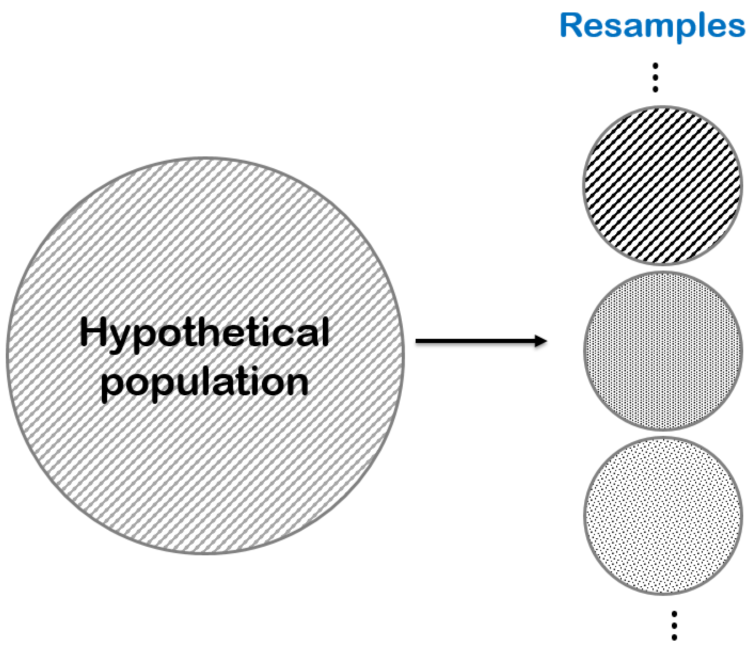
Sampling

A resampling procedure; pros and cons of different sampling schemes; bootstrap sampling; sampling bias; sample size



“Big data are not necessarily good data; well-designed small sample surveys can produce more accurate results than huge datasets that are just lying around.“

Nagiza F. Samatova, samatova@csc.ncsu.edu
Professor, Department of Computer Science
North Carolina State University

Learning Objectives: Sampling

- **Specify what is required for a simple random sample (SRS)**
- **Specify the resampling procedure to determine:**
  - the sampling distribution of a proportion
  - the sampling distribution of a mean
- **Understand pros and cons of different statistical sampling schemes:**
  - random, stratified, cluster, self-selection
- **Understand and use bootstrap and permutation sampling**
- **Understand the meaning of glossary terms:**
  - populations, samples, parameters, statistic, sampling frame, bias (see Glossary)
- **Understand sampling procedures:**
  - Explain the relationship between required sample size for different population sizes
  - Explain bias caused by self-selection and non-response in surveys

Sampling Packages in R

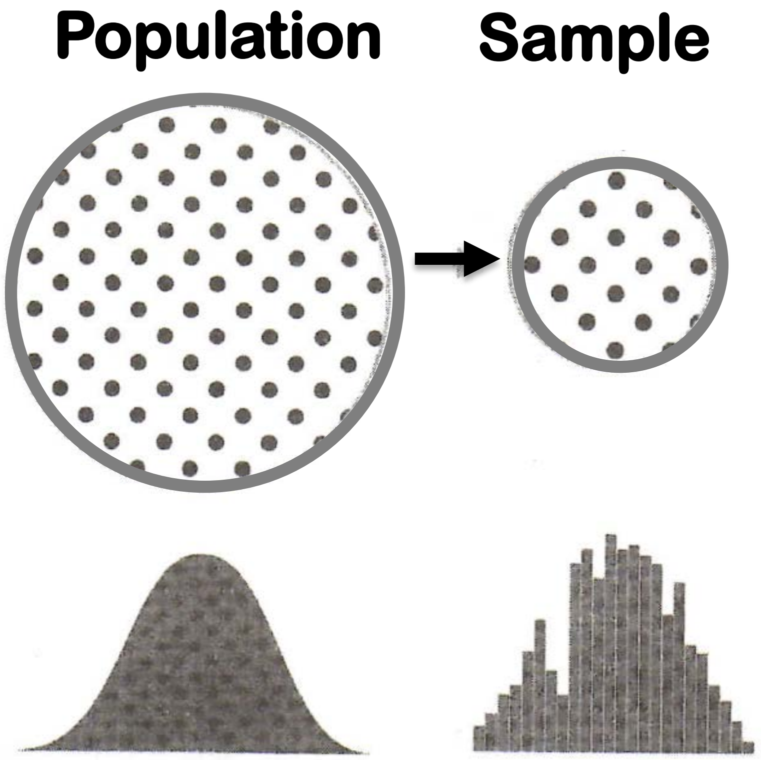
- **survey**:
  - install.packages ("survey")
  - library (survey)
  - r-survey.r-forge.r-project.org/survey/
- **sampling**:
  - install.packages ("sampling")
  - library (sampling)
  - cran.r-project.org/web/packages/sampling/sampling.pdf
- **bootstrap / boot: Bootstrap Sampling**
  - install.packages ("boot", "bootstrap")
  - library (boot)
  - library (bootstrap)
- **lmPerm / coin: Permutation Sampling**
  - install.packages ("coin", "lmPerm")
  - library (lmPerm): permutation tests for ANOVA and regression designs
  - library (coin): permutation tests to independence problems

Sampling: Basic Terminology

Term	Definition	Examples/Comments
Parameter	A measurable characteristic of the population	mean, proportion
Population	The target group of study	California voters (eligible to vote? vs. registered?)
Sample	A subset of the population. If drawn randomly, then it is a random sample	
Sampling frame	A practical representation of the population	Only registered voters
Statistic	A measurable characteristic of a sample used to estimate a population parameter	empirical mean is a statistic for a theoretical mean

Why Sampling?

- To learn about the population: population parameters
  - We don't get to measure/record/observe the *full population*, only a sample of it
- To allow greater attention to data exploration and data quality
  - For full data, it might be prohibitively expensive to:
    - Process missing values in data
    - Evaluate outliers
    - Meaningfully plot and visualize
- To provide scalability
  - Most algorithms scale non-linearly with data size
- To provide balanced group representations
  - Over-sampling of under-represented observations
  - Under-sampling of over-represented observation



How to Characterize a Sample?

Sample Statistic

- **Single sample:**
  - mean, median, standard deviation
  - proportions, ratio of proportions
- **Two samples:**
  - the difference in means
  - the difference in proportions
  - ratio of proportions
- **Proxy statistic:**
  - t-statistic
  - F-statistic
  - χ^2-statistic
  - Z-statistics

Sample Statistics vs. Population Parameters

S.S. vs. P.P.

Sample Statistics	S.S.	P.P.	Population Parameters
The mean of a quantitative variable within a sample	$\bar{x}$	μ	The mean of a quantitative variable in an entire population
The standard deviation of a quantitative variable within a sample	s	σ	The standard deviation of a quantitative variable in a population
The variance of a quantitative variable within a sample	s^2	σ^2	The variance of a quantitative variable in a population
The proportion of an outcome occurring within a sample	$\hat{p}$	p	The proportion of an outcome occurring in a population
The proportion of something not occurring within a sample	$\hat{q}$	q	The proportion of something not occurring in a population

Sample Statistics: Hats and Bars

Samples Drawn from Known Distributions

Distribution	Random Number Generator	Density	Distribution	Quantile
Normal	rnorm	dnorm	pnorm	qnorm
t	rt	dt	pt	qt
F	rf	df	pf	qf
χ^2	rchisq	dchisq	pchisq	qchisq

{dpqr}*distribution_abbreviation*()

- **d** = density
- **p** = distribution function
- **q** = quantile function
- **r** = random generation

- **pnorm(a)** $\equiv \boldsymbol{P(X \le a)}$: probability that a or smaller number occurs
- **pnorm(b) – pnorm(a)** $\equiv \boldsymbol{P(a \le X \le b)}$: probability that the variable falls between two points
- qnorm(): given the cumulative probability distribution, it returns the quantile

Population Parameters for Different Distributions

Distribution	Degrees of freedom	Mean	Variance
Normal		μ	σ^2
t	n	0	$n/(n-2)$
F	n_1 and n_2	$n_2/(n_2-2)$	a/b
χ^2	r	r	$2r$

$$a = 2n_2^2(n_1 + n_2 - 2)$$

$$b = n_1(n_2 - 2)^2\,(n_2 - 4)$$

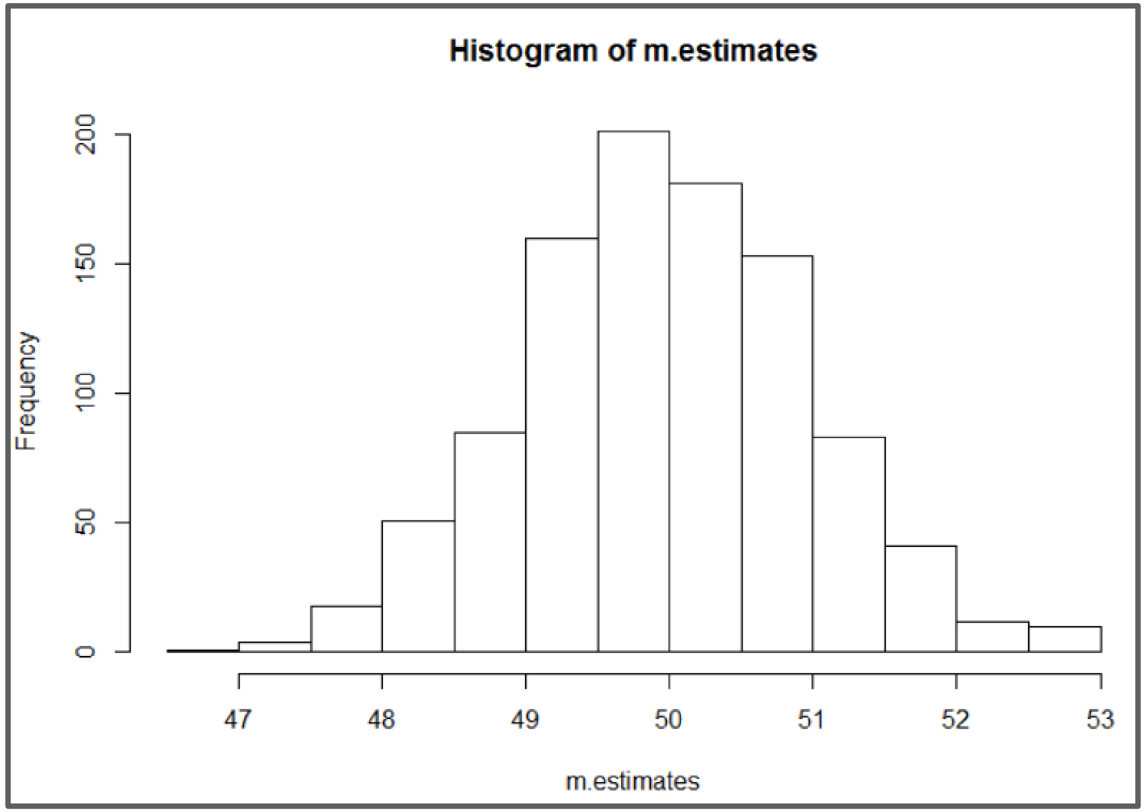
Is the Sample Mean the same as the Population Mean?

Population Parameters: mu and sd

```r
rand.normal <- rnorm (100, mean = 50, sd = 10)
mean (rand.normal)
sd (rand.normal)

m.estimates <- sapply (1:1000,
          FUN=function(iter) {
            mean(rnorm(100, mean=50, sd=10))
          })
hist(m.estimates)
mean (m.estimates)
var (m.estimates)
```

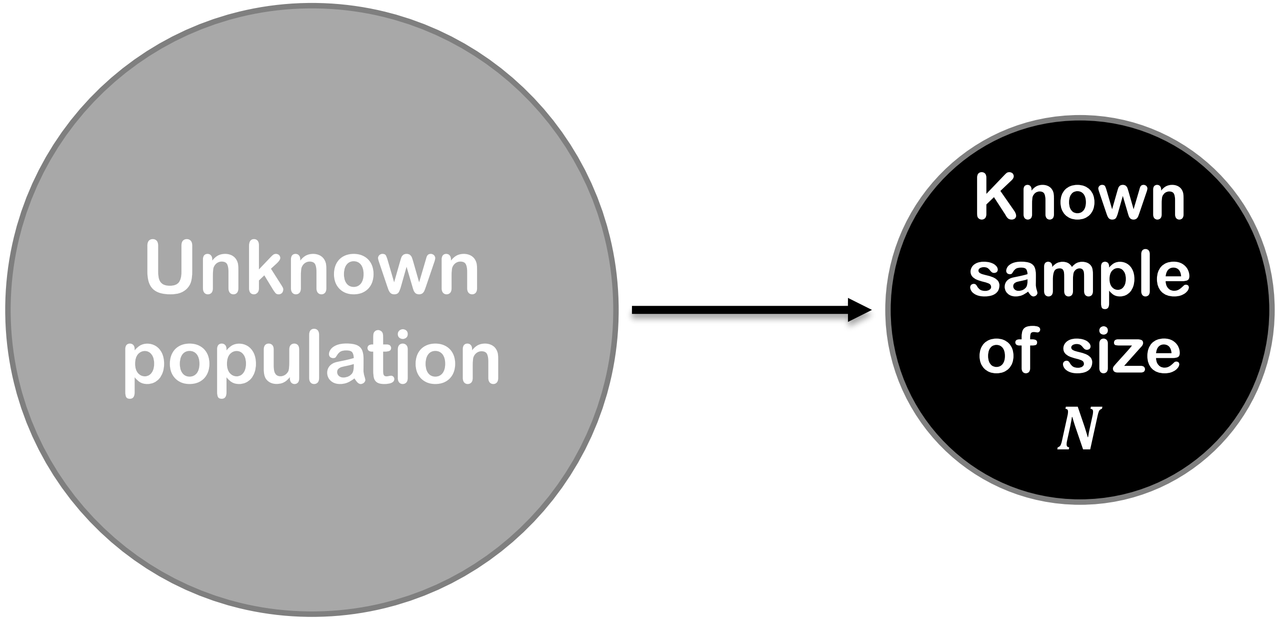
Sampling_normal_distribution.R

Sample Statistic



How sample statistic approximates population parameters for different sample sizes, n?

Sample Drawn from an Unknown Population



- How do samples drawn from an unknown population behave?
  - How different are they from one another?

Statistic & its Proxy: Hypothesis Testing

Aim	Model Statistic	Sample Statistic	Proxy Statistic	Formula for Proxy
Estimate the mean μ of a normal distribution with known variance σ^2	μ	m	Z-statistic	$Z \sim \frac{m-\mu}{\sigma/\sqrt{n}}$
Estimate the variance σ^2 of a normal distribution with known mean μ	σ^2	S^2	χ^2-statistic	$\chi^2_{n-1} \sim (n-1)\frac{S^2}{\sigma^2}$
Estimate the mean μ of a normal distribution with un-known variance σ^2	μ	m	t-statistic	$T_{n-1} \sim \frac{m-\mu}{S/\sqrt{n}}$

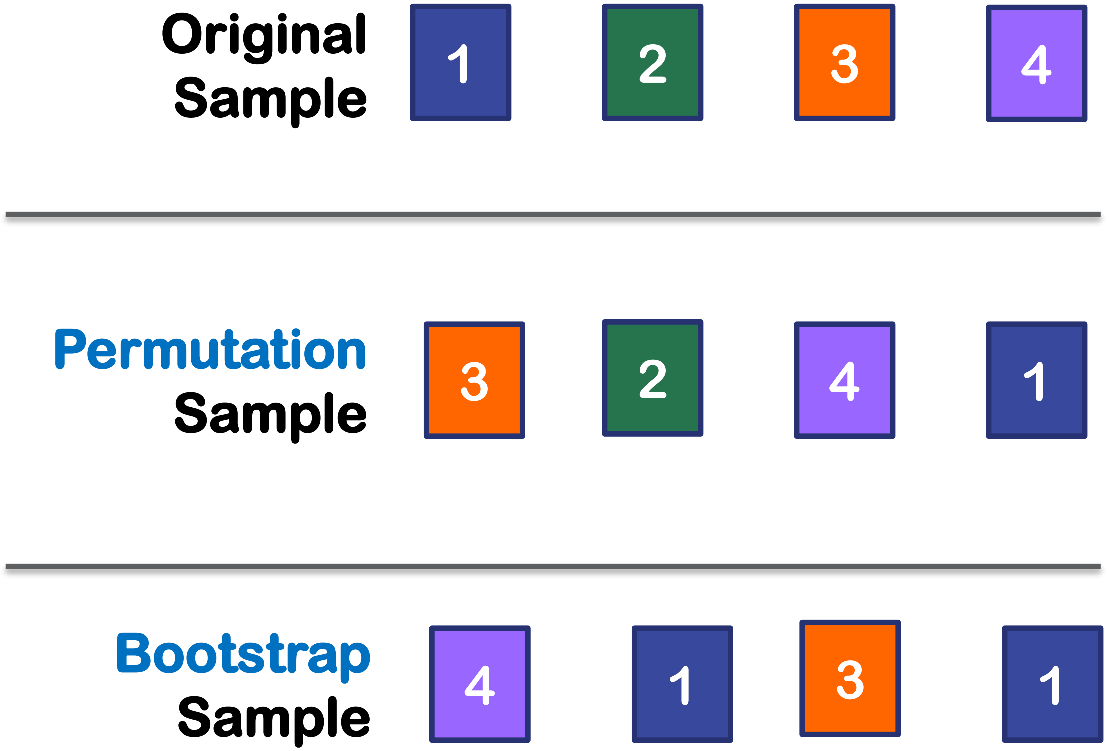
Ex.	Proxy Statistic	Distribution	Degrees of Freedom (df)
1	Z-statistic	$N(0,1)$	
2	χ^2-statistic	$\chi^2(n-1)$	$n-1$
3	t-statistic	T_{n-1}	$n-1$

Sampling Schemes

RESAMPLING, BOOTSTRAP & PERMUTATION SAMPLING

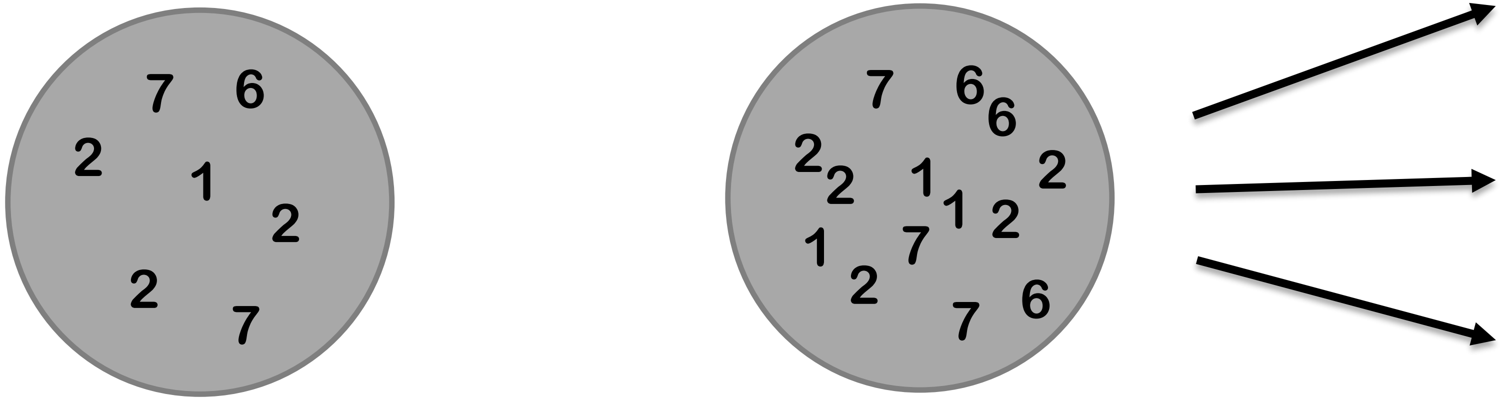
Resampling: Bootstrap and Permutation

- **Bootstrap Sampling**:
  - Sampling with replacement
  - Hypothesis Testing
  - Confidence Interval Estimation
  - R package: boot
- **Permutation Sampling**:
  - Sampling without replacement: shuffling
  - Permutation Tests: Independence Problems
    - Are responses independent of group labels?
    - Are two/k samples independent?
    - Are two categorical variables independent?
  - Permutation Tests: ANOVA & Regression Designs
    - Define later when we study regression
  - R package: coin and lmPerm
  - Remember:
    - set.seed(fixed_number) for reproducibility



Basic Bootstrap: Theory

Hypothetical Population

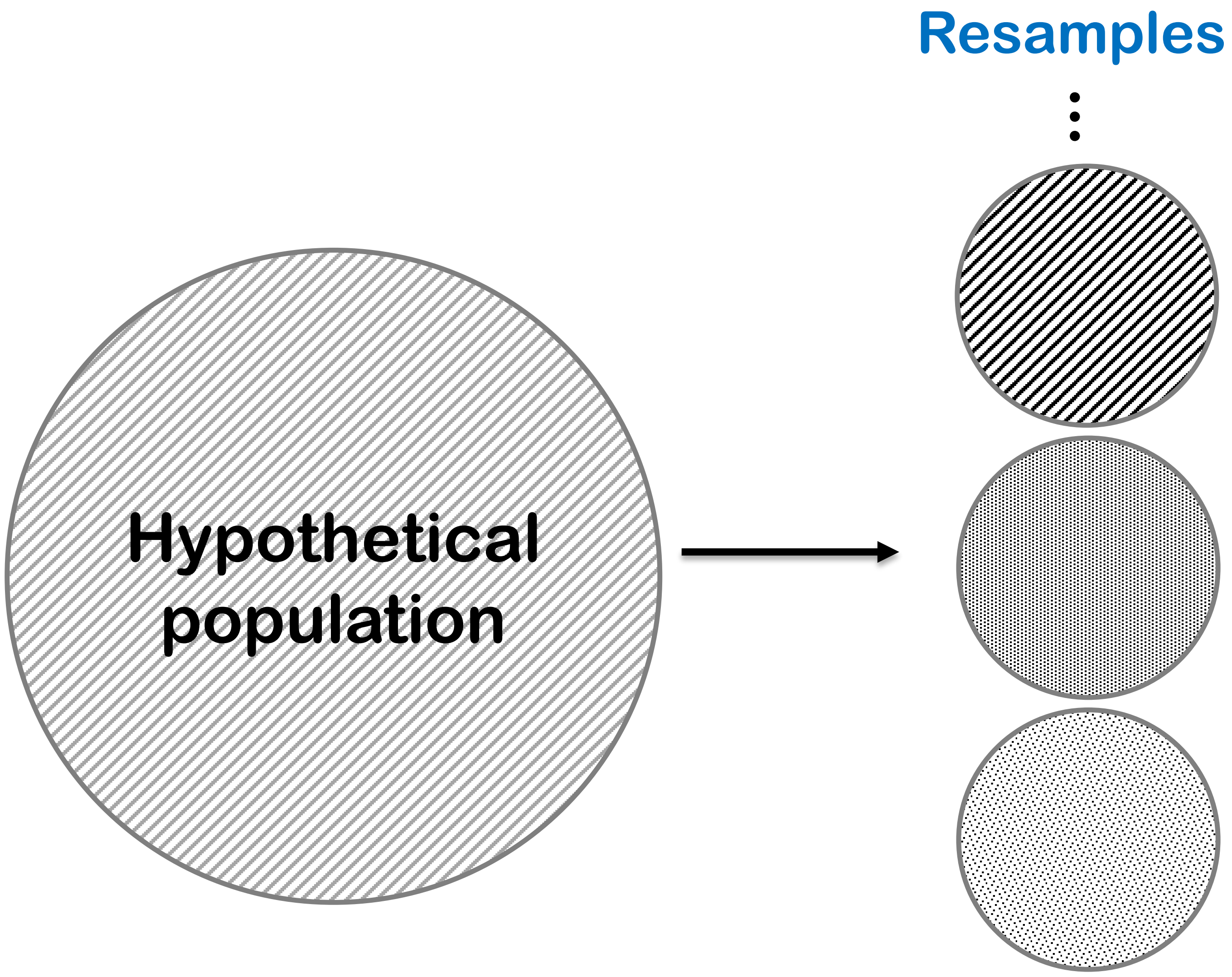


Draw lots of resamples

Original Sample

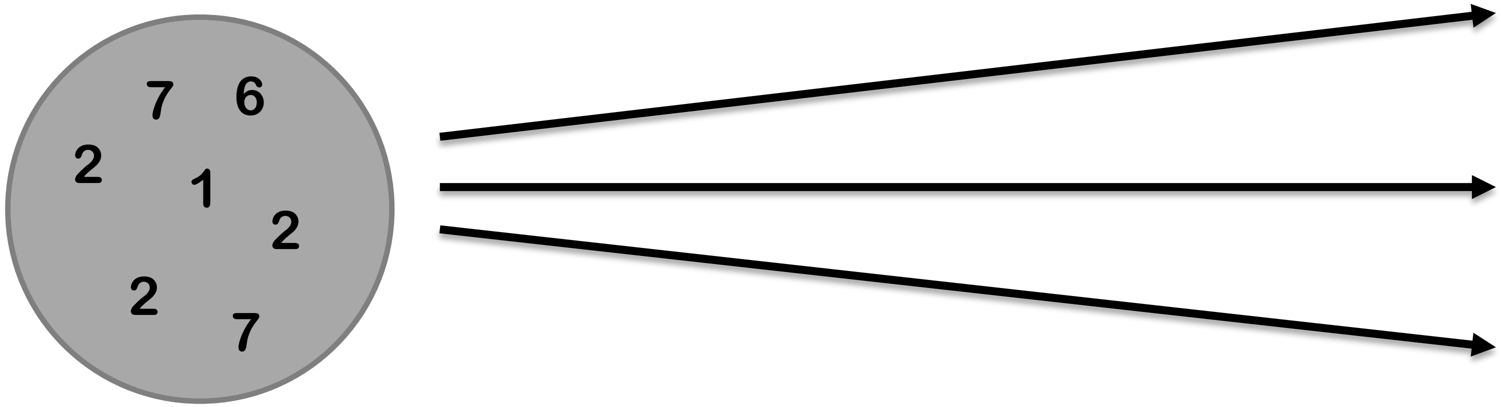
Sample replicated a huge number of times

Simulation: Bootstrap Sampling Procedure: In Theory



1. From the observed known sample, calculate a statistics to measure some attribute of the population (e.g., positive response rate, mean)
2. Create a hypothetical population using information from the sample
3. Draw a resample from the hypothetical population
4. Record the statistic of interest for the resample
5. Repeat steps 3 and 4 many times
6. Observe the sampling distribution of the statistic of interest to estimate an error or difference from the benchmark value of interest

Basic Bootstrap: Practice

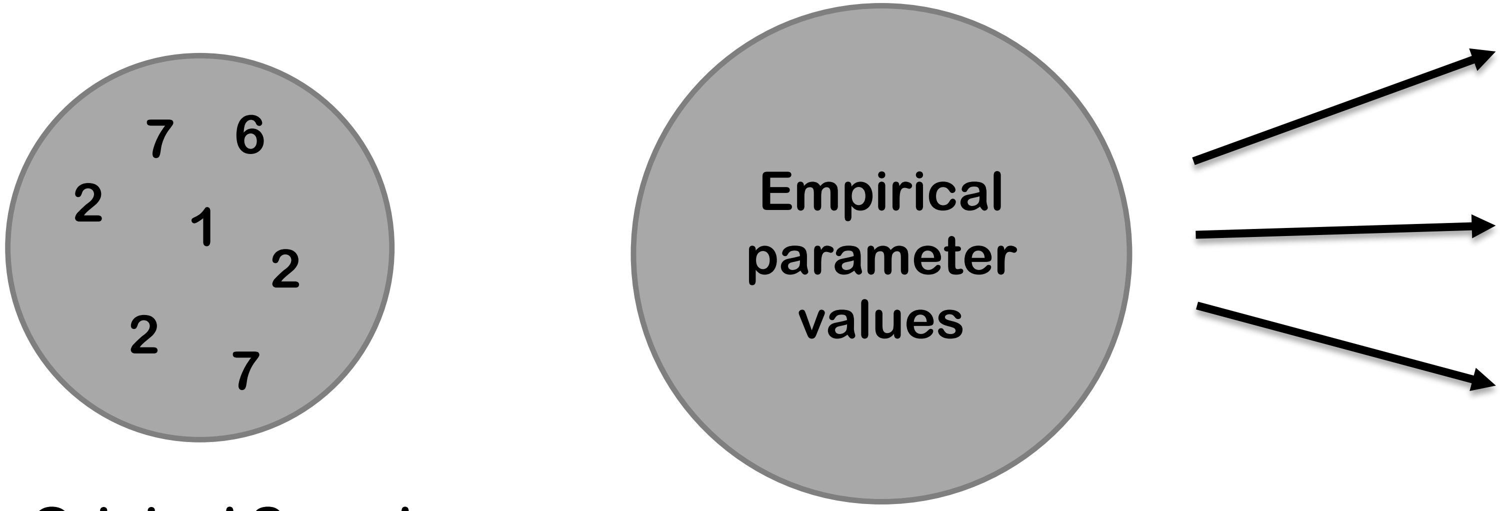


Original Sample

Draw lots of resamples, with replacement

Parametric Bootstrap

Known Distribution: Population



Draw lots of resamples,

Original Sample

Random number generator

- Normal distribution parameters
  - $\bar{x}$: mean from the sample
  - s : standard deviation from the sample

Bootstrapping with the boot::boot() in R

boot.obj = boot::boot (data = , R =, statistic = , …)

1. Write a function (e.g., statistic_funct()) that returns the statistic or statistics of interest
2. Pass this function to the boot() as statistic = statistic_function
3. Pass the number R of bootstrap replicates
4. Use boot.ci() function to obtain confidence intervals for the statistic(s) generated in Step 2

```r
library (boot)

loans_income <- read.csv(file = "../data_raw/loans_income.csv")[,1]

head (loans_income)

stat_fun <- function (x, idx) median (x [idx])

boot.obj <- boot (loans_income, R = 1000, statistic = stat_fun)
boot.obj

# estimate confidence interval on the obtained statistic
boot.ci (boot.obj, type="perc")
```

```
Bootstrap Statistics :
     original   bias     std. error
t1*     62000 -82.113     215.6994
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 1000 bootstrap replicates

CALL :
boot.ci(boot.out = boot.obj, type = "perc")

Intervals :
Level     Percentile
95%   (61200, 62000 )
Calculations and Intervals on Original Scale
```

Sampling Strategies

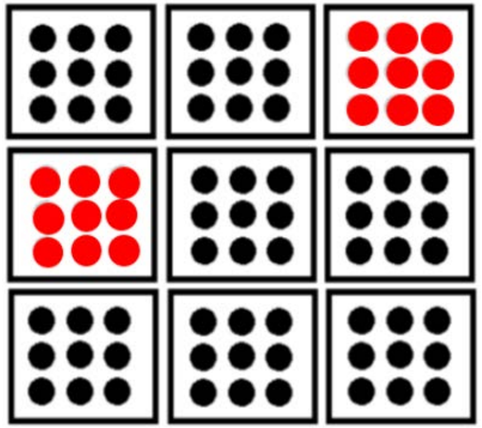
TYPES OF SAMPLING

Sampling Strategies

- Simple Random Sample
- Stratified Random Sample
- Cluster Sample
- Systematic Sample

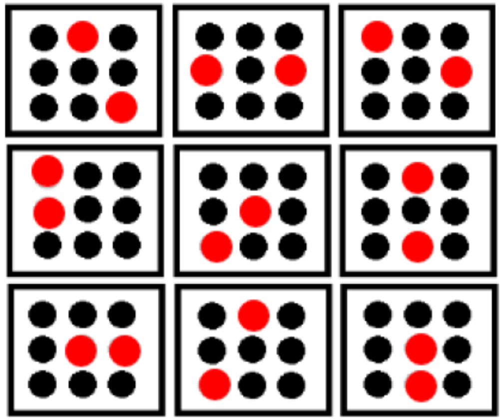
Sampling Strategies: Visual Illustration

Cluster



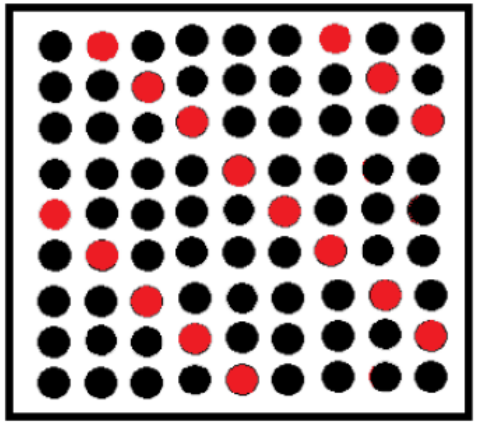
Randomly select 2 clusters and sample every individual in those

Stratified



Randomly select 2 individuals from each strata

Systematic



Randomly select 2nd individual, then select every 5th individual after that

Sampling Strategies

Term	Definition	Pros and Cons
Convenience Sampling	There is no effort to define a population or sampling frame: by inviting any one who saw the invite	(+) Easy and cheap (-) Non-representative sample, not well-designed
Cluster Sampling	Clusters of subsects or records selected, and the subjects or records within those clusters are surveyed and measured. Ensure that characteristics that define clusters do not introduce bias into the results	(+) Practical and efficient
Multi-stage Sampling	Randomly select groups and then apply systematic sampling within each group	(+) Minimize cost, sampling error, and bias
Self-Selection	The respondents themselves determine whether they participate in the survey	(-) Biased results
SRS: Simple Random Sample	Better known as a randomly drawn sample rather than random sample: each object in the population has an equal chance of being selected	(-) Does not guarantee a fully representative sample (-) Inefficient in practice
Stratified Sampling	The population is split into categories, or strata, and separate samples are drawn from each stratum.	
Systematic Sampling	Selection of every n^{th} record	

SRS: Simple Random Sample (sampling::srswor() and sample())

SRS_sampling.R

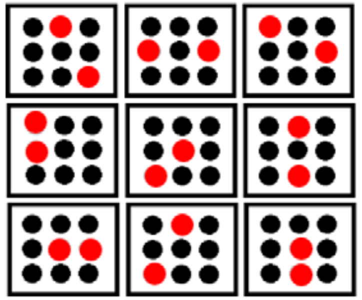
- **Assumptions**
  - population is homogeneous
- **Pros**
  - Simple in theory
  - Unbiased
  - Makes statistical inference possible
- **Cons**
  - Complex or inefficient in practice
  - Does not guarantee a completely random sample
- **R Examples:**
  - library (sampling)
  - srcwor (): without replacement
  - srcwr (): with replacement
  - sample()

```r
library (sampling)

data(belgianmunicipalities)
head(belgianmunicipalities)

population.size <- length (belgianmunicipalities$Tot04)
cat ("Population size: ", length (srs.sample), "\n")

# Note: vector of 0's and 1's of the same size as population
#       1: which observation to select
set.seed(2020)
sample.size <- 20
srs.bitmap <- srswor(n=sample.size,
                     N=population.size)
head (srs.bitmap)

# Access the records in the sample
#   using the sample bitmap
name <- belgianmunicipalities$Commune
as.vector ( name[srs.bitmap == 1] )
```

Bootstrap Sample: Sample with Replacement

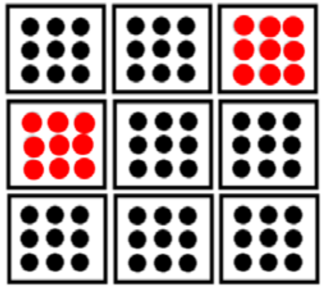
```r
my.data <- 1:20

# bootstrap sample
sample (my.data, replace = TRUE)
```

Stratified Sampling



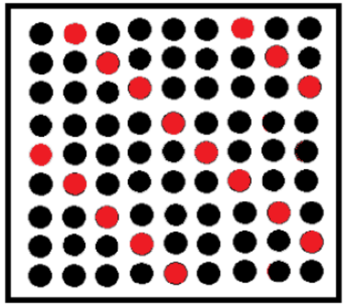
- **Assumptions**
  - population is divided into subgroups called strata
  - with important differences across strata
- **Pros**
  - usually increases precision
  - allows separate estimates per stratum
  - convenient/easier/cheaper
- **Cons**
  - requires knowledge of auxiliary variable
  - complicates analysis
- **Example**
  - Customer satisfaction:
    - Want to get input from different-sized customer orgs, different sectors, different regions

Cluster Sampling



- **Assumptions**
  - observational units are not directly accessible:
    - SRS of customer organizations
    - then SRS of employees within selected organizations
  - clusters are representative of populations
- **Pros**
  - cheaper, easier, more convenient than SRS
  - only need a list of clusters (not all observations)
- **Cons**
  - strong dependence within clusters may lead to inefficiency
  - more complex analysis than SRS

Systematic Sampling



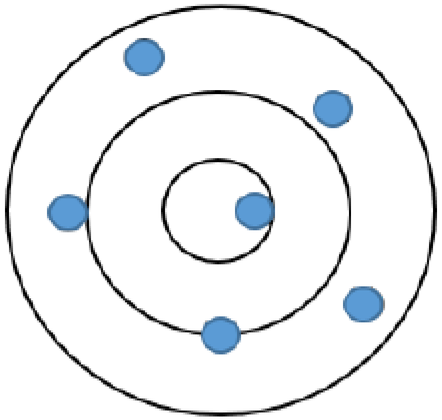
- **Assumptions**
  - population is homogenous or
  - strata/clusters are systematically arranged
- **Pros**
  - easy to implement
  - useful for data over time
  - convenient/cheap
- **Cons**
  - can be biased if not carefully selected
    - **seasonality, periodicity**
  - accuracy depends on the order of sampling units; never an SRS
- **Example**
  - Quality Control
    - Sample every 100^{th} item one item per hour from a continuous moving production line

Sampling

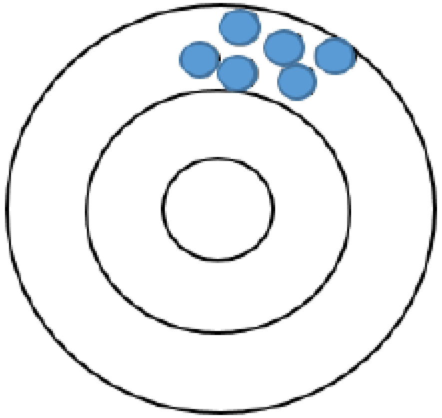
SAMPLE DESIGN

Representative Sample that leads to Accuracy & Precision

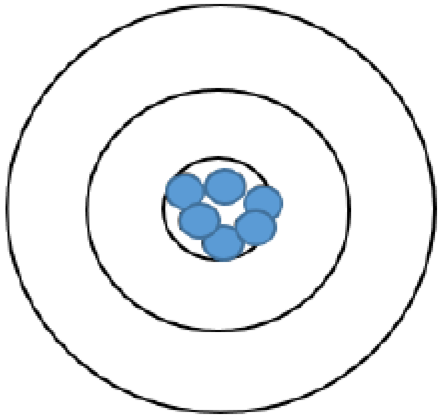
A small representative sample is more accurate and precise than a large sample that is not representative



Accurate
Not Precise



Not Accurate
Precise



Accurate
Precise

Sample Characteristics: Accuracy and Precision

- **Accuracy**
  - Mean
  - Median
  - Mode
- **Precision**
  - Variance
  - Interquartile range
  - Mean Absolute Deviation
- **Bounds on the Error of Population Parameter Estimation**
  - E.g., the probability that sample mean is different from the population mean within a given error is 0.95

Sample Design Goal & Criteria for Good Design

- Goal:
  - Maximize information while minimizing cost

- Criteria
  - Accuracy: how far is sample statistic from the corresponding population parameter (P.P.)
  - Precision: how small is standard error for a sample statistic
  - Error bounds: how small is the error on the P.P. estimation

Sample Design Procedure

- Design Process
  - Step-1: Decide on sampling strategy
  - Step-2: Select sample size
    - **Power Analysis slides on the Sample Size selection**

Step-1: Decide on Sampling Strategy

General Guidelines

- **Use stratified sampling**
  - To insure representation from particular groups
- **Use cluster sampling**
  - If individuals are spread out geographically or
  - If information/time/money is limited
- **Use systematic sampling**
  - If need to measure in real time
- **Context and pragmatism are key**
  - “Perfect” sampling plan no good if it cannot be implemented

Step-1: Decide on Sampling Strategy

Other Considerations

- How are individuals organized in the population?
  - What information is available?
  - Can I get a sampling frame for all individuals, or do I only have a list of clusters?
- How much time/money/resources can be devoted to collecting data?
- What do I want to learn about?
- Don't sample based on a response variable:
  - Want to measure customer satisfaction, but only sample from customers with historically high ratings is available

Sampling Bias

SELECTION BIAS AND RESPONSE NATURE

Biased samples are more likely to produce some outcomes than others… sample statistics may be consistently too high or too low

Bias due to Selection or the Nature of the Response

Term	Definition	Examples/Comments
Bias	A statistical procedure or measure is biased if applied to a sample from a population produces (under-)over-estimates of population characteristic	
Nonresponse Bias	A problem that occurs when non-responders do not show up in surveys	
Response Bias	Responses given differ from the truth	
Self-Selection	The respondents themselves determine whether they participate in the survey	(-) Biased results
Convenience Sampling	There is no effort to define a population or sampling frame: by inviting any one who saw the invite	(+) Easy and cheap (-) Non-representative sample, not well-designed
Selection Bias	Only a particular subset of people are selected or volunteer to be in the sample	
Volunteer response sample	Self-selected sample of people who responded to a general appeal	

Bias: Sample Selection

- **Selection bias**
  - Only a particular subset of people are selected or volunteer to be in the sample
- **Convenience samples**
  - Samples that are easy to take, based on a readily assembled group
    - **E.g., only selecting customers from a particular organization**
- **Volunteer response sample**:
  - Self-selected sample of people who responded to a general appeal
    - **Those who volunteer may be different from general population**
    - **Ex: Table cards in restaurants, online votes**
    - **Ex: Sending a general email blast to all customers**

Other Sources of Bias

- **Non-response bias**
  - Some part of the population may not respond or refuses to participate
  - Connection to missing data:
    - **If responses are MAR (Missing at random), could impute**
    - **If MNAR (Missing not at random), a small response rate could indicate a problem**
- **Response bias**
  - Responses given differ from the truth
  - Results from questions or people involved; could be intentional or unintentional
    - **Ex: Customer may not want to mention in person that they are not satisfied**

Other Things to Keep in Mind

- **It is important to pay attention to the sampling method used when considering the results of a survey**
- **If the sample is not random, proceed with extreme caution!**
  - You may not be able to make any conclusions about the full population
  - Instead, you have to think about what restricted/other population the sample is representative of

Acknowledgements

- Dr. Herle McGowan, NCSU
- Dr. Jacqueline M. Hughes-Oliver, NCSU