# Terminology, Definitions & Cheat Sheets

## STATISTICS

# Common Statistics: Various Types of Outcome Data

| Outcome Variable | Are the observations independent or correlated? | | Alternatives (if assumptions are violated) |
|---|---|---|---|
| | Independent | Correlated / Paired | |
| **Continuous** (e.g., profit, price) | Ttest<br>ANOVA<br>Linear correlation<br>Linear regression | Paired ttest<br>Repeated-measures ANOVA<br>Mixed models<br>GEE modeling | Wilcoxon sign-rank test<br>Wilcoxon rank-sum test<br>Kruskal-Wallis test<br>Spearman rank correlation |
| **Binary or categorical** (e.g., churn yes/no) | Risk difference<br>Relative risks<br>Chi-square test<br>Logistic regression | McNemar's test<br>Conditional logistic regression<br>GEE modeling | Fisher's exact test<br>McNemar's exact test |
| **Time-to-event** (e.g., time to part failure) | Rate ratio<br>Kaplan-Meier statistics<br>Cox regression | Frailty model (beyond the scope) | Time-varying effects (beyond the scope of this course) |

# Sampling

| Term | Definition | Examples/Comments |
|---|---|---|
| Bias | A statistical procedure or measure is biased if applied to a sample from a population produces (under-)over-estimates of population characteristic | |
| Nonresponse Bias | A problem that occurs when non-responders do not show up in surveys | |
| Parameter | A measurable characteristic of the population | mean, proportion |
| Population | The target group of study | California voters (eligible to vote? vs. registered?) |
| Sample | A subset of the population. If drawn randomly, then it is a random sample | |
| Sampling frame | A practical representation of the population | Only registered voters |
| SRS: Simple Random Sample | Better known as a randomly drawn sample rather than random sample: each object in the population has an equal chance of being selected | (-) Does not guarantee a fully representative sample<br>(-) Inefficient in practice |
| Statistic | A measurable characteristic of a sample used to estimate a population parameter | empirical mean is a statistic for a theoretical mean |

# Sampling Procedure

| Term | Definition | Examples/Comments |
|---|---|---|
| Bootstrap | A resample with replacement from an observed sample to observe a distribution of a sample statistic | |
| Randomization | The process of randomly assigning subjects to treatment | |
| Resample | A collection of observations drawn from the original sample, or generated randomly by a formula based on the original sample | |
| Sampling with replacement | Each item is replaced after it is drawn from a hat | |
| Sampling without replacement | Once an item is drawn, it is not eligible to be drawn again: also known as shuffling | |
| Simulation trial | Taking a resample and calculating the value of some statistic, such as mean, with it | |
| Simulation | A repeat of multiple single simulation trials and the collection of their calculation results | |

# Sampling Schemes

| Term | Definition | Examples/Comments |
|---|---|---|
| Convenience Sampling | There is no effort to define a population or sampling frame: by inviting any one who saw the invite | (+) Easy and cheap<br>(-) Non-representative sample, not well-designed |
| Cluster Sampling | Clusters of subsects or records selected, and the subjects or records within those clusters are surveyed and measured. Ensure that characteristics that define clusters do not introduce bias into the results | (+) Practical and efficient |
| Multi-stage Sampling | Randomly select groups and then apply systematic sampling within each group | (+) Minimize cost, sampling error, and bias |
| Self-Selection | The respondents themselves determine whether they participate in the survey | (-) Biased results |
| SRS: Simple Random Sample | Better known as a randomly drawn sample rather than random sample: each object in the population has an equal chance of being selected | (-) Does not guarantee a fully representative sample<br>(-) Inefficient in practice |
| Stratified Sampling | The population is split into categories, or strata, and separate samples are drawn from each stratum. | |
| Systematic Sampling | Selection of every $n^{th}$ record | |

# A/B Testing

| Term | Definition | Examples/Comments |
|---|---|---|
| Control group | A group of subjects exposed to no (or standard) treatment | |
| Randomization | The process of randomly assigning subjects to treatment | |
| Standard Error | The variability (standard deviation) of a sample statistic over many samples | Unlike standard deviation that refers to variability of values within a single sample |
| Subjects | The items that are exposed to treatments | web visitors, patients, .. |
| Test statistic | The metric used to measure the effect of treatment | t-statistic |
| Treatment | Something to which a subject is exposed | drug, price, web headline |
| Treatment group | A group of subjects exposed to a specific treatment | |

# Confidence Interval (CI)

| Term | Definition | Examples/Comments |
|---|---|---|
| Central Limit Theorem (CLT) | CLT says that the means drawn from multiple samples will be Normally distributed, even if the population is not Normally distributed if the sample size is "large enough" (20-30) and the departure from Normality is "not great" | Allows Normal-approximation formulas to be used in calculating sampling distributions for CIs and hypothesis tests |
| Confidence Interval | A 90% confidence interval for the mean (or other statistic) is a range that encloses the central 90% of the resampled means (or other statistic). 90% CI encloses the true statistic 90% of the time when constructed repeatedly in the same manner with the same population. | |
| Margin of Error | A plus or minus quantity attached to a point estimate, whereas a CI is the actual endpoints of the interval | If the mean of the positive rate is 36% and CI is [32,40], then margin of error is 36% +/-4% |
| Point Estimate | A statistic calculated from a sample | mean ($\bar{x}$), median, quantile |
| Standard Error | Standard error (s.e.) of a sample statistic is the standard deviation of the sample statistic (s.e. of the estimate) | sample estimate means sample statistic |

# Hypothesis Tests

| Term | Definition | Examples/Comments |
|---|---|---|
| Alternate Hypothesis | The theory you would like to accept, assuming that your results disprove the null hypothesis | Counterpoint to the null (what you hope to prove) |
| Alpha ($\alpha$) Level or Significance | The threshold level of statistical significance | Determined before a study is done: e.g. 5% is customary |
| Hypothesis Test, or Significance Test | Answers the questions, "Might this apparently interesting result have happened by chance owing to the luck of the draw in who/what gets selected in sample(s) or assigned to difference treatments?" | CI: "How much chance error might be in this measurement or estimate or model, owing to the luck …?" |
| Null Hypothesis<br>Null Model | An imaginary chance model representing the idea that nothing new is going on. The hypothesis that chance is to blame. | It nullifies what you are trying to prove (e.g., no difference between treatments A and B) |
| $p$-value | The frequency with which a result as extreme as observed results occurs just by chance, drawing from the null hypothesis model | $p$-value $\leq 0.05$ indicates statistical significance → rejects the Null Hypothesis. It should be low enough to meet the threshold established (alpha) for statistical significance |
| Statistical Inference | The process of accounting for random variation in data as you draw conclusions | |

# Hypothesis Tests (cont.)

| Term | Definition | Examples/Comments |
|---|---|---|
| One-tailed test or One-way test | The whole alpha $\alpha$ is allotted to only one direction of the distribution | Hypothesis test that counts chance results only in one direction |
| Test statistic | The metric used to measure the effect of treatment | t-statistic, F-statistic, $\chi^2$-statistic |
| Two-tailed test or Two-way test | half of alpha $\alpha$ is used to test the statistical significance in one direction of the distribution and the other half in the other direction | Hypothesis test that counts chance results only in two direction |
| Type I Error | Reject H0 when, in fact, H0 is true. Mis-takingly concluding an effect is real (when it is due to chance) | a jury convicted an innocent person for a crime that the person did not commit |
| Type II Error | Fail to Reject H0 when, in fact, H1 is true. Mis-takingly concluding an effect is due to chance (when it is real) | a guilty person escaped conviction |

# Statistical Distributions & Functions in R

| Distribution | Random Number Generator | Density | Distribution | Quantile |
|---|---|---|---|---|
| Normal | rnorm | dnorm | pnorm | qnorm |
| $t$ | rt | dt | pt | qt |
| $F$ | rf | df | pf | qf |
| $\chi^2$ | rchisq | dchisq | pchisq | qchisq |

**{dpqr}*distribution_abbreviation*()**

- **d** = density
- **p** = distribution function
- **q** = quantile function
- **r** = random generation

- **pnorm(a)** $\equiv P(X \leq a)$: probability that $a$ or smaller number occurs
- **pnorm(b) – pnorm(a)** $\equiv P(a \leq X \leq b)$: probability that the variable falls between two points
- qnorm(): given the cumulative probability distribution, it returns the quantile

# Statistical Distributions: Mean and Variance

| Distribution | Degrees of freedom | Mean | Variance |
|---|---|---|---|
| Normal | | $\mu$ | $\sigma^2$ |
| $t$ | $n$ | 0 | $n/(n-2)$ |
| $F$ | $n_1$ and $n_2$ | $n_2/(n_2-2)$ | $a/b$ |
| $\chi^2$ | $r$ | $r$ | $2r$ |

$$a = 2n_2^2(n_1 + n_2 - 2)$$
$$b = n_1(n_2 - 2)^2\,(n_2 - 4)$$

# Proxy Statistic: Hypothesis Testing & Confidence Intervals

| Aim | Model Statistic | Sample Statistic | Proxy Statistic | Formula for Proxy |
|---|---|---|---|---|
| Estimate the mean $\mu$ of a normal distribution with known variance $\sigma^2$ | $\mu$ | $m$ | $Z$-statistic | $Z \sim \frac{m-\mu}{\sigma/\sqrt{n}}$ |
| Estimate the variance $\sigma^2$ of a normal distribution with known mean $\mu$ | $\sigma^2$ | $S^2$ | $\chi^2$-statistic | $\chi^2_{n-1} \sim (n-1)\frac{S^2}{\sigma^2}$ |
| Estimate the mean $\mu$ of a normal distribution with un-known variance $\sigma^2$ | $\mu$ | $m$ | $t$-statistic | $T_{n-1} \sim \frac{m-\mu}{S/\sqrt{n}}$ |

| Ex. | Proxy Statistic | Distribution | Degrees of Freedom (df) |
|---|---|---|---|
| 1 | $Z$-statistic | $N(0,1)$ | |
| 2 | $\chi^2$-statistic | $\chi^2(n-1)$ | $n-1$ |
| 3 | $t$-statistic | $T_{n-1}$ | $n-1$ |

# Hypothesis Testing: Procedure

- Step 1: Define **a statistic** that obeys a certain **distribution** if the hypothesis is correct:
  - Ex-1: The mean $\mu$ from a normal distribution with known variance $\sigma^2$
  - Ex-2: The variance $\sigma^2$ from a normal distribution with known mean $\mu$
  - Ex-3: The mean $\mu$ from a normal distribution with unknown variance $\sigma^2$

- Step 2 (optional): Transform the statistic to a **proxy statistic** with the **proxy distribution** of better understood properties/characteristics:
  - Ex-1: $Z$-statistic from a uniform normal distribution, $N(0,1)$
  - Ex-2: $\chi_{n-1}$-statistic from a $\chi^2$ distribution with $n$ df
  - Ex-3: $T_{n-1}$-statistic from a $t$-distribution with $n-1$ df

- Step 3: Calculate the statistic (original/proxy) from the **sample**

- Step 4: Compute the **probability** (the **$p$-value**) of this sample with this statistic to be drawn from this distribution (original/proxy)
  - **Reject the hypothesis** if probability is **low** (e.g., **$p$-value < 0.05**)
  - **Fail to reject the hypothesis** otherwise (e.g., **$p$-value $\geq$ 0.05**)

# Hypothesis Tests

| Sample | Paired | Null Hypothesis | Assumptions | Test |
|---|---|---|---|---|
| One Sample | | $H_0 : \mu = \mu_0$ | i.i.d. $N(\mu, \sigma^2)$ | t.test() |
| | | | | |
| Two Samples | No | $H_0 : \sigma_1^2 = \sigma_2^2$ | Normally distributed | F-test: var.test()<br>Bartlett: bartlett.test() |
| Two Samples | No | $H_0 : \sigma_1^2 = \sigma_2^2$ | Non-parametric | Ansari-Bradley test: ansari.test() |
| Two Samples | No | $H_0 : \mu_1 = \mu_2$ | $\sigma_1^2 = \sigma_2^2$ | t.test(**var.equal=TRUE**) |
| Two Samples | No | $H_0 : \mu_1 = \mu_2$ | $\sigma_1^2 \neq \sigma_2^2$ | Welch t-test<br>t.test(var.equal=FALSE) |
| Two samples | No | $p_1(x) = p_2(x)$<br>$p$: probab. distr | Non-parametric | Wilcoxon rank sum<br>wilcox.test () |
| | | | | |
| Two Samples | Yes | $H_0 : \mu_1 = \mu_2$ | $\sigma_1^2 \neq \sigma_2^2$ | t.test(**paired=TRUE**) |
| Two samples | Yes | $p_1(x) = p_2(x)$<br>$p$: probab. distr | Non-parametric | wilcox.test (**paired=TRUE**) |

# Power and Sample Size

| Term | Definition | Examples/Comments |
|---|---|---|
| Effect size | The minimum size of the effect that you hope to be able to detect in a statistical test | 20% improvement in click rates |
| Statistical Power | The probability of detecting a given effect size with a given sample size | |
| Significance level | The statistical significance level at which the test will be conducted | |
| Type I Error | Reject H0 when, in fact, H0 is true. Mis-takingly concluding an effect is real (when it is due to chance) | a jury convicted an innocent person for a crime that the person did not commit |
| Type II Error | Fail to Reject H0 when, in fact, H1 is true. Mis-takingly concluding an effect is due to chance (when it is real) | a guilty person escaped conviction |

# Type I and II Errors, Power, Significance Level ($\alpha$)

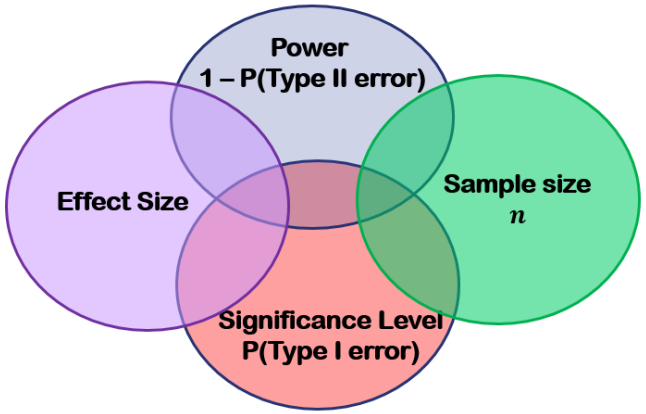
| Actual | Decision: Reject H0 | Decision: Fail to Reject H0 |
|---|---|---|
| H0 True | Type I Error | correct |
| H0 False | correct | Type II Error |

**Power = 1 – Probability (Type II Error)**

**$\alpha$ = Probability (Type I Error)**

- **Type I Error**
  - Probability of False Positives
  - Fail to find an effect that IS there
- **Type II Error**
  - The probability of False Negatives
  - Find an effect that is NOT there

- **Power** is defined as one minus the probability of making a Type II error
  - The probability of finding an effect that *IS* there
- **Significance level** ($\alpha$) is the probability of making a Type I error
  - The probability of finding an effect that is *NOT* there



# Multiple Testing

| Term | Definition | Examples/Comments |
|---|---|---|
| Type I Error | Mistakenly concluding that an effect is statistically significant | |
| False Discovery Rate (FDR) | Across multiple tests, the rate of making Type I error | |
| Adjustment of $p$-value | Accounting for doing multiple tests on the same data | |
| Overfitting | Fitting the noise | |

# Multiple Testing: Sources & Mitigation

- **Sources of multiplicity issues:**
  - Multiple comparisons in multiple tests of significance
  - Many variables
  - Many models
- **Multiplicity increases the risk of concluding that something is significant just by chance (Type I error)**
- **Mitigation strategies for multiple statistical comparisons**
  - Adjustment procedure: dividing alpha by the number of tests
  - Bonferroni adjustment: dividing alpha by the number of observations, *n*
- **Mitigation strategy for supervised modeling**
  - Cross-validation: holdout sample with labeled outcome variables

# Design #1: One-way Between-Groups ANOVA

- Examplar Study: Goal: To study two treatments of anxiety
  - **Treatment (Independent Variable)**: Two treatments
    - CBT: Cognitive Behavior Therapy
    - EMDR: Eye Movement Desensitization & Reprocessing Therapy
  - **Response (Dependent Variable)**: (collected after 5 weeks of treatment)
    - STAI: State-Trait Anxiety Inventory; a self-report measure of anxiety
  - **Subjects**:
    - Randomly divided between two ***independent groups***: CBT & EMDR

| | | Subjects |
|---|---|---|
| Treatment | CBT | $s_1$ |
| | | $s_2$ |
| | | ... |
| | | $s_{n/2}$ |
| | EMDR | $s_{n/2+1}$ |
| | | ... |
| | | ... |
| | | $s_n$ |

**One-way Between-Groups Balanced ANOVA**

- Treatment is a **between-groups** factor with two levels
- **Balanced design**: equal number of subjects in each treatment condition; otherwise, **unbalanced**
- **One-way**: because a single classification variable

**F-tests to assess the effects in ANOVA designs**

- If the F-test for Treatment is significant then reject the null hypothesis: $H_0$: the mean STAI scores are the same
- Conclude: the mean STAI scores changed over time for two therapies differed after five weeks of treatment

# Design #2: One-way Within-Groups ANOVA

- Examplar Study: Goal: To study ***longitudinally*** one treatment of anxiety
  - **Treatment (Independent Variable)**: One treatment
    - CBT: Cognitive Behavior Therapy
  - **Response (Dependent Variable)**: (collected after 5 weeks of treatment)
    - STAI: State-Trait Anxiety Inventory; a self-report measure of anxiety
  - **Subjects**:
    - The same subjects over different time points (**dependent groups**)
  - **Time**: Two different time points: 5 weeks & 6 months

**One-way Within-Groups Balanced ANOVA**

| | Time | |
|---|---|---|
| CBT Treatment | **5 weeks** | **6 months** |
| | $s_1$ | $s_1$ |
| | $s_2$ | $s_2$ |
| | ... | ... |
| | ... | ... |
| | $s_n$ | $s_n$ |

- Time is a **within-groups** factor with two levels: each subject is measured under both levels
  - **Repeated measures** ANOVA
- **One-way**: because a single classification variable

**Paired F-tests to assess effects in ANOVA designs**

- If the F-test for Treatment is significant then reject the null hypothesis: $H_0$: the mean STAI scores are the same
- Conclude: the mean STAI scores change over time: between 5 weeks and 6 months

# Design #3: Factorial (Mixed-Model) ANOVA

- Examplar Study: Goal: To study ***two treatments*** of anxiety ***longitudinally***
  - **Treatment (Independent Variable)**: Two treatments
    - CBT: Cognitive Behavior Therapy
    - EMDR: Eye Movement Desensitization & Reprocessing Therapy
  - **Response (Dependent Variable)**: (collected after 5 weeks of treatment)
    - STAI: State-Trait Anxiety Inventory; a self-report measure of anxiety
  - **Subjects**: Randomly assigned to two ***independent groups***: CBT & EMDR
  - **Time**: Two **dependent groups** over time: 5 weeks & 6 months

| | | 5 wks | 6 mo. |
|---|---|---|---|
| Treatment | CBT | $s_1$ | $s_1$ |
| | | $s_2$ | $s_2$ |
| | | ... | ... |
| | | $s_{n/2}$ | $s_{n/2}$ |
| | EMDR | $s_{n/2+1}$ | $s_{n/2+1}$ |
| | | ... | ... |
| | | ... | ... |
| | | $s_n$ | $s_n$ |

**Two-way Factorial ANOVA Design**

- **Main effects**: Impact of Therapy (averaged across Time) and Time (averaged across Therapy type)
- **Interaction effect**: Interaction of Therapy & Time
- **Factorial ANOVA**: cross 2 factors (**two-way**) or more

**Three F-tests to assess ANOVA design effects**

- F-test for Therapy: Significant→ CBI & EMDR differ
- F-test for Time: Significant → change over time
- F-test for Therapy × Time interaction: Significant → two treatments had a differential impact over time: different change from 5 wks to 6 mo. for 2 therapies

# Design #4: ANCOVA

- Goal: To study the treatments of anxiety with ***confounding factors***
- **Confounding Factor (Nuisance Variable)**: Other factors (than Treatment) that could explain the post-therapy differences on the dependent variable
  - Depression level:
    - A self-reported measure such as Beck Depression Inventory (BDI)
    - Although subjects were assigned randomly to treatment conditions, it is possible that two therapy groups differed in patient depression levels at the start of the study
    - → Any post-therapy differences might be due to the preexisting depression differences and not to experimental manipulations

**ANCOVA Design**

- Because depression could also explain the group difference on the dependent variable, it is a **confounding factor**
- Because the study is not interested in depression, this confounding factor is called a **nuisance variable**

# Design #5-6: MANOVA & MANCOVA

- Goal: To study the treatments of anxiety with ***multiple dependent variables*** and/or ***confounding factors***
- **Multiple Dependent Variables:** To increase the validity of the study
  - STAI: One dependent variable
  - Family ratings: Another dependent variable
  - Therapy ratings: Yet another dependent variable
  - A measure assessing the impact of anxiety on the daily functioning

- **MANOVA**: **Multivariate Analysis of Variance**
  - There is more than one dependent variable

- **MANCOVA**: **Multivariate Analysis of Covariance**
  - Besides multiple dependent variables, there are covariates present