An Efficient Parallel Implementation of a Perfect Hashing Method for Hypergraphs

Somesh Singh, INRIA and LIP, France
Bora Uçar, CNRS and LIP, France

15th Scheduling for Large Scale Systems Workshop

7–10 June 2022 | Fréjus, France

Problem of Interest

Given: A d-dimensional sparse tensor $\mathcal{T}$
Goal: To answer queries of the form — "Is $\mathcal{T}[i_1, \ldots, i_d]$ zero or nonzero?"

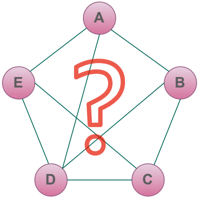
A desirable solution should have:

- $O(d)$ query response time
- *Small* memory overhead
- *Fast* preprocessing

Our focus: Hashing methods with worst-case optimal lookups

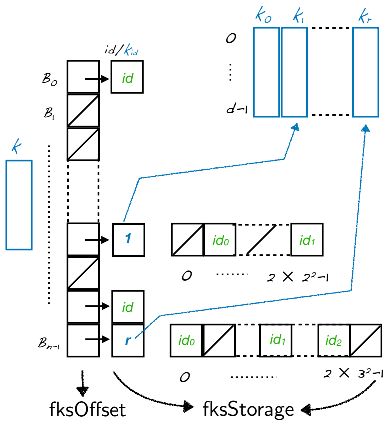
Motivating Applications

1. Kolda and Hong* propose an efficient algorithm for the decomposition of sparse tensors.
   - Sample the zeros and nonzeros of the given tensor.
   - For sampling zeros, a random set of indices is created, and those positions in the given tensor are checked for zero.
2. Checking for the presence of edges in a dense graph or subgraph (e.g. a quasi-clique).



*T. G. Kolda and D. Hong, "Stochastic gradients for large-scale tensor decomposition," SIAM Journal on Mathematics of Data Science, vol. 2, no. 4, pp. 1066–1095, 2020.

FKSLean[§] — A *Perfect Hashing* Method

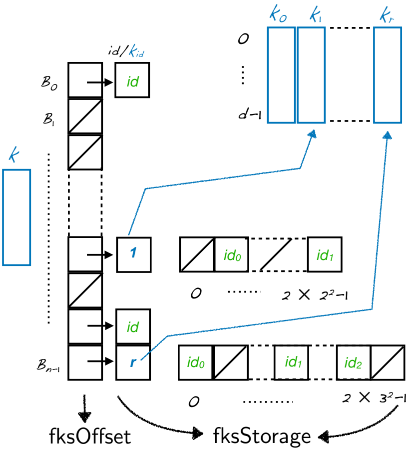


FKSLean data-structure

- FKSLean employs a two-level structure to obtain a perfect hashing.
- First level hash function:
  $h(\mathbf{k}, \mathbf{x}, p, n) := (\mathbf{k}^T\mathbf{x} \bmod p) \bmod n$
- Second level hash function:
  $h(\mathbf{k}_i, \mathbf{x}, p, 2b_i^2) := (\mathbf{k}_i{}^T\mathbf{x} \bmod p) \bmod 2b_i^2$

§ Bertrand *et al.*, "Algorithms and data structures for hyperedge queries," Inria Grenoble Rhône-Alpes, Research Report RR-9390v4, Apr. 2022.

FKSLean — Storage Requirements



FKSLean data-structure

- **K** is a set of d-tuples.

At each bucket B_i:

- b_i is the number of hyperedges.
- If $b_i = 0$, nothing is stored.
- If $b_i = 1$, a reference to the only hyperedge in B_i is stored.
- If $b_i \geqslant 2$,
  - A $\mathbf{k}_i \in \mathbf{K}$ which defines a perfect hashing for B_i is stored.
  - Storage space of size $2b_i^2$, which holds the references to the b_i hyperedges in B_i.

FKSLean — In Theory and Practice

A few theoretical results [Bertrand *et al.*]

1. One can quickly find a random $\mathbf{k} \in U$ guaranteeing $O(n)$ total storage space.
2. For every bucket B_i, one can quickly find a random $\mathbf{k}_i \in U$ that defines a second level perfect hash function for that bucket.
3. $O(\log n)$ different d-tuples in $\mathbf{K}$ are enough, in expectation, to supply each bucket with a suitable hash function.

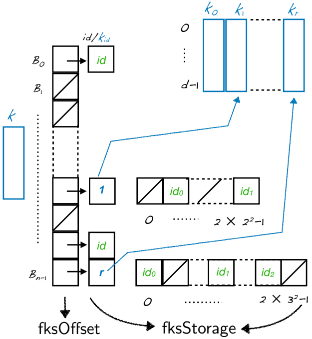
In practice [Bertrand *et al.*]

1. Total storage space required for the buckets is less than $5n$.
2. Less than $0.5 \log_2 n$ d-tuples in $\mathbf{K}$ suffice.

parFKSLean: Parallelize the Construction of FKSLean

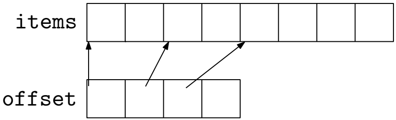
Parallel construction proceeds in two steps:

1. Setting up `fksOffset`, in parallel
   1. Bucketing
   2. Build hyperedge-lists for buckets
   3. Populate `fksOffset`
2. Populating `fksStorage`, in parallel



1 Setting-up `fksOffset`

1. Bucketing
   - Compute in parallel $h(\mathbf{k}, \mathbf{e}, p, n)$ for every hyperedge $\mathbf{e}$ and store it in $\mathtt{bucket\_ids}$ array.
2. Building hyperedge-lists for buckets
   - Maintain two arrays — $\mathtt{items}$ and $\mathtt{offset}$.



   - Populate $\mathtt{offset}$ array with histogram of $\mathtt{bucket\_ids}$ array.
   - Parallel prefix-sum on $\mathtt{offset}$ array using a two-pass algorithm.
   - Populate the $\mathtt{items}$ array in parallel.

1 Setting-up `fksOffset`

3. Populating `fksOffset`
   - Populate `fksOffset` in parallel, using the following relation:

$$\texttt{fksOffset}[i] := \begin{cases} b_i & \text{if } b_i \in \{0, 1\}, \\ 1 + 2b_i^2 & \text{otherwise.} \end{cases}$$

   where, $b_i := \texttt{offset}[i+1] - \texttt{offset}[i]$.

   - Parallel prefix-sum on `fksOffset` array.
   - Examine $\texttt{fksOffset}[n]$ for checking the storage requirement.

2 Populating `fksStorage`

1. Populate $\mathbf{K}$ with $2\log_2 n$ keys.
2. Coarse-grained parallelization for populating `fksStorage` — handle every bucket independently.
   - If $b_i = 0$, do nothing.
   - If $b_i = 1$, store the id of the hyperedge in `fksStorage` at position `fksOffset`$[i]$.
   - If $b_i \geqslant 2$,
     - Pick a $\mathbf{k}_i$ from $\mathbf{K}$ to effect a perfect hashing of the hyperedges mapped to B_i.
     - Place hyperedge $\mathbf{e}$ in the hyperedge-list of B_i at position $\left(h(\mathbf{k}_i, \mathbf{e}, p, 2b_i^2) + \texttt{fksOffset}[i]\right)$ in `fksStorage`.

Experimental Evaluation

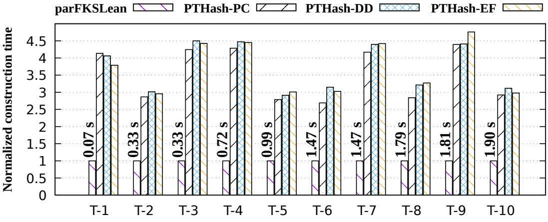
CPU	Intel Xeon Gold 5218 (64 cores, 2.3 GHz, 384 GB RAM)
Software	Debian GNU/Linux 10 (64 bit), GCC 8.3.0, OpenMP

State-of-the-art: `PTHash`[†] — nonminimal and minimal perfect hash function for static sets, with support for parallel construction.

Inputs: Tensors from the `FROSTT` (`http://frostt.io/`) dataset.

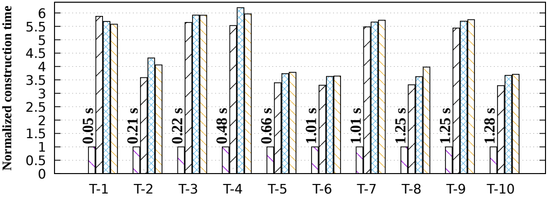
[†] G. E. Pibiri and R. Trani, "PTHash: Revisiting FCH minimal perfect hashing," in 44th SIGIR, International Conference on Research and Development in Information Retrieval. ACM, 2021, pp. 1339–1348

Construction Time



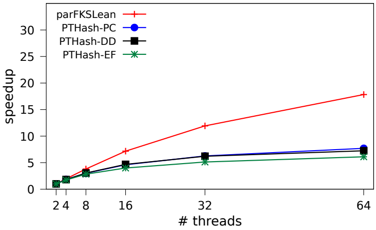
threads = 32

threads = 64

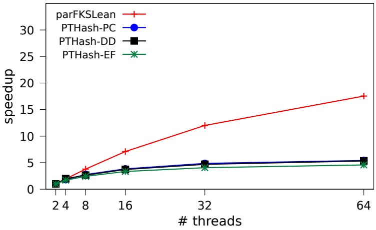


Takeaway: In the construction phase, parFKSLean is **always** faster than all the three variants of PTHash for all thread configurations.

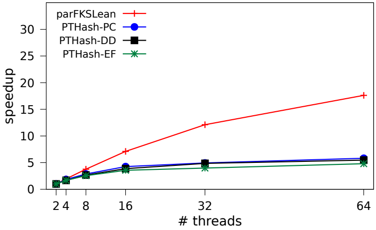
Scalability of Construction Phase



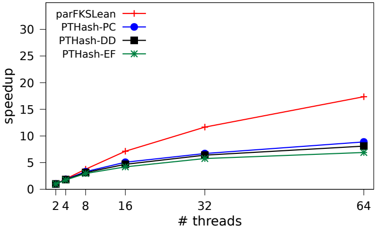
nell-2



flickr-4d



delicious-4d



nell-1

Takeaway: parFKSLean exhibits better parallel scaling.

Query Response Time

Tensor	#Threads	PTHash -PC	PTHash -DD	PTHash -EF	parFKSLean
nell-2	2	2.01	1.64	2.10	0.97
	4	1.01	0.95	1.06	0.53
	8	0.46	0.49	0.54	0.27
	16	0.25	0.27	0.29	0.15
	32	0.14	0.15	0.16	0.11
	64	0.08	0.09	0.11	0.07
flickr-4d	2	2.51	2.04	2.20	1.07
	⋮				
	64	0.11	0.09	0.09	0.08
delicious-4d	2	2.30	2.02	2.25	1.11
	⋮				
	64	0.10	0.09	0.15	0.08
nell-1	⋮				
	64	0.11	0.10	0.08	0.08

Execution time (in seconds) for 10^7 queries on four large tensors.

Takeaway: parFKSLean is at least as fast as the best performing variant of PTHash in all thread configurations for all inputs.

Conclusions

1. parFKSLean parallelizes the construction phase of FKSLean.
2. The construction phase of parFKSLean exhibits good parallel scaling.
3. parFKSLean outperforms the state-of-the-art both in construction, and query response.

Conclusions

1. `parFKSLean` parallelizes the construction phase of `FKSLean`.
2. The construction phase of `parFKSLean` exhibits good parallel scaling.
3. `parFKSLean` outperforms the state-of-the-art both in construction, and query response.

Thank You

(https://perso.ens-lyon.fr/somesh.singh/)

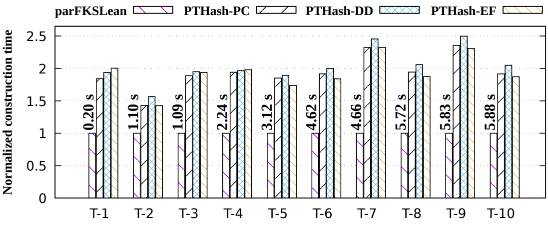
Backup Slides

Input Tensors

Tensor	d	Dimensions	n
chicago_crime (T-1)	4	6,186 × 24 × 77 × 32	5,330,673
vast-2015-mc1-3d (T-2)	3	165,427 × 11,374 × 2	26,021,854
vast-2015-mc1-5d (T-3)	5	165,427×11,374×2×100×89	26,021,945
enron (T-4)	4	6,066 × 5,699 × 244,268 × 1,176	54,202,099
nell-2 (T-5)	3	12,092 × 9,184 × 28,818	76,879,419
flickr-3d (T-6)	3	319,686 × 28,153,045 × 1,607,191	112,890,310
flickr-4d (T-7)	4	319,686 × 28,153,045 × 1,607,191 × 731	112,890,310
delicious-3d (T-8)	3	532,924 × 17,262,471 × 2,480,308	140,126,181
delicious-4d (T-9)	4	532,924 × 17,262,471 × 2,480,308 × 1,443	140,126,181
nell-1 (T-10)	3	2,902,330 × 2,143,368 × 25,495,389	143,599,552

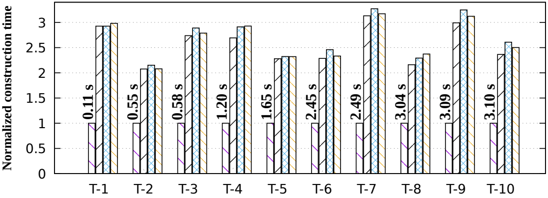
Input tensors from FROSTT dataset

Construction Time



threads = 8

threads = 16



Takeaway: In the construction phase, PARFKSLEAN is **always** faster than all the three variants of PTHash for all thread configurations.