Investigations on the use of Hashing for Parallel Graph and Hypergraph Processing

Somesh Singh, INRIA and LIP, France.

Bora Uçar, CNRS and LIP, France.

SIAM Conference on Parallel Processing for Scientific Computing (SIAM PP22)

Problem of Interest

Given: A d-dimensional sparse tensor $\mathcal{T}$
Goal: To answer queries of the form — "Is $\mathcal{T}[i_1, \ldots, i_d]$ zero or nonzero?"

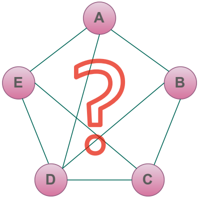
A desirable solution should have:

- $O(d)$ query response time
- *Small* memory overhead
- *Fast* preprocessing

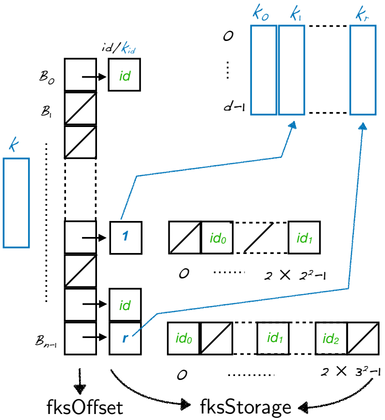
Our focus: Hashing methods with worst-case optimal lookups

Motivating Applications

1. Kolda and Hong* propose an efficient algorithm for decomposition of sparse tensors.
   - Sample the zeros and nonzeros of the given tensor.
   - For sampling zeros, a random set of indices is created, and those positions in the given tensor are checked for zero.
2. Checking for the presence of edges in a dense graph or subgraph (e.g. a quasi-clique).



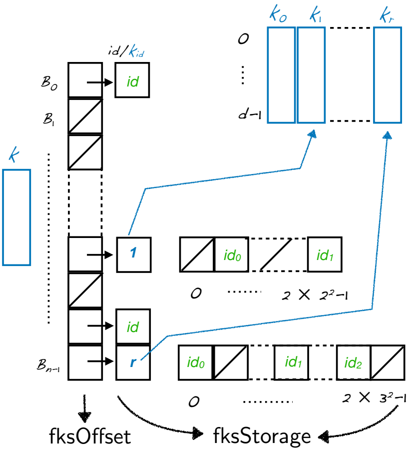
*T. G. Kolda and D. Hong, "Stochastic gradients for large-scale tensor decomposition," SIAM Journal on Mathematics of Data Science, vol. 2, no. 4, pp. 1066–1095, 2020.



FKSLean data-structure

- FKSLean employs a two-level structure to obtain a perfect hashing.
- First level hash function:
  $h(\mathbf{k}, \mathbf{x}, p, n) := (\mathbf{k}^T\mathbf{x} \bmod p) \bmod n$
- Second level hash function:
  $h(\mathbf{k}_i, \mathbf{x}, p, 2b_i^2) := (\mathbf{k}_i{}^T\mathbf{x} \bmod p) \bmod 2b_i^2$

Bertrand *et al.*, "Algorithms and data structures for hyperedge queries," Inria Grenoble Rhône-Alpes, Research Report RR-9390, Feb. 2021.



FKSLean data-structure

- **K** is a set of d-tuples.

At each bucket B_i:

- b_i is the number of hyperedges.
- If $b_i = 0$, nothing is stored.
- If $b_i = 1$, a reference to the only hyperedge in B_i is stored.
- If $b_i \geq 2$,
  - A $\mathbf{k}_i \in \mathbf{K}$ which defines a perfect hashing for B_i is stored.
  - Storage space of size $2b_i^2$, which holds the references to the b_i hyperedges in B_i.

FKSLean — In Theory and Practice

A few theoretical results [Bertrand *et al.*]

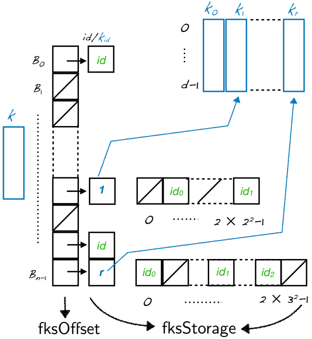
1. For a randomly chosen $\mathbf{k} \in U$ the probability that $\sum b_i^2 < 7n$ is more than $1/2$.
   - In expectation, one can find a $\mathbf{k}$ such $\sum b_i^2 < 7n$ in a few trials.
   - Such a $\mathbf{k}$ guarantees a total of $O(n)$ storage space for the buckets.
2. For a randomly chosen $\mathbf{k}_i \in U$ the probability that $\mathbf{k}_i$ defines a perfect hashing $h(\mathbf{k}_i, \mathbf{x}, p, 2b_i^2)$ for the hyperedges of B_i is more than $1/2$.
3. $O(\log_2 n)$ different d-tuples in $\mathbf{K}$ are enough, in expectation, to supply each bucket with a suitable hash function.
   - A total space of $O(d \log_2 n)$ for $\mathbf{K}$ suffices.

In practice [Bertrand *et al.*]

1. Total storage space required for the buckets is less than $5n$.
2. Less than $0.5 \log_2(n)$ d-tuples in $\mathbf{K}$ suffice.

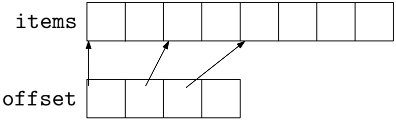
Parallel construction proceeds in two steps:

1. Setting up `fksOffset`, in parallel
   1. Bucketing
   2. Build hyperedge-lists for buckets
   3. Populate `fksOffset`
2. Populating `fksStorage`, in parallel



1 Setting-up `fksOffset`

1. Bucketing
   - Compute in parallel $h(\mathbf{k}, \mathbf{e}, p, n)$ for every hyperedge $\mathbf{e}$ and store it in *bucket_ids* array.
2. Building hyperedge-lists for buckets
   - Maintain two arrays — *items* and *offset*.



   - Populate *offset* array with histogram of *bucket_ids* array.
   - Parallel prefix-sum on *offset* array using a two-pass algorithm.
   - Populate the *items* array in parallel.

1 Setting-up `fksOffset`

3. Populating `fksOffset`
   - Populate `fksOffset` in parallel, using the following relation:

$$\texttt{fksOffset}[i] := \begin{cases} b_i & \text{if } b_i \in \{0, 1\}, \\ 1 + 2b_i^2 & \text{otherwise.} \end{cases}$$

   where, $b_i := \texttt{offset}[i+1] - \texttt{offset}[i]$.

   - Parallel prefix-sum on `fksOffset` array.
   - Examine $\texttt{fksOffset}[n]$ for checking the storage requirement.

2 Populating fksStorage

1. Populate $\mathbf{K}$ with $2\log_2(n)$ keys.
2. Coarse-grained parallelization for populating `fksStorage` — handle every bucket independently.
    - If $b_i = 0$, do nothing.
    - If $b_i = 1$, store the id of the hyperedge in `fksStorage` at position `fksOffset`$[i]$.
    - If $b_i \geq 2$,
        - Pick a $\mathbf{k}_i$ from $\mathbf{K}$ to effect a perfect hashing of the hyperedges mapped to B_i.
        - Place hyperedge $\mathbf{e}$ in the hyperedge-list of B_i at position $h(\mathbf{k}_i, \mathbf{e}, p, 2b_i^2) + \texttt{fksOffset}[i]$ in `fksStorage`.

Experimental Evaluation

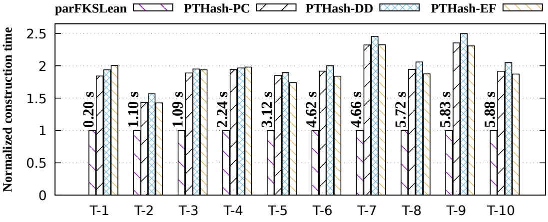
CPU	Intel Xeon Gold 5218 (64 cores, 2.3 GHz, 384 GB RAM)
Software	Debian GNU/Linux 10 (64 bit), GCC 8.3.0, OpenMP

State-of-the-art: `PTHash`[†] — nonminimal and minimal perfect hash function for static sets, with support for parallel construction.

Inputs: Tensors from the `FROSTT` (`http://frostt.io/`) dataset.

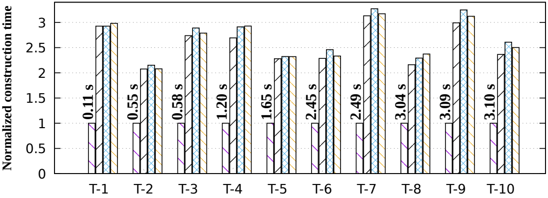
[†] G. E. Pibiri and R. Trani, "PTHash: Revisiting FCH minimal perfect hashing," in 44th SIGIR, International Conference on Research and Development in Information Retrieval. ACM, 2021, pp. 1339–1348

Construction Time



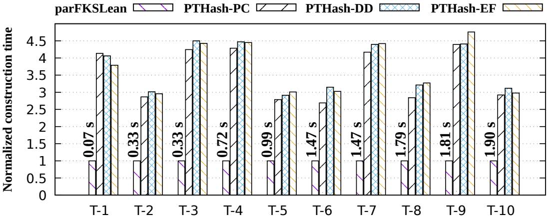
threads = 8

threads = 16

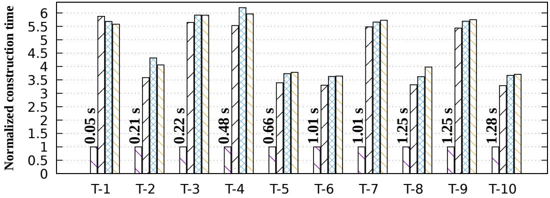


Takeaway: In the construction phase, parFKSLean is **always** faster than all the three variants of PTHash for all thread configurations.

Construction Time



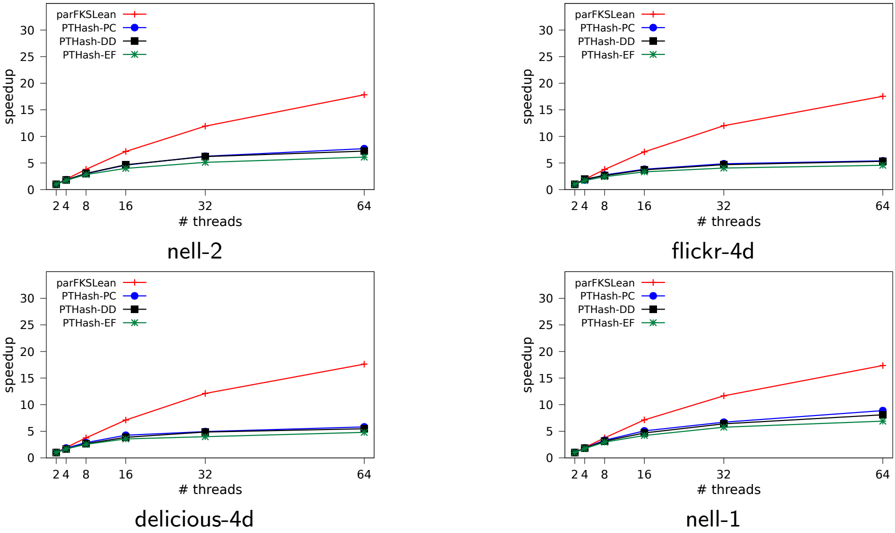
threads = 32



threads = 64

Takeaway: In the construction phase, parFKSLean is **always** faster than all the three variants of PTHash for all thread configurations.

Scalability of Construction Phase



Takeaway: `parFKSLean` exhibits better parallel scaling.

Query Response Time

Tensor	#Threads	PTHash -PC	PTHash -DD	PTHash -EF	parFKSLean
nell-2	2	2.01	1.64	2.10	0.97
	4	1.01	0.95	1.06	0.53
	8	0.46	0.49	0.54	0.27
	16	0.25	0.27	0.29	0.15
	32	0.14	0.15	0.16	0.11
	64	0.08	0.09	0.11	0.07
flickr-4d	2	2.51	2.04	2.20	1.07
	⋮				
	64	0.11	0.09	0.09	0.08
delicious-4d	2	2.30	2.02	2.25	1.11
	⋮				
	64	0.10	0.09	0.15	0.08
nell-1	⋮				
	64	0.11	0.10	0.08	0.08

Execution time (in seconds) for 10^7 queries on four large tensors.

Takeaway: parFKSLean is at least as fast as the best performing variant of PTHash in all thread configurations for all inputs.

Conclusions

1. PARFKSLEAN parallelizes the construction phase of FKSLean.
2. The construction phase of PARFKSLEAN exhibits good parallel scaling.
3. PARFKSLEAN outperforms the state-of-the-art both in construction, and query response.

Conclusions

1. parFKSLean parallelizes the construction phase of FKSLean.
2. The construction phase of parFKSLean exhibits good parallel scaling.
3. parFKSLean outperforms the state-of-the-art both in construction, and query response.

Thank You

https://perso.ens-lyon.fr/somesh.singh/

Backup Slides

Input Tensors

Tensor	d	Dimensions	n
chicago_crime (T-1)	4	$6{,}186 \times 24 \times 77 \times 32$	5,330,673
vast-2015-mc1-3d (T-2)	3	$165{,}427 \times 11{,}374 \times 2$	26,021,854
vast-2015-mc1-5d (T-3)	5	$165{,}427 \times 11{,}374 \times 2 \times 100 \times 89$	26,021,945
enron (T-4)	4	$6{,}066 \times 5{,}699 \times 244{,}268 \times 1{,}176$	54,202,099
nell-2 (T-5)	3	$12{,}092 \times 9{,}184 \times 28{,}818$	76,879,419
flickr-3d (T-6)	3	$319{,}686 \times 28{,}153{,}045 \times 1{,}607{,}191$	112,890,310
flickr-4d (T-7)	4	$319{,}686 \times 28{,}153{,}045 \times 1{,}607{,}191 \times 731$	112,890,310
delicious-3d (T-8)	3	$532{,}924 \times 17{,}262{,}471 \times 2{,}480{,}308$	140,126,181
delicious-4d (T-9)	4	$532{,}924 \times 17{,}262{,}471 \times 2{,}480{,}308 \times 1{,}443$	140,126,181
nell-1 (T-10)	3	$2{,}902{,}330 \times 2{,}143{,}368 \times 25{,}495{,}389$	143,599,552

Input tensors from FROSTT dataset