

1. Suppose that the money supply process has the form  $m_t = m + \rho m_{t-1} + \varepsilon_t$ , where  $m$  is a constant and  $0 < \rho < 1$ .

a. Show that it is possible to express  $m_{t+n}$  in terms of the known value  $m_t$  and the sequence  $\{\varepsilon_{t+1}, \varepsilon_{t+2}, \dots, \varepsilon_{t+n}\}$ .

$$m_t = m + \rho m_{t-1} + \varepsilon_t, \quad m \text{ is constant}, \quad 0 < \rho < 1$$

$$m_1 = m + \rho m_0 + \varepsilon_1$$

$$m_2 = m + \rho m_1 + \varepsilon_2 = m + \rho \cdot (m + \rho m_0 + \varepsilon_1) + \varepsilon_2 = m + \rho m + \rho^2 m_0 + \rho \varepsilon_1 + \varepsilon_2$$

$$m_3 = m + \rho m_2 + \varepsilon_3 = m + \rho \cdot (m + \rho m + \rho^2 m_0 + \rho \varepsilon_1 + \varepsilon_2) + \varepsilon_3$$

$$= m + \rho m + \rho^2 m + \rho^3 m_0 + \rho^2 \varepsilon_1 + \rho \varepsilon_2 + \varepsilon_3 = m(\rho^0 + \rho^1 + \rho^2) + \rho^3 m_0 + \rho^2 \varepsilon_1 + \rho \varepsilon_2 + \varepsilon_3$$

$$\therefore m_t = m \sum_{i=0}^{t-1} \rho^i + \rho^t m_0 + \sum_{i=0}^{t-1} \rho^i \varepsilon_{t-i}$$

If  $0 < \rho < 1$  the term  $\rho^t$  approaches zero as  $t$  approaches infinity.

Also, the infinite sum  $[1, \rho, \rho^2, \dots]$  converges to  $1/(1-\rho)$

$$\therefore m_t = m/(1-\rho) + \sum_{i=0}^{\infty} \rho^i \varepsilon_{t-i}$$

b. Suppose that all values of  $\varepsilon_{t+i}$  for  $i > 0$  have a mean value of zero. Explain how you could use your result in part a to forecast the money supply  $n$  periods into the future.

$$m_t = m + \rho m_{t-1} + \varepsilon_t$$

$$m_{t+1} = m + \rho m_t + \varepsilon_{t+1}$$

$$E[m_{t+1}] = m + \rho m_t$$

$$m_{t+2} = m + \rho m_{t+1} + \varepsilon_{t+2}$$

$$E[m_{t+2}] = m + \rho \cdot E[m_{t+1}] = m + \rho \cdot (m + \rho m_t) = m + \rho m + \rho^2 m_t$$

$$m_{t+3} = m + \rho m_{t+2} + \varepsilon_{t+3}$$

$$E[m_{t+3}] = m + \rho \cdot E[m_{t+2}] = m + \rho \cdot (m + \rho m + \rho^2 m_t) = m + \rho m + \rho^2 m + \rho^3 m_t$$

$$\therefore m_{t+n} = m \sum_{i=0}^{n-1} \rho^i + \rho^n m_t$$

If  $0 < \rho < 1$  the term  $\rho^n$  approaches zero as  $n$  approaches infinity.

Also the infinite sum  $[1, \rho, \rho^2, \dots]$  converges to  $1/(1-\rho)$

$$\therefore m_{t+n} = m/(1-\rho)$$

2. Consider the stochastic difference equation

$$y_t = 0.8y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$$

a. Suppose that the initial conditions are such that  $y_0 = 0$  and  $\varepsilon_0 = \varepsilon_{-1} = 0$ . Now suppose that  $\varepsilon_1 = 1$ . Determine the values  $y_1$  through  $y_5$  by forward iteration.

if  $t=1$

$$y_1 = 0.8y_0 + \varepsilon_1 - 0.5\varepsilon_0 = \varepsilon_1 = 1$$

$$\therefore y_1 = 1$$

if  $t=2$

$$y_2 = 0.8y_1 + \varepsilon_2 - 0.5\varepsilon_1 = 0.8 + \varepsilon_2 - 0.5$$

$$\therefore y_2 = 0.3 + \varepsilon_2$$

if  $t=3$

$$y_3 = 0.8y_2 + \varepsilon_3 - 0.5\varepsilon_2$$

$$= 0.8(0.3 + \varepsilon_2) + \varepsilon_3 - 0.5\varepsilon_2$$

$$= 0.24 + 0.8\varepsilon_2 + \varepsilon_3 - 0.5\varepsilon_2$$

$$\therefore y_3 = 0.24 + 0.3\varepsilon_2 + \varepsilon_3$$

if  $t=4$

$$y_4 = 0.8y_3 + \varepsilon_4 - 0.5\varepsilon_3$$

$$= 0.8(0.24 + 0.3\varepsilon_2 + \varepsilon_3) + \varepsilon_4 - 0.5\varepsilon_3$$

$$= 0.192 + 0.24\varepsilon_2 + 0.8\varepsilon_3 + \varepsilon_4 - 0.5\varepsilon_3$$

$$\therefore y_4 = 0.192 + 0.24\varepsilon_2 + 0.3\varepsilon_3 + \varepsilon_4$$

if  $t=5$

$$y_5 = 0.8y_4 + \varepsilon_5 - 0.5\varepsilon_4$$

$$= 0.8(0.192 + 0.24\varepsilon_2 + 0.3\varepsilon_3 + \varepsilon_4) + \varepsilon_5 - 0.5\varepsilon_4$$

$$= 0.536 + 0.192\varepsilon_2 + 0.24\varepsilon_3 + 0.8\varepsilon_4 + \varepsilon_5 - 0.5\varepsilon_4$$

$$\therefore y_5 = 0.536 + 0.192\varepsilon_2 + 0.24\varepsilon_3 + 0.3\varepsilon_4 + \varepsilon_5$$

$$= 0.536\varepsilon_1 + 0.192\varepsilon_2 + 0.24\varepsilon_3 + 0.3\varepsilon_4 + \varepsilon_5$$

b. Trace out the time path of an  $\varepsilon_t$  shock on the entire time path of the  $\{y_t\}$  sequence.

$$y_t = 0.3 \times 0.8^2 \varepsilon_1 + 0.3 \times 0.8^1 \varepsilon_2 + 0.3 \times 0.8^0 \varepsilon_3 + \varepsilon_t$$

$$y_t = 0.3 \times 0.8^3 \varepsilon_1 + 0.3 \times 0.8^2 \varepsilon_2 + 0.3 \times 0.8^1 \varepsilon_3 + 0.3 \times 0.8^0 \varepsilon_4 + \varepsilon_t$$

$$\therefore y_t = 0.3 \sum_{i=1}^{t-1} \varepsilon_i \cdot 0.8^{t-i-1} + \varepsilon_t$$

Shock이  $y_t$ 에 영향을 미친다.

3. Consider the second-order autoregressive process  $y_t = a_0 + a_2 y_{t-2} + \varepsilon_t$  where  $|a_2| < 1$ . Find:

- $E_{t-2} y_t$

$$\begin{aligned}
 E_{t-2}[a_0 + a_2 y_{t-2} + \varepsilon_t] &= E_{t-2}[a_0] + E_{t-2}[a_2 y_{t-2}] + E_{t-2}[\varepsilon_t] = a_0 + a_2 E_{t-2}[y_{t-2}] \\
 &= a_0 + a_2 E_{t-2}[a_0 + a_2 y_{t-4} + \varepsilon_{t-2}] = a_0 + a_2(a_0 + a_2 E_{t-2}[y_{t-4}]) = a_0 + a_2 a_0 + a_2^2 E_{t-2}[y_{t-4}] \\
 &= a_0 + a_2 a_0 + a_2^2 E_{t-2}[a_0 + a_2 y_{t-6} + \varepsilon_{t-4}] = a_0 + a_2 a_0 + a_2(a_0 + a_2 E_{t-2}[y_{t-6}]) = a_0 + a_2 a_0 + a_2^2 a_0 + a_2^3 E_{t-2}[y_{t-6}] \\
 \therefore a_0(1 + a_2 + a_2^2 + \dots + a_2^{n-1}) + a_2^n E_{t-2}[y_{t-2n}]
 \end{aligned}$$

If  $|a_2| < 1$  the term  $a_2^n$  approaches zero as  $n$  approaches infinite  
 Also the infinite sum  $[1 + a_2 + a_2^2 + \dots]$  converges  $1/(1-a_2)$   
 $\therefore a_0/(1-a_2)$

- $E_{t-1} y_t$

$$\begin{aligned}
 E_{t-1} y_t &= E_{t-1}[a_0 + a_2 y_{t-2} + \varepsilon_t] = a_0 + a_2 E_{t-1}[y_{t-2}] = a_0 + a_2 E_{t-1}[a_0 + a_2 y_{t-4} + \varepsilon_{t-2}] = a_0 + a_2 a_0 + a_2^2 E_{t-1}[y_{t-4}] \\
 &\dots a_0(1 + a_2 + a_2^2 + \dots + a_2^{n-1}) + a_2^n E_{t-1}[y_{t-2n}]
 \end{aligned}$$

If  $|a_2| < 1$  the term  $a_2^n$  approaches zero as  $n$  approaches infinite  
 Also the infinite sum  $[1 + a_2 + a_2^2 + \dots]$  converges  $1/(1-a_2)$   
 $\therefore a_0/(1-a_2)$

- $E_t y_{t+2}$

$$\begin{aligned}
 E_t y_{t+2} &= E_t[a_0 + a_2 y_{t-2} + \varepsilon_{t+2}] = a_0 + a_2 E_t[y_{t-2}] = a_0 + a_2 a_0 E_t(a_0 + a_2 y_{t-4} + \varepsilon_{t-2}) \\
 &= a_0 + a_2 a_0 + a_2^2 E_t[y_{t-2}] = a_0 + a_2 a_0 + a_2^2 E_t(a_0 + a_2 y_{t-4} + \varepsilon_{t-2}) \\
 &= a_0 + a_2 a_0 + a_2^2 a_0 + a_2^3 E_t[y_{t-4}] \\
 \therefore a_0(1 + a_2 + a_2^2 + \dots + a_2^{n-1}) + a_2^n E_t[y_{t-2n+2}]
 \end{aligned}$$

If  $|a_2| < 1$  the term  $a_2^n$  approaches zero as  $n$  approaches infinite  
 Also the infinite sum  $[1 + a_2 + a_2^2 + \dots]$  converges  $1/(1-a_2)$   
 $\therefore a_0/(1-a_2)$

- $\text{cov}(y_t, y_{t-1})$

$$\begin{aligned}
 \text{Cov}(y_t, y_{t-1}) &= \text{Cov}(a_0 + a_2 y_{t-2} + \varepsilon_t, y_{t-1}) = \text{Cov}(a_0, y_{t-1}) + \text{Cov}(a_2 y_{t-2}, y_{t-1}) + \text{Cov}(\varepsilon_t, y_{t-1}) \\
 &= \text{Cov}(a_2 y_{t-2}, y_{t-1}) = a_2 \cdot \text{Cov}(y_{t-2}, y_{t-1}) \\
 &= a_2 \cdot \text{Cov}(a_0 + a_2 y_{t-4} + \varepsilon_{t-4}, a_0 + a_2 y_{t-3} + \varepsilon_{t-3}) = a_2 \text{Cov}(a_2 y_{t-4}, a_2 y_{t-3}) \\
 \therefore a_2^i \cdot \text{Cov}(y_t, y_i) &= 0
 \end{aligned}$$

- $\text{cov}(y_t, y_{t-2})$

$$\begin{aligned}
 \text{Cov}(y_t, y_{t-2}) &= \text{Cov}(a_0 + a_2 y_{t-2} + \varepsilon_t, y_{t-2}) = \text{Cov}(a_2 y_{t-2}, y_{t-2}) = a_2 \text{Var}(y_{t-2}) \\
 &= a_2 \text{Var}(y_{t-2})
 \end{aligned}$$

$$\begin{aligned}
 \text{Var}(y_{t-2}) &= \text{Var}(a_0 + a_2 y_{t-4} + \varepsilon_{t-2}) = a_2^2 \text{Var}(y_{t-4}) + \sigma^2 \\
 &= a_2^5 \cdot \text{Var}(y_{t-6}) + a_2 \sigma^2 + a_2^3 \sigma^2 + \dots \\
 &= a_2^i \text{Var}(y_t) + a_2 \sigma^2 + \dots
 \end{aligned}$$

$$\therefore a_2^i \text{Var}(y_t) + a_2 \sigma^2 + a_2^i \sigma^2 + \dots$$

If  $|a_2| < 1$  the term  $a_2^i$  approaches zero as  $i$  approaches infinity  
 Also the infinite sum  $[a_2 + a_2^3 + a_2^5 + \dots]$  converges to  $a_2/(1-a_2^2)$   
 $\therefore \sigma^2 a_2 / (1-a_2^2)$

4. [bonus points] There are often several representations for the identical time-series process.

In the next, the standard equation for an AR(1) model is given by  $y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$ .

- a. Show that equivalent representations are i.  $(y_t - \bar{y}) = a_1(y_{t-1} - \bar{y}) + \varepsilon_t$  where  $\bar{y}$  is the unconditional mean of the  $\{y_t\}$  series and ii.  $y_t = a_0/(1-a_1) + \mu_t$  where  $\mu_t = a_1 \mu_{t-1} + \varepsilon_t$ .

i.  $\bar{y}$  is the unconditional mean of the  $\{y_t\}$  series

$$\begin{aligned}\bar{y} &= E[y_t] = E[a_0 + a_1 y_{t-1} + \varepsilon_t] = a_0 + a_1 E[y_{t-1}] \\ &= a_0 + a_1 E[a_0 + a_1 y_{t-2} + \varepsilon_{t-1}] = a_0 + a_1 a_0 + a_1^2 E[y_{t-2}] \\ \therefore a_0(1+a_1+a_1^2+\dots+a_1^n) + a_1^n E[y_{t-n+1}] \\ \therefore a_0/(1-a_1)\end{aligned}$$

$$y_t - (a_0/(1-a_1)) = a_1(y_{t-1} - (a_0/(1-a_1))) + \varepsilon_t$$

$$y_t - (a_0/(1-a_1)) = a_1 y_{t-1} - a_0 a_1 / (1-a_1) + \varepsilon_t$$

$$y_t = a_1 y_{t-1} - a_0 a_1 / (1-a_1) + a_0 / (1-a_1) + \varepsilon_t$$

$$y_t = a_1 y_{t-1} + a_0 \left( \frac{-a_1}{1-a_1} + \frac{1}{1-a_1} \right) + \varepsilon_t$$

$$y_t = a_1 y_{t-1} + a_0 \left( \frac{1-a_1}{1-a_1} \right) + \varepsilon_t$$

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$$

ii.  $y_t = a_0 / (1-a_1) + a_1 \mu_{t-1} + \varepsilon_t$

$$\begin{aligned}\mu_t &= a_1 \mu_{t-1} + \varepsilon_t \\ \mu_{t-1} &= a_1 \mu_{t-2} + \varepsilon_{t-1} \\ \mu_{t-2} &= a_1 \mu_{t-3} + \varepsilon_{t-2}\end{aligned}$$

$$\begin{aligned}y_t &= a_0 / (1-a_1) + a_1 \mu_{t-1} + \varepsilon_t \\ &= a_0 / (1-a_1) + a_1 (a_1 \mu_{t-2} + \varepsilon_{t-1}) + \varepsilon_t \\ &= a_0 / (1-a_1) + a_1^2 \mu_{t-2} + a_1 \varepsilon_{t-1} + \varepsilon_t \\ &= a_0 / (1-a_1) + a_1^3 \mu_{t-3} + a_1^2 \varepsilon_{t-2} + a_1 \varepsilon_{t-1} + \varepsilon_t \\ \therefore y_t &= a_0 / (1-a_1) + a_1^n \mu_{t-n} + \sum_{i=0}^{n-1} a_1^i \varepsilon_{t-i} \\ \text{if } |a_1| < 1 \text{ the term } a_1^n \text{ approaches zero as } n \text{ approaches infinite} \\ y_t &= a_0 / (1-a_1) + \sum_{i=0}^t a_1^i \varepsilon_{t-i} \\ \therefore y_t &= a_0 / (1-a_1) + \sum_{i=0}^t a_1^i \varepsilon_{t-i}\end{aligned}$$

b. In Chapter 1, we considered several models with a deterministic time trend. For example,

$y_t = a_0 + a_1 y_{t-1} + a_2 t + \varepsilon_t$  where  $|a_1| < 1$ . Explain why the  $y_t$  sequence is not stationary. Also, explain why the  $y_t$  sequence is stationary about the trend line  $a_0 + a_1 t$ . What does it mean to say that the  $y_t$  sequence is trend stationary?

$$y_t = a_0 + a_1 y_{t-1} + a_2 t + \varepsilon_t$$

$$y_1 = a_0 + a_1 y_0 + a_2 + \varepsilon_1$$

$$y_2 = a_0 + a_1 y_1 + 2a_2 + \varepsilon_2 = a_0 + a_1(a_0 + a_1 y_0 + a_2 t_1) + 2a_2 + \varepsilon_2 = a_0 + a_1 a_0 + a_1^2 y_0 + a_1 a_2 + a_2 t_1 + a_2 + \varepsilon_2$$

$$y_3 = a_0 + a_1 y_2 + 3a_2 + \varepsilon_3 = a_0 + a_1(a_0 + a_1 a_0 + a_1^2 y_0 + a_1 a_2 + a_2 t_1 + a_2 + \varepsilon_1 + 2a_2 + \varepsilon_2) + 3a_2 + \varepsilon_3$$

$$= a_0 + a_1 a_0 + a_1^2 a_0 + a_1^3 y_0 + a_1^2 a_2 + a_1^3 t_1 + 2a_2 a_1 + a_2 \varepsilon_1 + 3a_2 + \varepsilon_3$$

$$= (a_0 + a_1 a_0 + a_1^2 a_0) + a_1^3 y_0 + (a_1^3 \varepsilon_1 + a_2 \varepsilon_1 + \varepsilon_2) + (1 \cdot a_1^2 a_2 + 2 \cdot a_1 a_2 + 3 \cdot a_2 \cdot a_2)$$

$$y_t = a_0 + \sum_{i=0}^{t-1} a_1^i + a_1^t y_0 + \sum_{i=0}^{t-1} a_1^i \varepsilon_{t-i} + a_2 (1 \cdot a_1^2 + 2 \cdot a_1 + 3 \cdot a_0) \quad \therefore a_2 \sum_{i=1}^t a_1^{t-i} \text{ is not finite}$$

$$y_t = a_0 + \sum_{i=0}^{t-1} a_1^i + a_1^t y_0 + \sum_{i=0}^{t-1} a_1^i \varepsilon_{t-i} + a_2 \sum_{i=1}^t a_1^{t-i}$$

if  $|a_1| < 1$  the term  $a_1^t$  approaches zero as  $t$  approaches infinite

$$y_t = a_0 / (1-a_1) + \sum_{i=0}^{t-1} a_1^i \varepsilon_{t-i} + a_2 \sum_{i=1}^t a_1^{t-i}$$

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_t + \varepsilon_t, |\alpha_i| < 1$$

$y_t$ 의 trend line  $\alpha_0 + \alpha_2 y_t$ 는  $y_t = \alpha_1 y_{t-1} + \varepsilon_t$

$y_t = \alpha_1 y_{t-1} + \varepsilon_t, |\alpha_1| < 1$  은 stationary이다.

$y_t$ 가 covariance stationary인지를 proof

$$y_t = \alpha_1 y_{t-1} + \varepsilon_t, |\alpha_1| < 1$$

$$\text{Lag 1: } y_{t-1} = \alpha_1 y_{t-2} + \varepsilon_{t-1}$$

$$\begin{aligned} y_t &= \alpha_1 (\alpha_1 y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t = \alpha_1^2 y_{t-2} + \alpha_1 \varepsilon_{t-1} + \varepsilon_t = \alpha_1^2 (\alpha_1 y_{t-3} + \varepsilon_{t-2}) + \alpha_1 \varepsilon_{t-2} + \varepsilon_t \\ &= \alpha_1^3 y_{t-3} + \alpha_1^2 \varepsilon_{t-2} + \alpha_1 \varepsilon_{t-1} + \varepsilon_t \end{aligned}$$

$$\therefore y_t = \alpha_1^i y_{t-i} + \sum_{i=0}^{t-1} \alpha_1^i \varepsilon_{t-i}$$

if  $|\alpha_1| < 1$  the term  $\alpha_1^i$  approaches zero as  $i$  approaches infinity.

$$\therefore y_t = \sum_{i=0}^{t-1} \alpha_1^i \varepsilon_{t-i}$$

$$\textcircled{1} E[y_t] = E\left[\sum_{i=0}^{t-1} \alpha_1^i \varepsilon_{t-i}\right] = \sum_{i=0}^{t-1} \alpha_1^i E[\varepsilon_{t-i}] = 0$$

$$\begin{aligned} \textcircled{2} \text{ Cov}(y_t, y_{t-s}) &= E[(y_t - \mu)(y_{t-s} - \mu)] \quad \therefore \mu = E[y_t] = 0 \\ &= E[y_t \cdot y_{t-s}] = E\left[\left(\sum_{i=0}^{t-1} \alpha_1^i \varepsilon_{t-i}\right) \left(\sum_{j=0}^{t-s-1} \alpha_1^j \varepsilon_{t-s-j}\right)\right] \\ &= E[(\varepsilon_t + \alpha_1 \varepsilon_{t-1} + \alpha_1^2 \varepsilon_{t-2} + \dots)(\varepsilon_{t-s} + \alpha_1 \varepsilon_{t-s-1} + \dots)] \\ &= \beta^2 \cdot \alpha_1^s \cdot (1 + \alpha_1^2 + \alpha_1^4 + \dots) = \frac{\beta^2 \cdot \alpha_1^s}{1 - \alpha_1^2} \end{aligned}$$

$$\textcircled{3} \text{ Var}(y_t) = \text{Cov}(y_t, y_t) = \frac{\beta^2}{1 - \alpha_1^2}$$

$\therefore$   $y_t$ 의 trend line은  $\alpha_0 + \alpha_2 y_t$ 는  $y_t = \alpha_1 y_{t-1} + \varepsilon_t$ 를 stationary로 한다.

따라서  $y_t$ 는 trend stationary이다.

**Lab 1.** The file QUARTERLY.CSV contains a number of series including the U.S. index of industrial production (*indprd*). unemployment rate (*urate*), and producer price index for industrial goods (*finished*). All of the series run from 1960Q1 to 2012Q4.

- Construct the growth rate of the series as  $y_t = \log(indprod_t) - \log(indprod_{t-1})$ . Since the first few autocorrelations suggest an AR(1), estimate  $y_t = \alpha + \beta y_{t-1} + \varepsilon_t$ .
- Estimate *urate* as an AR(2) process including an intercept. By defining  $y_t = urate_t$ , you should find values of  $\alpha$ ,  $\beta$ , and  $\gamma$  of  $y_t = \alpha + \beta y_{t-1} + \gamma y_{t-2} + \varepsilon_t$ .

### Data Load

```
In [2]: import pandas as pd
import numpy as np
import statsmodels.api as sm
data = pd.read_csv("QUARTERLY.csv")
```

Data 8211

### Data preprocessing

```
In [3]: data['IndProd_t-1'] = data['IndProd'].shift(1)
data['yt_inprod'] = np.log(data['IndProd']) - np.log(data['IndProd_t-1'])
```

```
In [4]: data.head(5)
```

```
Out[4]:
```

	Date	FFR	Tbill	Tb1yr	r5	r10	PPINSA	Finished	CPI	CPCORE	...	M2SA	M2NSA	Unemp	IndProd	RGDP	Potent	Deflator	Curr	IndProd_t-1
0	1960-01-01	3.93	3.87	4.57	4.64	4.49	31.67	33.20	29.40	18.92	...	896.1	299.40	5.13	23.93	2845.3	2824.2	18.521	31.830	NaN
1	1960-04-01	3.70	2.99	3.87	4.30	4.26	31.73	33.40	29.57	19.00	...	903.3	300.03	5.23	23.41	2832.0	2851.2	18.579	31.862	23.93
2	1960-07-01	2.94	2.36	3.07	3.67	3.83	31.83	33.43	29.59	19.07	...	919.4	305.50	5.53	23.02	2836.6	2878.7	18.648	32.217	23.41
3	1960-10-01	2.30	2.31	2.99	3.75	3.89	31.70	33.67	29.78	19.14	...	932.8	312.30	6.27	22.47	2800.2	2906.7	18.700	32.624	23.02
4	1961-01-01	2.00	2.35	2.87	3.64	3.79	31.80	33.63	29.84	19.17	...	948.9	317.10	6.80	22.13	2816.9	2934.8	18.743	32.073	22.47

5 rows x 21 columns

a

coef const가 알파, coef arL1이 베타  
 $y_t = \text{알파} + \text{베타 } y_{t-1} + \text{잡실론 } t$

```
In [5]: model = sm.tsa.arima.ARIMA(data['yt_inprod'], order = (1,0,0))
result = model.fit()
result.summary()
```

```
Out[5]: SARIMAX Results
```

Dep. Variable:	yt_inprod	No. Observations:	212
Model:	ARIMA(1, 0, 0)	Log Likelihood:	621.539
Date:	Wed, 29 Mar 2023	AIC:	-1237.079
Time:	23:08:49	BIC:	-1227.009
Sample:	0	HQIC:	-1233.009
- 212			
Covariance Type:	opg	<0.05	
coef	std err	z	P> z  [0.025 0.975]
const	0.0065	0.002	2.607 0.009
arL1	0.6066	0.041	14.746 0.000
sigma2	0.0002	1e-05	16.110 0.000
Ljung-Box (L1) (Q): 0.92 Jarque-Bera (JB): 77.49			
Prob(Q): 0.34		Prob(JB): 0.00	
Heteroskedasticity (H): 0.37		Skew: -0.09	
Prob(H) (two-sided): 0.00		Kurtosis: 5.96	

$$y_t = 0.0065 + 0.6066 y_{t-1} + \hat{\varepsilon}_t$$

b

```
In [46]: model = sm.tsa.arima.ARIMA(data['Unemp'], order = (2,0,0))
result = model.fit()
result.summary()
```

```
Out[46]: SARIMAX Results
```

Dep. Variable:	Unemp	No. Observations:	212
Model:	ARIMA(2, 0, 0)	Log Likelihood:	-5.541
Date:	Wed, 22 Mar 2023	AIC:	19.082
Time:	13:13:49	BIC:	32.508
Sample:	0	HQIC:	24.508
- 212			
Covariance Type:	opg	<0.05	
coef	std err	z	P> z  [0.025 0.975]
const	6.0881	0.568	10.724 0.000
arL1	1.6404	0.037	43.886 0.000
arL2	-0.6780	0.039	-17.568 0.000
sigma2	0.0604	0.005	12.972 0.000
Ljung-Box (L1) (Q): 0.05 Jarque-Bera (JB): 79.56			
Prob(Q): 0.83		Prob(JB): 0.00	
Heteroskedasticity (H): 0.65		Skew: 0.89	
Prob(H) (two-sided): 0.07		Kurtosis: 5.41	

$$y_t = 6.0881 + 1.6404 y_{t-1} - 0.6780 y_{t-2} + \hat{\varepsilon}_t$$