

Apply (SSN, college, name,  
HSCode, HSCity, sports)

SSN  $\rightarrow$  name

HSCode  $\rightarrow$  HSCity

K	S	
✓	✓	A. { SSN, college, HSCode, <del>HSCity</del> , sports }
-	✓	B. { SSN, college, HSCode, HSCity, name, sports }
-	ⓔ	C. { SSN, college, name }

$$S(A, B, C, D, E)$$

$$AB \rightarrow D$$

$$C \rightarrow E$$

$$DE \rightarrow BC$$

1. You can "remove" attributes on the right-hand side (RHS)

$DE \rightarrow BC$  is equivalent to

$$\begin{bmatrix} DE \rightarrow B \\ DE \rightarrow C \end{bmatrix}$$


2. You cannot "remove"  
attributes on the LHS  
movies (title, year, length)  
title, year  $\rightarrow$  length





The closure of  $C$  is  $CEDB$

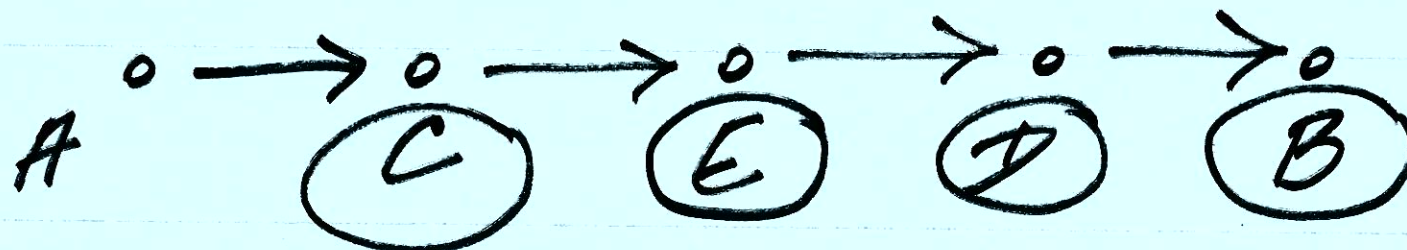
$U(A, B, C, D, E)$

- ✓  $A \rightarrow C (-)$
- ✓  $C \rightarrow E \text{ III}$
- ✓  $E \rightarrow D \text{ III}$
- ✓  $D \rightarrow B \text{ III}$

Does

$C \rightarrow B$    
  $\equiv$  hold? yes

Does  $C \rightarrow A$    
  $\equiv$  hold? no



The closure algorithm  
for computing closure of  
given set of attributes

0. Make  $X$  the initial closure set
1. Iterate
  - if there is a FD whose LHS is a subset of  $X$
  - $\Rightarrow$  add the FD's RHS to  $X$
2. Stop when  $\begin{matrix} \text{= no more unused attributes} \\ \text{or no more attributes} \end{matrix}$
3. Return  $X$





$R(A, B, C)$  $A \rightarrow B$  $B \rightarrow C$ 

Does  $A \rightarrow C$  hold? Yes

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Does  $C \rightarrow B$  hold? No

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The "closure" answers:

(1) The closure of A is ABC

(2) The closure of C is C



$R(ABCD)$  $AB \rightarrow C$  $C \rightarrow D$  $D \rightarrow A$ 

Does  $CD \rightarrow A$   
hold?

Does  $CD \rightarrow B$   
hold?



With the closure algorithm: We can

1. Prove whether a candidate FD holds
2. Find all derived FDs





$R(ABCD)$ 
 $AB \rightarrow C$ 
 $C \rightarrow D$ 
 $D \rightarrow A$ 

"Does candidate  
FD hold?

- for instance,  
 $CD \rightarrow A$

or  
 $CD \rightarrow B$

 $A^+ = A$ 
 $B^+ = B$ 
 $B \rightarrow B$ 
 $C^+ = CDA$ 
 $D^+ = DA$ 
 $AB^+ = ABCD$ 
 $AC^+ = ACD$ 
 $BC^+ = BCDA$ 
 $AD^+ = AD$ 
 $BD^+ = BDAC$ 
 $CD^+ = CDA$ 
 $ABC^+ = ABCD$ 
 $ABD^+ = ABCD$ 
 $ACD^+ = ACD$ 
 $BCD^+ = ABCD$ 
 $ABCD^+ = ABCD$ 


Keys:  $AB, BC, BD$

Superkeys:  $AB, BC, BD, ABC,$   
 $ABD, BCD, ABCD$

Derived FDs:

$$C \rightarrow A$$

$$CD \rightarrow A$$

$$AB \rightarrow D$$

$$ABC \rightarrow D$$

$$AC \rightarrow D$$

$$ABD \rightarrow C$$

$$BC \rightarrow D$$

$$BCD \rightarrow A$$

$$BC \rightarrow A$$

$$BD \rightarrow A$$

$$BD \rightarrow C$$





$$U(ABCDE)$$

$$A \rightarrow C$$

$$C \rightarrow E$$

$$E \rightarrow D$$

$$D \rightarrow B$$

Pr.

Find all FDs  
in the projection  
of  $U$  onto  
 $CD B$

$$CD B$$

$$C \rightarrow D$$

$$C \rightarrow B$$

$$D \rightarrow B$$


$$CD \rightarrow B$$

$$R(CDB)$$


$$CB \rightarrow D$$

$$C^+ = \underline{CEDB}$$

$$D^+ = \underline{DB}$$

$$B^+ = B$$

$$CD^+ = CEDB$$

$$CB^+ = CEDB$$

$$DB^+ = DB$$

$$CDB^+ = CDBE$$



You never need  
to submit  
trivial FVS  
as part of any  
assignment



$R(ABCDE)$  $AB \rightarrow DE$  $C \rightarrow E$  $D \rightarrow C$  $E \rightarrow A$ 

Project the FDs onto  
 $\pi(ABC)$

