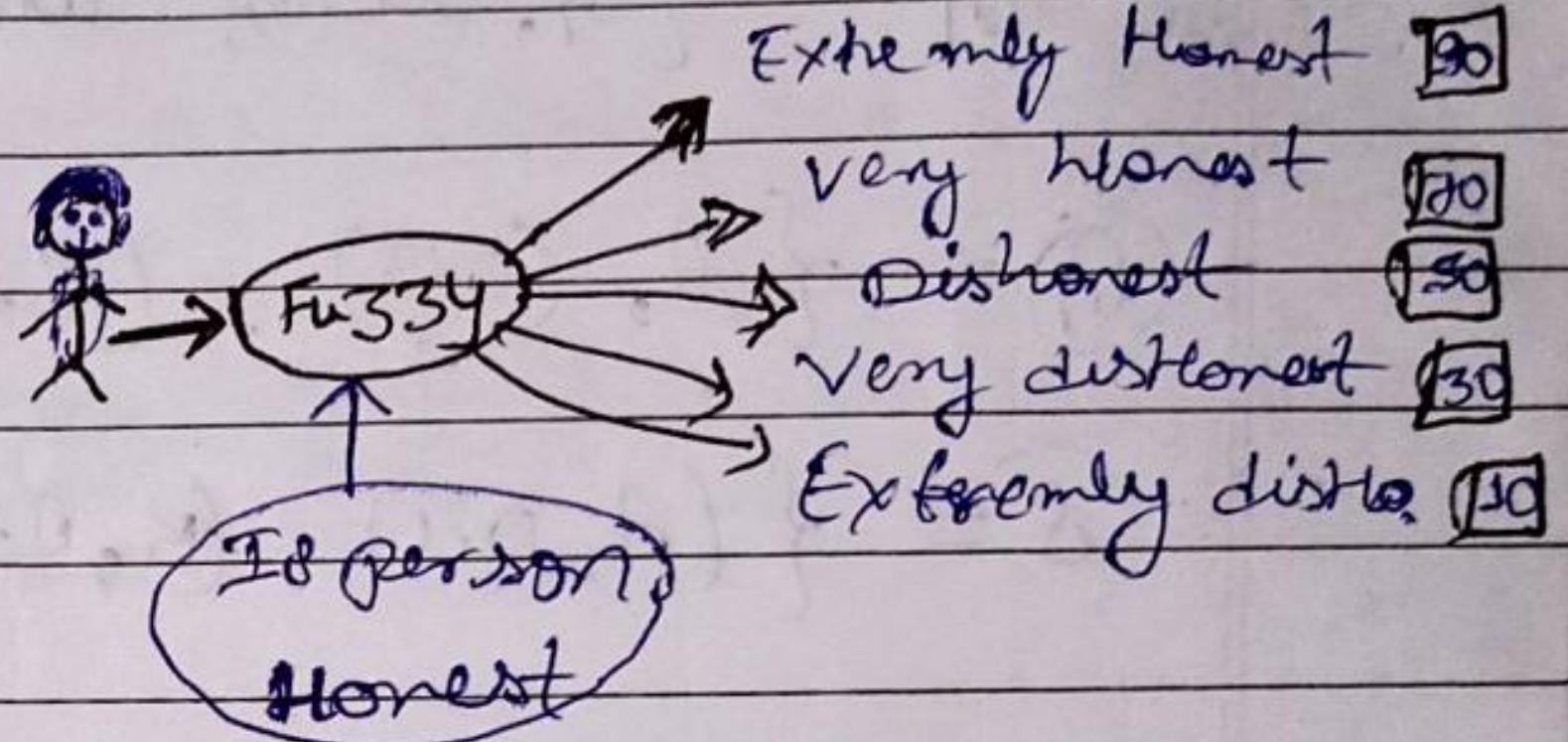
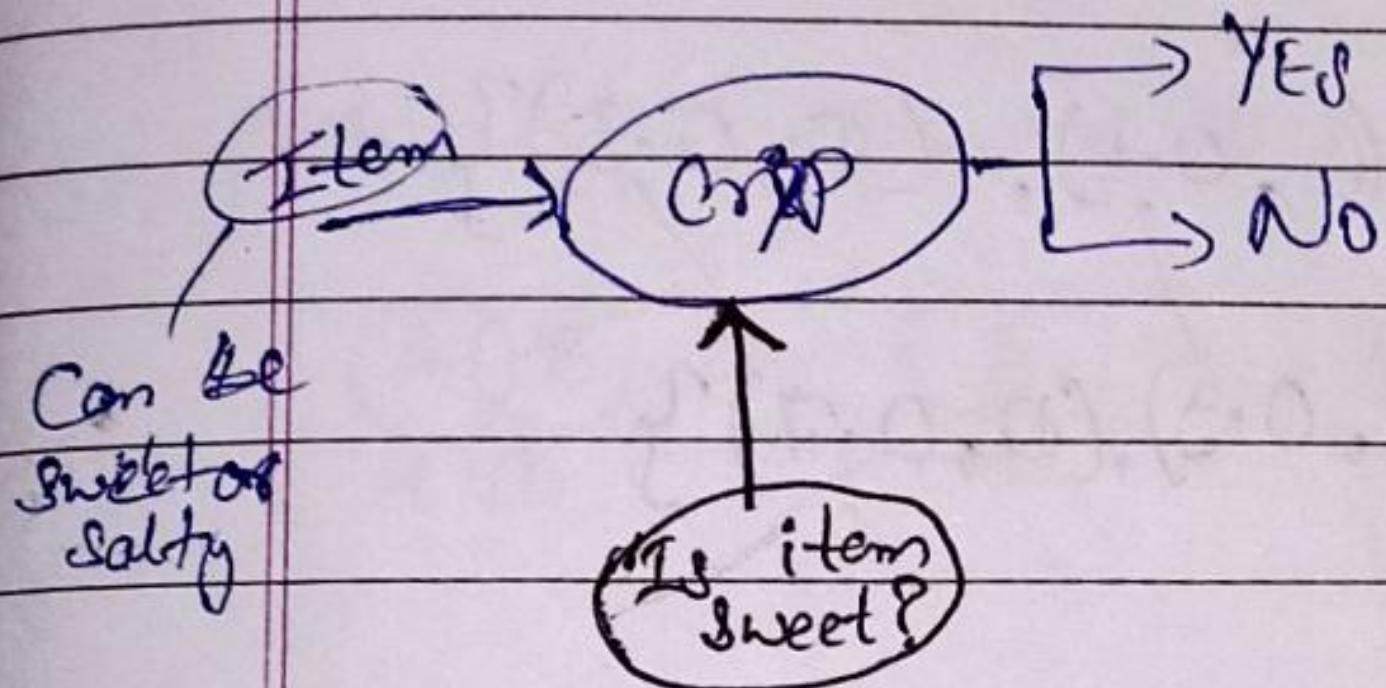
→ may be

→ may not be

→ Absolutely

→ partially.

Score

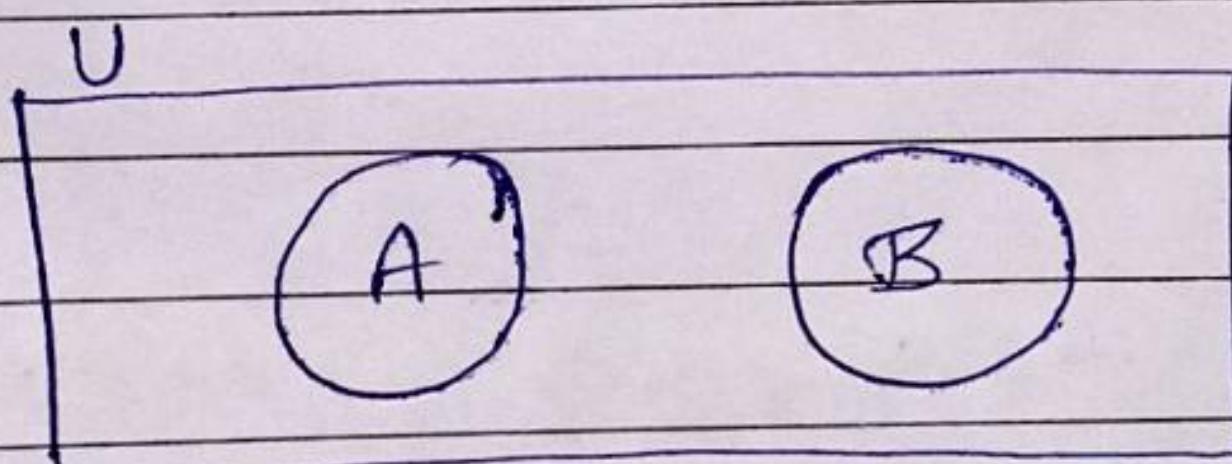


## \* Crisp P. Set :-

U : All students

A : In 10<sup>th</sup>

B : In 12<sup>th</sup>



{ \*↑ In this not a single individual  
 who available in Both, & its }  
 is Crisp set (yes or no).

→ An element only can belongs to one set, Not more than one.

# Fuzzy SETS

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U: All students ; G: Good students.  
S: Bad students.

Membership function  $G = \{G_i, M_i\}$  ( $M_i$ ) degree of Goodness.

$$G = \{A, (0.9), (B, 0.7), (C, 0.1), (D, 0.3)\}$$

$$S = \{(A, 0.1), (B, 0.3), (C, 0.9), (D, 0.7)\}$$

- Note :- Membership value are ranging from 0 to 1 (also  $0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ )
- ↳ If m.value = 0 then we ignore that variable.
  - ↳ Sets only contain ( $0 < \text{values} \leq 1$ ) or element.
  - ↳ IN this A element can present in two sets.

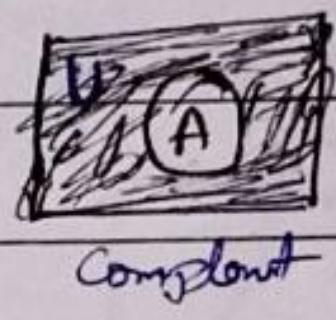
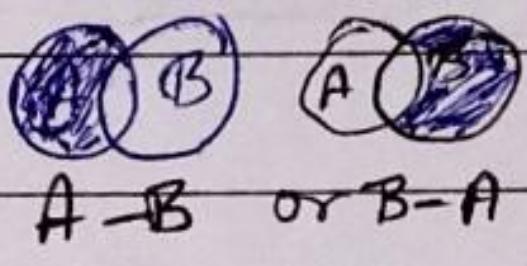
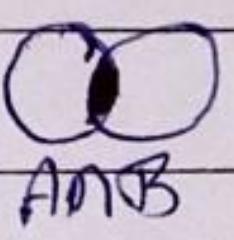
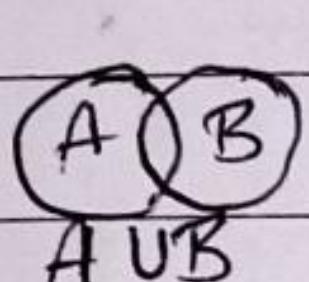
# \* Classical Sets

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► Classical set is a collection of distinct objects. Ex:- A set of students passing grade

## # Operations on Cl. set:-

Union, Intersection, Difference, Complement of set, Cartesian Product.



## \* Property of Classical sets:-

① Commutative Property :-  $[A \cup B = B \cup A]$ , or  
 ~~$A \cap B = B \cap A$~~

② Associative Property :-  $[A \cup (B \cup C) = (A \cup B) \cup C]$   
 $[A \cap (B \cap C) = (A \cap B) \cap C]$

③ Distributive Property :-  $[A \cup (B \cap C) = (A \cup B) \cap (A \cup C)]$   
 $[A \cap (B \cup C) = (A \cap B) \cup (A \cap C)]$

④ Idempotency Property :-  $[A \cup A = A]$  and  $[A \cap A = A]$

⑤ Identity Property :-  $[A \cup \emptyset = A]$   $[A \cap X = A]$   $[A \cap \emptyset = \emptyset]$   
 $[A \cup X = X]$   
X: Universal set

⑥ Transitive Property :- if  $A \subseteq B \subseteq C$ , then  $A \subseteq C$

⑦ Involution :-  $\bar{\bar{A}} = A$

⑧ De Morgan's Law :-  $\overline{A \cap B} = \bar{A} \cup \bar{B}$   $\overline{A \cup B} = \bar{A} \cap \bar{B}$

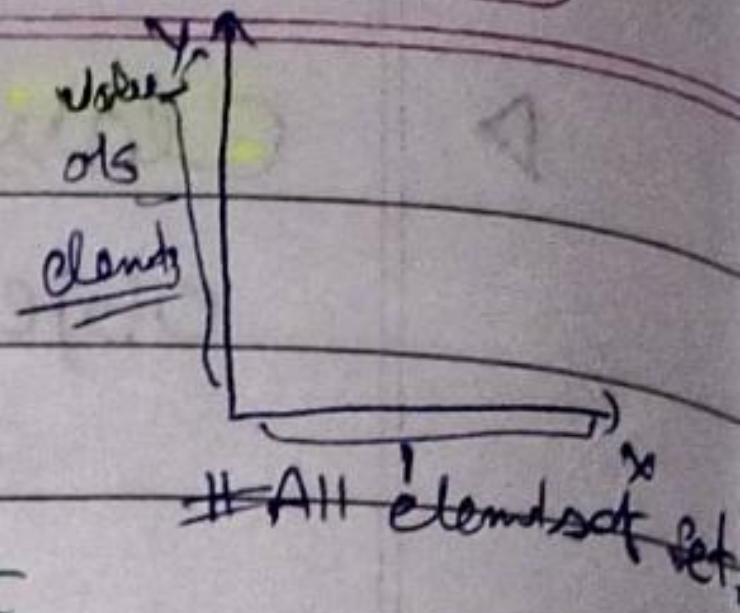
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# MEMBERSHIP FUNCTION

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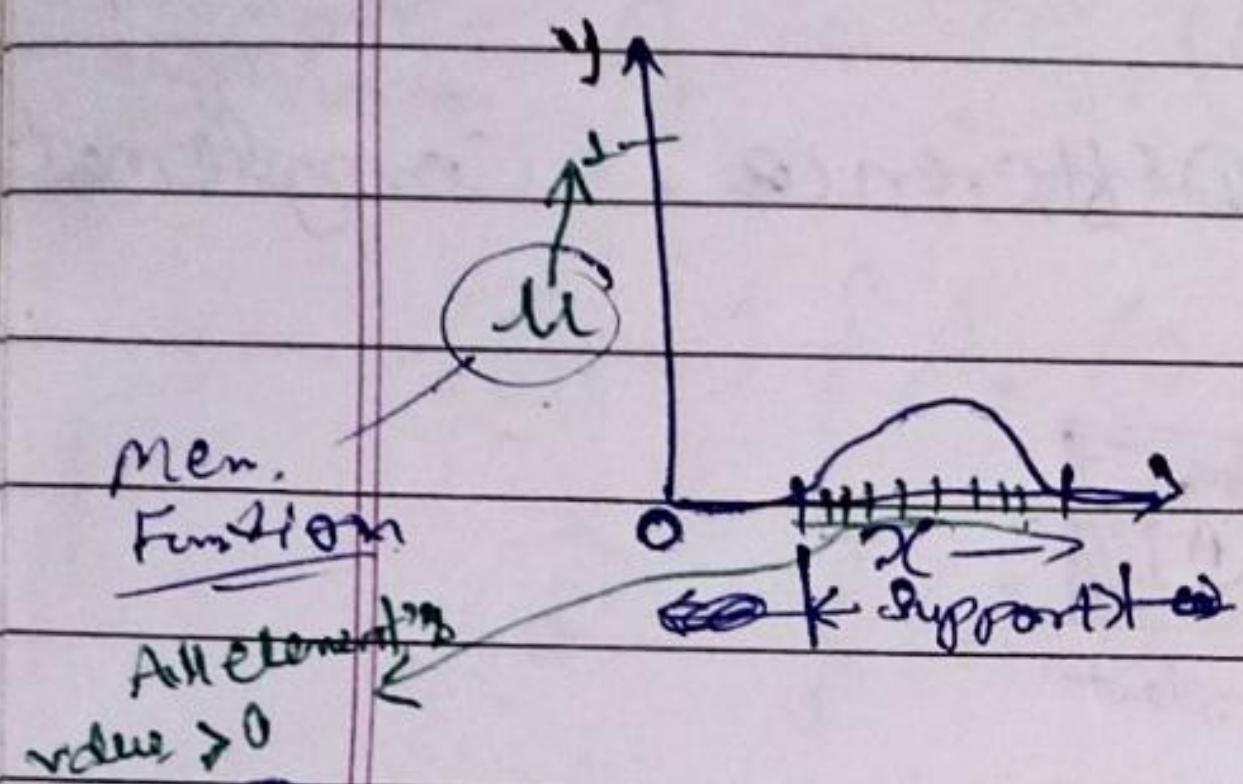


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- (I) **Support** :- 'Set' of all points such that

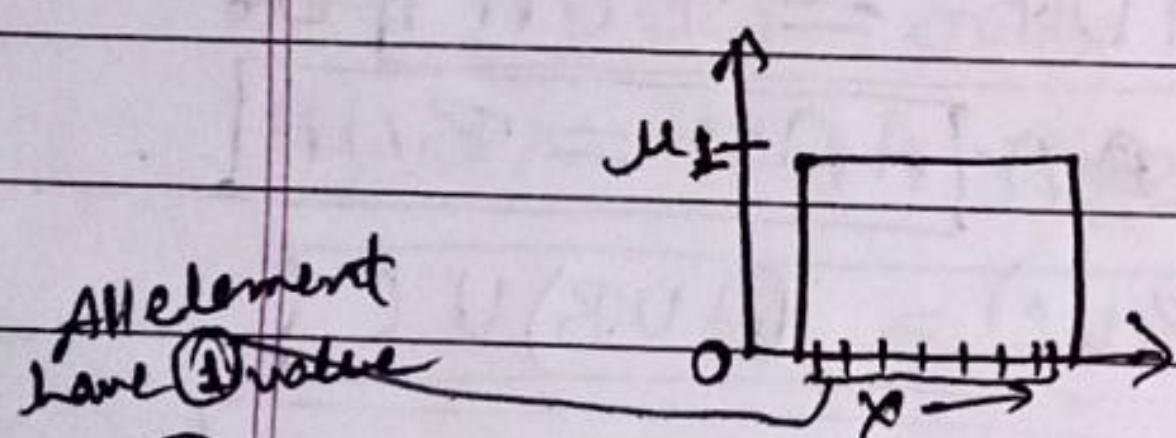
$$\{\mu_A(x) > 0\}$$



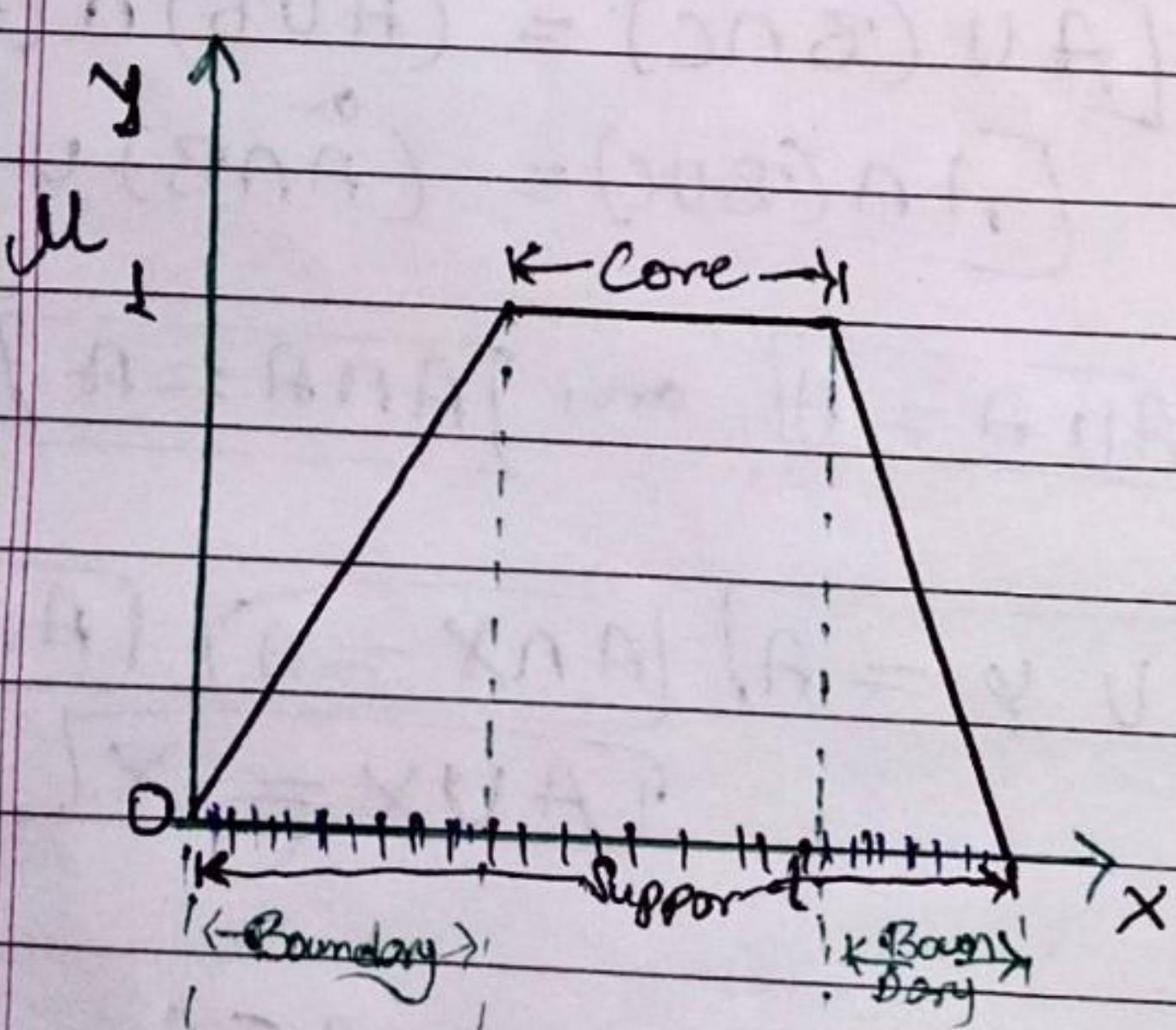
$$\text{Support}(A) = \{x | \mu_A(x) > 0\}$$

- (II) **Core** :-  $\{\mu_A(x) = 1\}$

$$\text{Core}(A) = \{x | \mu_A(x) = 1\}$$



- (III) **Boundary** :-  $(1 > \mu_A(x) > 0)$

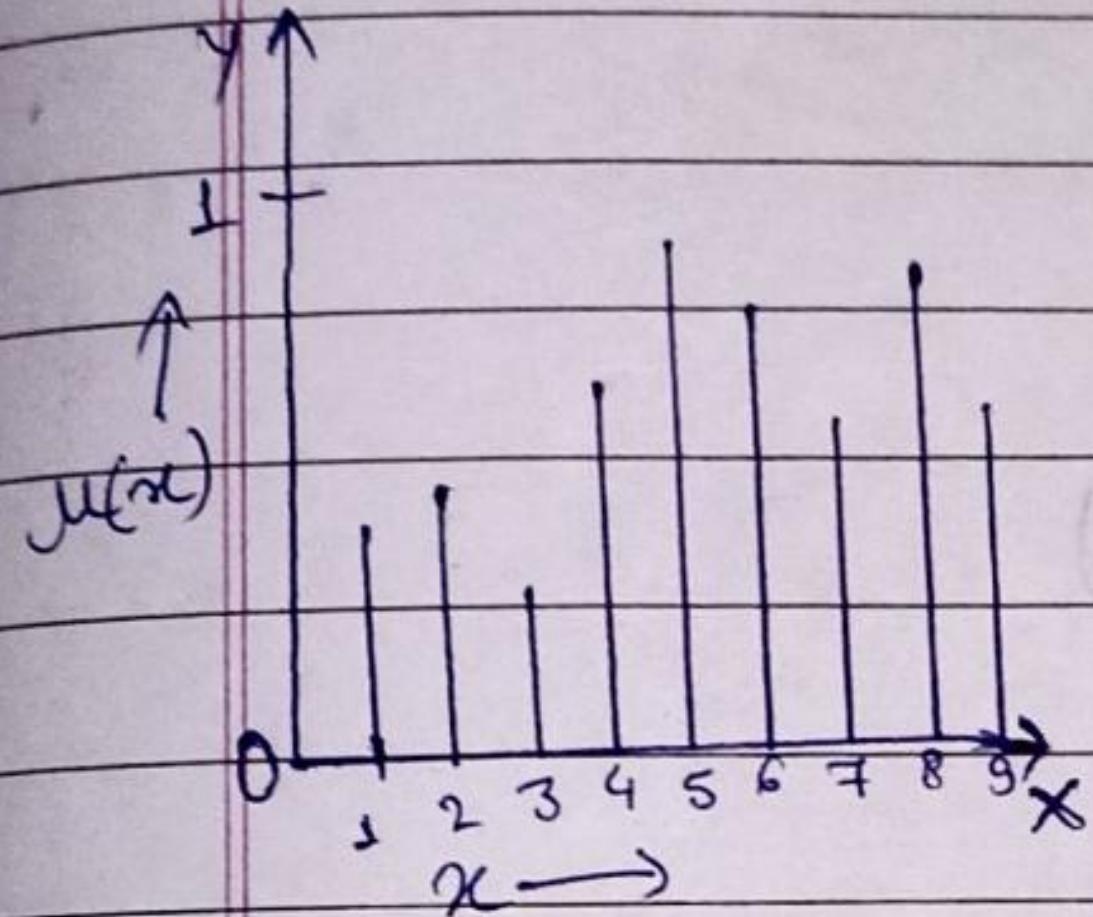


**Note** \* Those elements who belongs to core, never belongs to boundary

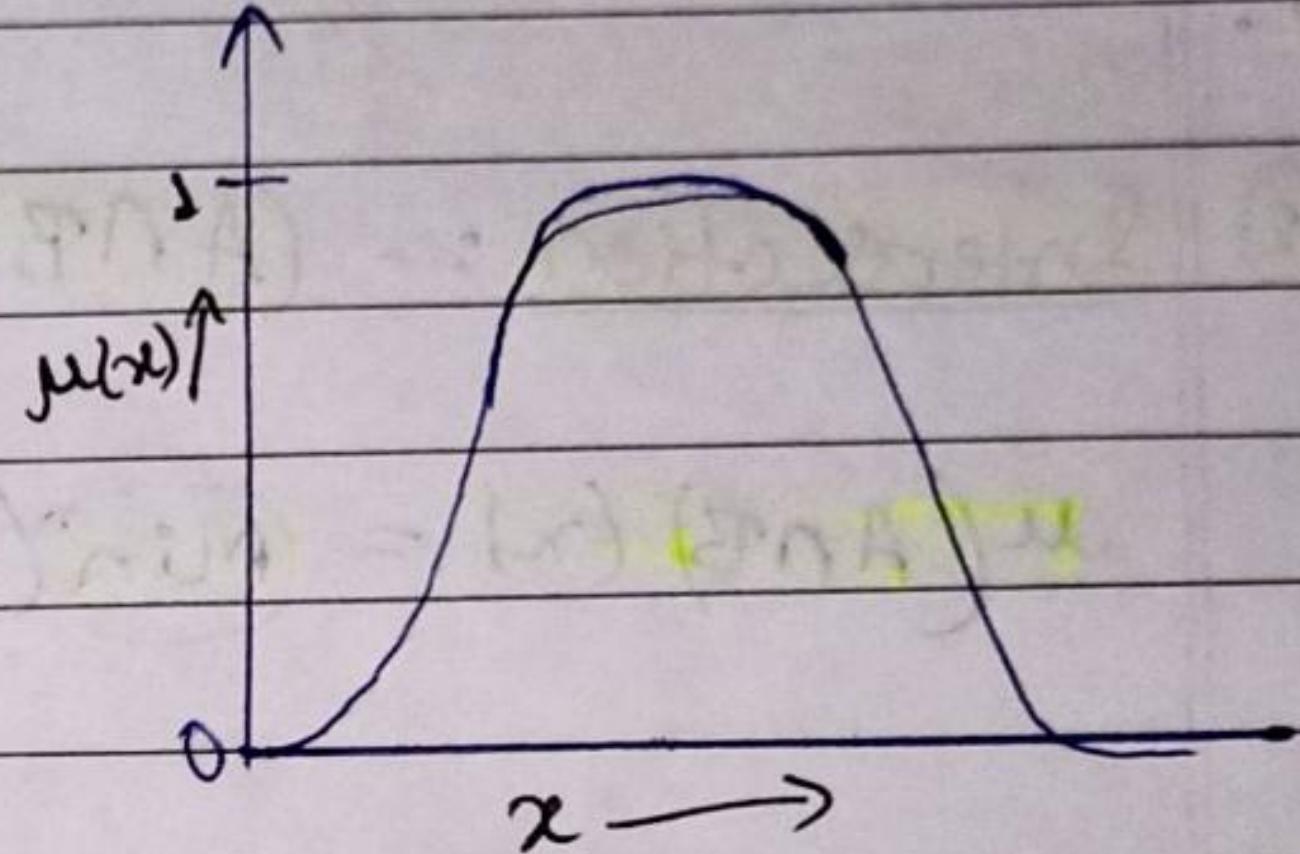
# Membership Function

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## \* Discrete M.F :-



## \* Continuous M.F :-

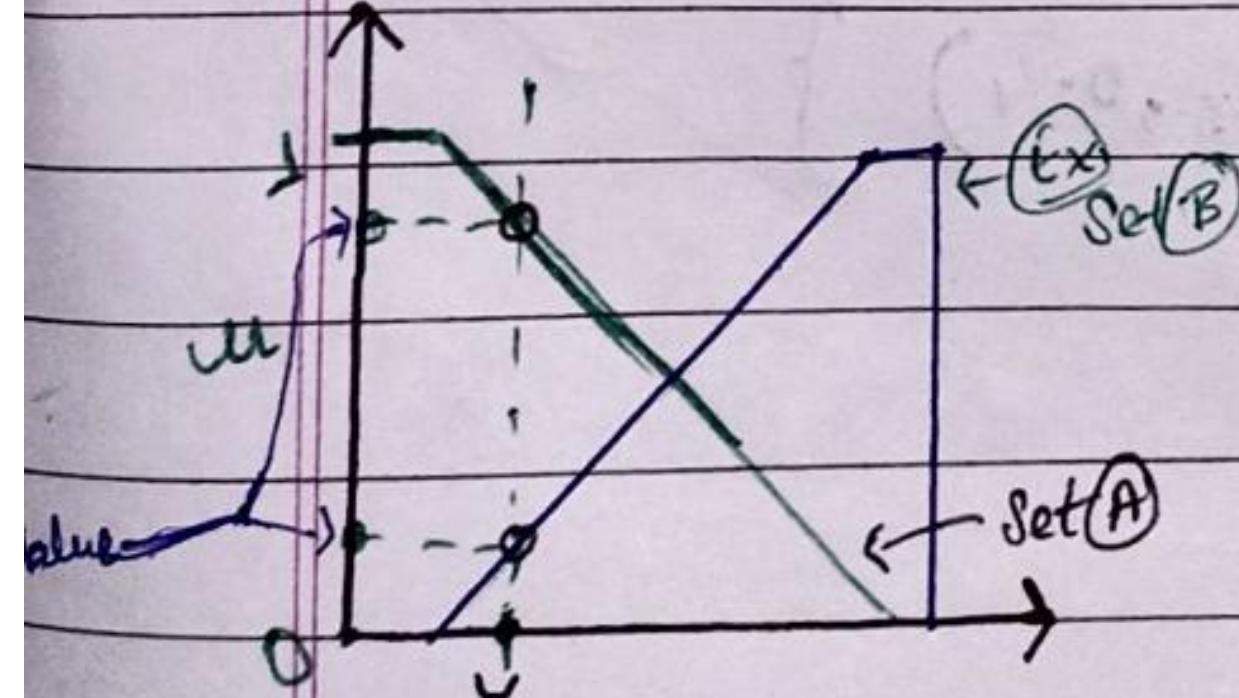


① If both  $x$  &  $\mu(x)$  are discrete then follow this M.F ↑.

② if  $x$  or  $\mu(x)$  are discrete then also follow D.M.F.

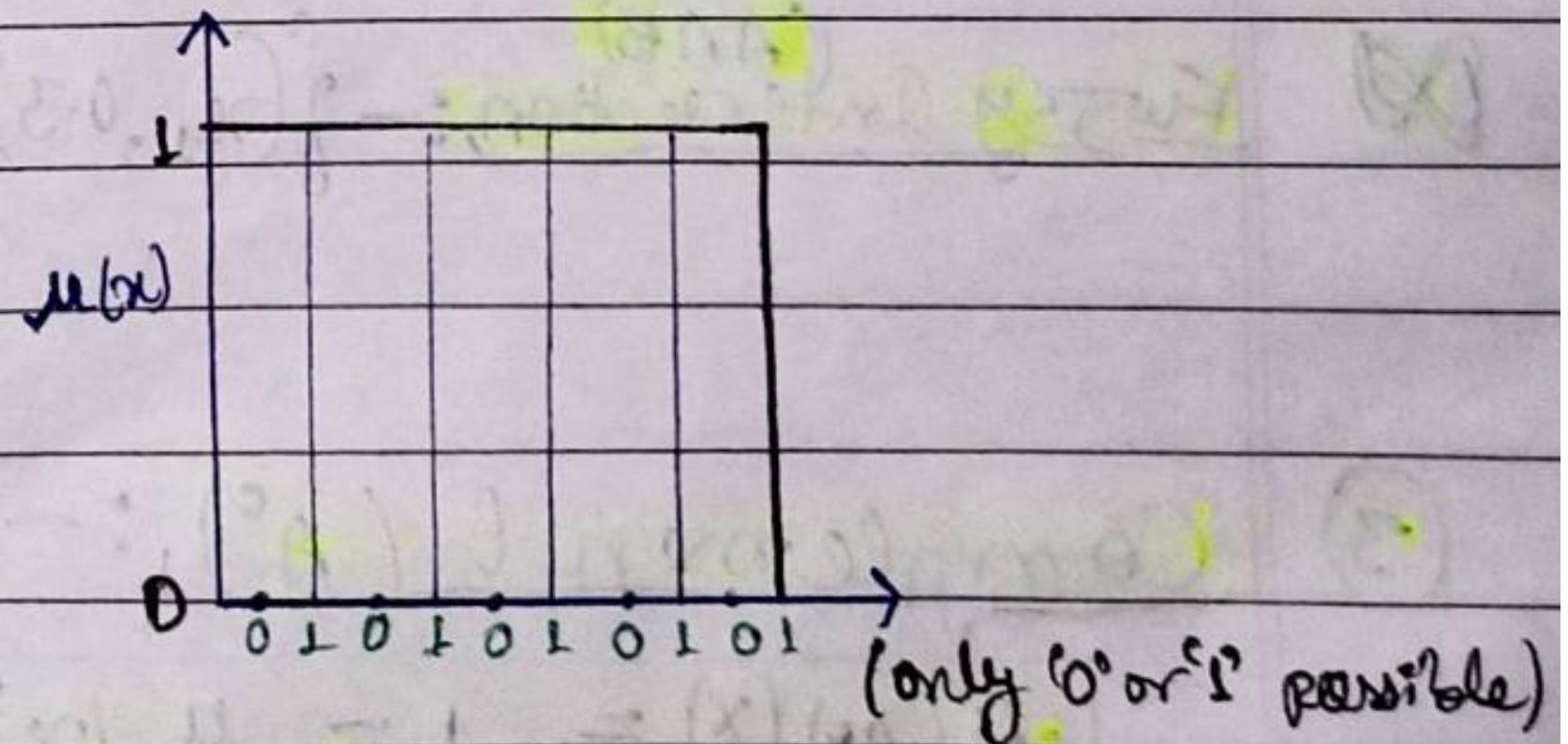
① If  $x$  &  $\mu(x)$  both continuous, ② if  $x$  or  $\mu(x)$  continuous ③ if any one or (both) is continuous then use this f(x)

## \* Fuzzy set :-



This element intersects two points. So this element belongs to A & B Both with 'different value'.

## \* Crisp set :-



① Union :-  $(A \cup B)$ 

\* To get membership value.  $\mu_{(A \cup B)}(x) = \max(\mu_A(x), \mu_B(x))$  :- Formula to get mem. val.

② Intersection :-  $(A \cap B)$ 

$$\mu_{(A \cap B)}(x) = \min(\mu_A(x), \mu_B(x))$$

Note :- In fuzzy set operation elements will remain same as Normal set (union, intersection etc.) But, only mem. value ( $\mu(x)$ ) are taken according to formula.

△ Example :- Two sets are assumed -

$$A = \{(x_1, 0.6), (x_2, 0.7), (x_3, 0.4)\}$$

$$B = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.5)\}$$

❖ Fuzzy Union  $(A \cup B)$  :-  $\{(x_1, 0.6), (x_2, 0.7), (x_3, 0.5)\}$

❖ Fuzzy Intersection  $(A \cap B)$  :-  $\{(x_1, 0.3), (x_2, 0.2), (x_3, 0.4)\}$

③ Complement  $(A^c)$  :-

$$\mu_{(A^c)}(x) = 1 - \mu_A(x)$$

used  
Above set

Ex :- Fuzzy  $A^c = \{(x_1, 0.4), (x_2, 0.3), (x_3, 0.6)\}$

Fuzzy  $B^c = \{(x_1, 0.7), (x_2, 0.8), (x_3, 0.5)\}$

(4) Vector product ( $A \cdot B$ ) :-

$$[\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)]$$

(5) Scalar Product ( $\alpha \times A$ )

$$[\mu_{\alpha A}(x) = \alpha \times \mu_A(x)]$$

(6) Equality :- ( $A = B$ )

$$[\mu_A(x) = \mu_B(x)]$$

(7) Power ( $A^\alpha$ )

$$[\mu_{A^\alpha}(x) = (\mu_A(x))^\alpha]$$

Ex :-  $A = \{(x_1, 0.6), (x_2, 0.7), (x_3, 0.4)\}$

$$B = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.5)\}$$

∞ Fuzzy Vector product ( $A \cdot B$ ) :-  $\{(x_1, 0.18), (x_2, 0.14), (x_3, 0.20)\}$

∞ Scalar product ( $\alpha \times A$ ) :-  $\{(x_1, 0.6), (x_2, 0.7), (x_3, 0.4)\}$

Range

Range is 0 to 1  
including (0, 1)  
we assumed  
(0.1),  $x_1 = (x_1 \times \alpha)$

∞ Equality ( $A = B$ ) :-  $[A(x_1) \neq B(x_1)]$  [example]

∞ Power ( $A^\alpha$ )

if  $\alpha$  is less than 1,

① its called as 'dilution'

② if  $\alpha$  is greater than 1,  
its called 'con'

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# Combination of Operation ADVANCED operators.

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VIII

Sum (A+B) :-

$$[\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)]$$

↓  
Vector product

IX

Difference (A-B) :-

$$\mu_{A-B}(x) = \mu_{A \cap B^c}(x)$$

↓  
① ②

[Complement + Intersection]

X

Disjunctive sum (A ⊕ B) [XOR]

$$[\mu_{(A \oplus B)}(x) = ((A^c \cap B) \cup (A \cap B^c))]$$

↓    ↓    ↓    ↓    ↓  
① ② ③ ④ ⑤

XI

Cartesian Product :-  $(A \times B)$

$$[\mu_{A \times B}(x, y) = \min(\mu_A(x), \mu_B(y))]$$

\* Example :-

$$A(x) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$$

$$B(y) = \{(y_1, 0.8), (y_2, 0.6), (y_3, 0.3)\}$$

→ Fuzzy Cartesian (A × B) :-

	$y_1$	$y_2$	$y_3$
$x_1$	0.2	0.2	0.2
$x_2$	0.3	0.3	0.3
$x_3$	0.5	0.5	0.3

# Fuzzy Set Properties

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- These all is followed by Fuzzy set :-

(I) Commulative :-  $A \cap B = B \cap A$ ;  $B \cup A = A \cup B$

(II) Associative :-  $A \cup (B \cup C) = (A \cup B) \cup C$ ;  $A \cap (B \cap C) = (A \cap B) \cap C$

(III) Distributive :-  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ ;  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(IV) Idempotence :-  $A \cup A = A$ ;  $A \cap A = \phi$  (null);  $A \cup \phi = A$ ;  $A \cap \phi = \phi$

(V) Transitive :- If  $A \subseteq B$ ;  $B \subseteq C$  then  $A \subseteq C$   
Proper subset/equal

(VI) De Morgan's law :-  $(A \cap B)^c = A^c \cup B^c$ ;  $(A \cup B)^c = A^c \cap B^c$

- Crisp Relation :- Also known as Classical.

Ex:-  $A = \{1, 2, 3\}$ ;  $B = \{4, 7, 8\}$

$A \times B = \{(1, 4), (1, 7), (1, 8), (2, 4), (2, 7), (2, 8), (3, 4), (3, 7), (3, 8)\}$

$$R = \begin{bmatrix} & 4 & 7 & 8 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$$

$R = \{(1, 4), (2, 7), (3, 8)\}$

$R_1 = \{(a, b) | a \leq b, (a, b) \in A \times B\}$

$R_1 = \{(1, 4), (1, 7), (1, 8), (2, 4), (2, 7), (2, 8), (3, 7), (3, 8), (3, 4)\}$

# Operations on crisp/ classic

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(I)

Union ( $A \cup B$ ) :-

- We choose 'Max' value of any particular position  $(a_{ij}, b_{ij})$  if  $(a_{ij} > b_{ij})$  we choose  $a_{ij}$  value.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A \cup B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(II)

Intersection ( $A \cap B$ ) :-

- Here, we choose the 'MIN' value of compared two elements of in a particular sets.  $(a_{ij}, b_{ij})$ ,  $b_{ij}$  will be chosen.

$$A \cap B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(III)

Complement ( $A^c$ ) :-

- Just complement of the elements.  $(0) = 1$ ,  $1^c = 0$ .

$$A^c = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Sc  
M=4

# Fuzzy If-then Rule

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- \* Fuzzy Implication
- \* Fuzzy Rule
- \* Fuzzy Conditional Statement

If  $x$  is A, then  $y$  is B.

- \* It makes the Relation b/w two expression.  
Establish

- o  $x$  is 'A' :- Antecedent  
called as ↓  
} premise
- o  $y$  is 'B' :- consequence  
called as ↓  
} conclusion

Fuzzy Rule 'R' denoted  
as  $R: A \rightarrow B$   
↑  
Implicate  
Symbol.

Q) Representation of Relation is ('best way') :- [Matrix]

# Example :- { IF Temp is high - (Premise)  
then pressure is low - (Conclusion).

Rules

$R: T_{high} \rightarrow P_{low}$

$$T_{high} = \{(25, 0.1), (30, 0.2), (35, 0.5), (40, 0.8)\}$$

All cells represent  
the strength value

25	0.1	0.1	0.1
30	0.2	0.2	0.2
35	0.3	0.5	0.4
40	0.3	0.5	0.4

$$P_{low} = \{(2, 0.3), (5, 0.5), (6, 0.4)\}$$

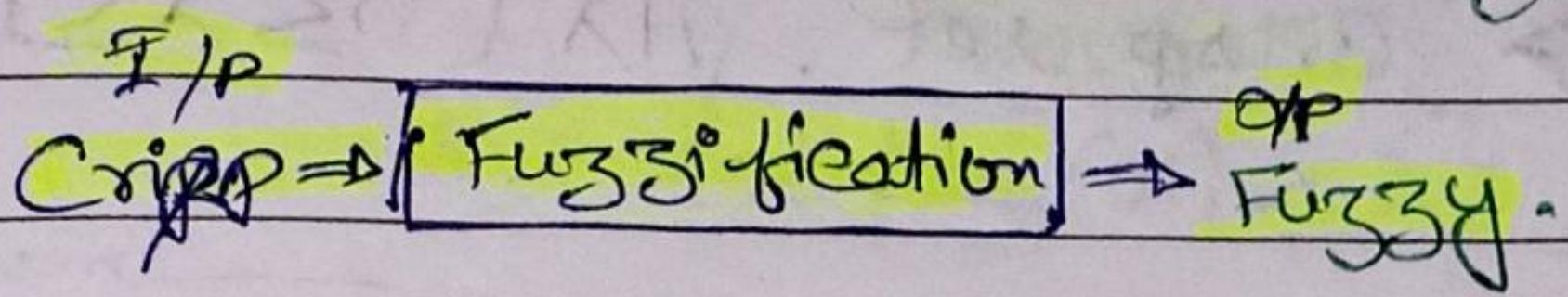
$$R: A \rightarrow B \quad R = (A \times B) = \begin{bmatrix} 2 & 5 & 6 \\ 25 & 0.1 & 0.1 & 0.1 \\ 30 & 0.2 & 0.2 & 0.2 \\ 35 & 0.3 & 0.5 & 0.4 \\ 40 & 0.3 & 0.5 & 0.4 \end{bmatrix}$$

Note! → To represent the relation

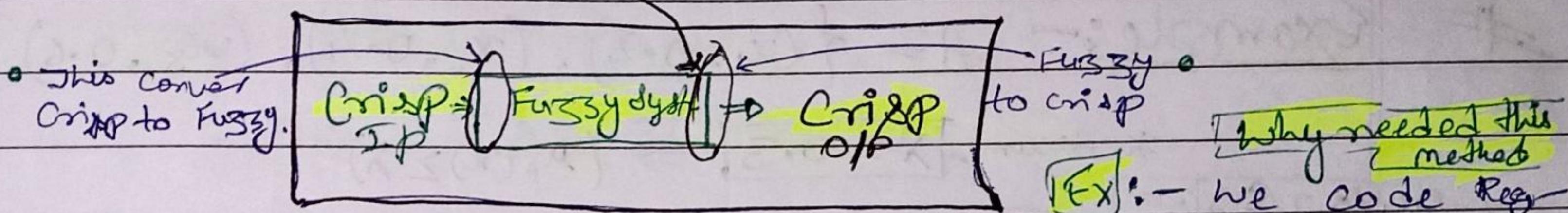
b/w Two sets, best way is  
Matrix by using 'Cartesian product'  
 $(A \times B)$  = of both sets.

## De-fuzzification

\* **Fuzzification** :- To convert the crisp into fuzzy is called fuzzification.



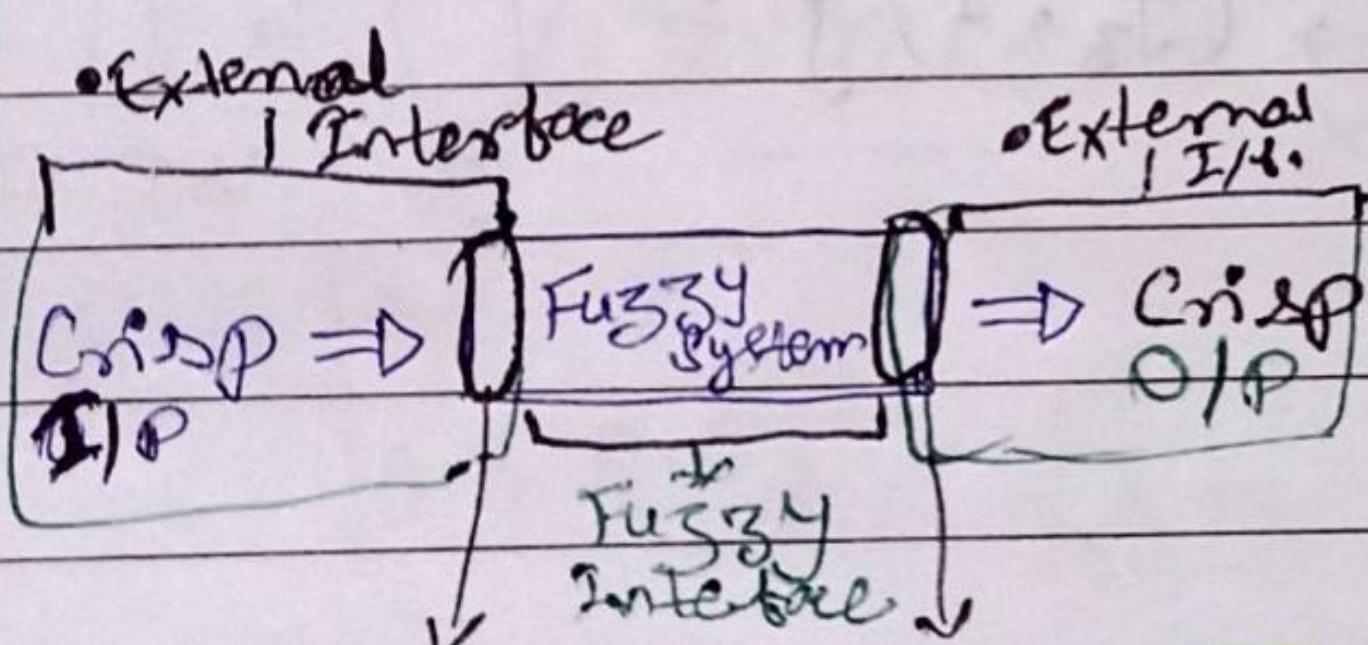
► **De-fuzzification** :- Fuzzy to crisp conversion.



- 1). Lambda - cut Method
- 2). Maxima - Method.
- 3). Weighted sum Method
- 4). Centroid Method

manly and gives the input in Highlevel language, but Machine don't understand it, so by help of compiler it change [HLL to LL] & execution the program And at end again Compiler change that [LL to HLL]

~~Because we don't understand machine language~~  
Same this applied in Fuzzy.



\* Fuzzy fire \* defuzzy fire.

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M=9

①

Lambda  $\rightarrow$  Cut (Fuzzy set) method

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• Fuzzy sets  $\rightarrow$  crisp set!

• Fuzzy set  $A \xrightarrow{\text{To}} \text{Crisp set } A_\lambda \quad (0 \leq \lambda \leq 1)$

element/variable

•  $A_\lambda = \{x | \mu_A(x) \geq \lambda\}$  must satisfy by  $(x)$   
The element who belongs to Crisp set

II Example:-  $A = \{(x_1, 0.2), (x_2, 0.4), (x_3, 0.6)\}$

\* Assumed  $\lambda = 0.3 \Rightarrow (\mu_A(x) \geq \lambda)$

\*  $\{x_1, 0.2\} \not\geq 0.3$

Not satisfy  
(or put.)  
else  $x_1$

$A_{0.3} = \{(x_2, 0), (x_2, 1), (x_3, 1)\} = \{x_2, x_3\}$

$x_1$  doesn't belong to A set.

Grip set - 0  
{ } contain element will be selected

$B = \{(y_1, 0.5), (y_2, 0.4), (y_3, 0.7)\}$ .

$B_{0.7} = \{(y_1, 0), (y_2, 0), (y_3, 1)\} = \{y_3\}$   
(B) crisp set.

$\lambda$  value  
For this example

# Lambda Cut For

X Fuzzy rel  $\rightarrow$  Crisp relation X

① Fuzzy set A  $\xrightarrow{\text{to}}$  Crisp set  $A\lambda$  ( $0 \leq \lambda \leq 1$ ).

$$A\lambda = \{x \mid u_A(x) \geq \lambda\}$$

\* Crisp set.

Ex Examples -

$$[A] = \{(x_1, 0.2), (x_2, 0.4), (x_3, 0.6)\}$$

$$\text{Taken: } [\lambda = 0.3]$$

$$[A_{0.3}] = \{(x_1, 0), (x_2, 1), (x_3, 1)\} = \{x_2, x_3\}$$

$$[B] = \{(y_1, 0.5), (y_2, 0.4), (y_3, 0.7)\}$$

$$\text{Assumed for } [B] \quad B_{0.7} = \{(y_1, 0), (y_2, 0)\}$$

## Lambda = Cut Method (Fuzzy Relation)

$$R = \begin{bmatrix} 0.2 & 1 \\ 0.3 & 0.6 \end{bmatrix}$$

Fuzzy Relation  $\xrightarrow{\text{to}}$  Crisp relation

$$\lambda = 1, 0.5, 0.1, 0$$

\* Crisp relation depends on ' $\lambda$ ' (Lambda)

$$R_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$R_{0.5} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix},$$

$$R_{0.1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Consider  $\lambda = 1$

$$R_0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Sc.  
M = 4

## ② Maxima Methods

+ height of fuzzy set.

\* crisp value denoted here by  $x^*$ .

(I)

First OF Maxima (FOM) :-

(II)

Last OF Maxima (LOM) :-

(III)

Mean OF Maxima (MOM) :-

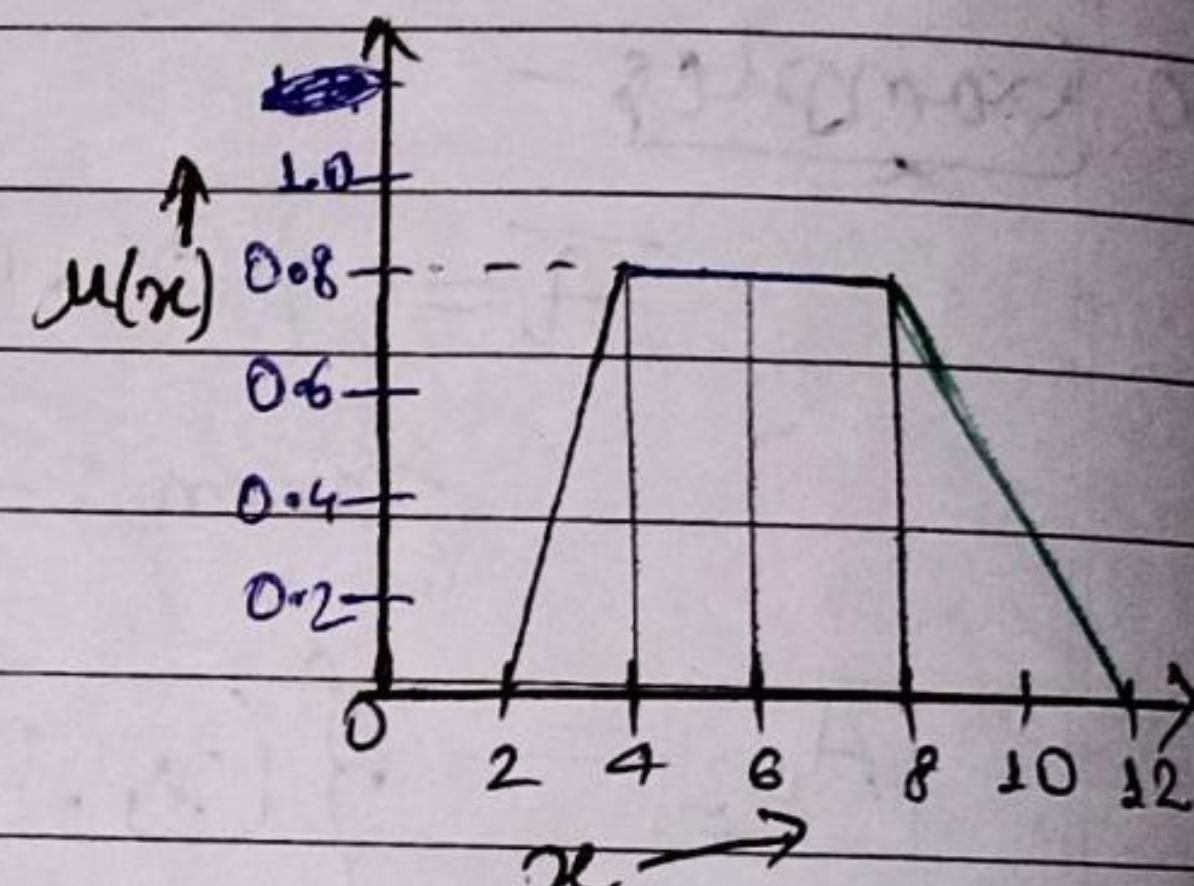
► Maxima :- is a 'height of Fuzzy set'

(I)

FOM :- Just observe the Graph:-

& take first max element :-

$$x^* = 4$$



(II)

LOM :- Just observe the Graph &

take last 'Max' element :-

$$\text{Max} = \{4, 6, 8\}$$

According to example

$$x^* = 8$$

↳ \* Height is 1.0

(III)

MOM :- Just find the 'mean' of maximum elements,

$$x^* = \frac{\sum_{i \in M} x_i}{|M|}$$

$$x = 4, 6, 8$$

$$M = \{x \mid u_A(x) = \text{height of fuzzy set}\}$$

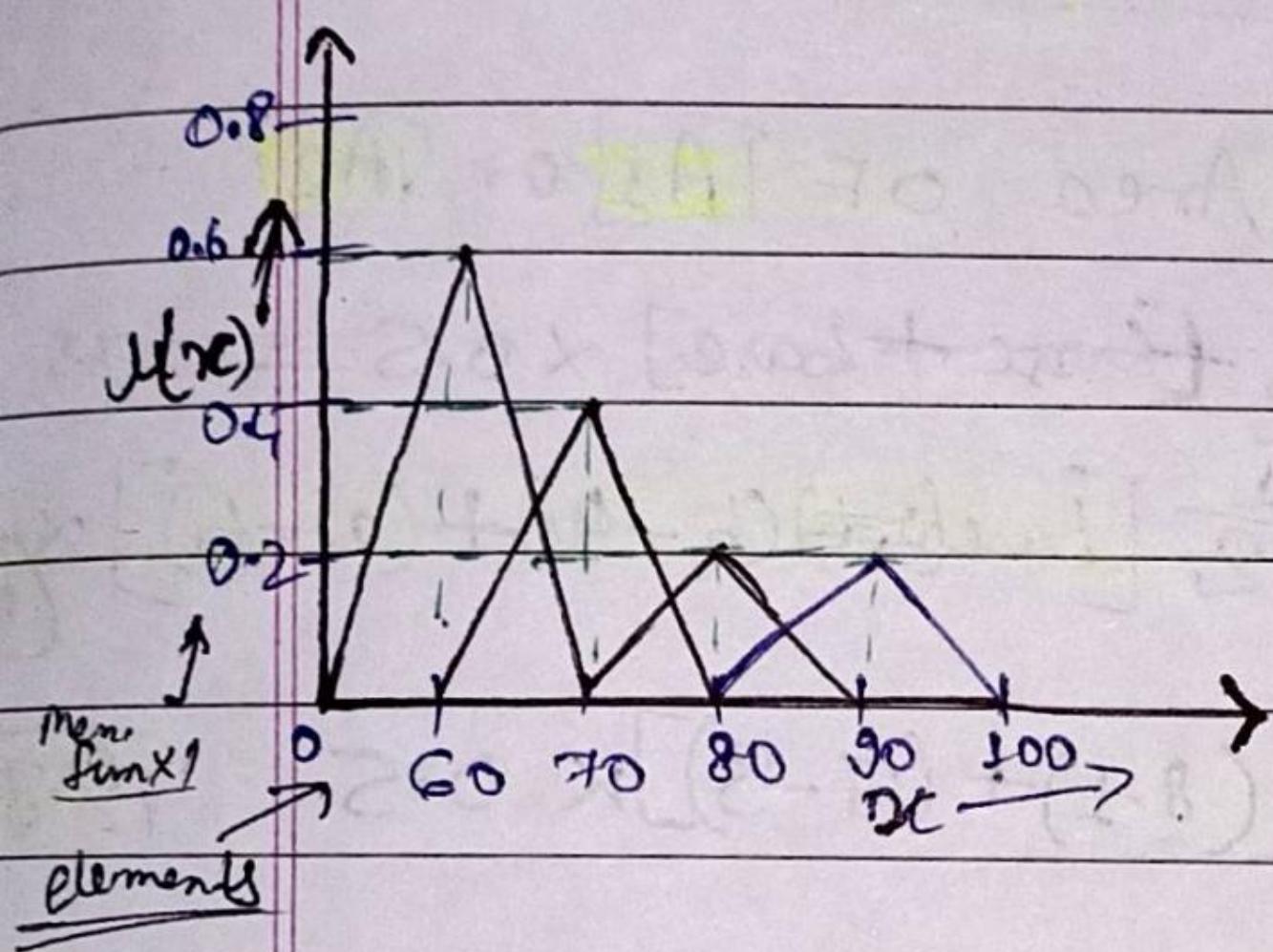
$$x^* = \frac{4+6+8}{3} = 6$$

|M| = Cardinality of set M.

(3) Weighted Average Method

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$$x^* = \frac{\sum u(x) \cdot x}{\sum u(x)}$$



$$x^* = \frac{(60 \times 0.6) + 70 \times 0.4 + 80 \times 0.2 + 90 \times 0.2}{0.6 + 0.4 + 0.2 + 0.2}$$

$$x^* = 70$$

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④

Debunkification

## ④ Centroid Method.

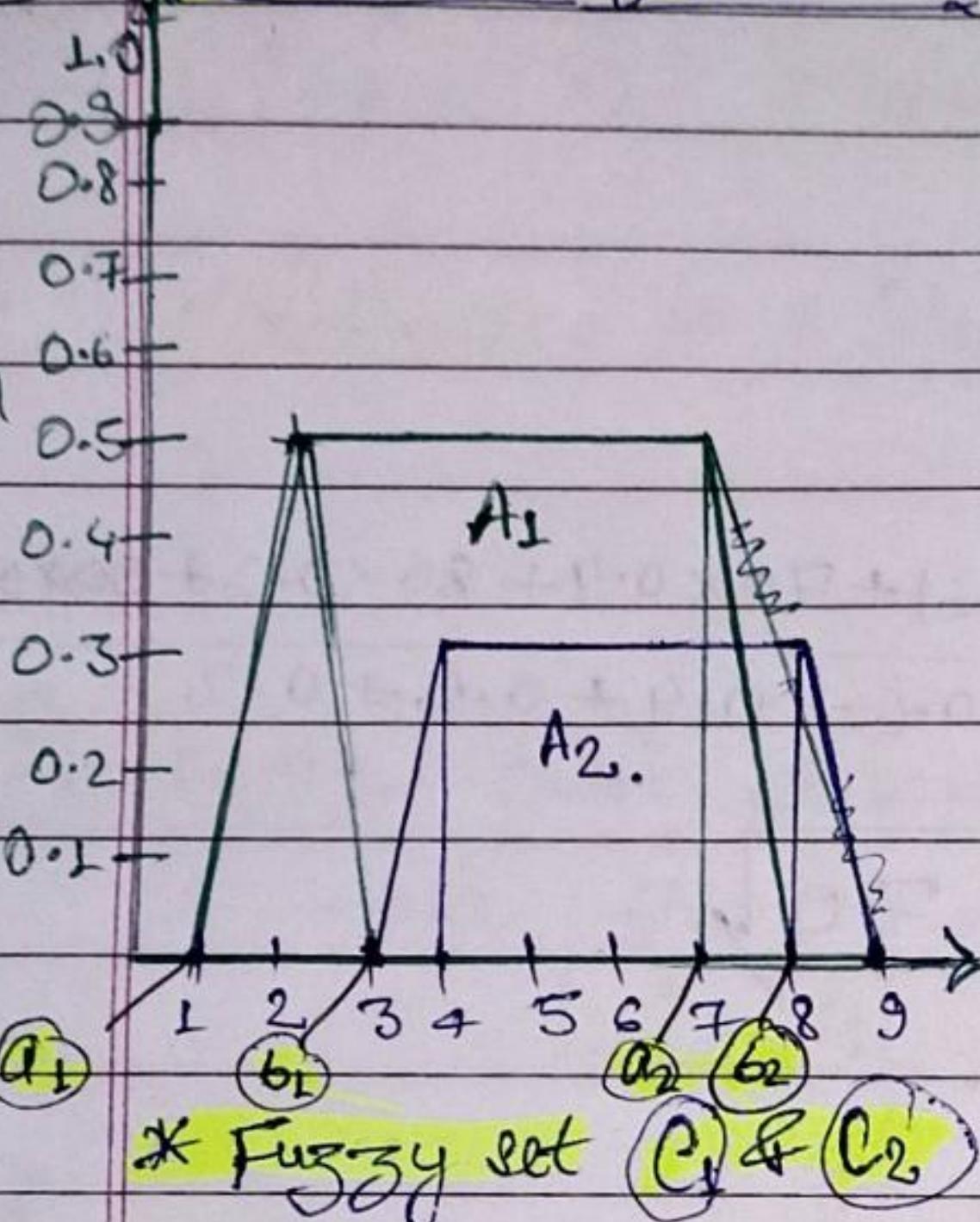
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①

Center of Sum (COS) :-



\* Fuzzy set C<sub>1</sub> & C<sub>2</sub>

$$x^* = \frac{\sum A_i x_{c_i}}{\sum A_i}$$

\* To Find Area of A<sub>1</sub> or A<sub>2</sub>

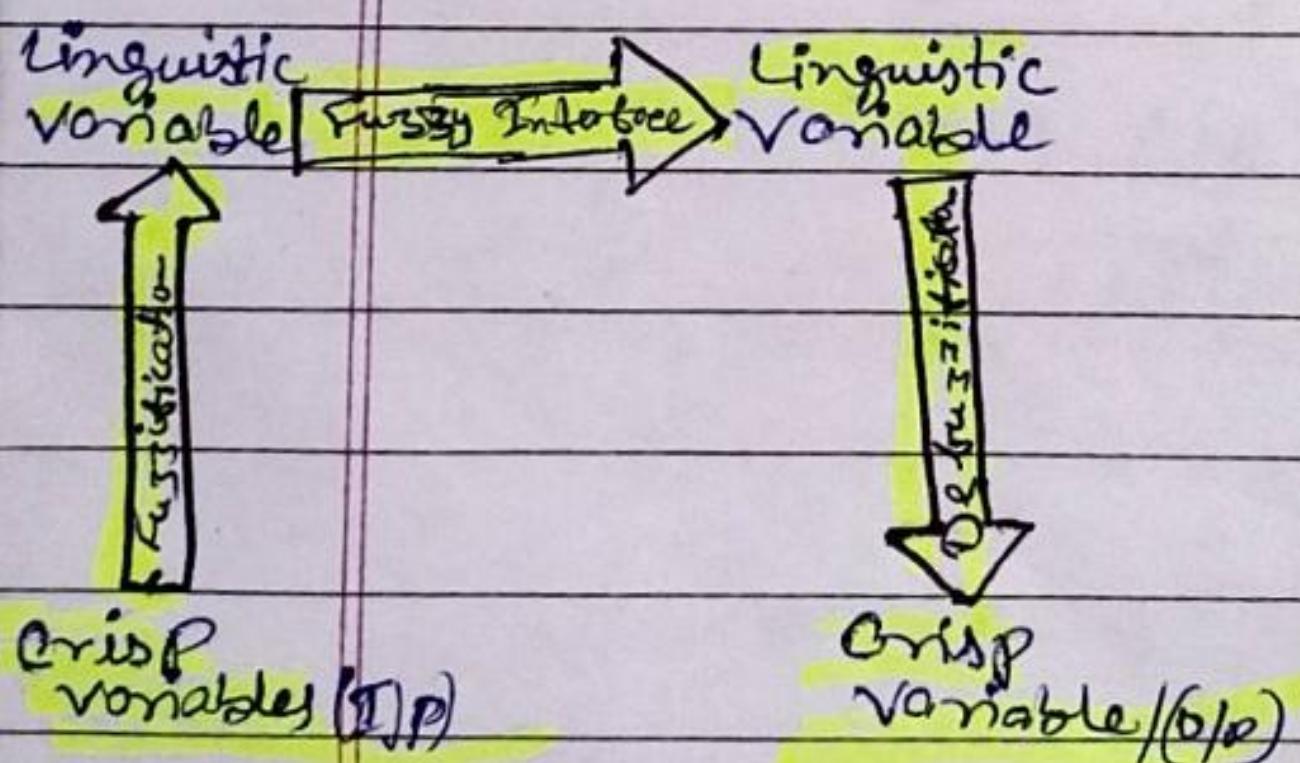
$$A_1 = \frac{1}{2} [base + base] \times 0.5 = 2.75$$

$$\begin{aligned} [A_1] &= \frac{1}{2} [base] (b_2 - a_1) + (a_2 - b_1) \times \\ &= \frac{1}{2} [(8-1) + (7-3)] \times 0.5 = 12.5 \end{aligned}$$

$$[A_2] = \frac{1}{2} [(9-3) + (8-4)] \times 0.3 = 10.5$$

$$\begin{aligned} x^* &= \frac{A_1 \cdot (x_{c_1}) + A_2 \cdot (x_{c_2})}{A_1 + A_2} \\ &= \frac{2.75 \times 5 + 1.50 \times 6}{2.75 + 10.5} \end{aligned}$$

$$x^* = 5.35$$



②

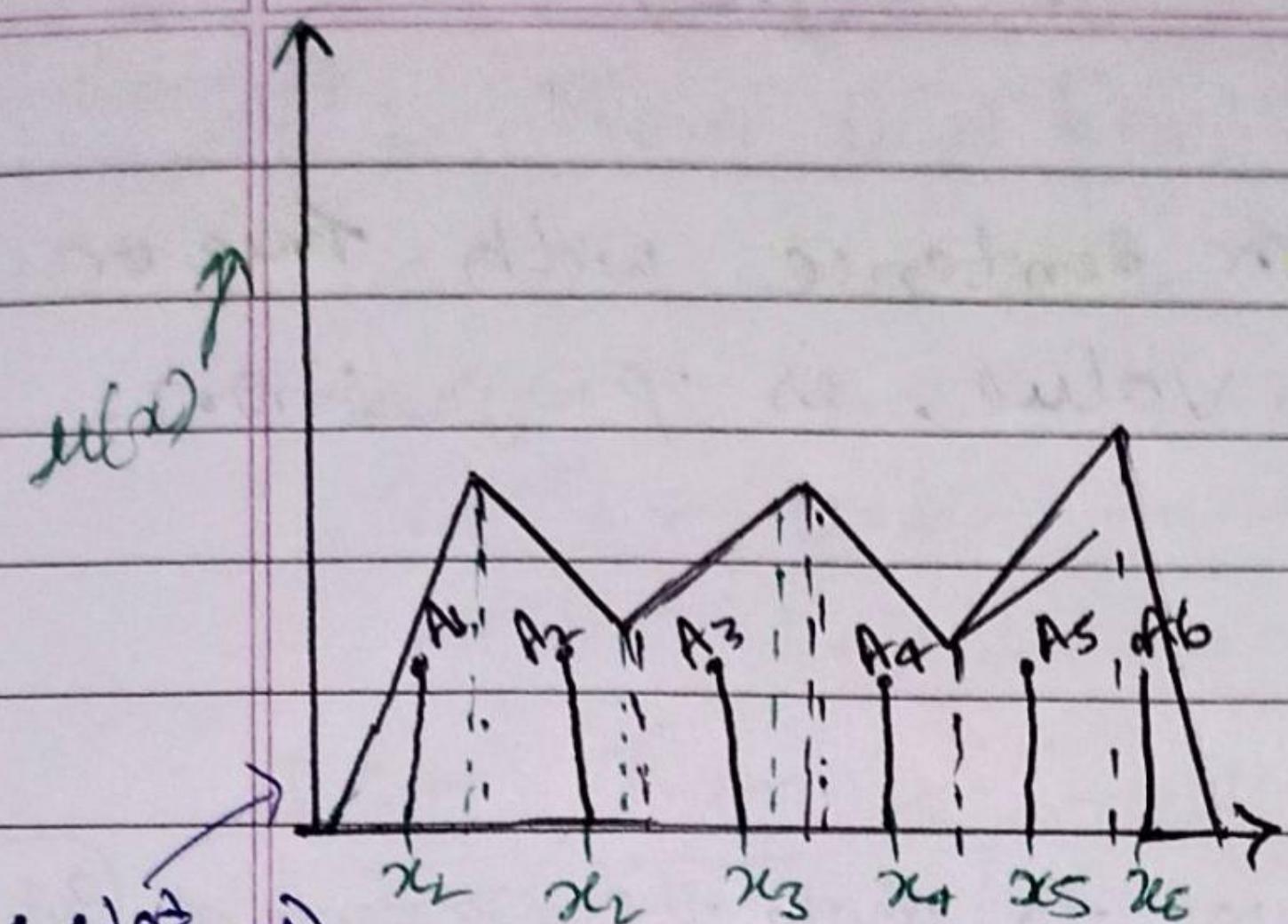
Center of Gravity :- (COG) :- 1st step :- Partition

$$x^* = \frac{\sum_{i=1}^n x_i \cdot \mu(x_i)}{\sum_{i=1}^n \mu(x_i)}$$

$$x^* = \frac{\sum_{i=1}^n x_i \cdot A_i}{\sum_{i=1}^n A_i}$$

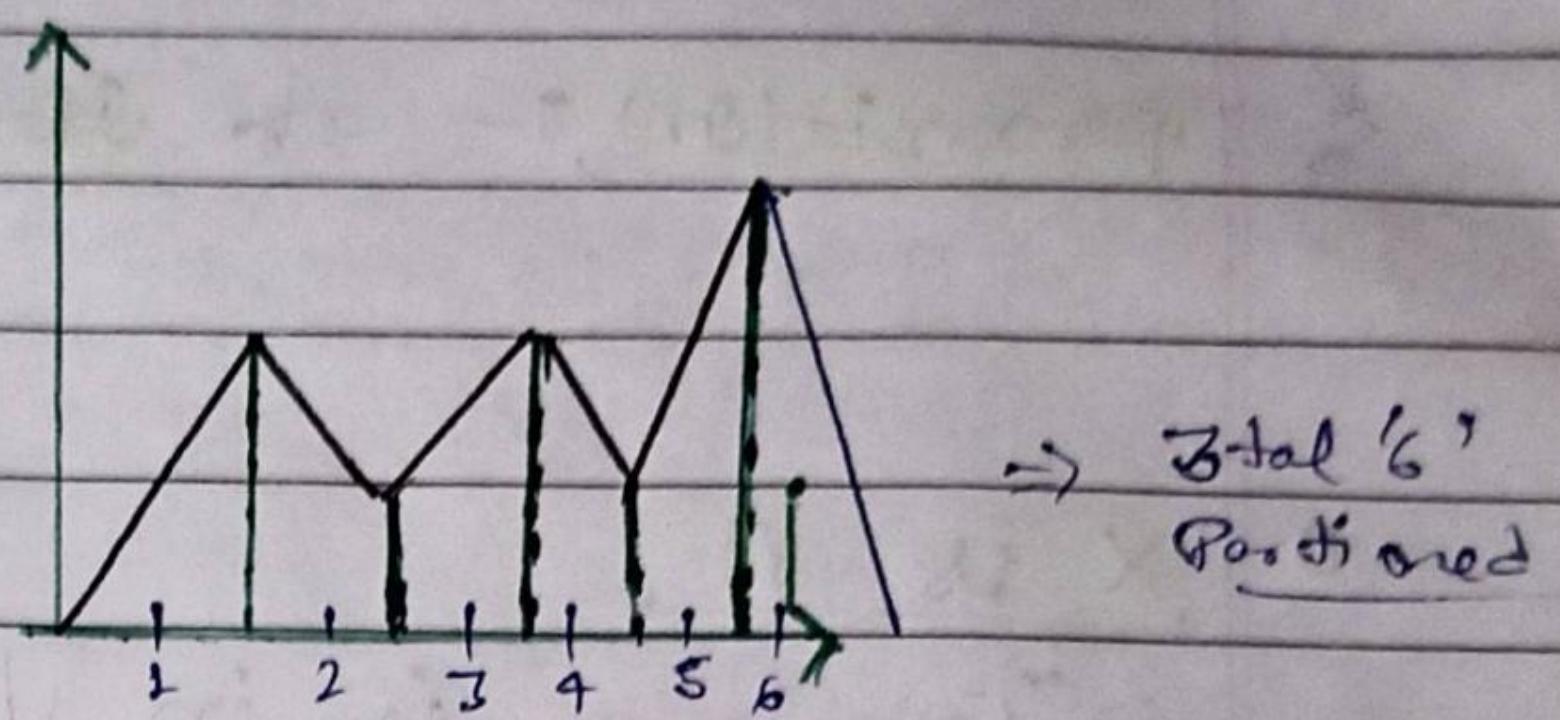
- # ② Get Area of each element
- [A<sub>i</sub> to n.]
- ③ Apply formula
- ④ COG of each element

COG



(Divided with dotted line)

$$X = \frac{\sum_{i=1}^n x_i \cdot A_i}{\sum_{i=1}^n A_i}$$



$x_i$  =  $i^{th}$  Position of COG.  
 $A_i$  = Area of  $i^{th}$  Position.

# Truth Value of Tables IN Fuzzy logic

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\* Proposition :- A statement, or sentence with true or false value. is proposition.

o  $x$  is A,

$\Rightarrow$  "London is in UK,"

Subject

Predicate used to show the Property of Subject

# Truth Table Defines logic function of 2 propositions.

① Conjunction ( $\wedge$ ) :  $x \text{ AND } y$

② Disjunction ( $\vee$ ) :  $x \text{ OR } y$

③ Implication ( $\rightarrow$ ) : If  $x$  then  $y$

④ Bi-directional ( $\leftrightarrow$ ) :  $x \text{ IFF } y$

'If and only if'

o Truth value of proposition in fuzzy logic are in range  $[0, 1]$  -  $(0.1, 0.2, \dots, 1)$ .

Eg :- P: Ram is Boy.

$$T(P) = 0.8$$

Truth value

$$\triangleright T(x \text{ AND } y) = T(x) \wedge T(y) = \min[T(x), T(y)]$$

$$\triangleright T(x \text{ OR } y) = T(x) \vee T(y) = \max[T(x), T(y)]$$

$$\triangleright T(\text{NOT } x) = 1 - T(x)$$

$$\triangleright T(x \rightarrow y) = T(x) \rightarrow T(y) = \max[1 - T(x), \min[T(x), T(y)]]$$

## Fuzzy Proposition

## Crisp Proposition

P: Ram is a boy.

\*only True or False in crisp

$T(P) = 0.0$  — Abs. False

$T(P) = 0.2$  — Partially False

$T(P) = 0.8$  — Partially True

$T(P) = 1.0$  — Abs. True

Q: — Ram is Intelligent

$$T(Q) = 0.6$$

⇒ Ram is not Intelligent

$$T(\bar{Q}) = 1 - T(Q) = 0.4$$

⇒ Ram is Boy and so is Intelligent

$$\begin{aligned} T(P \wedge Q) &= \min(T(P), T(Q)) \\ &= \min(0.8, 0.6) \\ &= 0.6. \end{aligned}$$

• Fuzzy predicates :- (Shivam is Tall) Fuzzy Predicate,  
 (tall, short, quick) Because not mention [Sat, 4ft]

• Fuzzy Predicate Modifier :- "Aligator is slightly shiny."  
 (very, fairly, moderately, rather, slightly)

• Fuzzy Quantifiers :- "many element is greater than".  
 (most, several, many)

• Fuzzy Quantifiers :-

↳ Based on Truth ( $x$  is 1)

↳ Based on probability ( $x$  is  $\frac{1}{2}$ ) Probability

↳ Based on possibility

( $x$  is  $\frac{1}{2}$ ) Possibility

8c

# Decomposition of Rules

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(I)

Multiple Conjunctive Antecedents:-

If  $x$  is  $A_1, A_2, A_3 \dots A_n$  Then  $y$  is  $B_m$

$$A_m = A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n$$

$$\rightarrow M_{A_m}(x) = \min [M_{A_1}(x), M_{A_2}(x), \dots, M_{A_n}(x)]$$

⇒ [If  $A_m$  then  $B_m$ ]

(II)

Multiple Disjunctive Antecedent :-

If  $x$  is  $A_1, A_2, A_3, \dots, A_n$  Then  $y$  is  $B_m$

$$A_m = A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n$$

$$M_{A_m}(x) = \max [M_{A_1}(x), M_{A_2}(x), \dots, M_{A_n}(x)]$$

(III)

Conditional statements:-

If  $A_1$  then  $B_1$  else  $B_2$ .

⇒ { If  $A_1$  then  $B_1$   
OR  
If NOT  $A_1$  then  $B_2$  }

(IV)

Nested If-then rules:- (use AND, atmost one)

If  $A_1$  Then [If  $A_2$  then  $B_1$ ]

⇒ If  $A_1$  AND  $A_2$  Then  $B_1$

# Aggregation of Fuzzy Rule

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## I 'Conjunctive' System of Rules :-

- Rules to be jointly satisfied.
- Using AND (Bcos it True when both I/p true).
- Using Intersection.

$$y = y_1 \text{ AND } y_2 \text{ AND } y_3 \dots \text{ AND } y_n$$

$$y = y_1 \cap y_2 \cap y_3 \dots \cap y_n$$

$$m_y(y) = \min [m_{y_1}(y), m_{y_2}(y), m_{y_3}(y) \dots m_{y_n}(y)]$$

## II 'Disjunctive' System of Rule :-

- The satisfaction of at least one Rule (Best suited for gate logic)
- 'OR' is used.

$$y = y_1 \text{ OR } y_2 \text{ OR } y_3 \dots \text{ OR } y_n$$

$$y = y_1 \cup y_2 \cup y_3 \cup \dots \cup y_n$$

$$m_y(y) = \max [m_{y_1}(y), m_{y_2}(y), m_{y_3}(y) \dots m_{y_n}(y)]$$

# Linguistic Variable:-

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- \* We studied Fuzzy logic uses linguistic variables which are the words or sentences in a natural language. Ex., If we say 'Temperature', it is linguistic variable; the values of which are very hot, or cold, slightly hot or cold, very warm, or slightly warm etc.
- \* The words very slightly are the linguistic hedges.

## Characterization of Linguistic variable

Four terms

- Name of the variable, generally represented by  $x$ .
- Term set of the variable, generally represented
- Syntactic rules for generating [by  $t(x)$ ]  
the value of the variable  $x$ .
- Semantic rules for matching linking every value of  $x$  and its significance.

## Reasoning Fuzzy :- It mainly used in Control System analysis.

premise 1 (fact)  $\rightarrow X$  is  $A'$

premise 2 (rule)  $\rightarrow$  if  $X$  is  $A$  then  $Y$  is  $B$

(Conclusion)  $\Rightarrow$   $[Y \text{ is } B']$

(eg) = where  $A'$  is close to  $A$  and  $B'$  is close to  $B$ .  
when  $A'$ , and  $B'$  are fuzzy sets of approx.  
Universes, the foregoing Inference procedure is  
called Approximate Reasoning or 'Fuzzy Reasoning'

## # Different modes of Reasoning:-

- ① Categorical Reasoning:
- ② Qualitative Reasoning:
- ③ Syllogistic "
- ④ Dispositional "

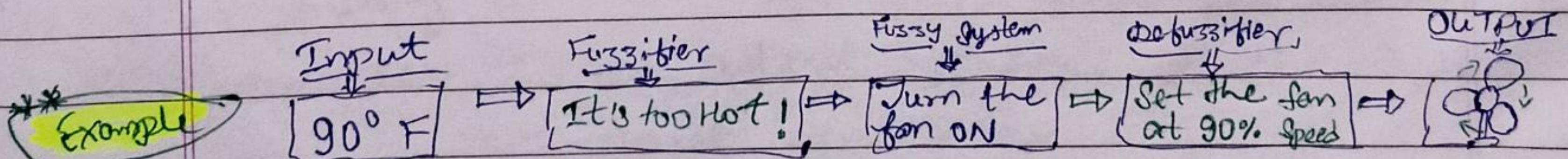
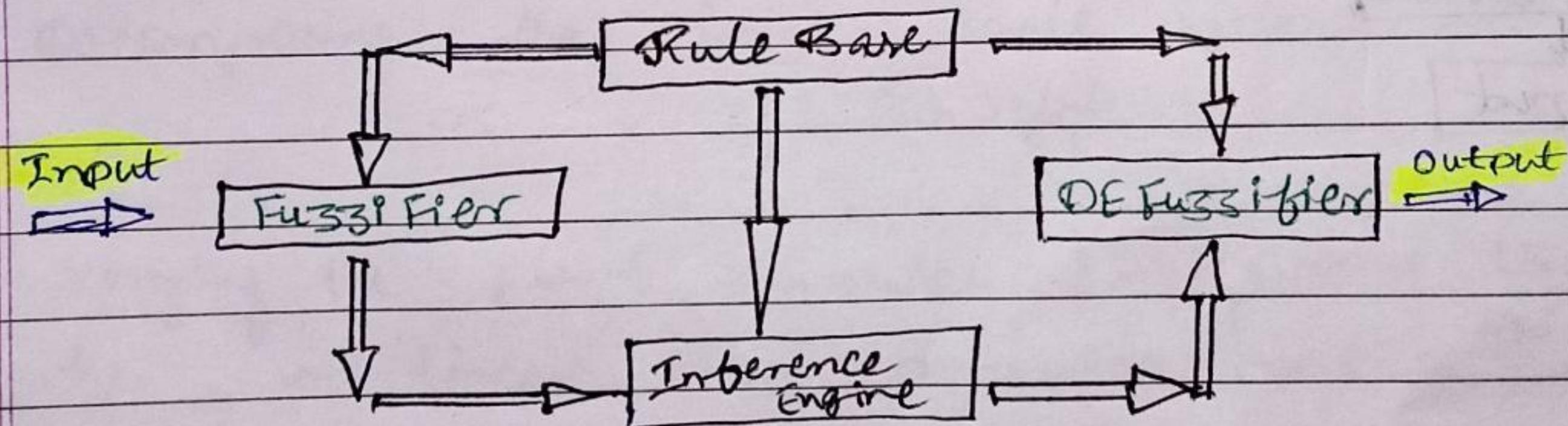
# Application OF Fuzzy Logic

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DATE: / /

- (1) It is used in the aerospace field for altitude control of Spacecraft and satellite.
- (2) It has been used in Automotive System for speed control, traffic control.
- (3) Used for decision-making support system and personal evolution in the large company business.
- (4) Application in the chemistry industry for controlling the PH, drying, distillation process.
- (5) Electronics:- • Humidity in a clean room • Vacuum cleaner  
• washing machine • microwave oven • A.C
- (6) Pattern Recognition & classification.
- (7) Medical:- • Medical diagnostic support system  
• Multivariable control of Anesthesia
- (8) Radiology diagnoses.
- (9) Security, Transportation (Railway, Brake & stop), Psychology.

## \* Architecture 'Fuzzy Logic' \*



## \* Working of Fuzzy System \*

## INFERENCE ENGINE

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8  
Mod

\* It determines the matching degree of the current Fuzzy input with respect to each rule and decides which rules are to be fired according to the Input field. Next, the fired rules are connected combined to form the control action.

\* Fuzzification :- It used to convert inputs i.e. crisp<sup>num</sup> to fuzzy set.

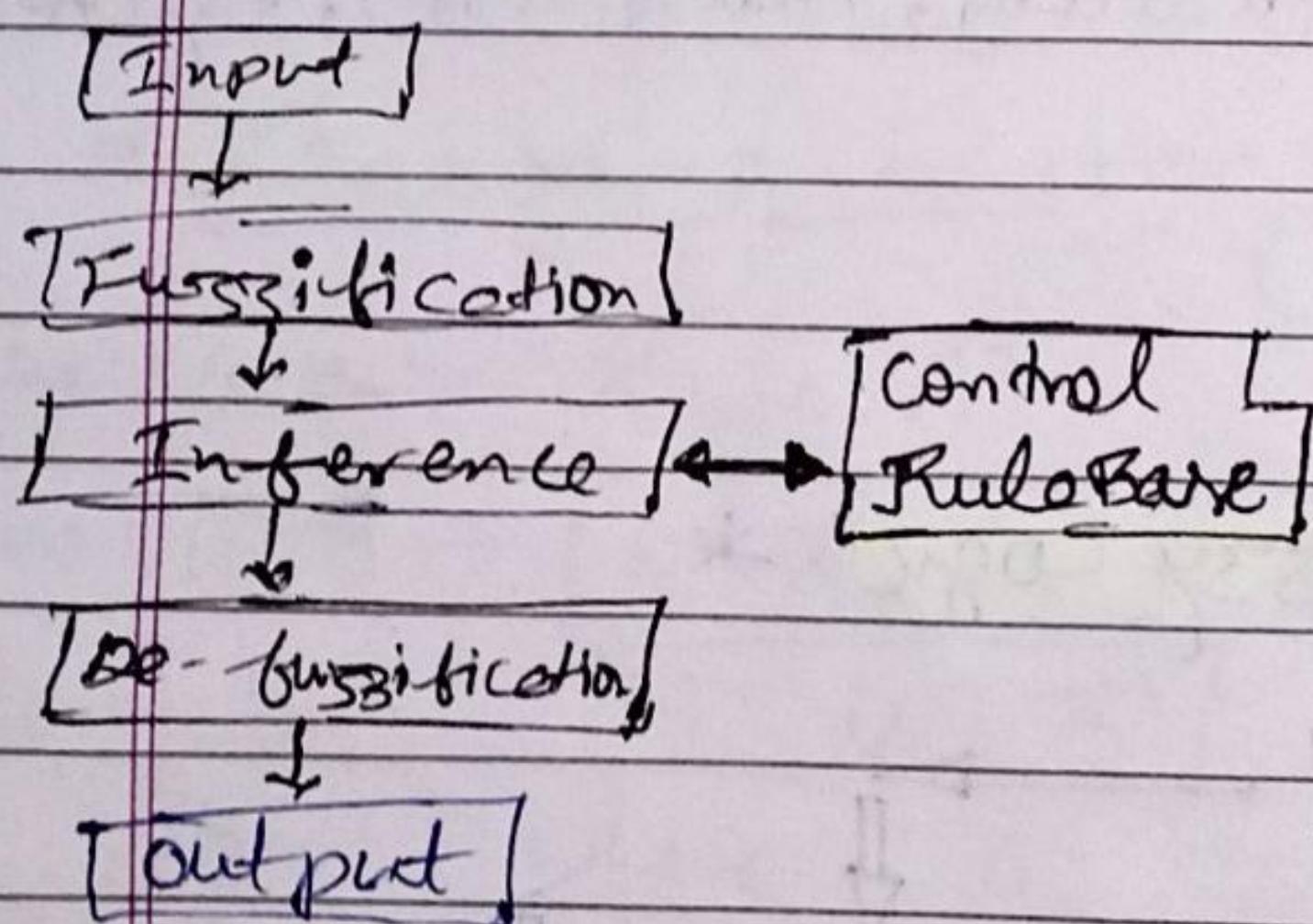
D Three largely types of fuzzifier :-

① Singleton

② Gaussian

③ Trapezoidal  
triangular

\* Working (flow-chart Fuzzy system)



# Fuzzy Control (FLC)

## Controller Design

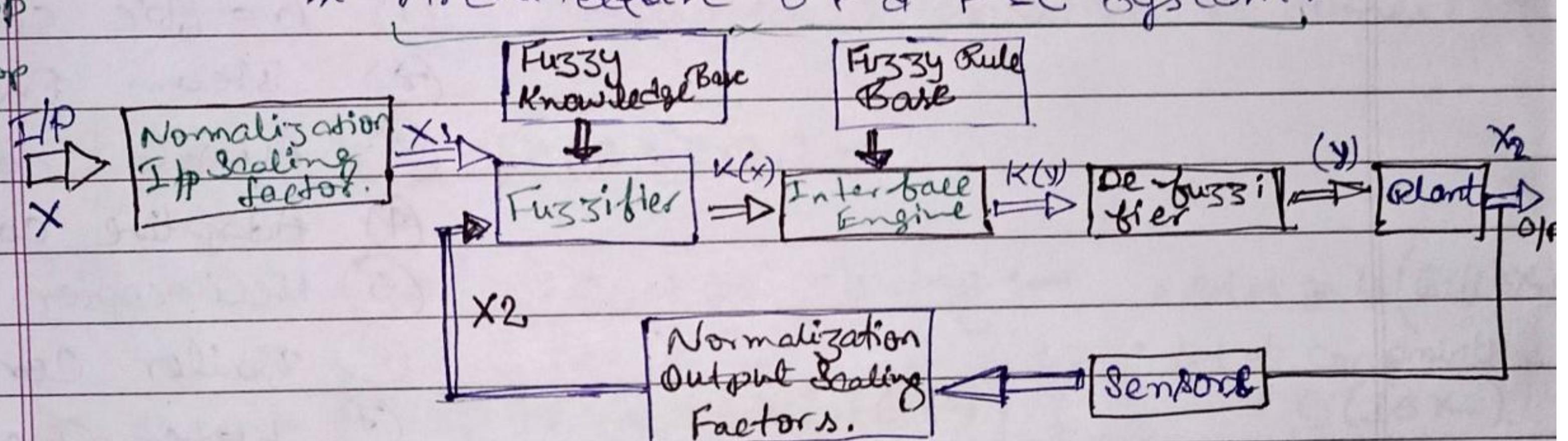
■ Fuzzy logic control (FLC) is most active research area in the application of fuzzy set theory, fuzzy reasoning, and fuzzy logic.

○ The application of FLC extends from industrial process control to biomedical instrumentation and securities.

Two (c-s)

\* open loop

\* close loop



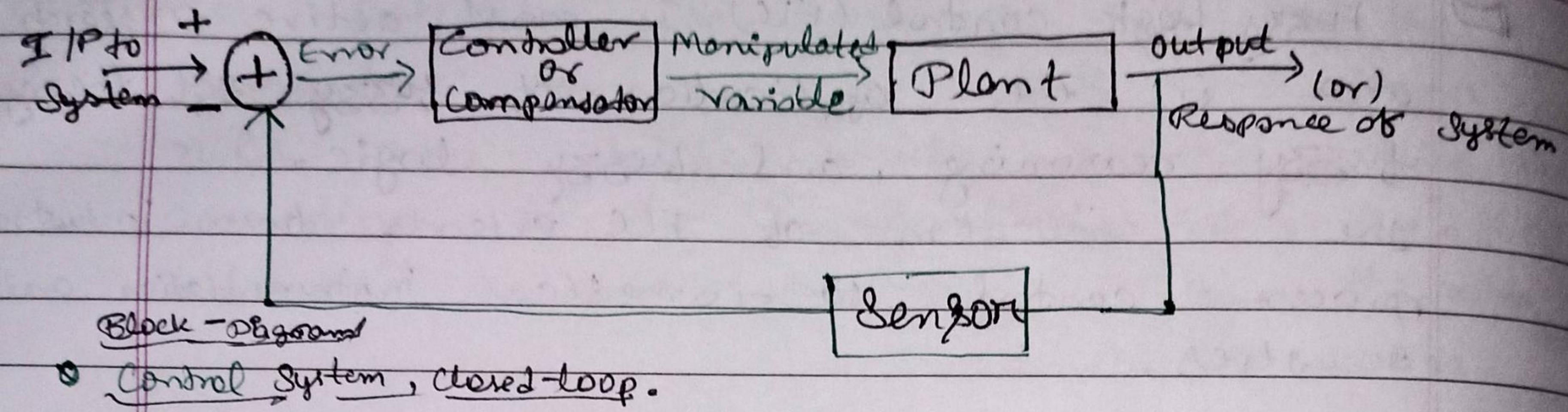
\* Control System Design:- Designing a controller for a complex physical system involves the following steps

- (1) Decomposing the large - scale system into a collection of various Subsystems.
- (2) Varying the plant dynamics slowly and linearizing the non-linear plant dynamics about a set of operation points.
- (3) Organising a set of state variables, control variables, or output features for the system under consideration.
- (4) Designing simple, P, PD, PID controllers for the subsystems. Optimal controllers can also be designed.

# FLC / Fuzzy logic Controller

Control system

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\* Application [FLC] :-

- ① Traffic control
- ② Steam Engine
- ③ Missile control
- ④ Adaptive control
- ⑤ Heli-copter Model
- ⑥ Boiler Control.
- ⑦ Water Treatment
- ⑧ Cooling Plant contr.

# Fuzzy Decision Making.

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\* Steps for Decision Making :-

- Determining the set of Alternatives:- In this, the alternatives from which the decision has to be taken must be determined.
- Evaluating Alternatives:- Here, The alternatives must be evaluated so that the decision can be taken about one of the alternatives.
- Comparison b/w Alternatives:- A comparison b/w the evaluated alternative is done.

# Types of Decision:-

(1) Individual Decision Making :-      • set of goals ( $G_i : i \in X_n$ )

$$F_D = \min \left[ \sum_{i \in X_n} f_{G_i}(a), \sum_{j \in X_m} f_{C_j}(a) \right] \quad \begin{array}{l} \bullet \text{Set of constraints} \\ \downarrow \\ = C_j : j \in X_m \end{array}$$

(2) Multi-person Decision Making :-

• Number of persons preferring  $x_i$  to  $x_j = N(x_i, x_j)$

• Total number of decision makers =  $n$

$$\text{then, } SC(x_i, x_j) = \frac{N(x_i, x_j)}{n}$$

- (3) Multi-objective Decision Making :-

Soft  
Computing  
M0.5

Fuzzy

Decision Making

(100)

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④ Multi-attribute Decision Making :- It is comes out on the basis of linear equation as follows.

$$\{ Y = A_1 \cdot X_1 + A_2 \cdot X_2 + \dots + A_i \cdot X_i + \dots + A_n \cdot X_n \}$$