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Q 1)

Q. 1.

since,

$$P(\mu|x) \propto P(\mu) \cdot P(x|\mu)$$

$$P(\mu|x) \propto P(\mu) \cdot \prod_{i=1}^n P(x_i|\mu)$$

$$= \frac{1}{\sqrt{2\pi}\sigma_0} \cdot e^{-\frac{(\mu-\mu_0)^2}{2\sigma_0^2}} \cdot \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi}^{n+1} \cdot \sqrt{\sigma_0^2} \cdot \sigma^n} \cdot e^{-\frac{\mu^2 + 2\mu\mu_0 + \mu_0^2}{2\sigma_0^2} - \sum_{i=1}^n \frac{x_i^2 - 2\mu x_i + \mu^2}{2\sigma^2}}$$

$$\propto e^{-\frac{-\mu^2\sigma^2 + 2\mu\mu_0\sigma^2 - \mu_0^2\sigma^2 - \sigma^2 \sum x_i^2 + 2\mu\sigma^2 \sum x_i - n\mu^2\sigma^2}{2\sigma_0^2\sigma^2}}$$

$$= e^{-\frac{[-\mu^2(\sigma^2 + n\sigma_0^2) + 2\mu(\mu_0\sigma^2 + \sigma_0^2 \sum x_i) - (\mu_0^2\sigma^2 + \sigma_0^2 \sum x_i^2)]}{2\sigma_0^2\sigma^2}}$$

$$P(\mu|x) = \frac{1}{\sqrt{\frac{2\sigma_0^2\sigma^2}{\sigma^2 + n\sigma_0^2}}} \cdot e^{-\frac{(\mu - \frac{\mu_0\sigma^2 + \sigma_0^2 \sum x_i}{\sigma^2 + n\sigma_0^2})^2}{\frac{2\sigma_0^2\sigma^2}{\sigma^2 + n\sigma_0^2}}}$$

Q. 4

$$P(\mu|x) \propto e^{-\frac{(\mu - \frac{\mu_0\sigma^2 + \sigma_0^2 \sum x_i}{\sigma^2 + n\sigma_0^2})^2}{\frac{2\sigma_0^2\sigma^2}{\sigma^2 + n\sigma_0^2}}}$$

Q. 6.

Q 2)

From the equation, A) the derived posterior distribution is the quadratic and a Gaussian distribution as it is proportional to the Normal form.

Q. 2.

$$e^{-\frac{(\mu - \mu_n)^2}{2\sigma_n^2}} \sim N(\mu_n, \sigma_n^2)$$

where, $\sigma_n^2 = \frac{\sigma_0^2 \sigma^2}{\sigma^2 + n\sigma_0^2}$ and

$$\mu_n = \frac{\mu_0 \sigma^2 + \sigma_0^2 \sum x_i}{\sigma^2 + n\sigma_0^2}$$

Q 3)

Q. 3)

$$\sigma_n^2 = \frac{\sigma_0^2 \sigma^2}{\sigma^2 + n\sigma_0^2} = \frac{1}{\left(\frac{\sigma^2}{\sigma_0^2 \sigma^2} + \frac{n}{\sigma^2}\right)}$$

$$= \frac{1}{\left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}\right)}$$

$$= \left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}\right)^{-1}$$

$$\boxed{\frac{1}{\sigma_n^2} = \frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}} \quad \dots (B)$$

$$\mu_n = \frac{\mu_0 \sigma^2 + \sigma_0^2 \sum x_i}{\sigma^2 + n\sigma_0^2}$$

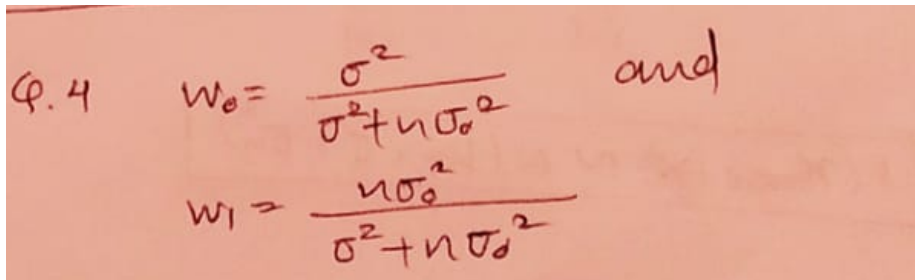
$$= \frac{\mu_0 \sigma^2}{\sigma^2 + n\sigma_0^2} + \frac{\sigma_0^2 \sum x_i}{\sigma^2 + n\sigma_0^2}$$

$$= \frac{\mu_0 \sigma^2}{\sigma^2 + n\sigma_0^2} + \frac{\sigma_0^2 n \cdot \frac{\sum x_i}{n}}{\sigma^2 + n\sigma_0^2}$$

$$\boxed{\mu_n = \frac{\mu_0 \sigma^2}{\sigma^2 + n\sigma_0^2} + \frac{n \times \sigma_0^2}{\sigma^2 + n\sigma_0^2}} \quad \dots (C)$$

Q 4)

From equation C) weights can be derived as below



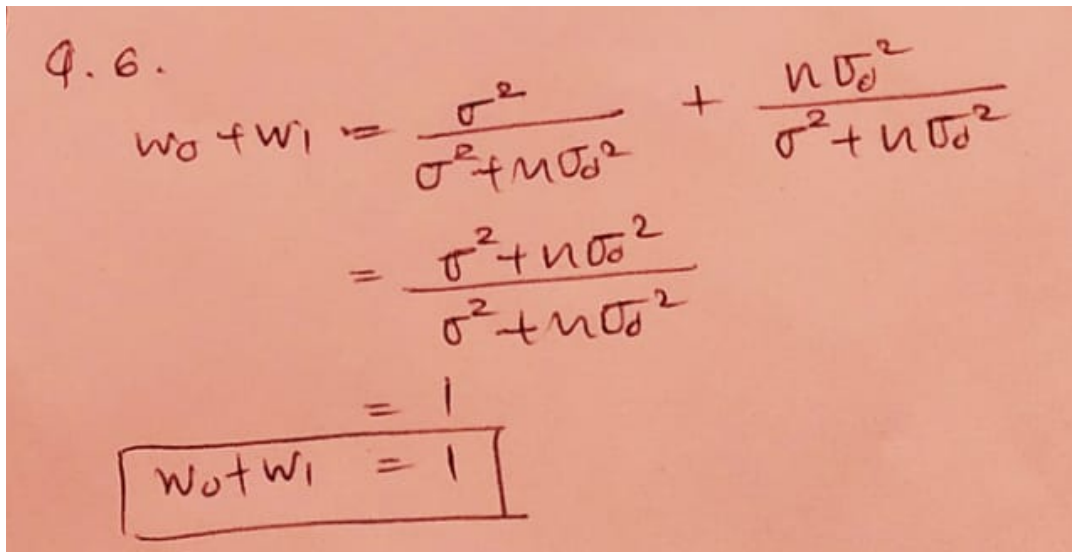
Q.4 $w_0 = \frac{\sigma^2}{\sigma^2 + n\sigma_d^2}$ and $w_1 = \frac{n\sigma_d^2}{\sigma^2 + n\sigma_d^2}$

Q 5)

Above equations, w_0 and w_1 indicate the weights are inversely proportional to their variances.

Q 6)

The sum of weights w_0 and w_1 is equal to the



Q.6.
$$w_0 + w_1 = \frac{\sigma^2}{\sigma^2 + n\sigma_d^2} + \frac{n\sigma_d^2}{\sigma^2 + n\sigma_d^2}$$
$$= \frac{\sigma^2 + n\sigma_d^2}{\sigma^2 + n\sigma_d^2}$$
$$= 1$$

$w_0 + w_1 = 1$

Q 7)

Q.7

$$w_0 = \frac{\sigma^2}{\sigma^2 + n\sigma_d^2} = \frac{1}{\left(1 + \frac{n\sigma_d^2}{\sigma^2}\right)}$$
$$w_1 = \frac{n\sigma_d^2}{\sigma^2 + n\sigma_d^2} = \frac{1}{1 + \frac{\sigma^2}{n\sigma_d^2}}$$

As the weight w_0 's numerator is 1 and denominator is one plus something so anyhow it is going to be between 0 and 1.

Similarly, for w_1 , its values lie between 1 and 0.

Hence, both weight's values lie between 1 and 0.

Q 8)

From previous answers, it can be inferred weight values lie between 1 and 0. The maximum values of $\mu_n = \mu_0 + \bar{x}$. But since both weights are inversely dependent on n , the edge case of 0 and 1 are not possible. Hence, μ_n will always lie between μ_0 and \bar{x} .

Q 9)

Q. 9

$$P(x_{\text{new}} | X) = \int P(x_{\text{new}} | \mu) P(\mu | X) d\mu$$

since,

$$x_{\text{new}} = (x_{\text{new}} - \mu) + \mu$$

where, $x_{\text{new}} - \mu \sim N(0, \sigma^2)$ (Normal distribution)

and $\mu \sim N(\mu_n, \sigma_n^2)$ (Normal distribution)

$$x_{\text{new}} = (x_{\text{new}} - \mu) + \mu$$

$$P(x_{\text{new}} | X) \sim N(0, \sigma^2) + N(\mu_n, \sigma_n^2)$$

$$\text{Hence, } N\left(\sum_{i=1}^n c_i \mu_i, \sum_{i=1}^n c_i^2 \sigma_i^2\right)$$

$$\boxed{P(x_{\text{new}} | X) \sim N(\mu_n, \sigma^2 + \sigma_n^2)}$$

Q 10)

```
import numpy as np
import scipy.stats as st
from matplotlib import pyplot as plt
sample = 20
line = np.linspace(0, 10, sample)
m0 = 4
sd0 = 0.8
d_prior = st.norm(m0, sd0).pdf(line)
mx = 6 sdx = 1.5

d_sample = st.norm(mx, sdx).pdf(line)
x_t = st.norm(mx,sdx).rvs(sample)
variance = 1/((1/sd0**2)+(sample/sdx**2))
mean = variance * ((m0/sd0**2)+(np.mean(x_t)*sample/sdx**2) )
print("Posterior distribution:\nMean =",round(mean,3)) print("Variance =",round(variance,3))
d_posterior = st.norm(mean, np.sqrt(variance)).pdf(line)
plt.figure(figsize=(20,20)) plt.plot(line,d_prior,"r-",label='Prior Distribution')
plt.plot(line,d_sample,"g-",label='Sample Distribution') plt.plot(line,d_posterior,"b-
",label='Posterior Distribution')
plt.legend(loc='upper right')
plt.title('Probability Density Plot') plt.ylabel('Probability Density')
plt.xlabel('X')
plt.show()
```

Posterior distribution:

Mean = 5.727

Variance = 0.096

