Apply(SSN, college, name, HSCode, HScity, sports)

SSN → name

HSCode → HSCity

K	S	
✓	✓	A. {SSN, college, HSCode, ~~HScity~~ sports}
—	✓	B. {SSN, college, HSCode, HScity, name, sports}
—	—	C. {SSN, college, name}

$S(A, B, C, D, E)$

$AB \to D$

$C \to E$

$DE \to BC$

1. You can "remove" attributes on the right-hand side (RHS)
   $\underline{DE \to BC}$ is equivalent to
   $$\begin{bmatrix} DE \to B \\ DE \to C \end{bmatrix}$$

2. You cannot "remove" attributes on the LHS

Movies(title, year, length)

title, year $\rightarrow$ length

The closure of C is CEDB

$U(A, B, C, D, E)$

✓ $A \to C$ (–)

✓ $C \to E$ |||

✓ $E \to D$ |||

✓ $D \to B$ |||

Does $\underline{C} \to B$ hold? yes

Does $\underline{C} \to A$ hold? no

$A \longrightarrow C \longrightarrow E \longrightarrow D \longrightarrow B$

The closure algorithm for computing closure of given set of attributes

0. Make X the initial closure set

1. Iterate
   - if there is a FD whose LHS is a subset of X
     ⇒ add the FD's RHS to X

2. Stop when
   - no more unused FDs
   - or no more attributes

3. Return X

$R(A, B, C)$

$A \to B$

$B \to C$

Does $A \to C$ hold? Yes

Does $C \to B$ hold? No

The "closure" answers:

(1) The closure of A is ABC

(2) The closure of C is C

$R(ABCD)$

$AB \to C$

$C \to D$

$D \to A$

Does $CD \to A$ hold?

Does $CD \to B$ hold?

With the closure algorithm: We can

1. Prove whether a candidate FD holds
2. Find all derived FDs

$R(ABCD)$

$AB \to C$

$C \to D$

$D \to A$

"Does candidate FD hold?
– for instance,
$CD \to A$
or
$CD \to B$

$A^+ = A$

$B^+ = B$ $\quad \boxed{B \to B}$

$C^+ = CDA$

$D^+ = DA$

$AB^+ = ABCD$

$AC^+ = ACD$

$BC^+ = BCDA$

$AD^+ = AD$

$BD^+ = BDAC$

$CD^+ = CDA$

$ABC^+ = ABCD$

$ABD^+ = ABCD$

$ACD^+ = ACD$

$BCD^+ = ABCD$

$ABCD^+ = ABCD$

Keys: AB, BC, BD

Superkeys: AB, BC, BD, ABC, ABD, BCD, ABCD

Derived FDs:

$C \to A$	$CD \to A$
$AB \to D$	$ABC \to D$
$AC \to D$	$ABD \to C$
$BC \to D$	$BCD \to A$
$BC \to A$	
$BD \to A$	
$BD \to C$	

U(ABCDE)

$A \to C$

$C \to E$

$E \to D$

$D \to B$

~~Pr.~~

Find all FDs in the projection of U onto CDB

$C \to D$
$C \to B$
$D \to B$
$CD \to B$

R(CDB)

$CB \to D$

$C^+ = CEDB$
$D^+ = DB$
$B^+ = B$

$CD^+ = CEDB$
$CB^+ = CEDB$
$DB^+ = DB$
$CDB^+ = CDBE$

You never need
to submit
trivial FDs
as part of any
assignment

R(ABCDE)

AB → DE

C → E

D → C

E → A

Project the FDs onto

S(ABC)