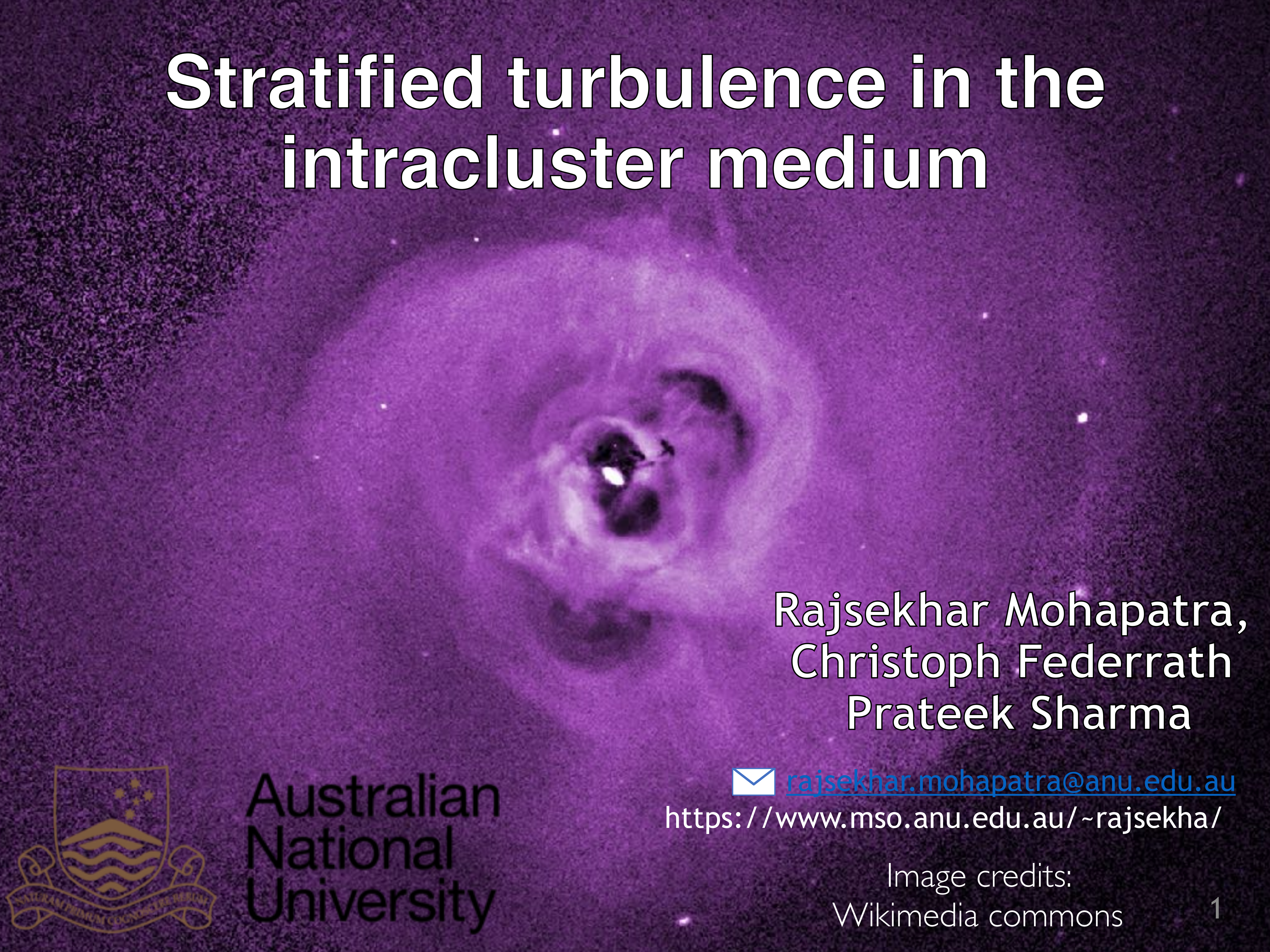
Stratified turbulence in the intracluster medium



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https://www.mso.anu.edu.au/~rajsekha/

Image credits:
Wikimedia commons

Who am I?


Astronomy department at ANU
PhD student from India
Stratified Turbulence in the ICM
Motorcycling
Hiking

Galaxy cluster

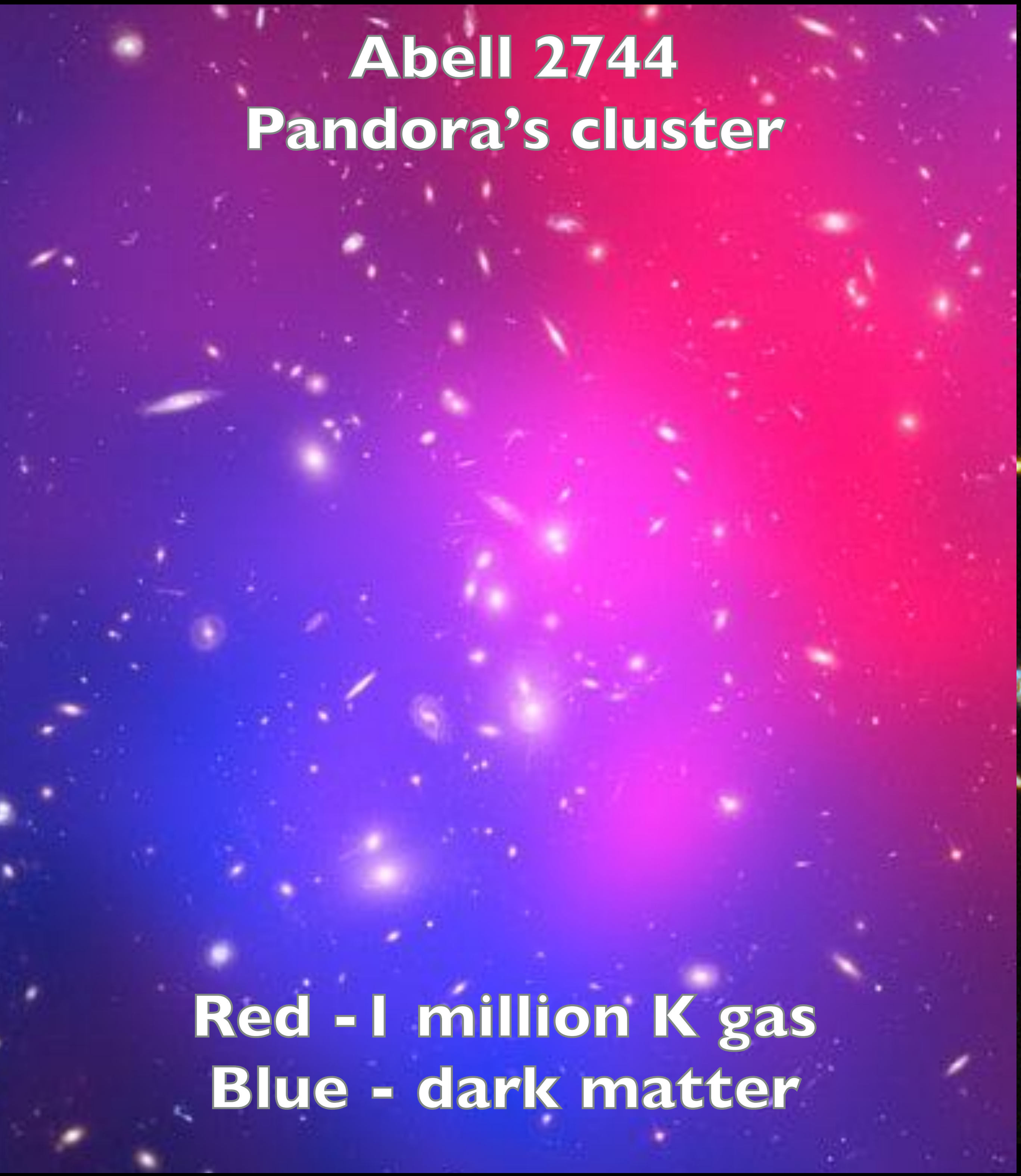
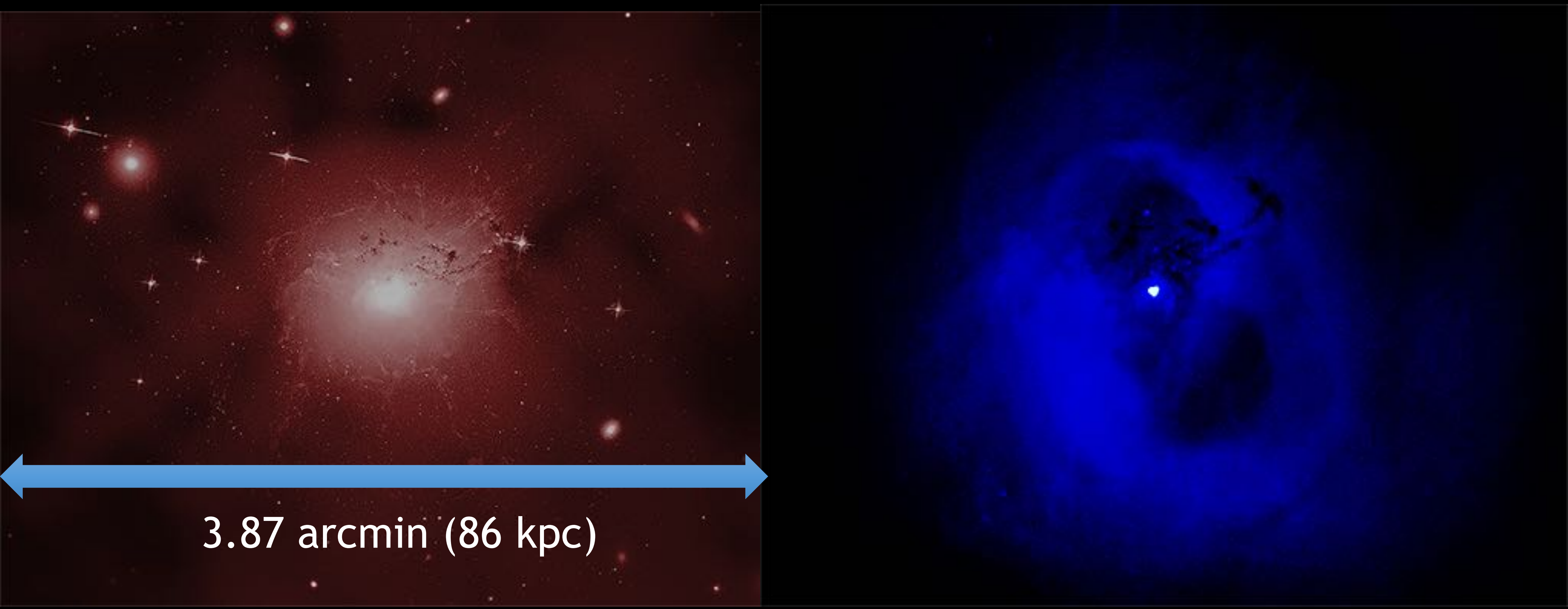


Image sources: Hubble space telescope, Chandra X-ray telescope, Wikimedia commons

The intracluster medium



Perseus cluster in visible (left) and X-ray (right)

Image credits: X-ray: NASA/CXO/Oxford University/J. Conlon et al. Optical: NASA/ESA/IoA/A. Fabian et al.; DSS

Cooling rate $\propto n^2\lambda(T)$

The cooling flow problem

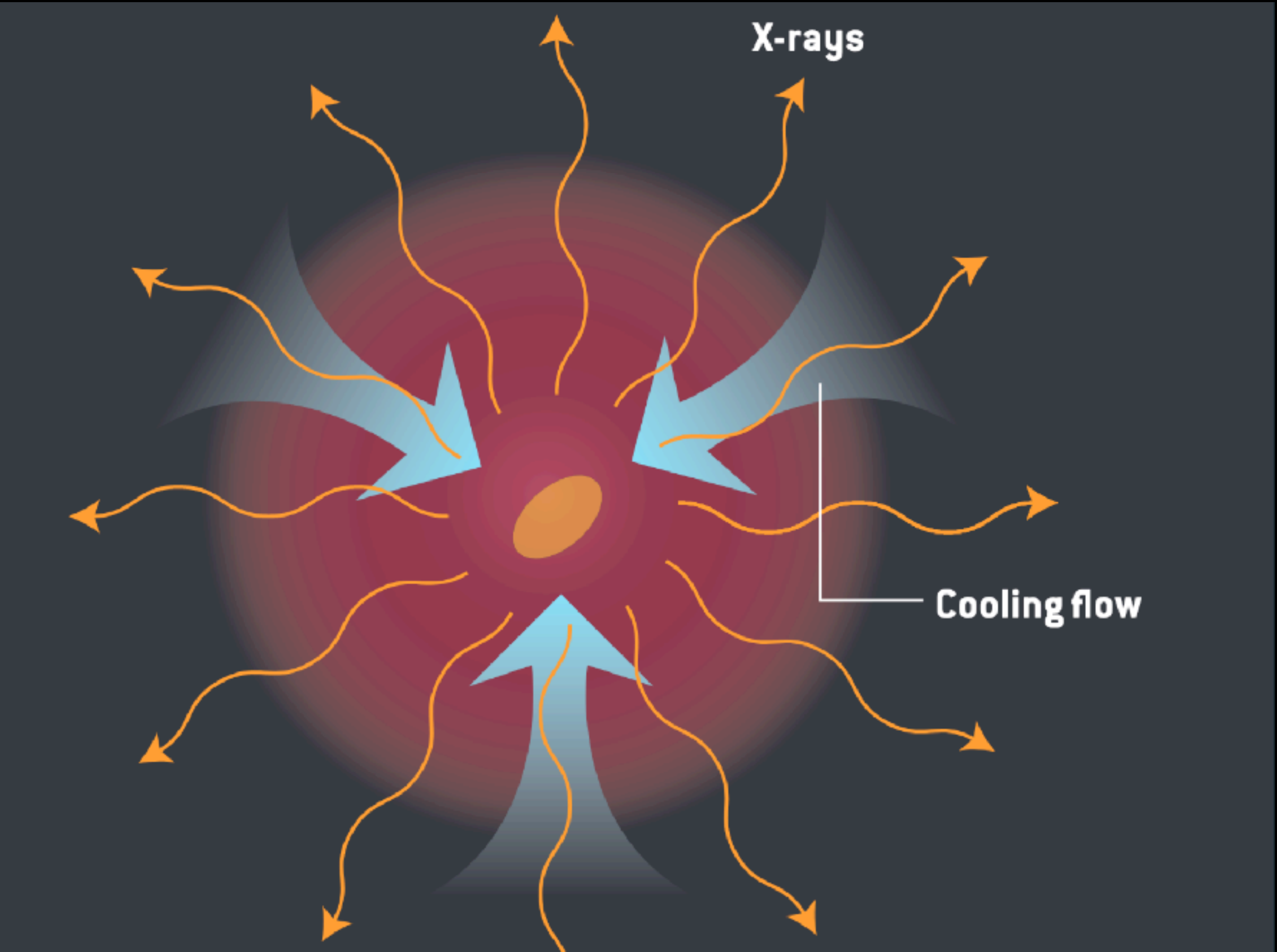
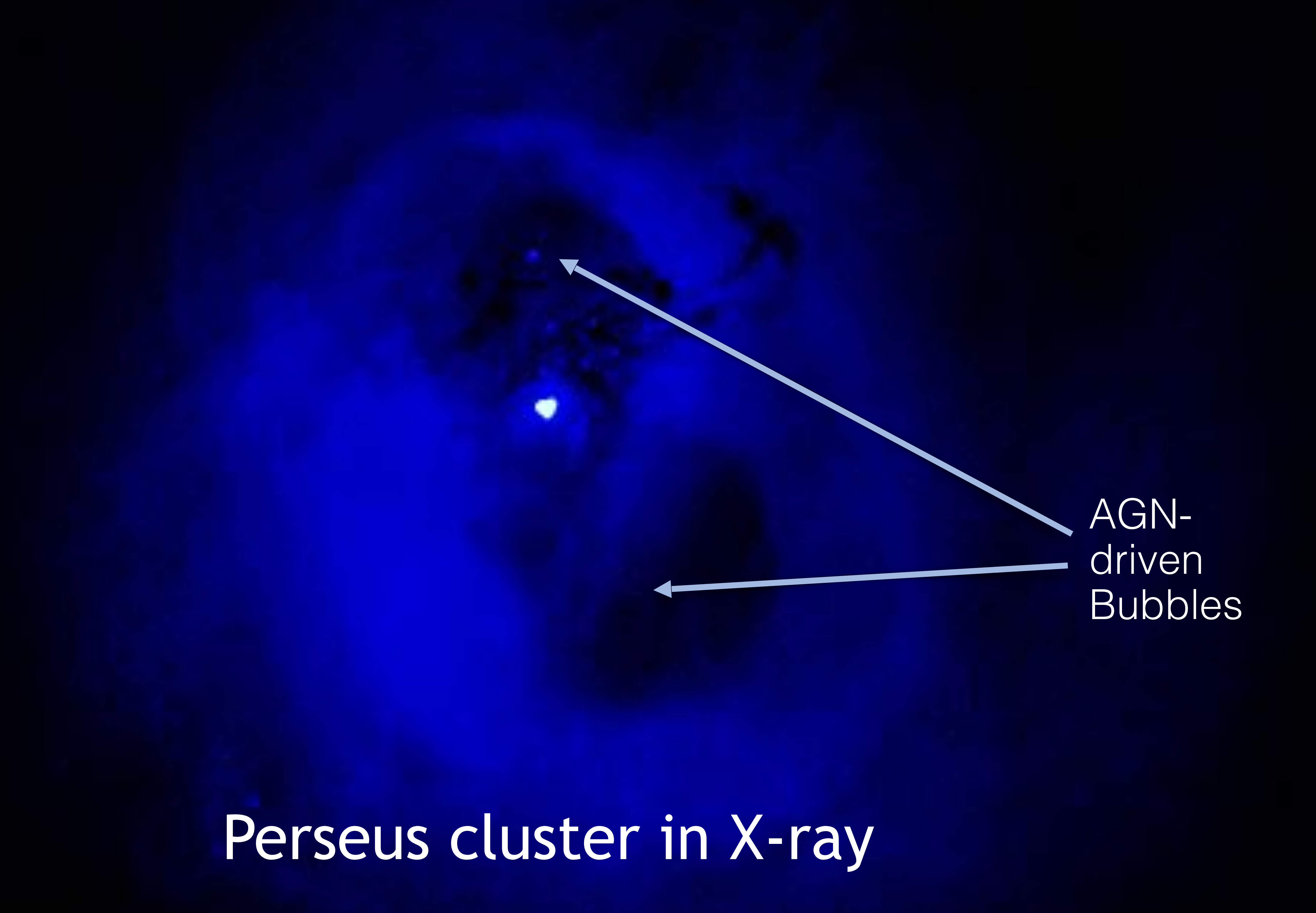


Image credits: Tucker et al 2007

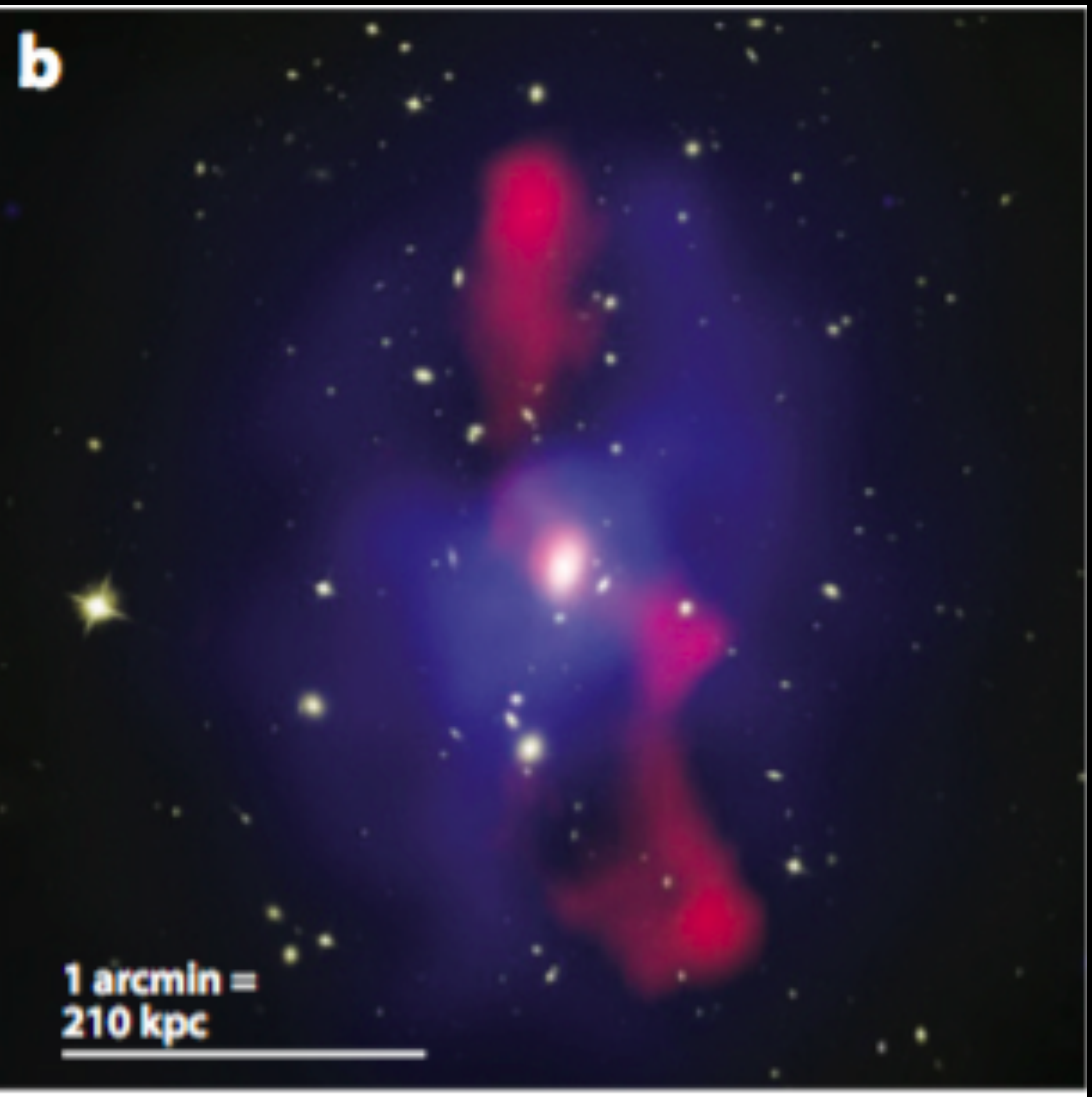
AGN-driven turbulence



Perseus cluster in X-ray

Image credits: X-ray: NASA/CXO/Oxford University/J. Conlon et al.

The feedback model

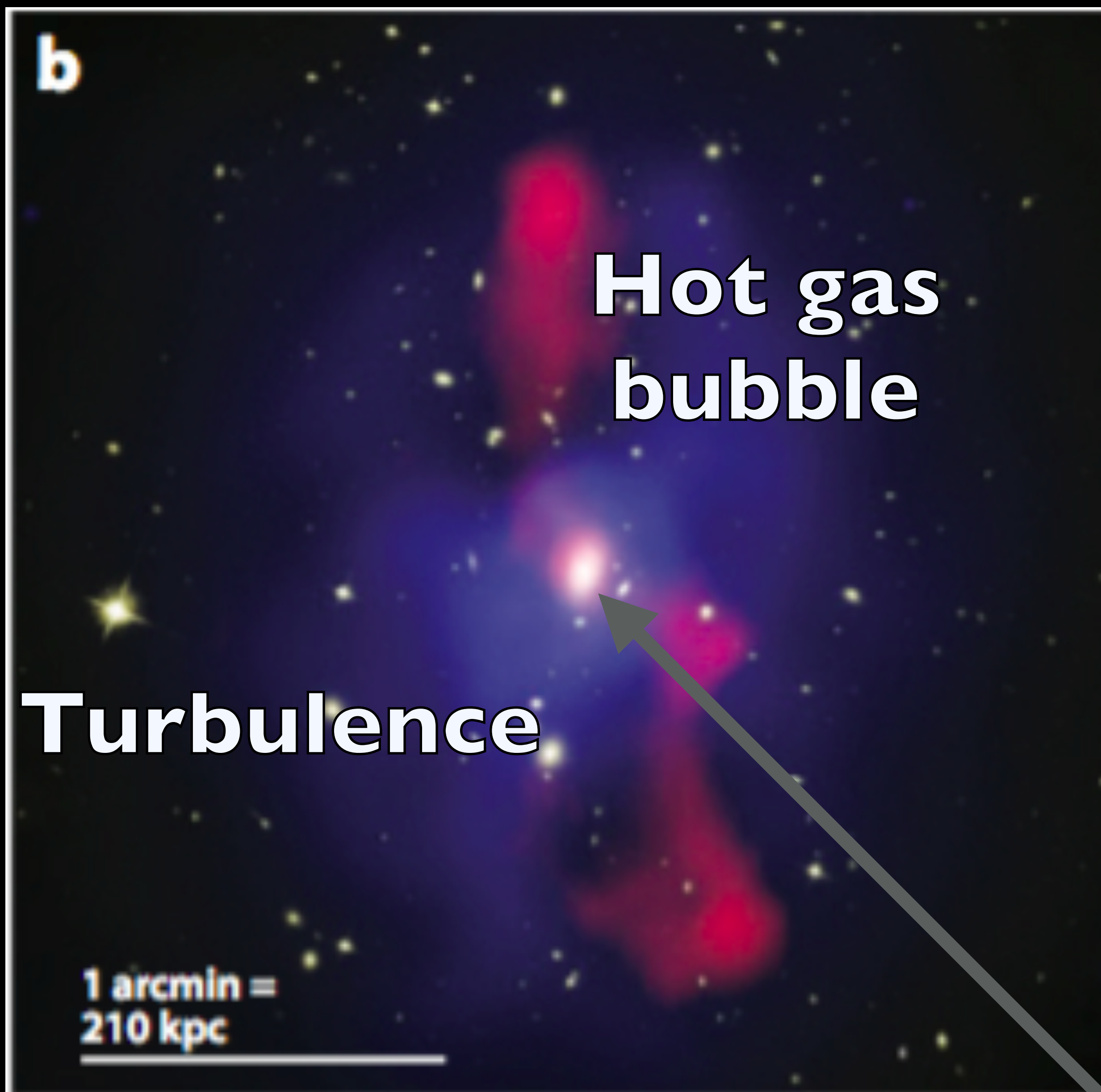


X-ray luminosity~jet power
=> ~thermal balance

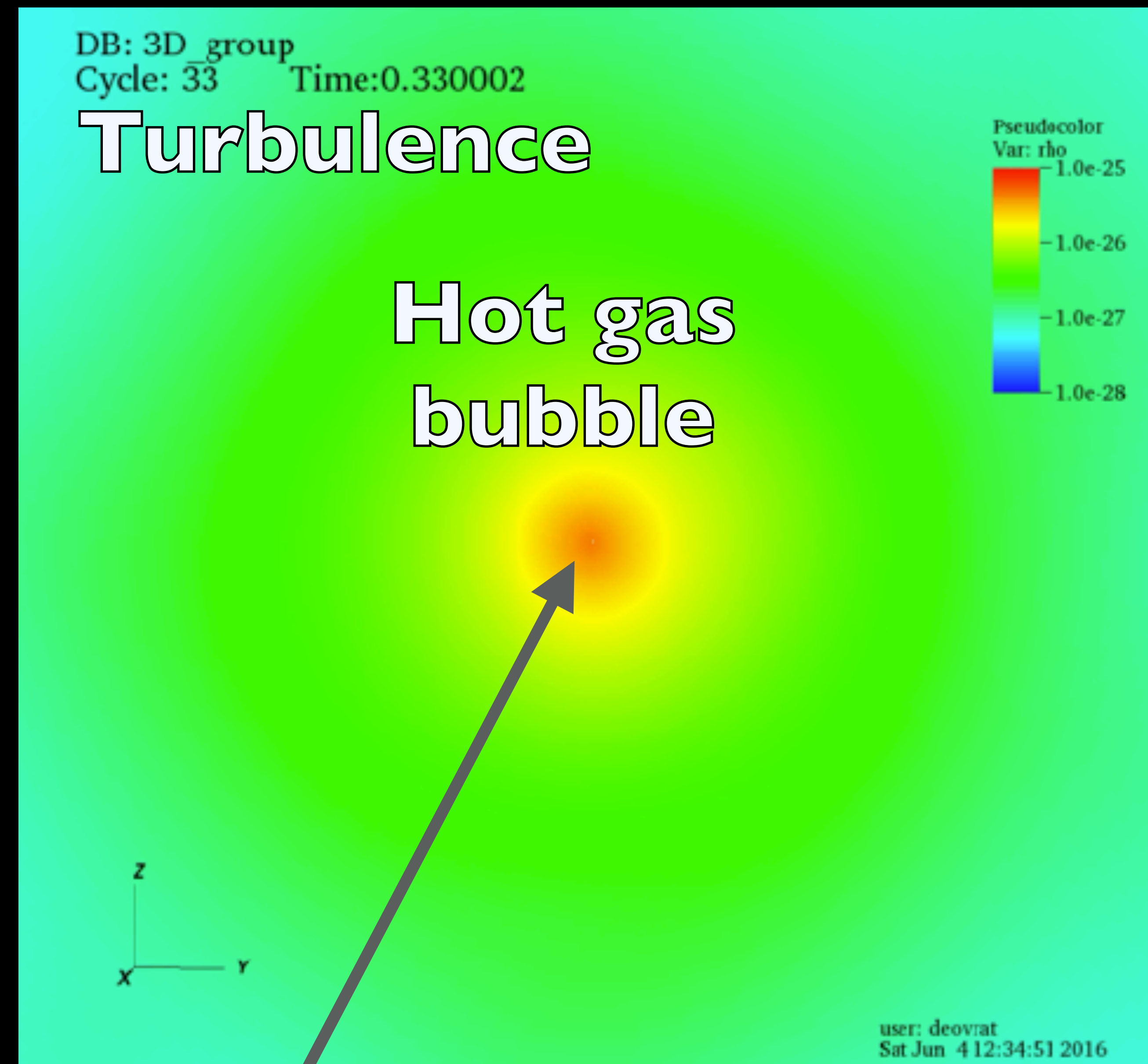
Image credits:
McNamara+ 2009

But how do AGN jets feed energy to the ICM? Turbulence?

Stirring big galaxies



MS0735.6+7421 cluster

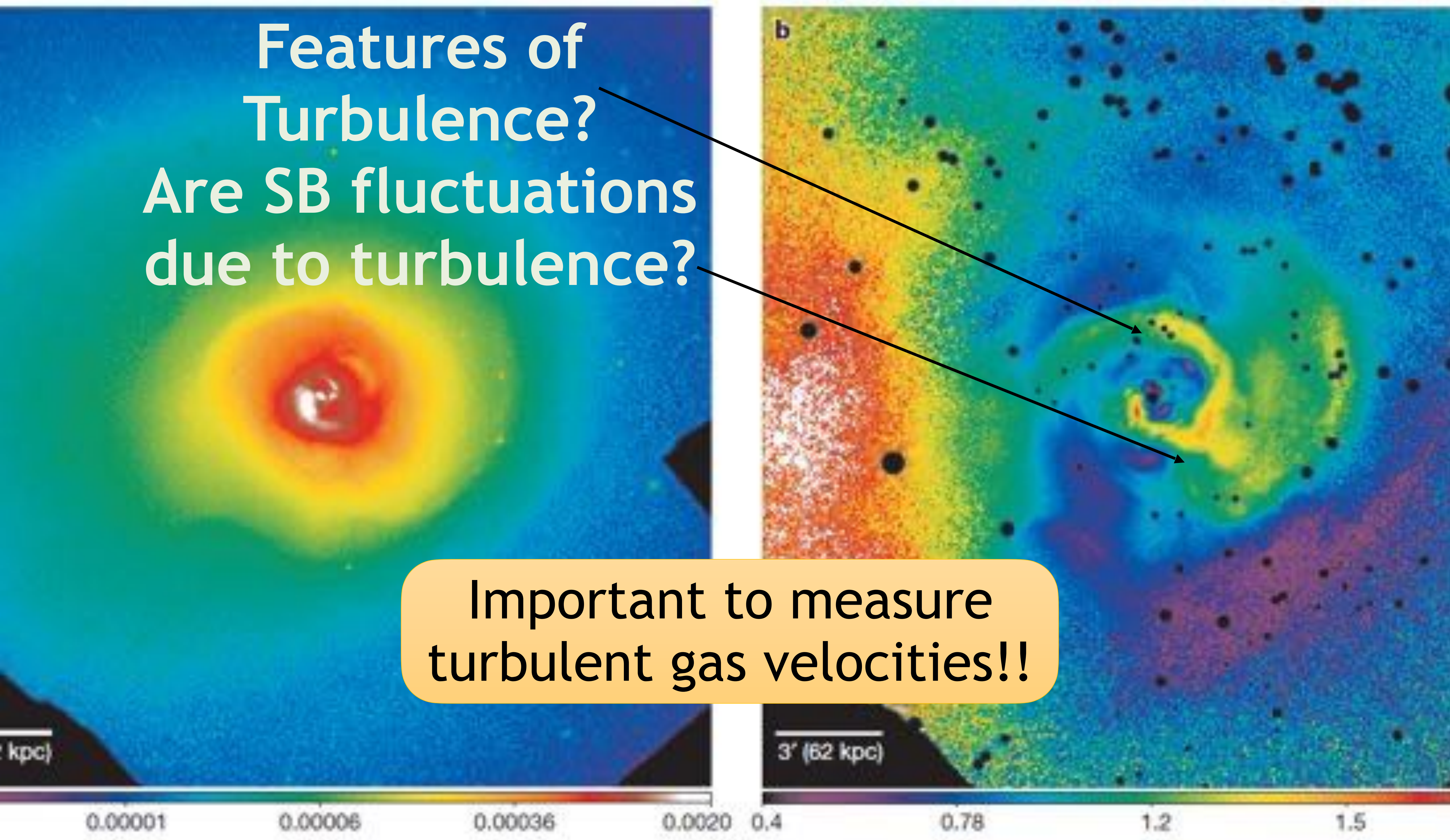


Supermassive black hole in the centre

Image credits: McNamara+ 2009

Video credits: Deovrat Prasad

Turbulence in the ICM



X Ray surface brightness image of Perseus cluster (counts/s/pixel)
Image credits: Zhuravleva et al, 2014

Measuring turbulence - I

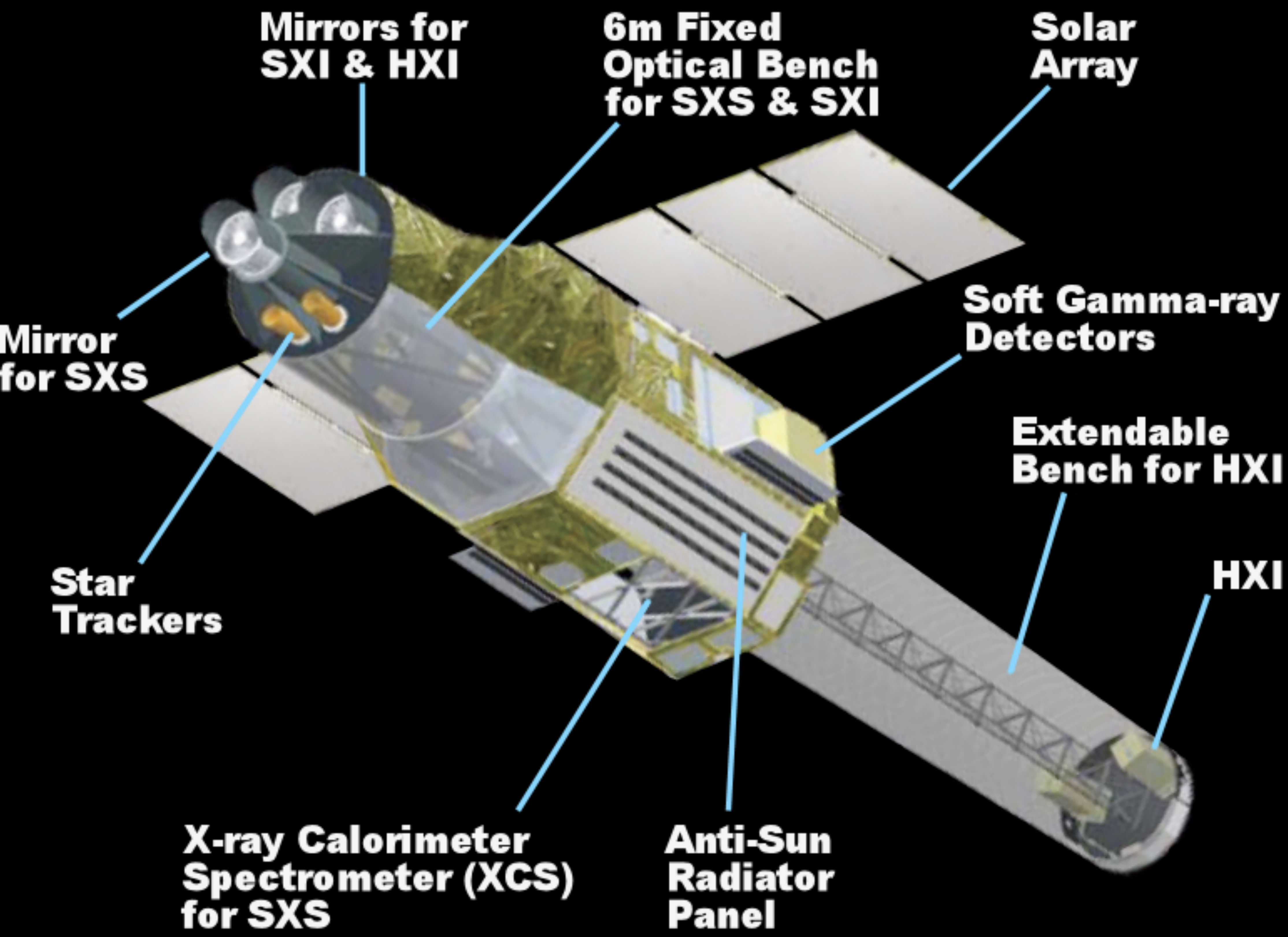
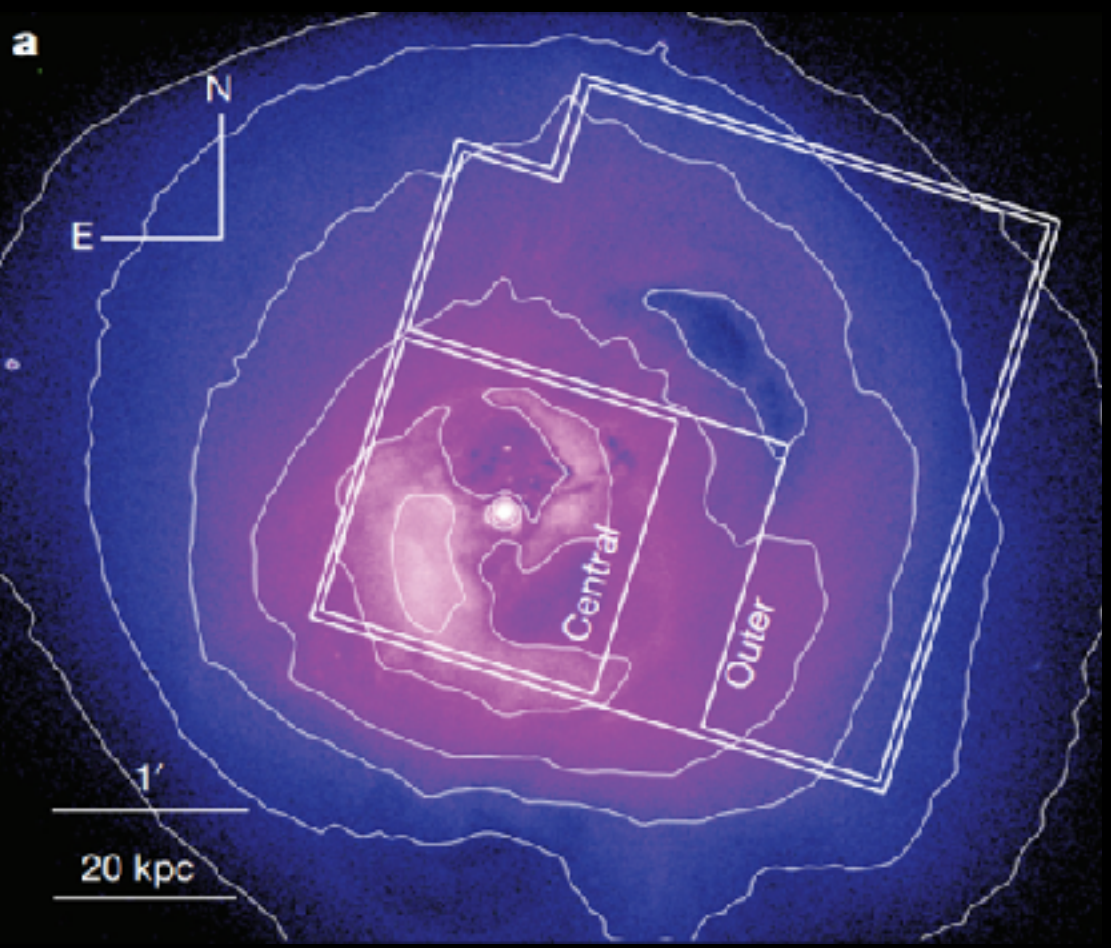


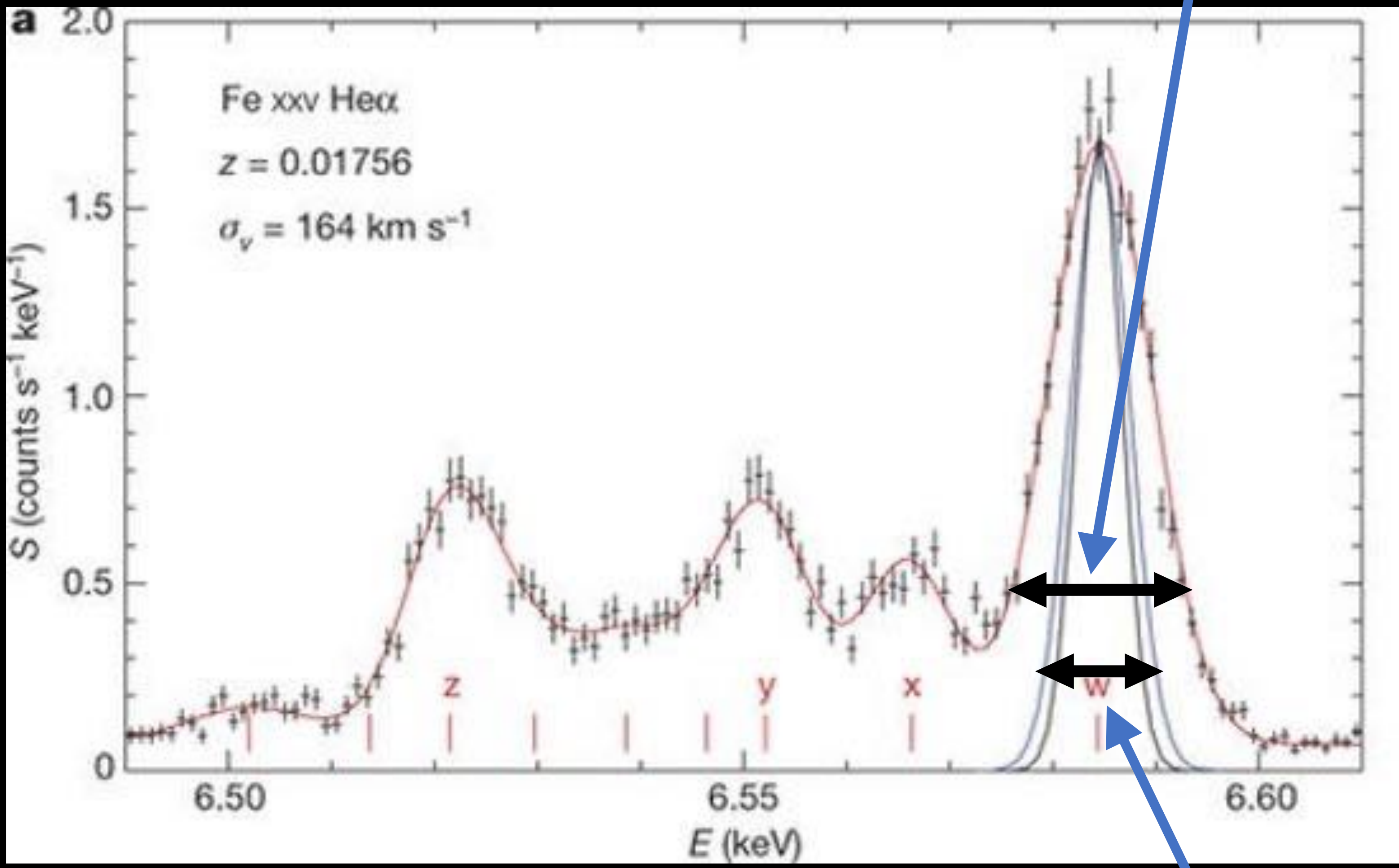
Image credits: NASA, Hitomi schematic

Measuring turbulence - I

Look at line broadening of heavy ions (such as Fe XXV)



Turbulent



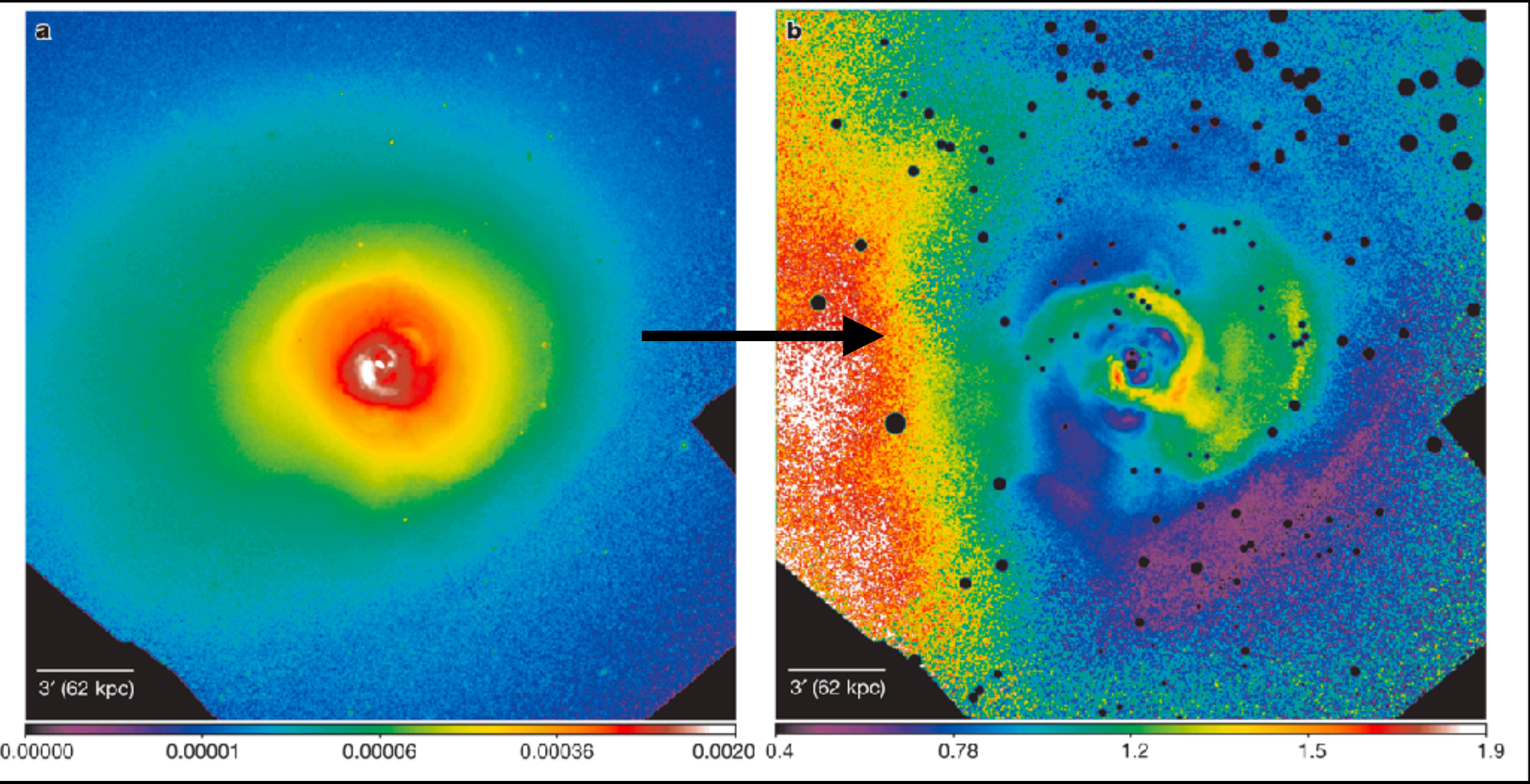
Thermal

Image credits: Hitomi collaboration, 2016

$$\sigma_v \approx 164 \text{ km/s}$$

Measuring turbulence - II

Step 1: SB/SB map -> SB fluctuations



X Ray surface brightness fluctuations
Image credits: Zhuravleva+, 2014b

Measuring turbulence - II

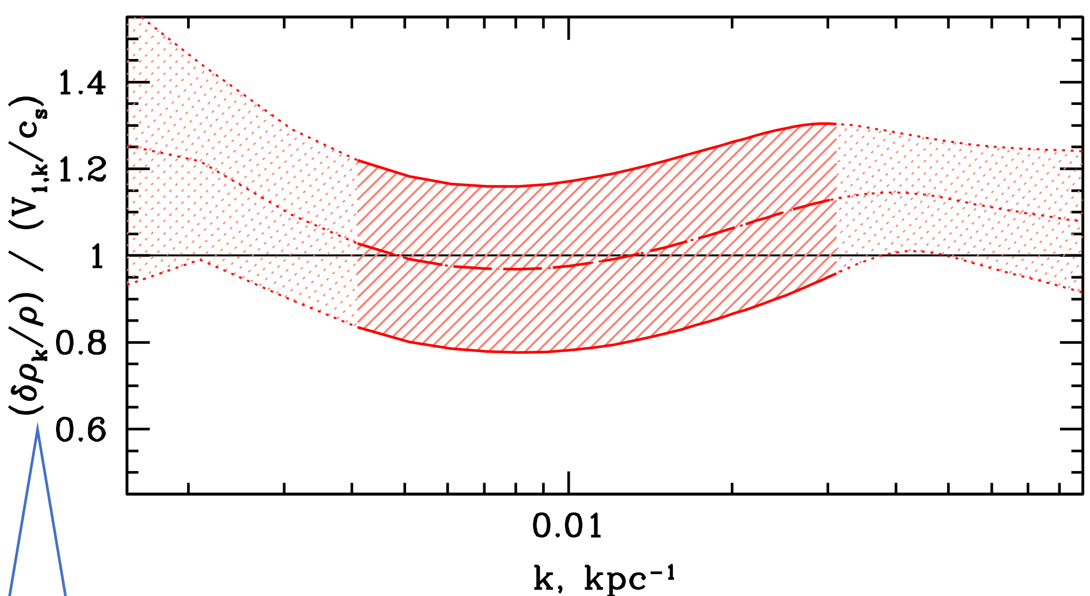
Step 2:

SB fluctuations -> Density power spectrum (Churazov+, 2012)

Step 3:

Density power spectrum -> Velocity power spectrum

Good match with Hitomi measured gas velocities!!



$\eta_k \approx 1$

Not well calibrated

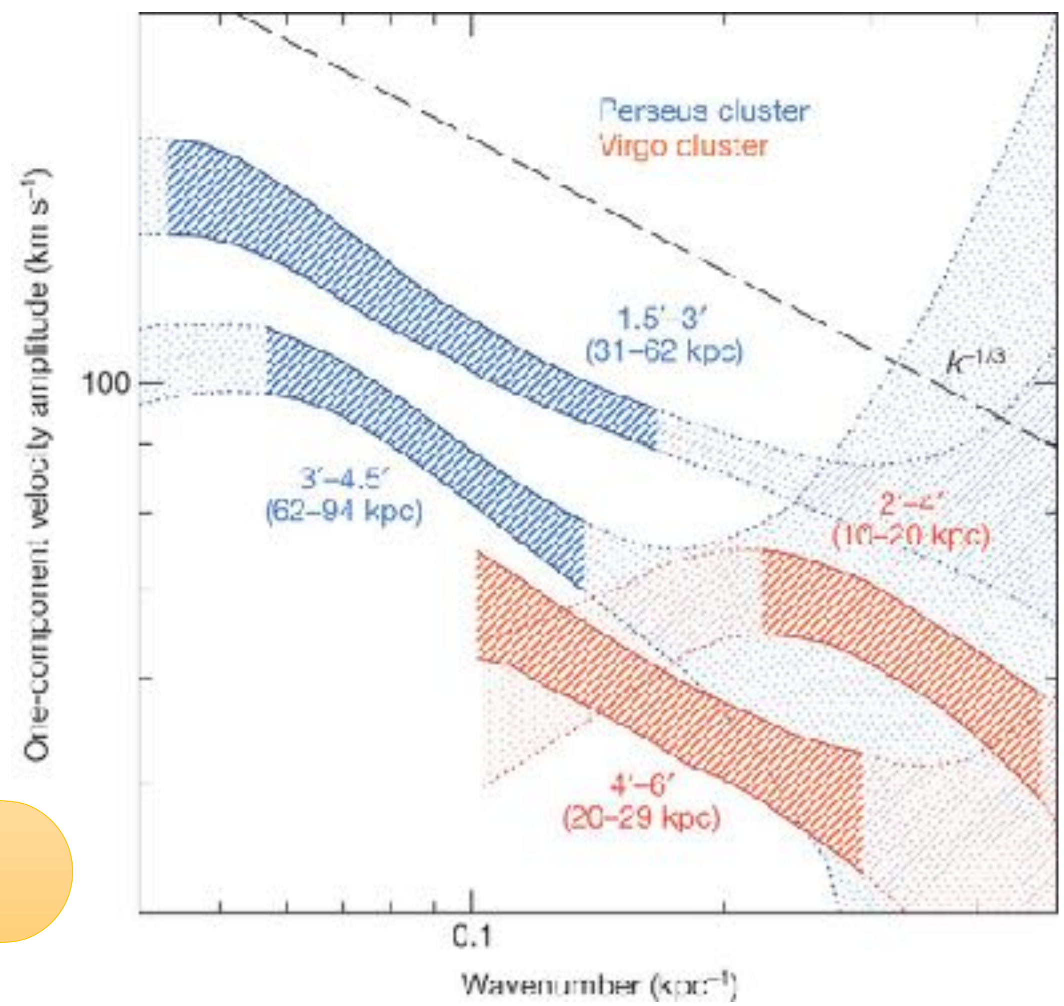


Image credits: Zhuravleva+, 2014a, 2014b

Measuring turbulence - III

Thermal SZ effect: LOS integral of pressure

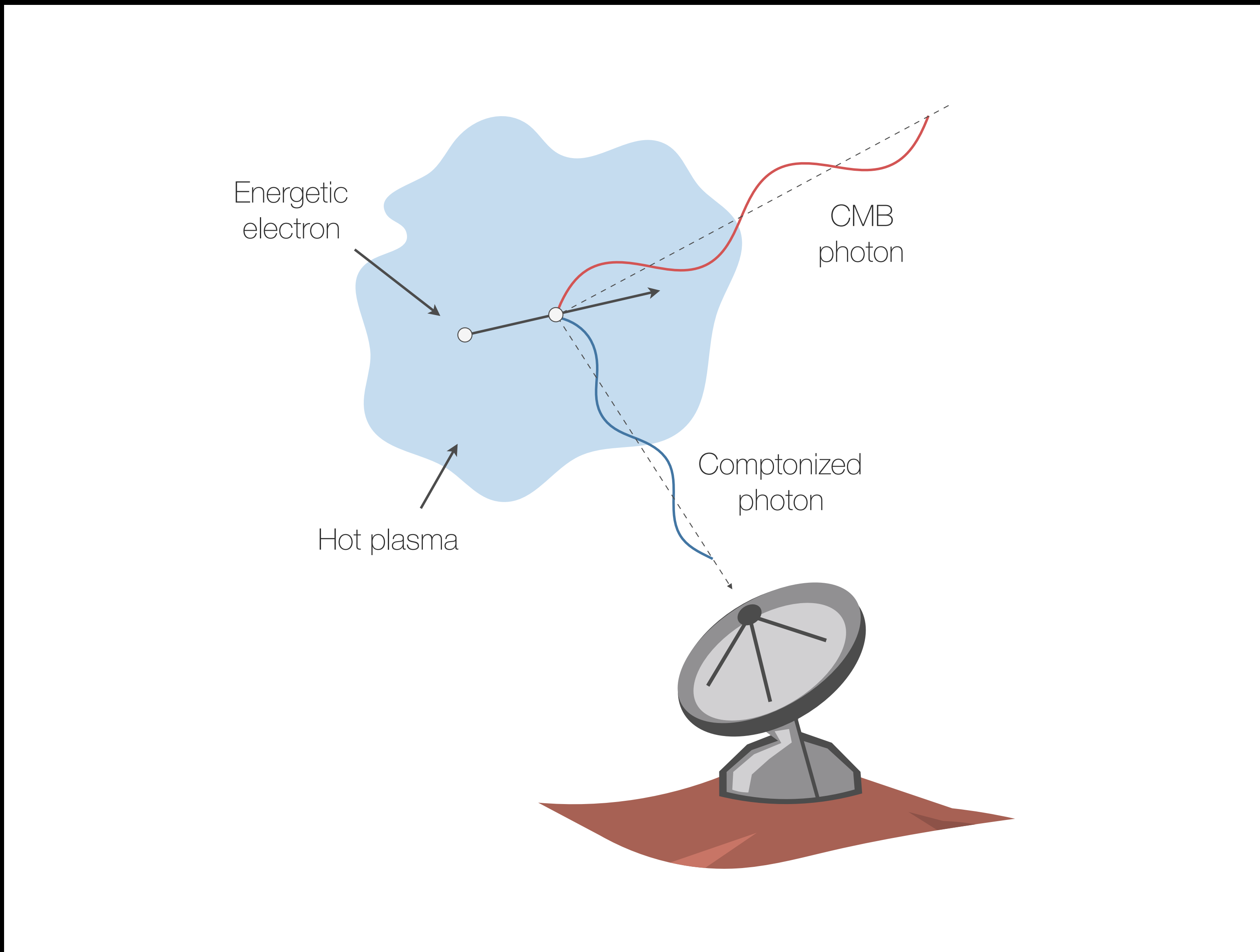
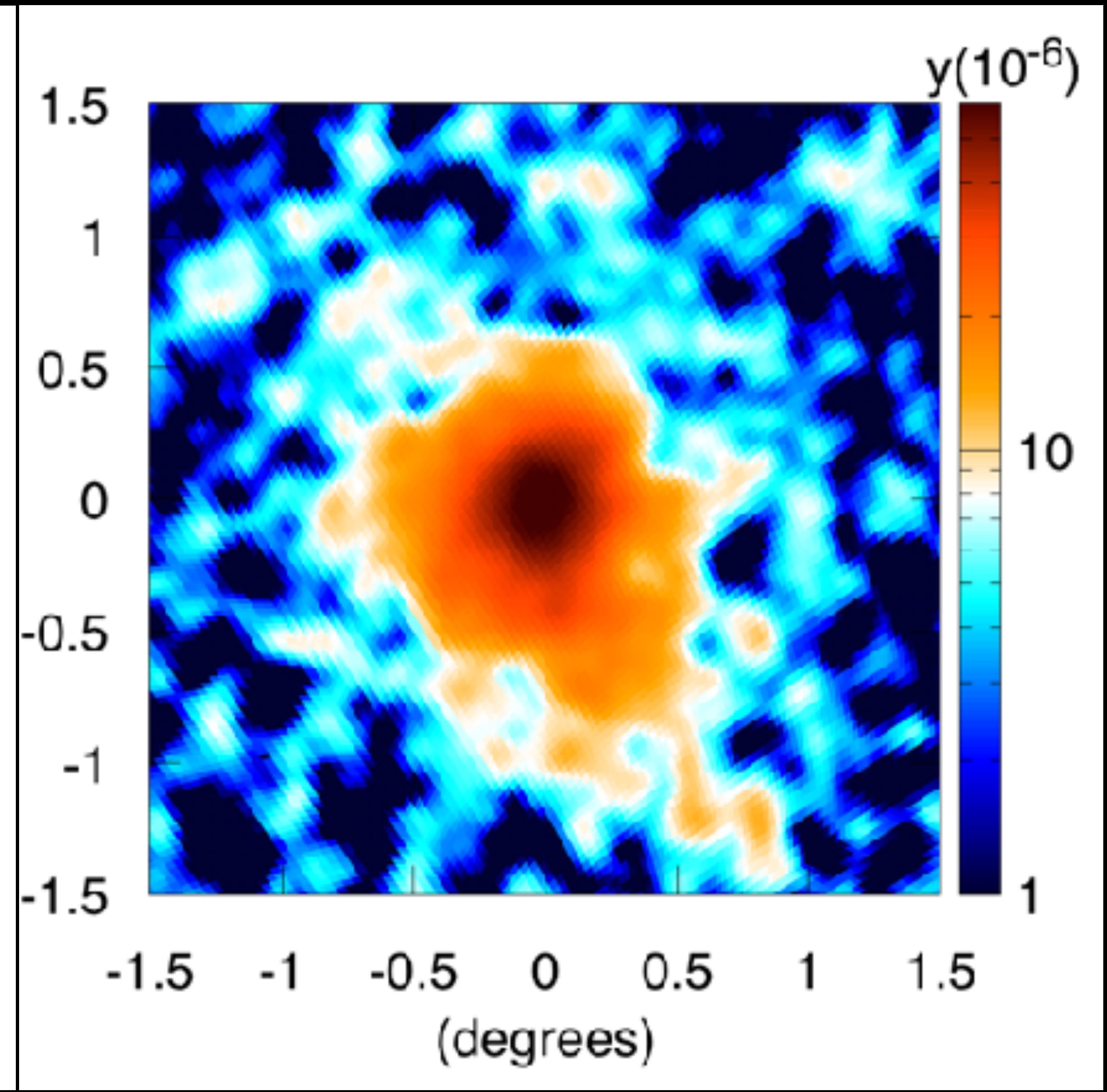


Image credits: Mroczkowski+, 2018

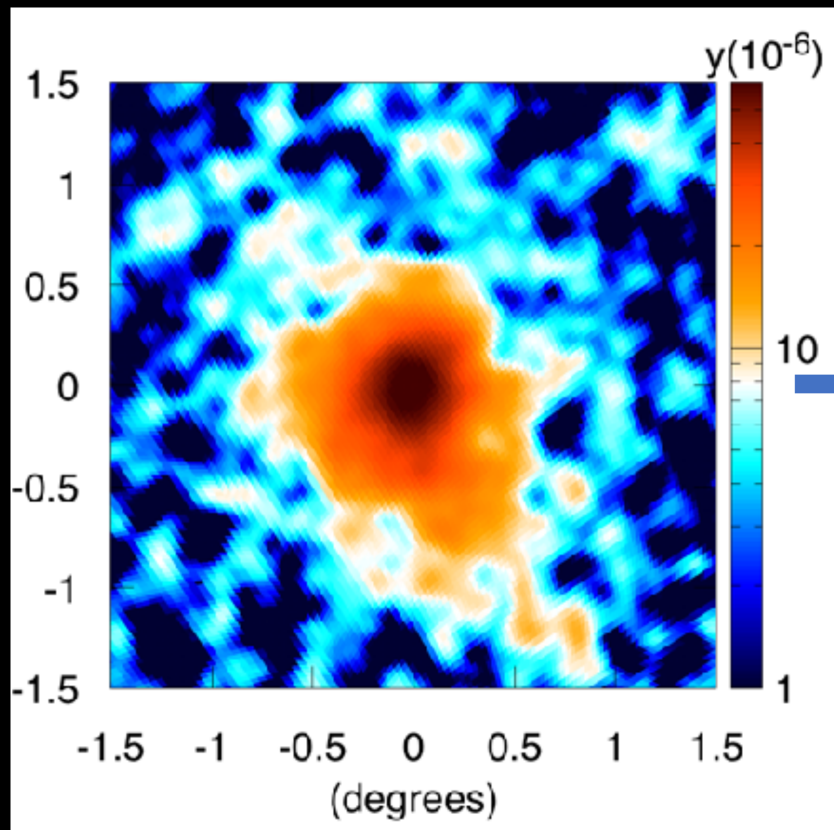
Compton-y profile of Coma cluster

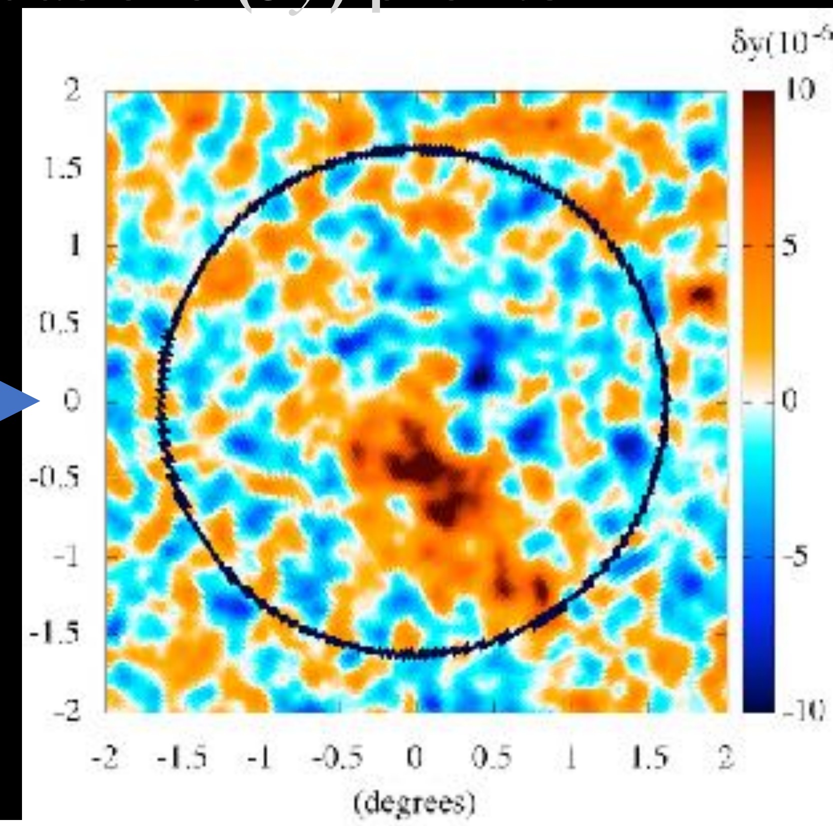


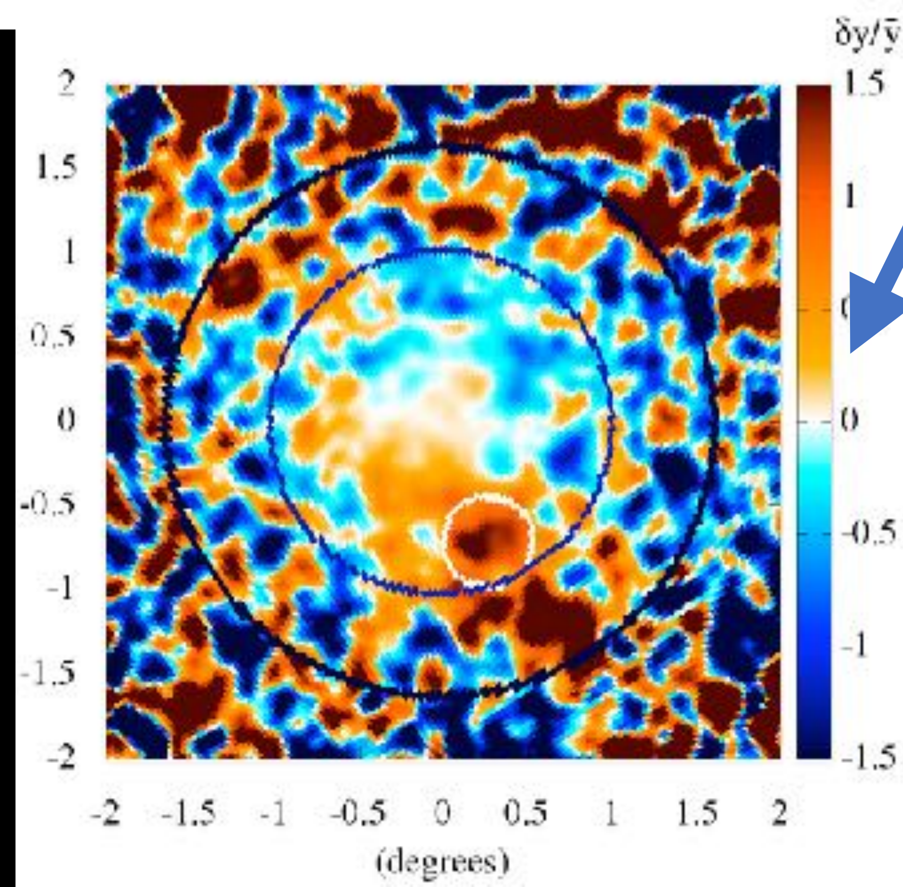
Khatri & Gaspari, 2016

Measuring turbulence - III

Compton-y profile -> Fluctuations (δy) profile





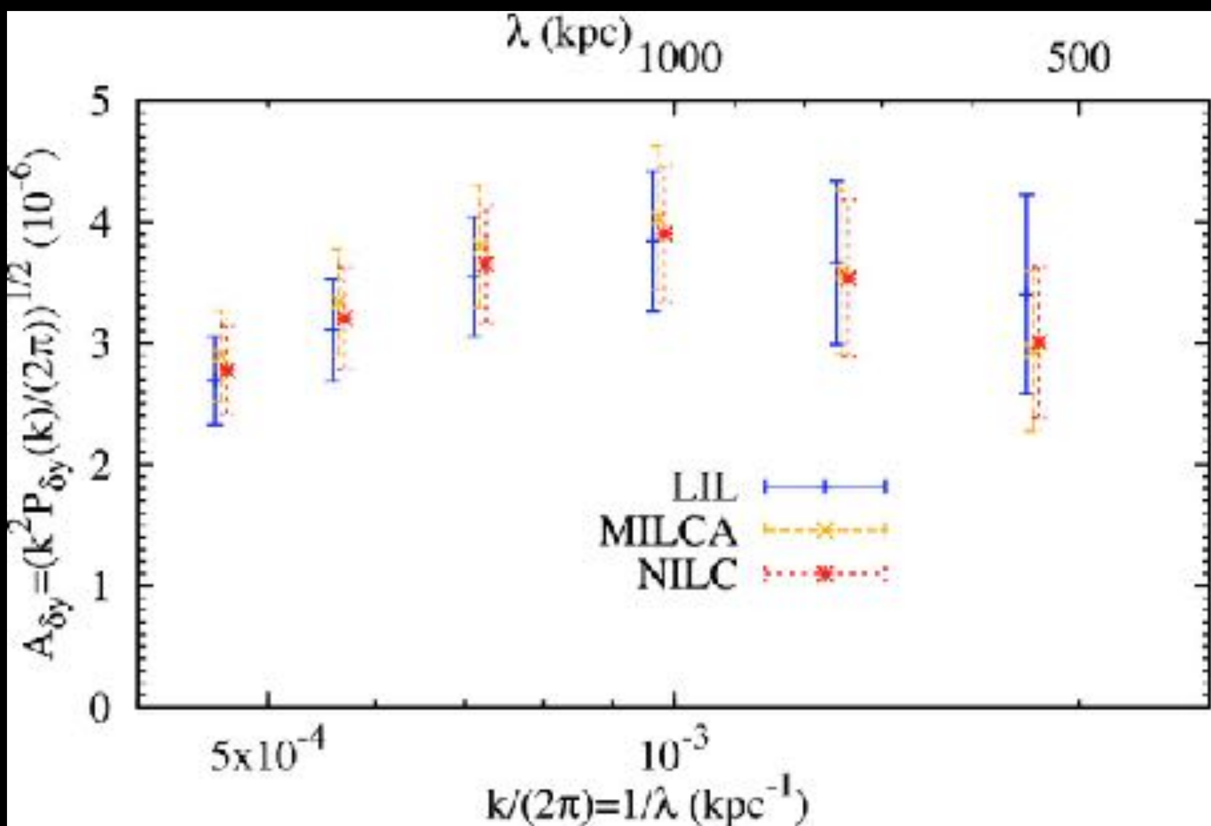


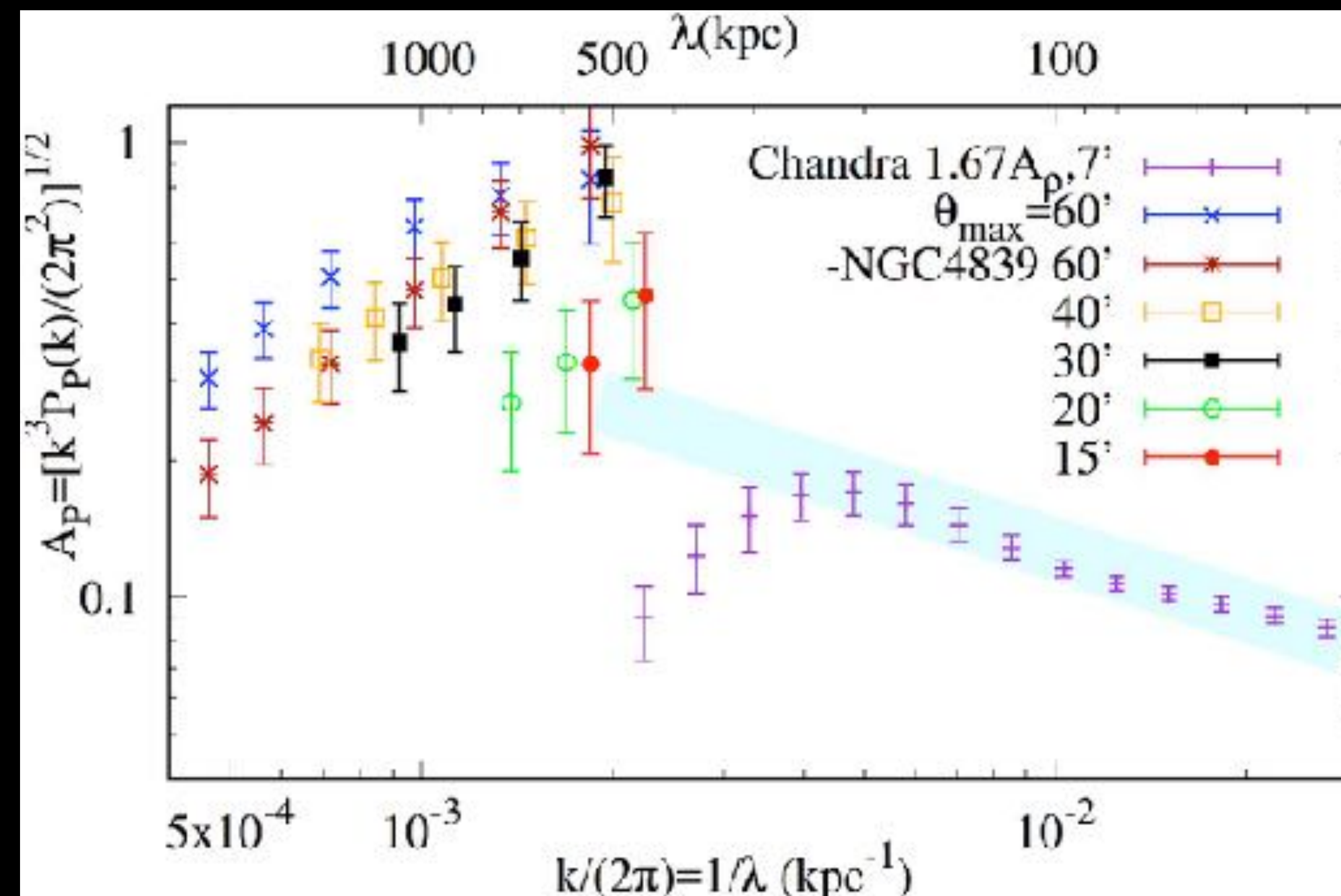
Normalised fluctuations ($\delta y/y$)

Image credits: Khatri & Gaspari, 2016

Measuring turbulence - III

$(\delta y/y)$ **Fourier amplitude** → **Pressure Fourier amplitude**





Currently lacks resolution to go down to sub-100 kpc!

Image credits: Khatri & Gaspari, 2016

Measuring turbulence - IV

10^4 K Cold gas (H alpha) velocities -> 10^7 K Hot gas velocities

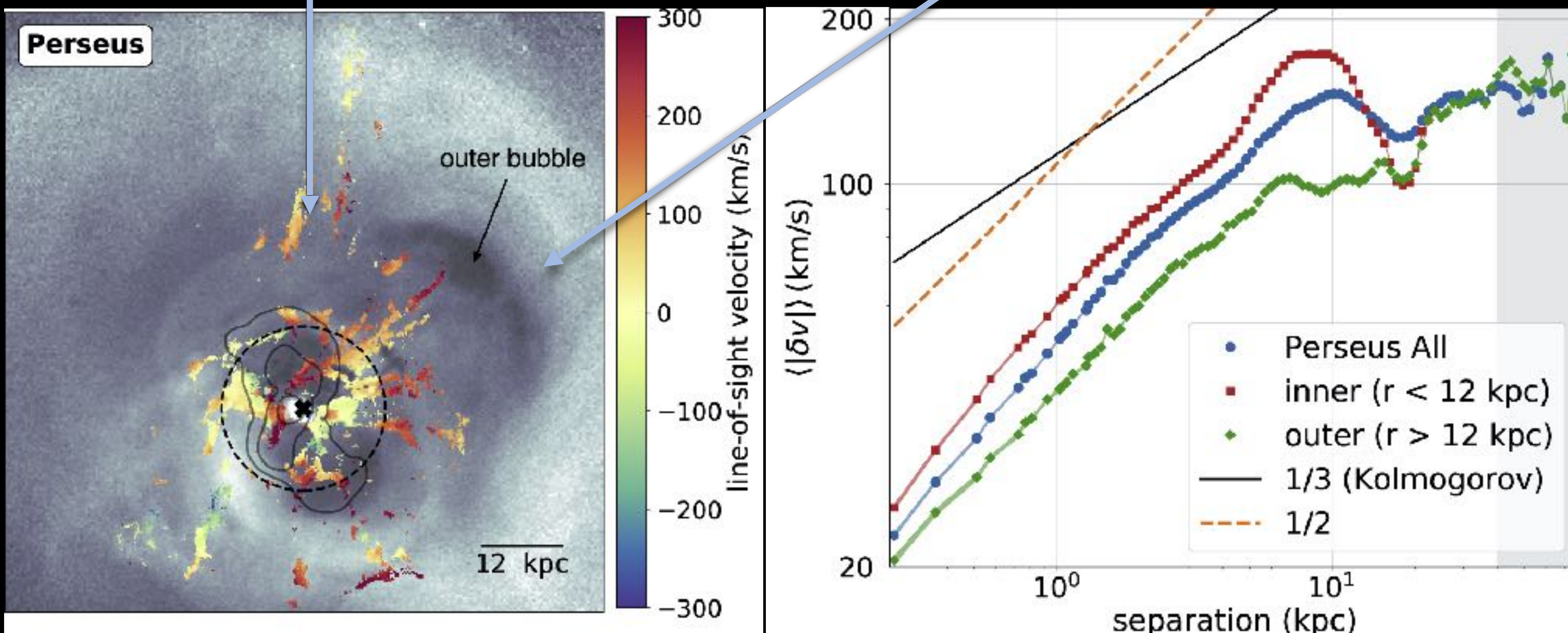


Image credits: Li+, 2020

Stratified turbulence

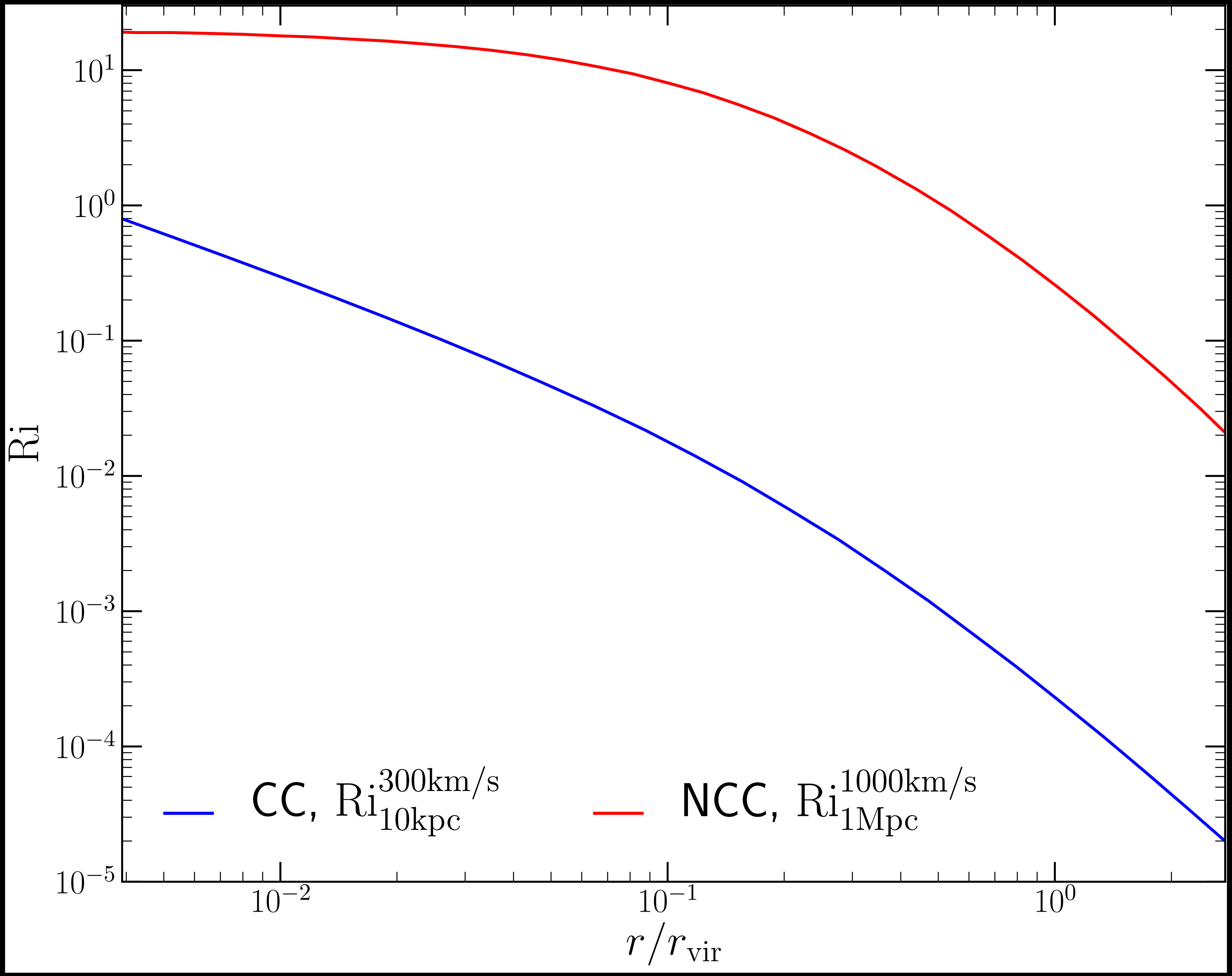
- Seen in earth's oceans, atmosphere
- Atmospheres of other planets and stars and clusters

- Characterised by Richardson number (or Froude number)

$$\mathrm{Ri}_L = \frac{\text{buoyancy force}}{\text{turbulent force}}$$

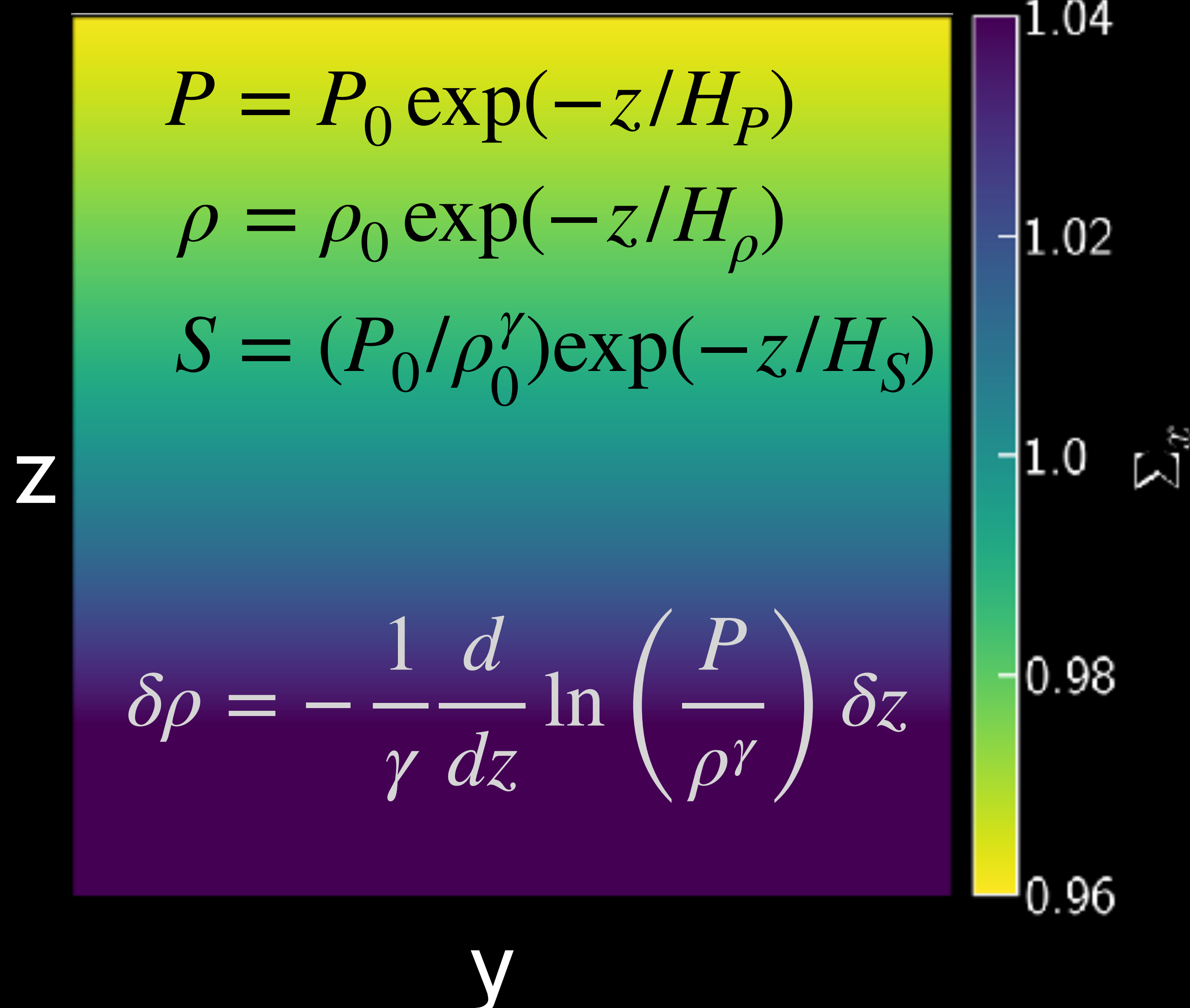
Image credits: (1) Image Credit: NASA Endeavour space shuttle (2)NASA/Goddard Space Flight Center Scientific Visualization Studio, the Cassini Imaging Team, CICLOPS, and Cosmos Studios

Stratified turbulence in galaxy clusters



Relevant time scales

- Brunt Vaisala Oscillation time scale



$$\frac{\mathrm{d}^2\delta z}{\mathrm{d}t^2} = -\underbrace{\frac{g}{\gamma}\frac{d}{dz}\ln\left(\frac{P}{\rho^\gamma}\right)}_{N^2}\delta z = -N^2\delta z$$

Condition for stable stratification:

$$\left(\frac{\gamma}{H_\rho} - \frac{1}{H_P}\right) > 0, \text{ or } H_\rho/H_P < \gamma$$

- Buoyant velocity $u_b = g(\delta\rho/\rho)/N$

- Eddy turnover time scale

$$t_{\text{eddy}} = \frac{\ell}{u_\ell}$$

Model equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) + \nabla P = \mathbf{F} + \rho \mathbf{g}$$

Forcing term

Gravitational acceleration

$$\frac{\partial E}{\partial t} + \nabla \cdot ((E + P)\mathbf{v}) = \mathbf{F} \cdot \mathbf{v} + \rho (\mathbf{v} \cdot \nabla) \Phi$$

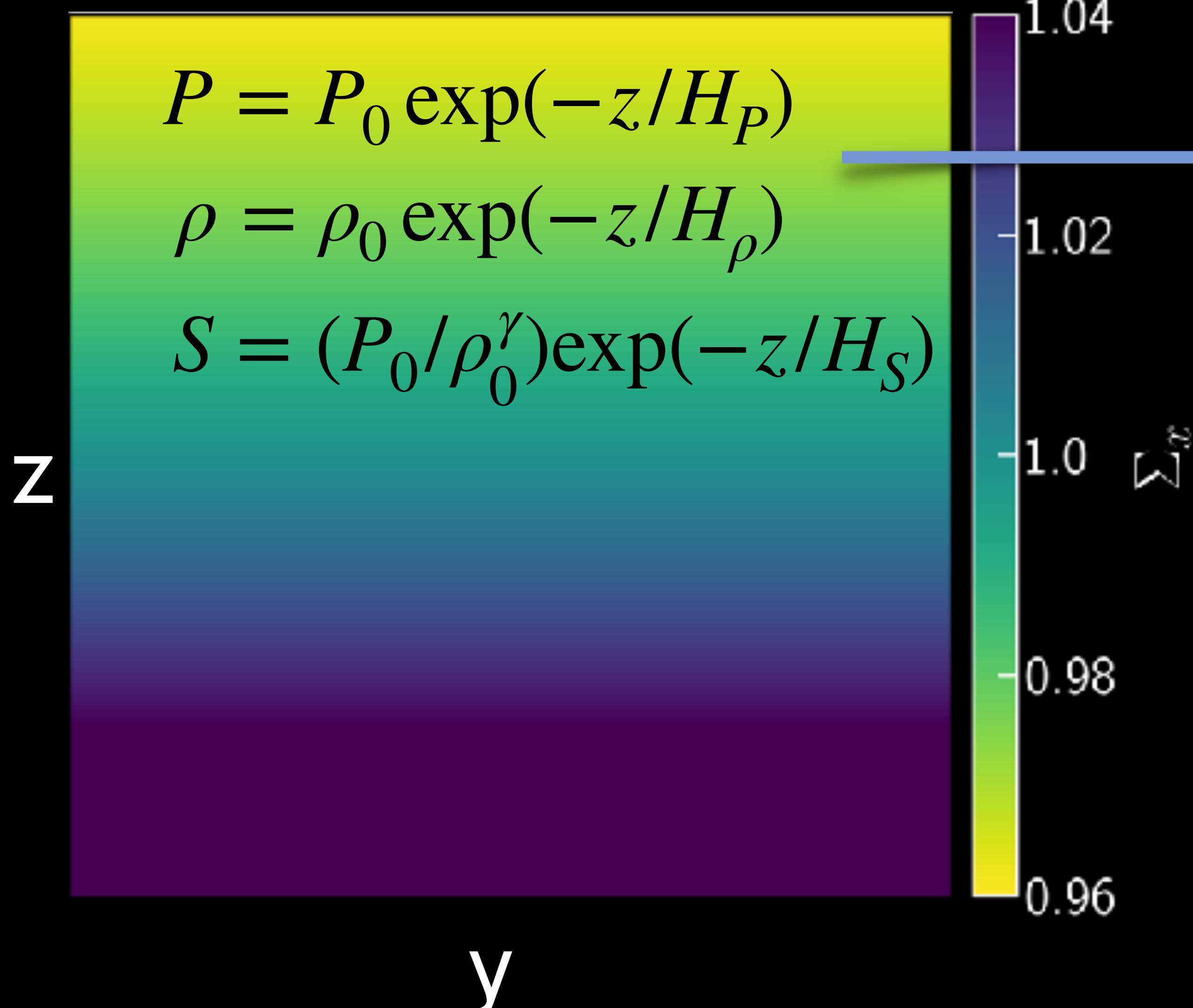
Gravitational potential

Turbulent forcing power

$$E = \frac{P}{\gamma - 1} + \frac{\rho \mathbf{v} \cdot \mathbf{v}}{2}$$

$$P = \frac{\rho k_B T}{\mu}$$

Project 1-Setup



$$P = P_0 \exp(-z/H_P)$$

$$\rho = \rho_0 \exp(-z/H_\rho)$$

$$S = (P_0/\rho_0^\gamma)\exp(-z/H_S)$$

Exponential profiles in a box
1024^3

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) + \nabla P = \mathbf{F} + \rho \mathbf{g}$$

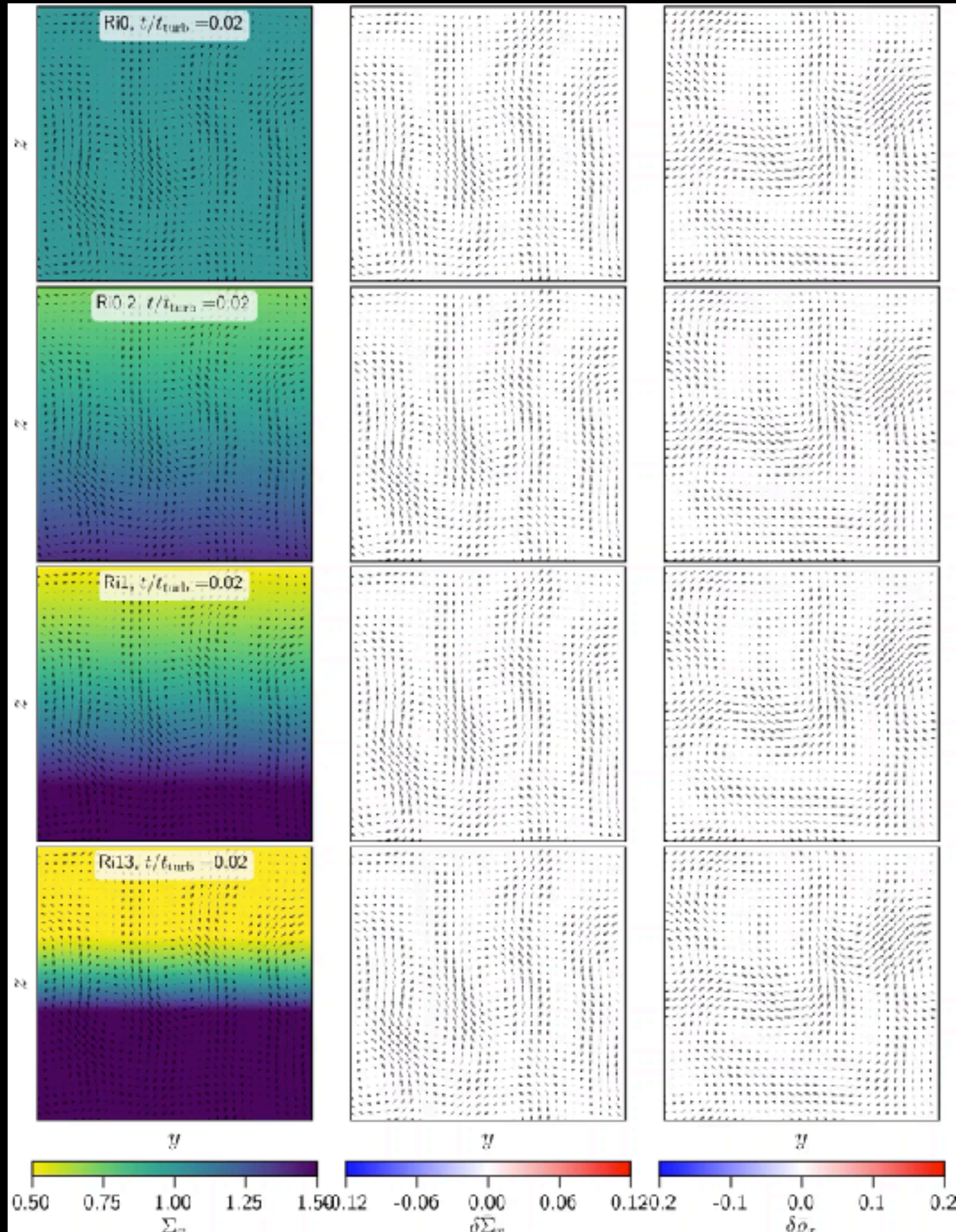
Drive subsonic turbulence
$(\mathscr{M} \approx 0.25)$

$$\mathrm{Ri}_L \approx \frac{\mathrm{N}^2}{(v_L/L)^2} = \frac{\text{buoyancy force}}{\text{turbulent force}}$$

Density projection maps

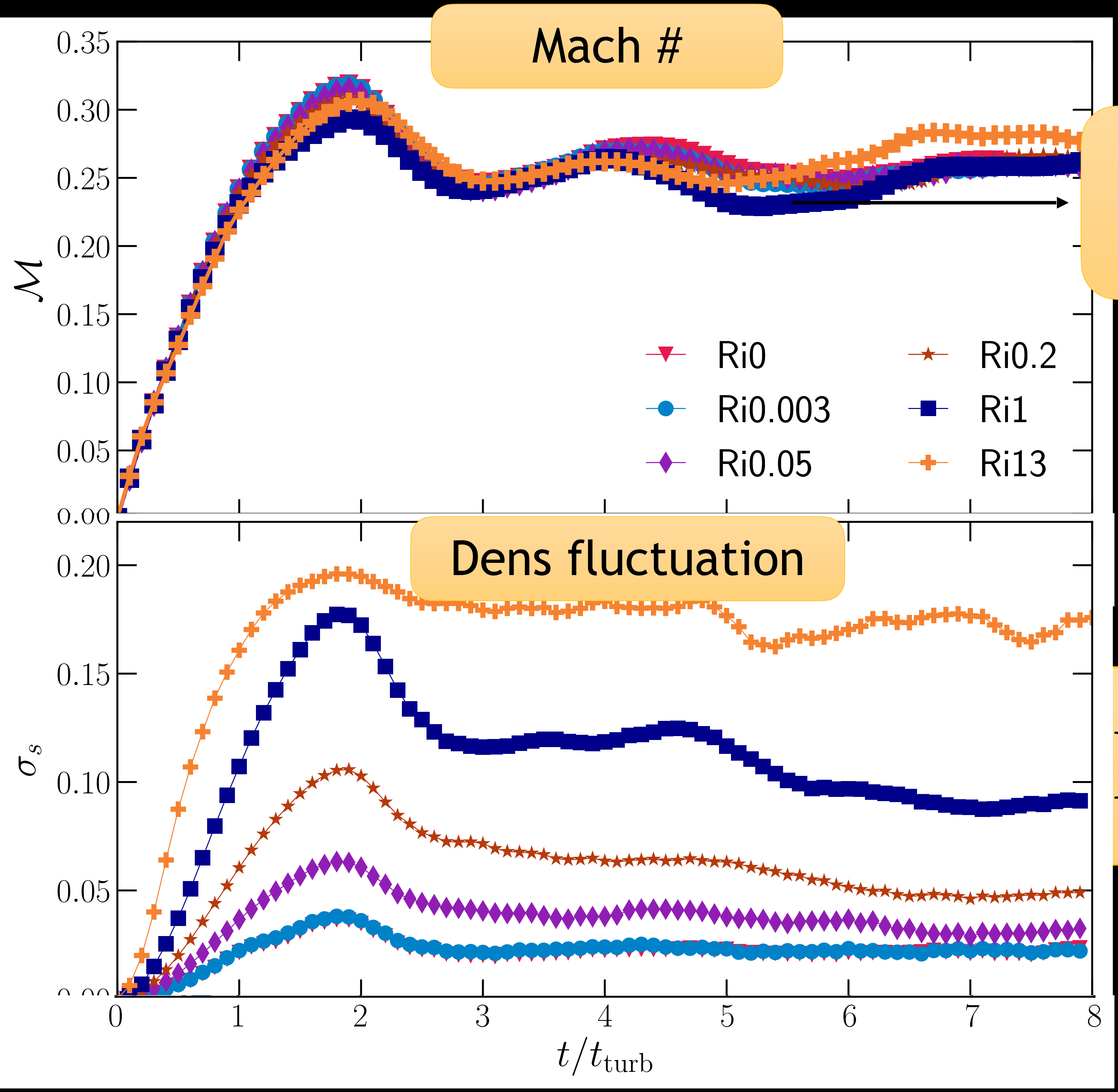
$\mathrm{Ri}\uparrow \;=>\; \delta\bar{\rho}\uparrow$

Mohapatra, Federrath and Sharma, 2020



Column density | **Column density fluctuations** | **Density (sliced) fluctuations - $\bar{\rho}_x$**

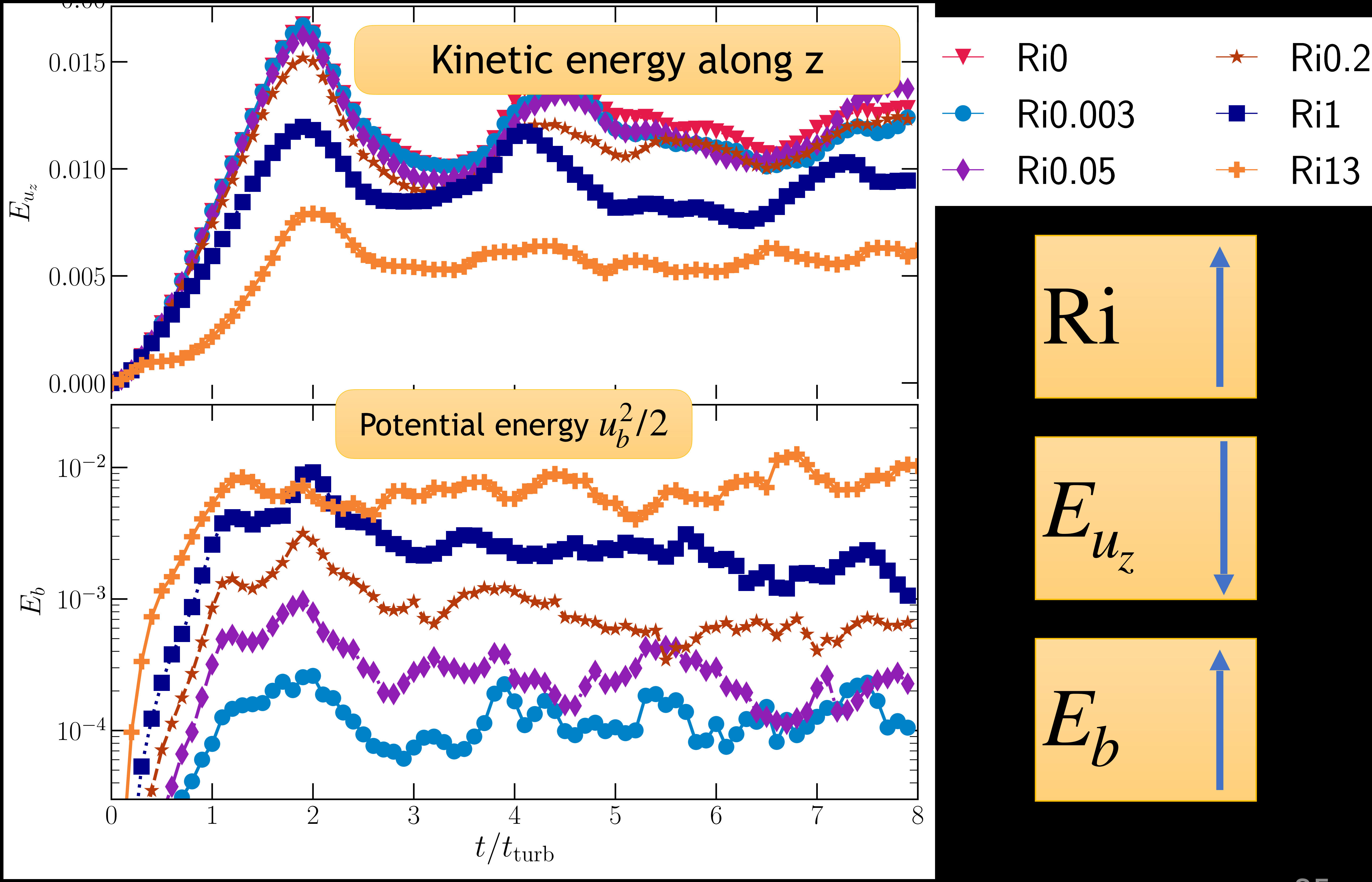
Time evolution



Different profiles, same driving

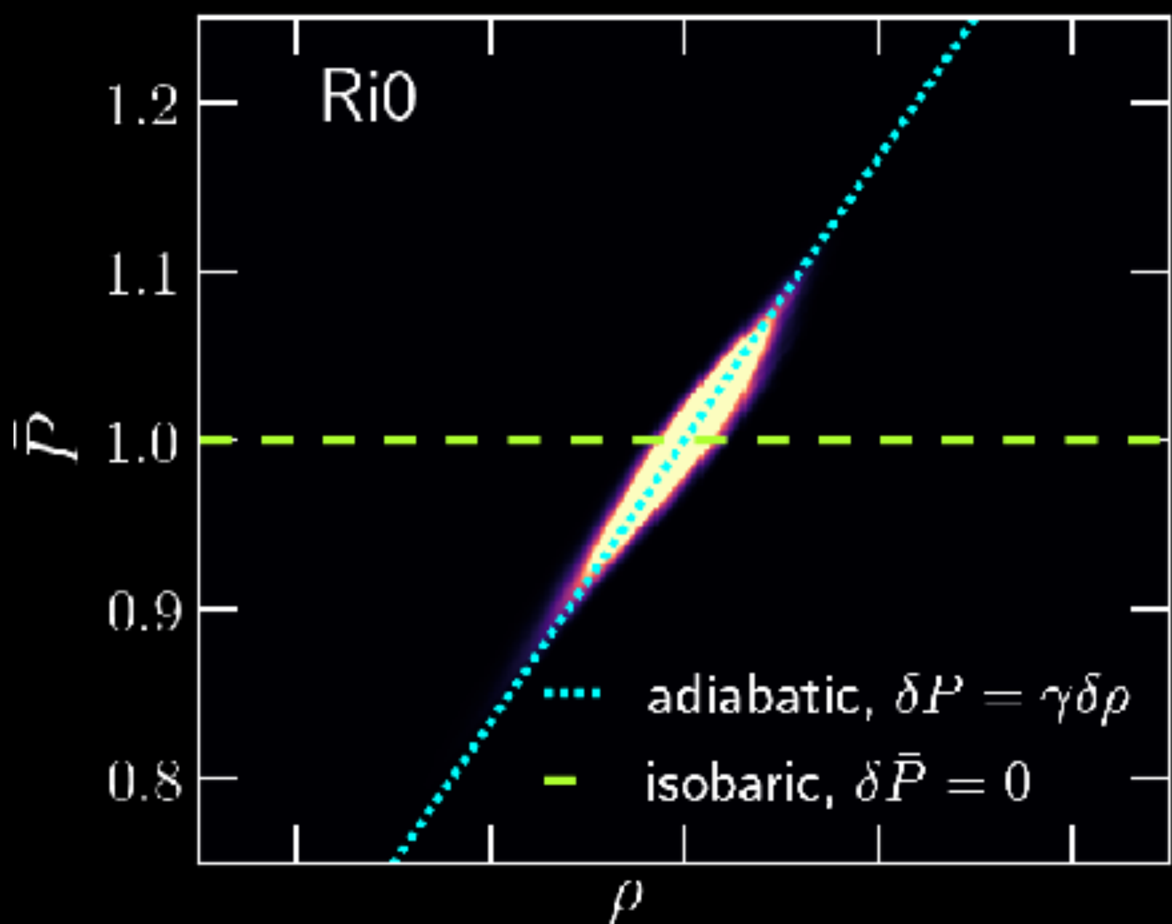
$\mathrm{Ri} \uparrow \Rightarrow \delta\bar{\rho} \uparrow$

Time evolution

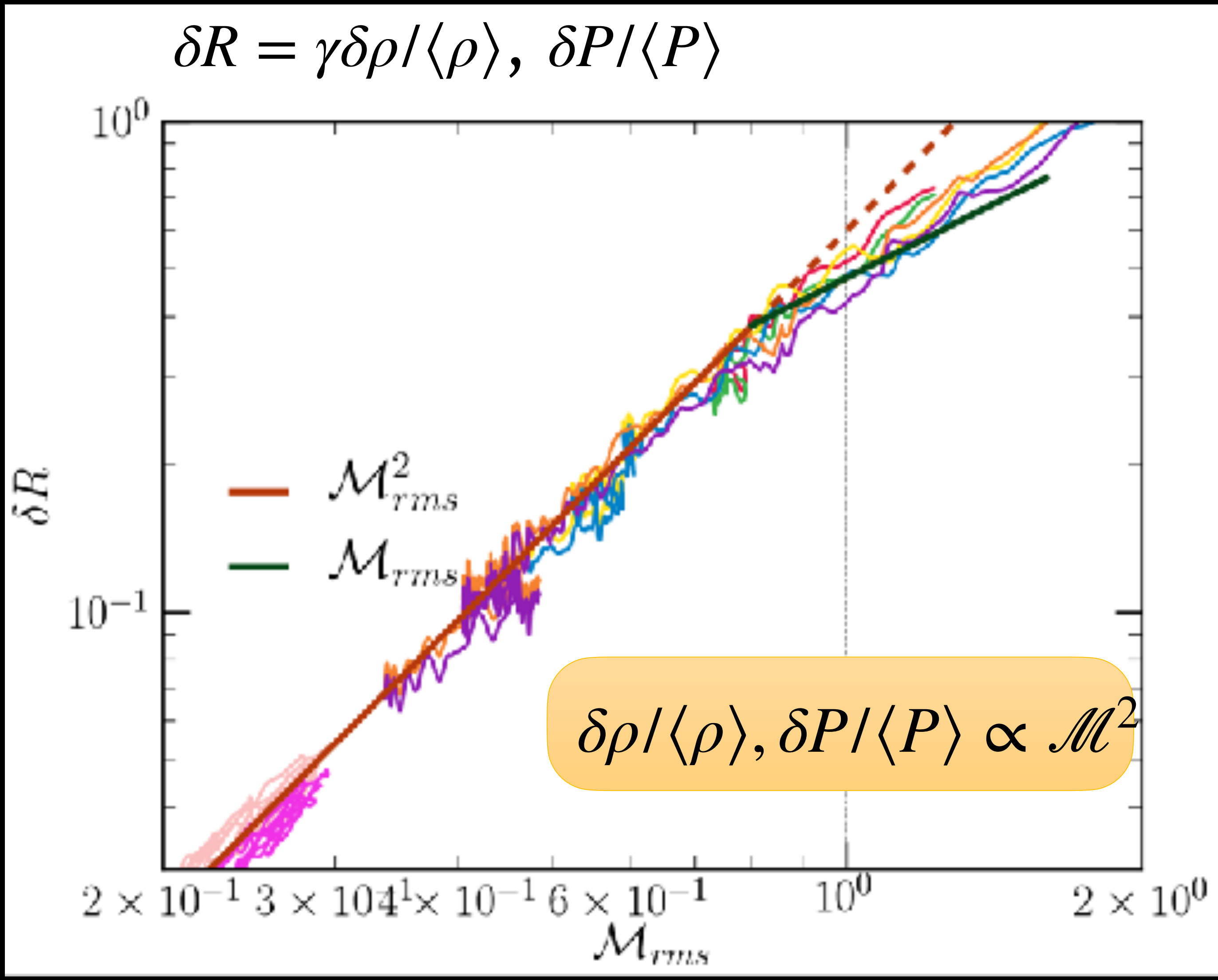


Density and pressure fluctuations - without stratification

P vs ρ



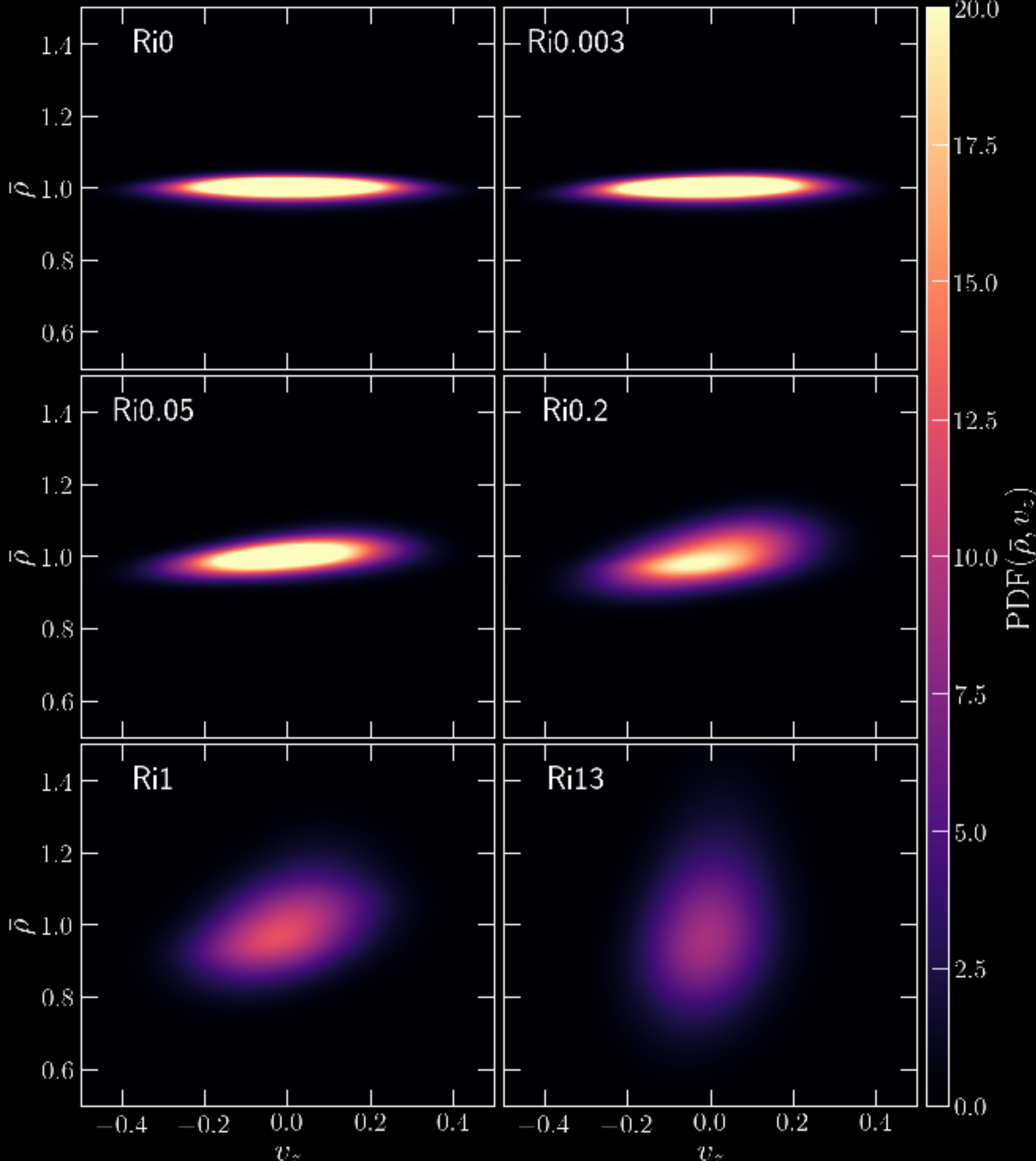
Adiabatic



$\delta\rho/\langle\rho\rangle, \delta P/\langle P\rangle \propto \mathcal{M}^2$

Mohapatra and Sharma, 2019

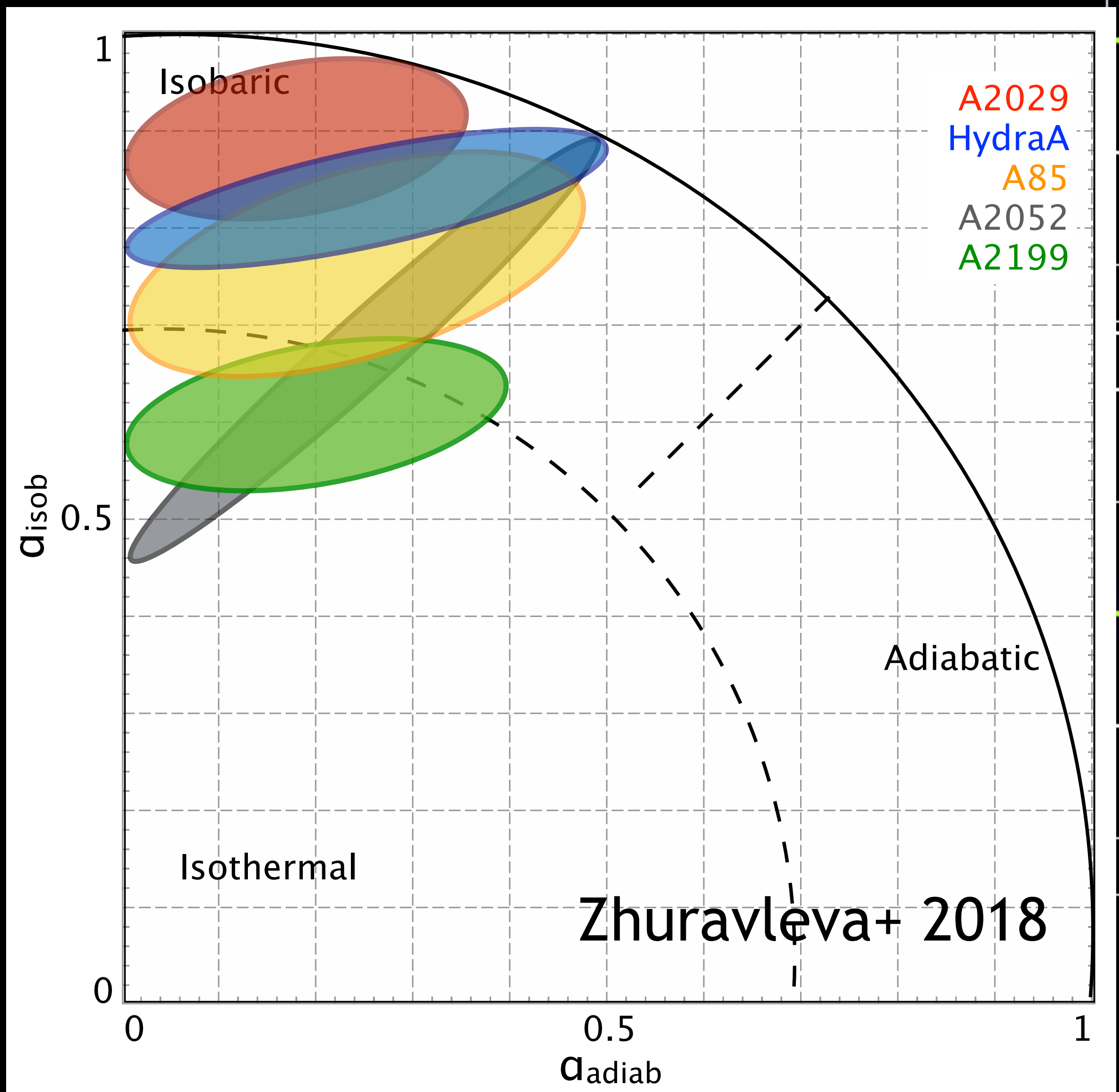
Density vs v_z PDF

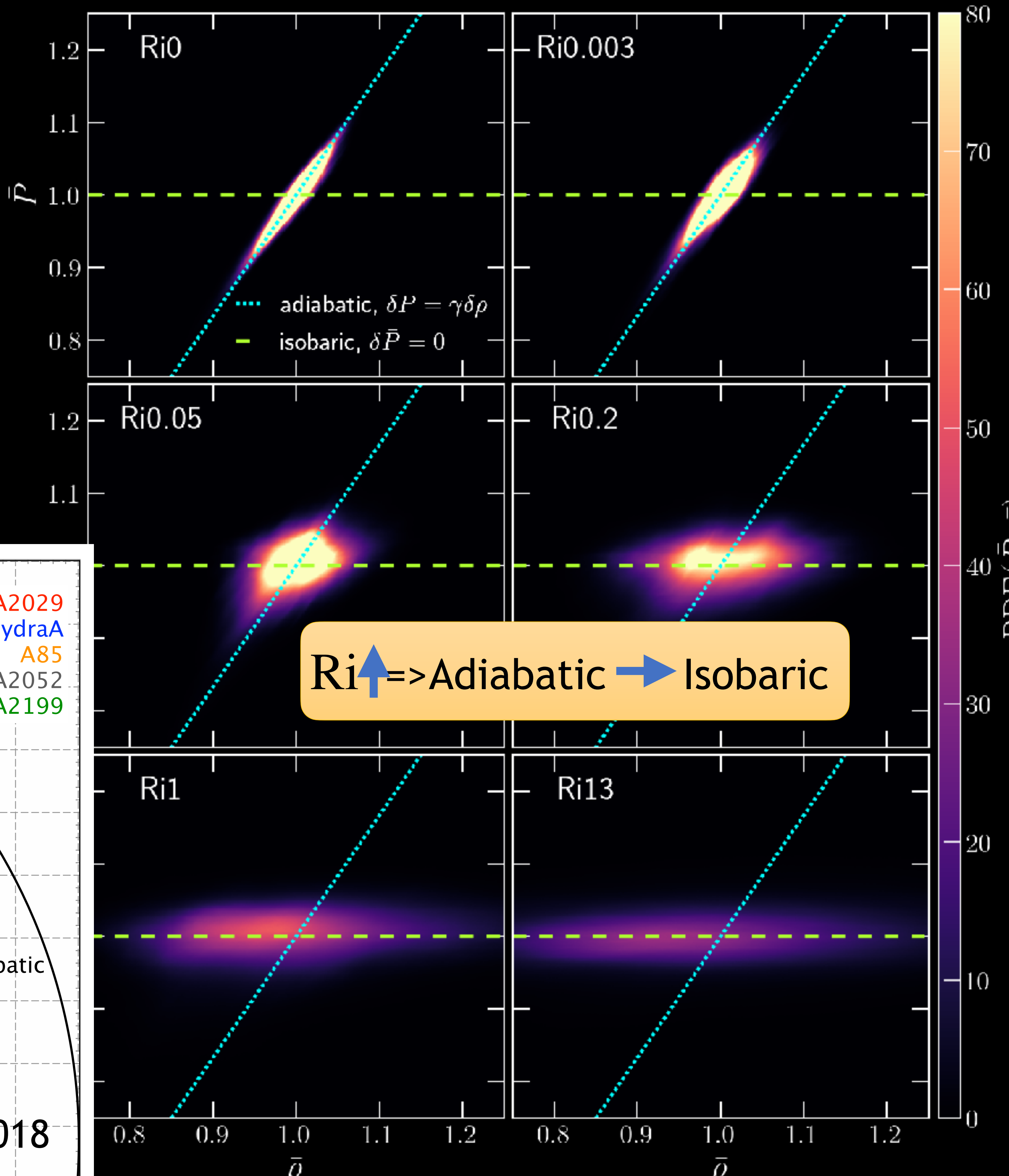


Ri↑ =>PDF rotates ACW

Mohapatra+, 2020

Pressure vs density PDF



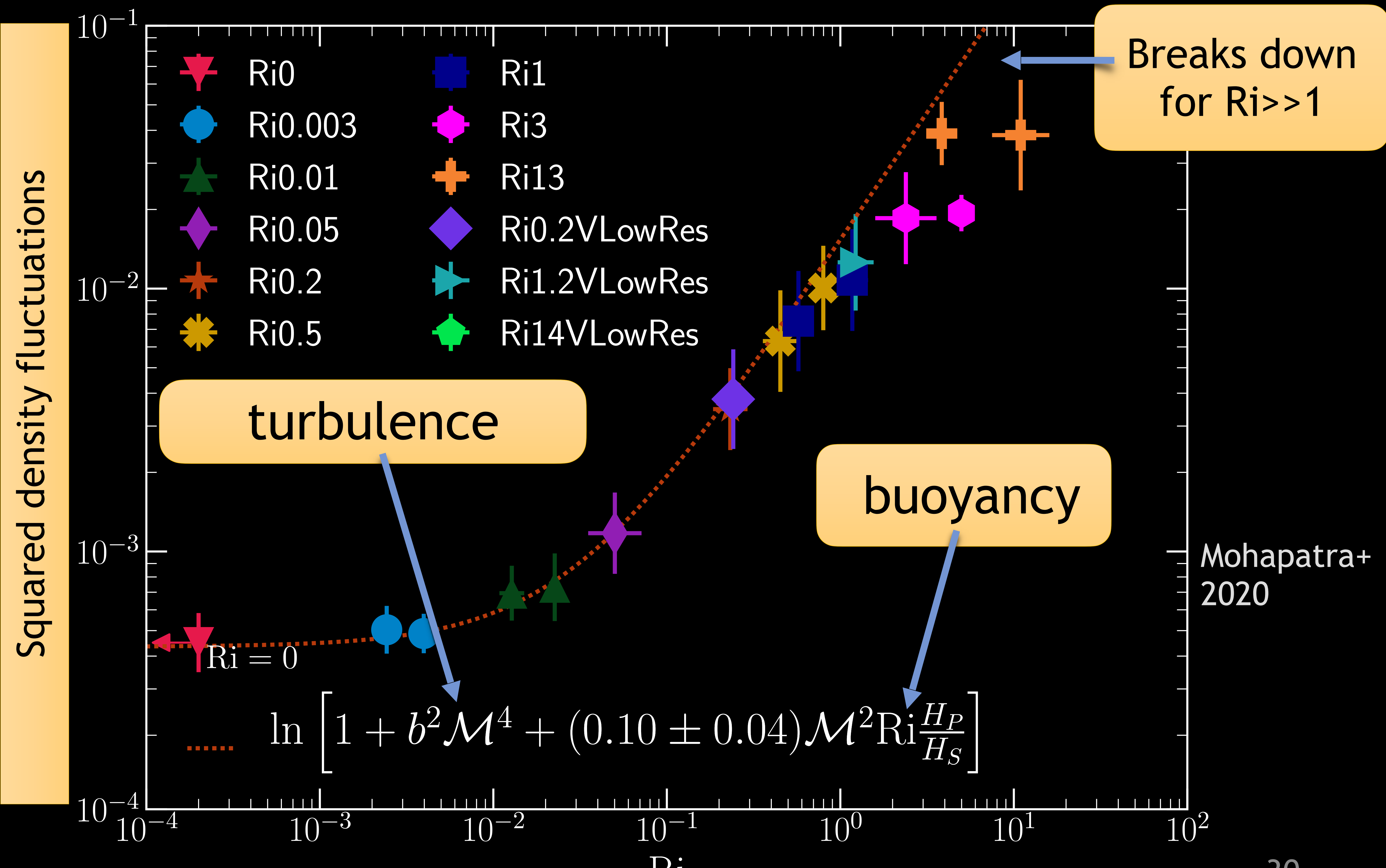


Mohapatra+ 2020

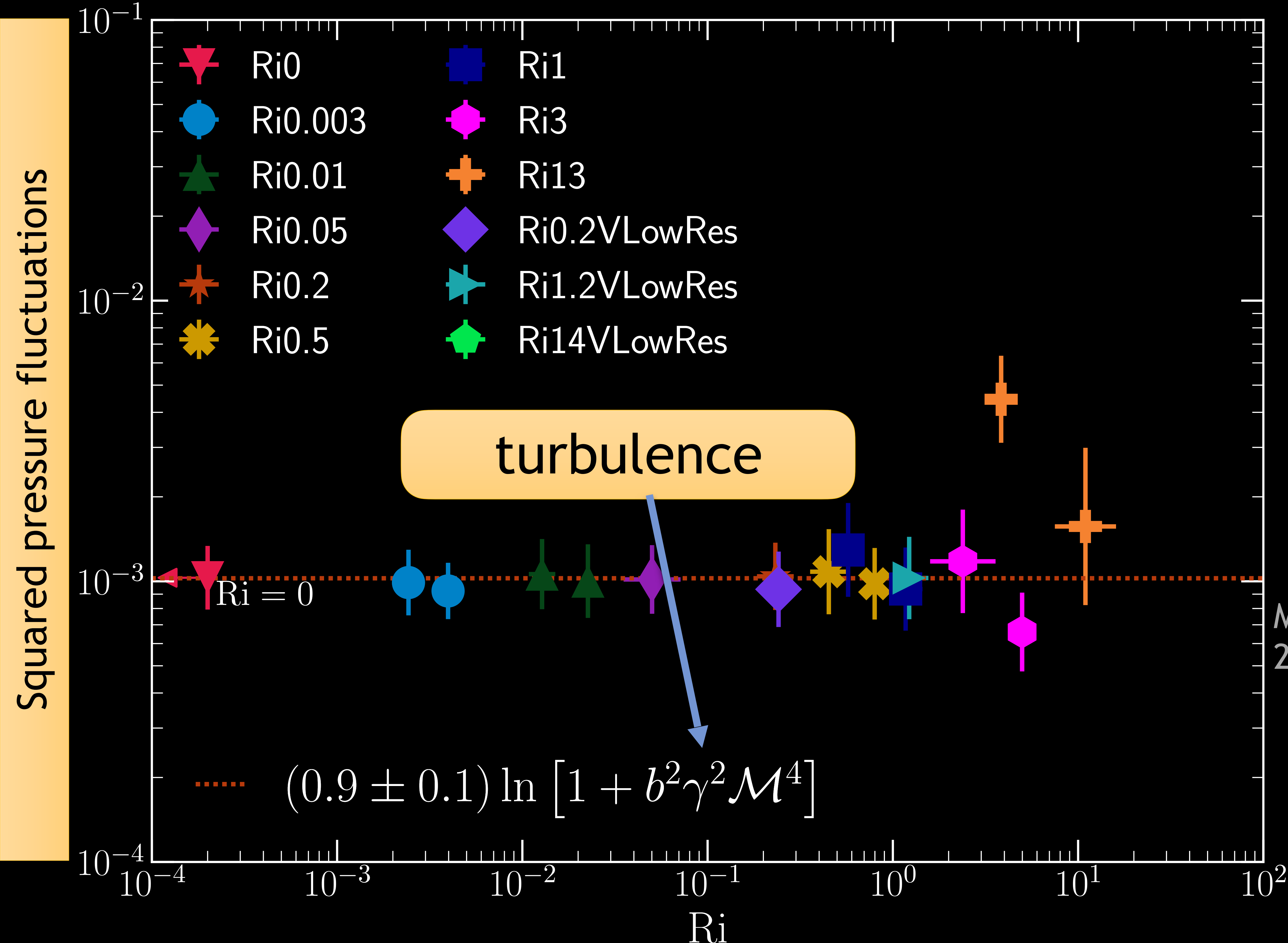
Deriving the scaling relation

- No stratification - $\delta\rho_{\text{turb}}, \delta P_{\text{turb}} \propto \mathscr{M}^2$
- With stratification - $\delta P_{\text{buoy}} = 0, \delta\rho_{\text{buoy}} > 0$
  - δP should be unaffected by stratification
  - $\delta\rho_{\text{buoy}}$ correlated with u_z- is there some way to write $\delta\rho_{\text{buoy}}$ in terms of δz?
- Now let's derive the scaling relation for density fluctuations
  - $\delta\bar{\rho}^2_{\text{buoy}} = \dfrac{N^4}{g^2}\delta z^2,\ N^2 = \dfrac{g}{\gamma}\dfrac{\mathrm{d}S}{\mathrm{d}z} = \dfrac{g}{\gamma H_S},\ \delta z^2 = \zeta^2 L^2_{\text{driv}}$
  - $\delta\bar{\rho}^2_{\text{buoy}} = \dfrac{\zeta^2 L^2_{\text{driv}}}{\gamma^2 H_S^2} == \dfrac{\zeta^2 \mathrm{Ri}\mathscr{M}^2 c_s^2}{N^2\gamma^2 H_S^2} = \zeta^2\mathscr{M}^2\mathrm{Ri}\dfrac{H_P}{H_S},$
    using $c_s^2 = \gamma g H_P$

Density fluctuations vs Ri

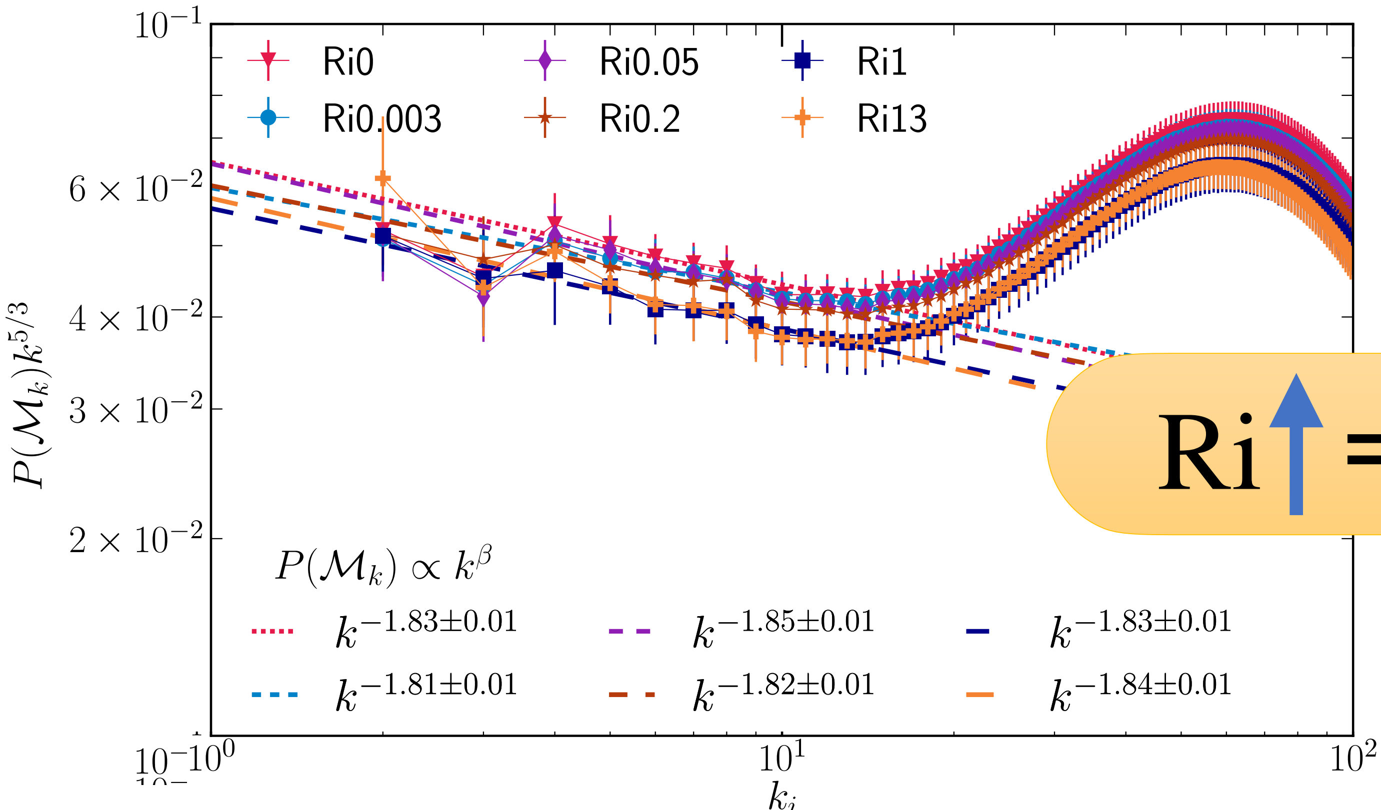


Pressure fluctuations vs Ri



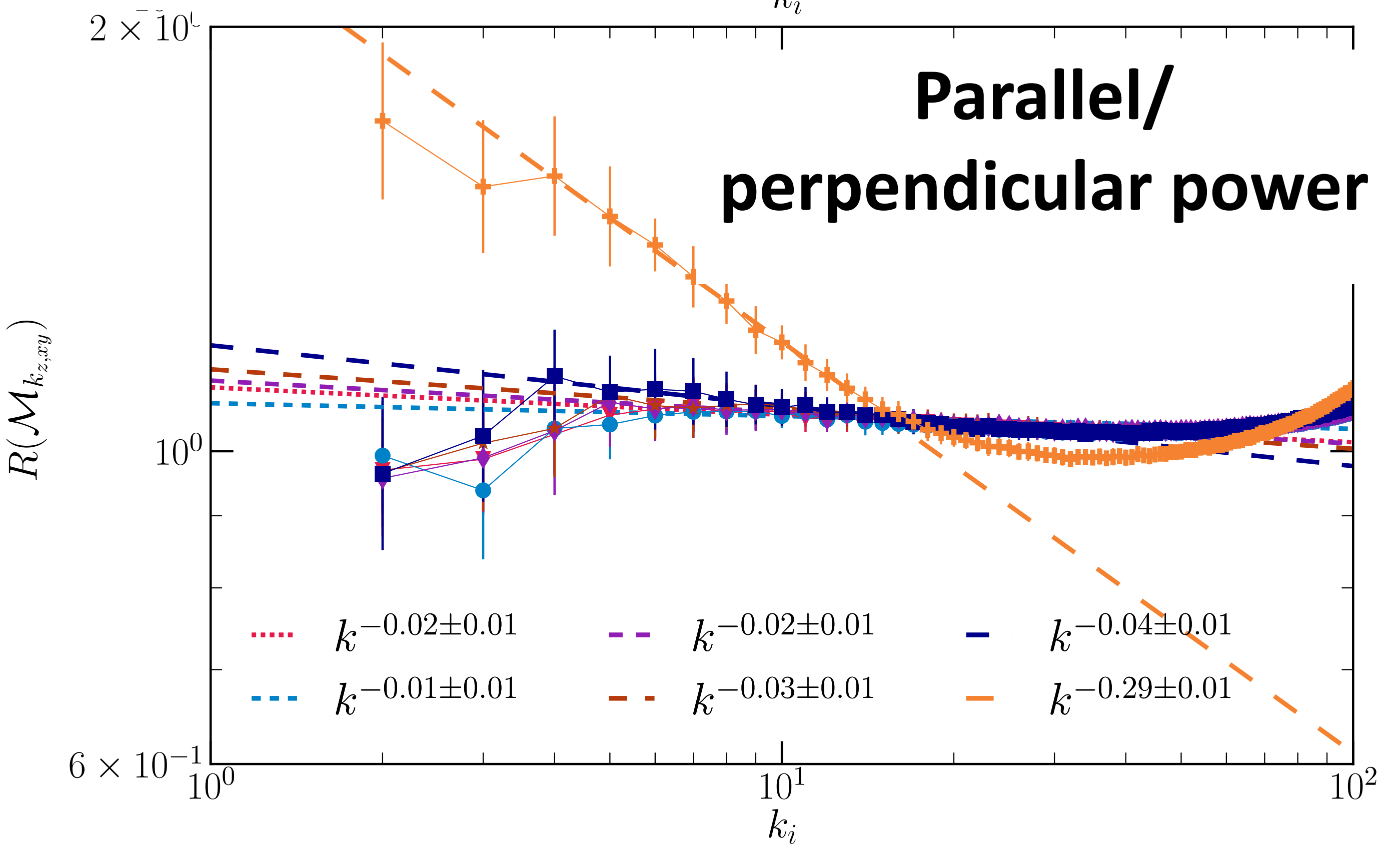
Mohapatra+ 2020

Velocity power spectra



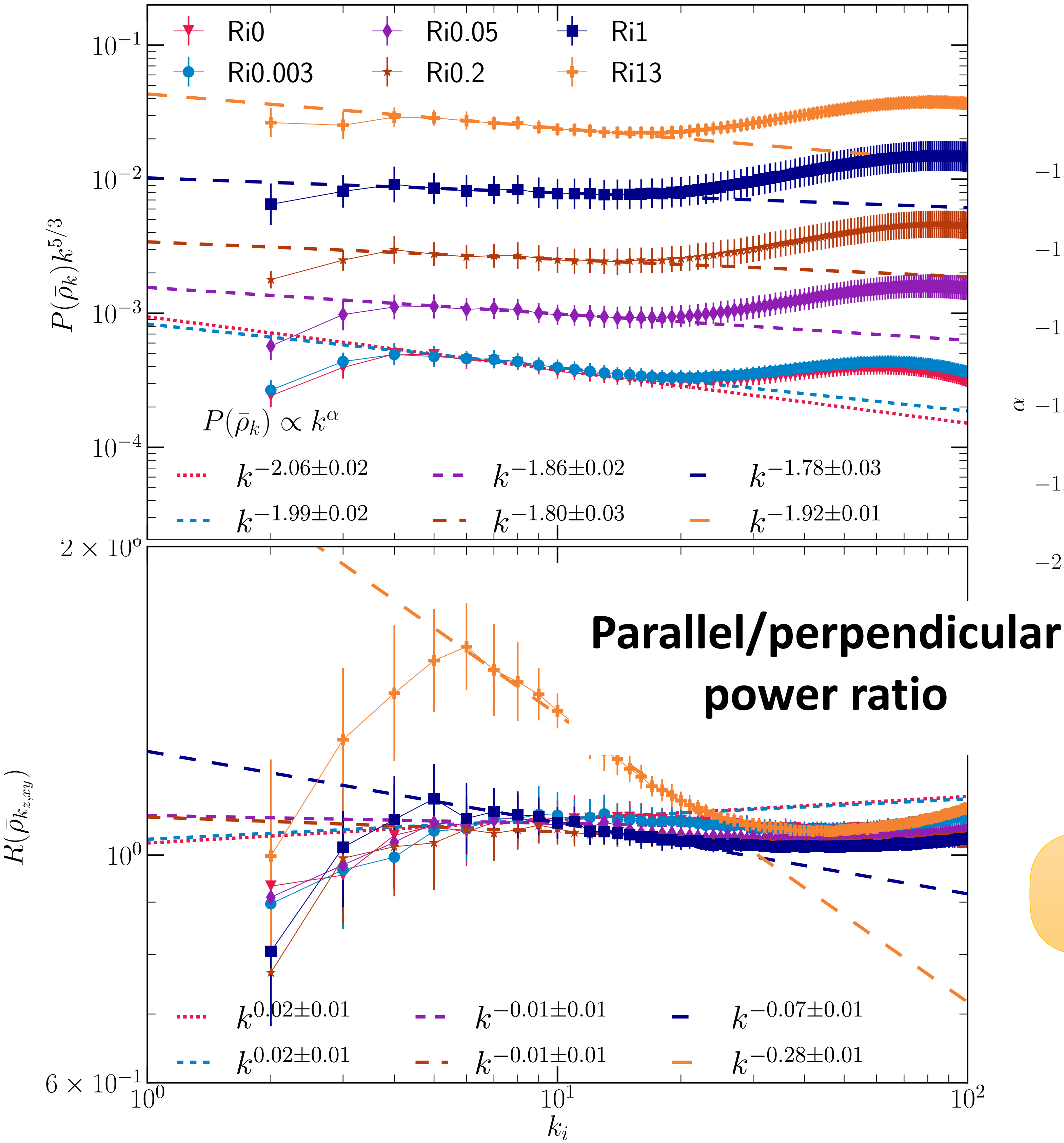
$$P(v_k/\langle c_s \rangle)$$

Ri↑ => Slope → invariant!



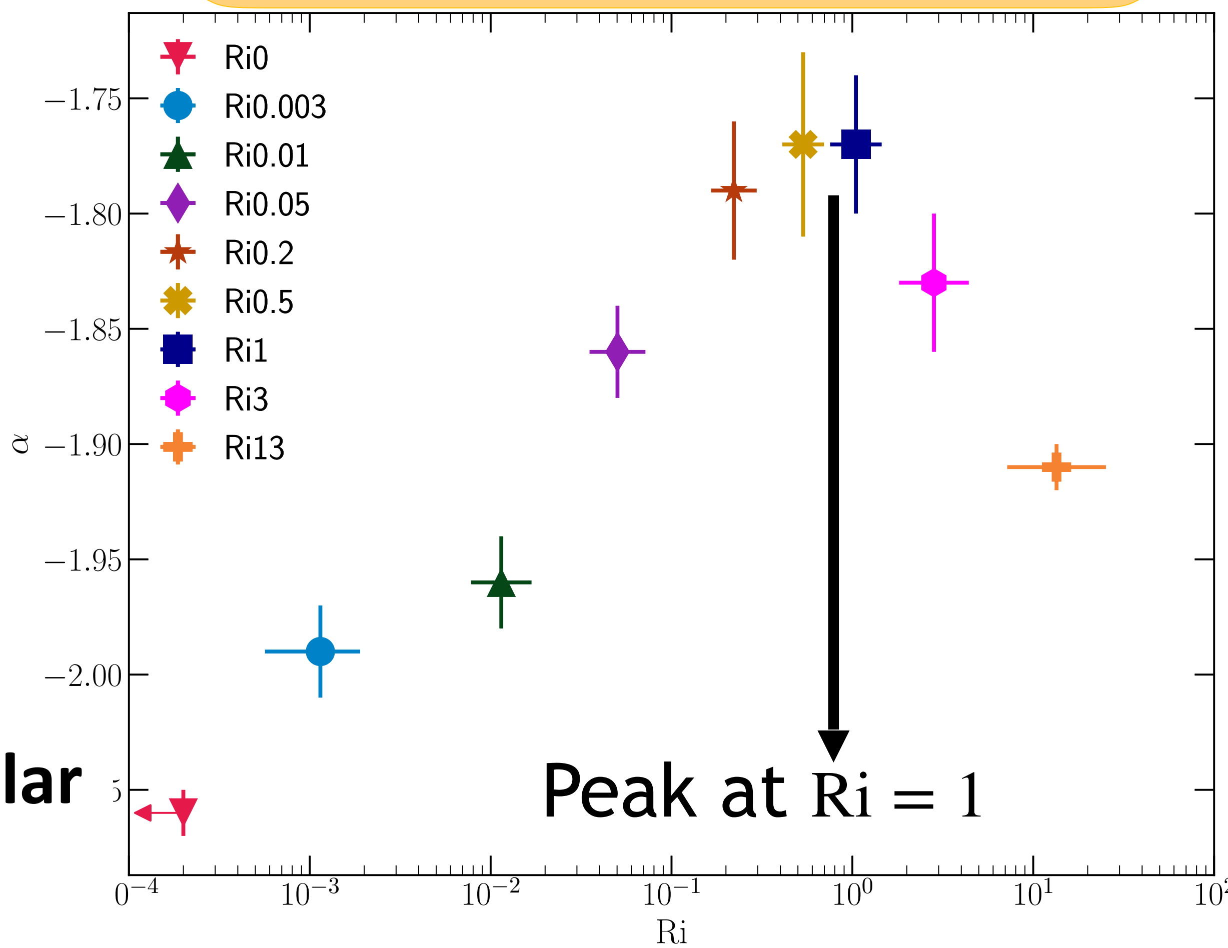
Ri>1 → anisotropy!

Density power spectra



Parallel/perpendicular power ratio

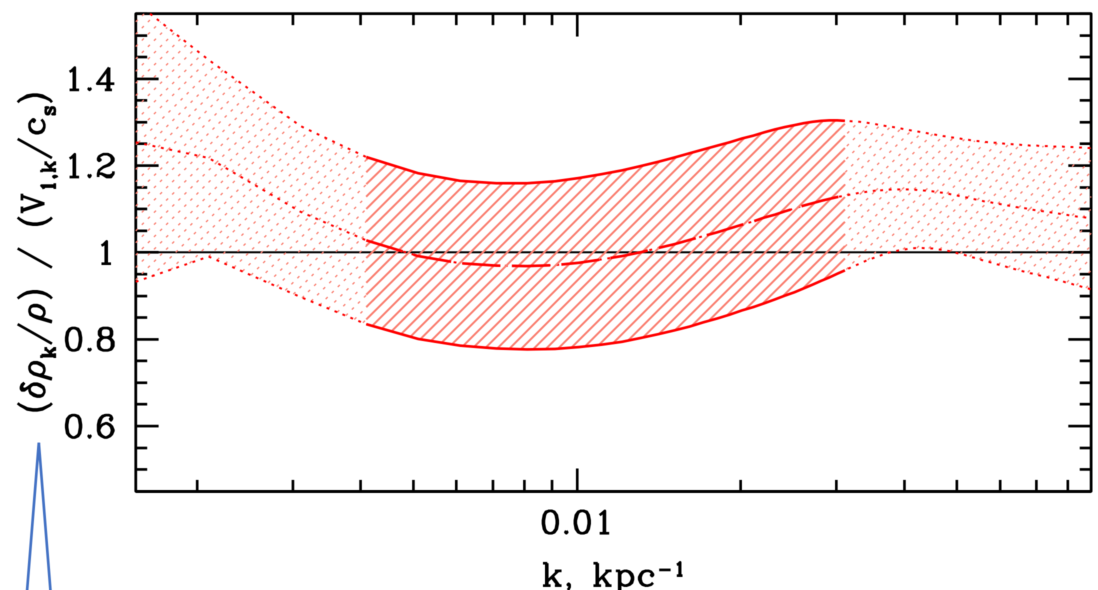
Slope vs Ri



Ri>1 → anisotropy!

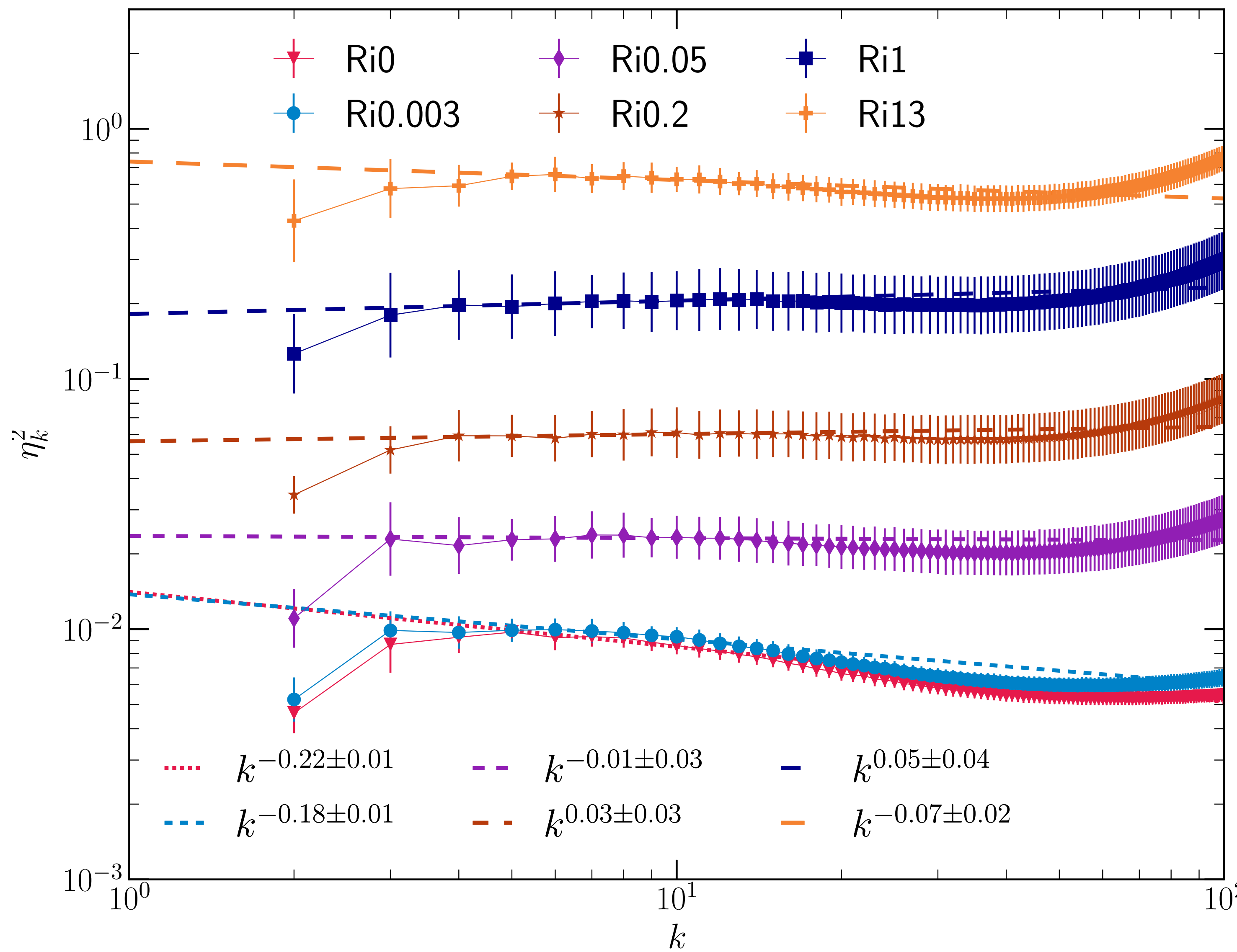
Ratio between density and velocity η_k

Current literature



$\eta_k \approx 1$

What we find

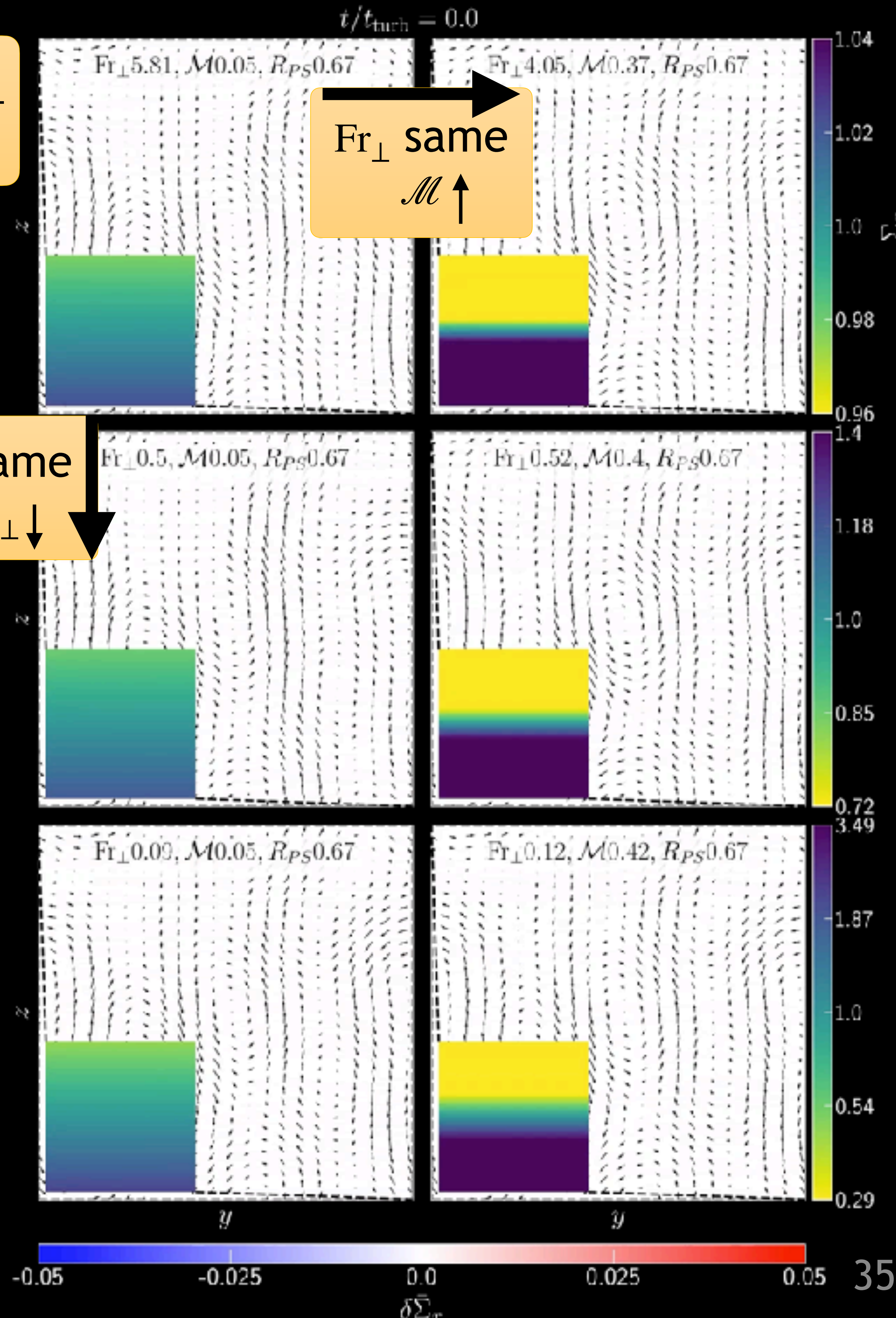


$\eta_k^2 \propto \mathrm{Ri}$

Project-1 follow-up

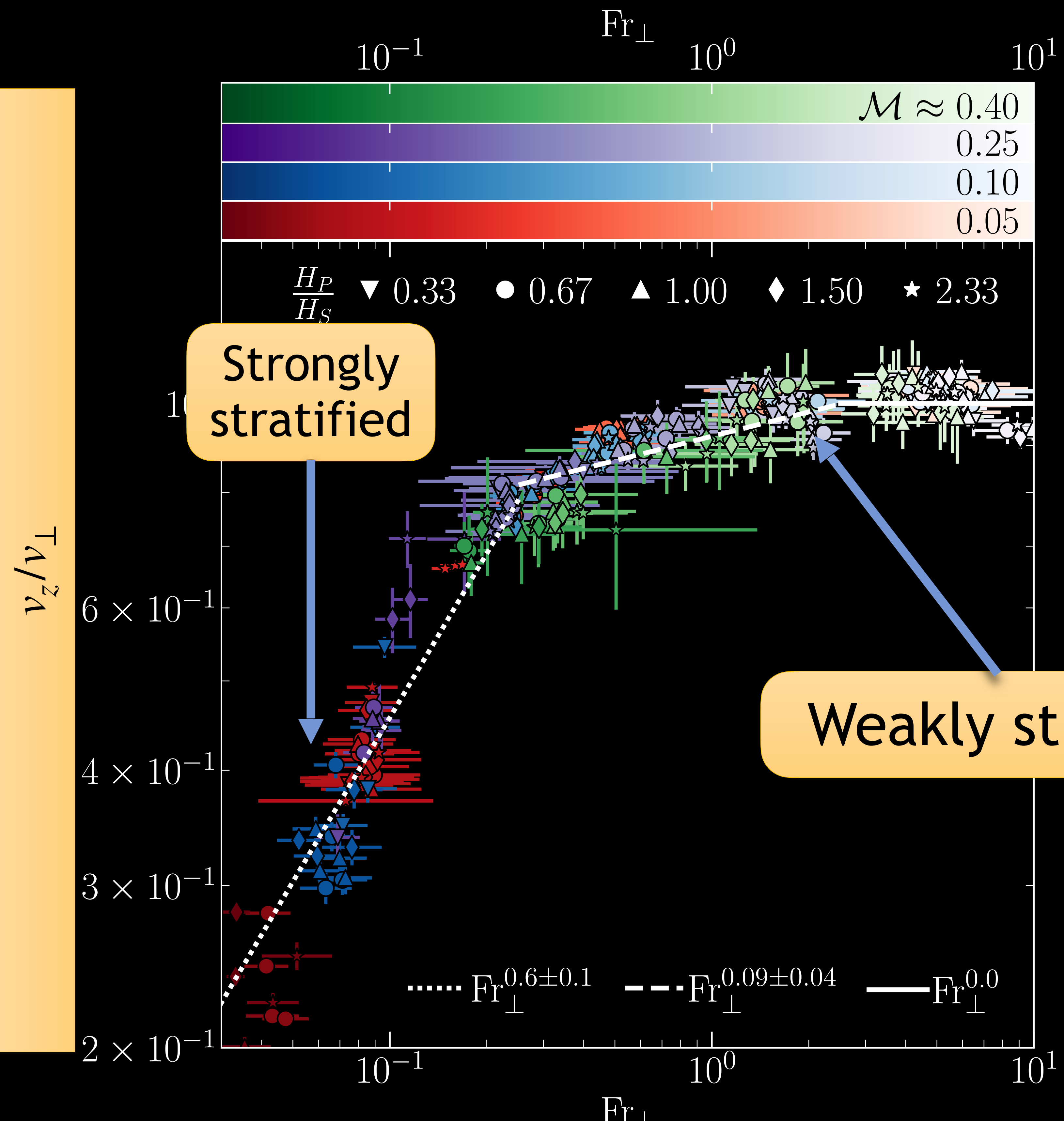
$$\mathrm{Fr}_\perp \approx \frac{1}{\sqrt{\mathrm{Ri}_\perp}}$$

- Scan parameter space of the three parameters - Mach number, Froude number and H_P/H_S
  - Same setup, 96 simulations
  - Resolution - 256^3-1024^3
  - Density and pressure fluctuations vs Mach number and Froude number
  - Extend relation to low Froude number 0.1-0.01 (Ri~100-1000)



Mohapatra, Federrath and Sharma, submitted

Velocity anisotropy

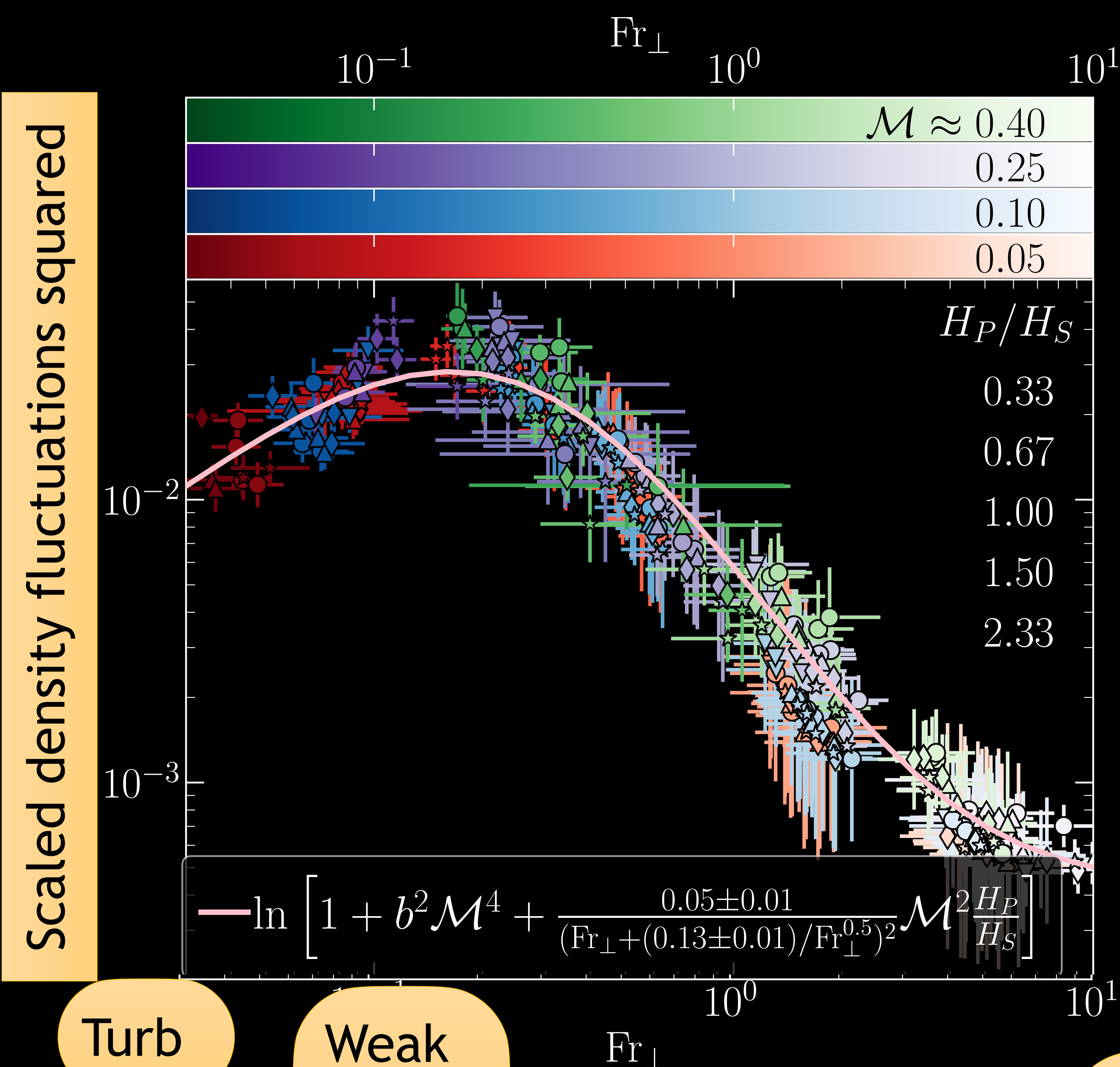


$$\mathrm{Fr}_\perp = \frac{u_\perp}{NL} = \frac{\text{buoyancy time scale}}{\text{turbulent time scale}}$$

$$\mathrm{Fr}_\perp \approx \frac{1}{\sqrt{\mathrm{Ri}_\perp}}$$

Mohapatra+, submitted

Density fluctuations - strong stratification



Turb

Weak Strat

Strong Strat

$$\ln\left[1 + b^2\mathscr{M}^4 + \frac{0.05}{(\mathrm{Fr}_\perp + 0.13/\sqrt{\mathrm{Fr}_\perp})^2}\mathscr{M}^2\frac{H_P}{H_S}\right]$$

- What happens in strong stratification limit?
- δz not proportional to L anymore, $\delta z \approx v_z/N$
- Velocity distribution is no longer isotropic
- $V_z \approx V_\perp \mathrm{Fr}_\perp^{0.5}$ - make use of this relation
- In this limit,

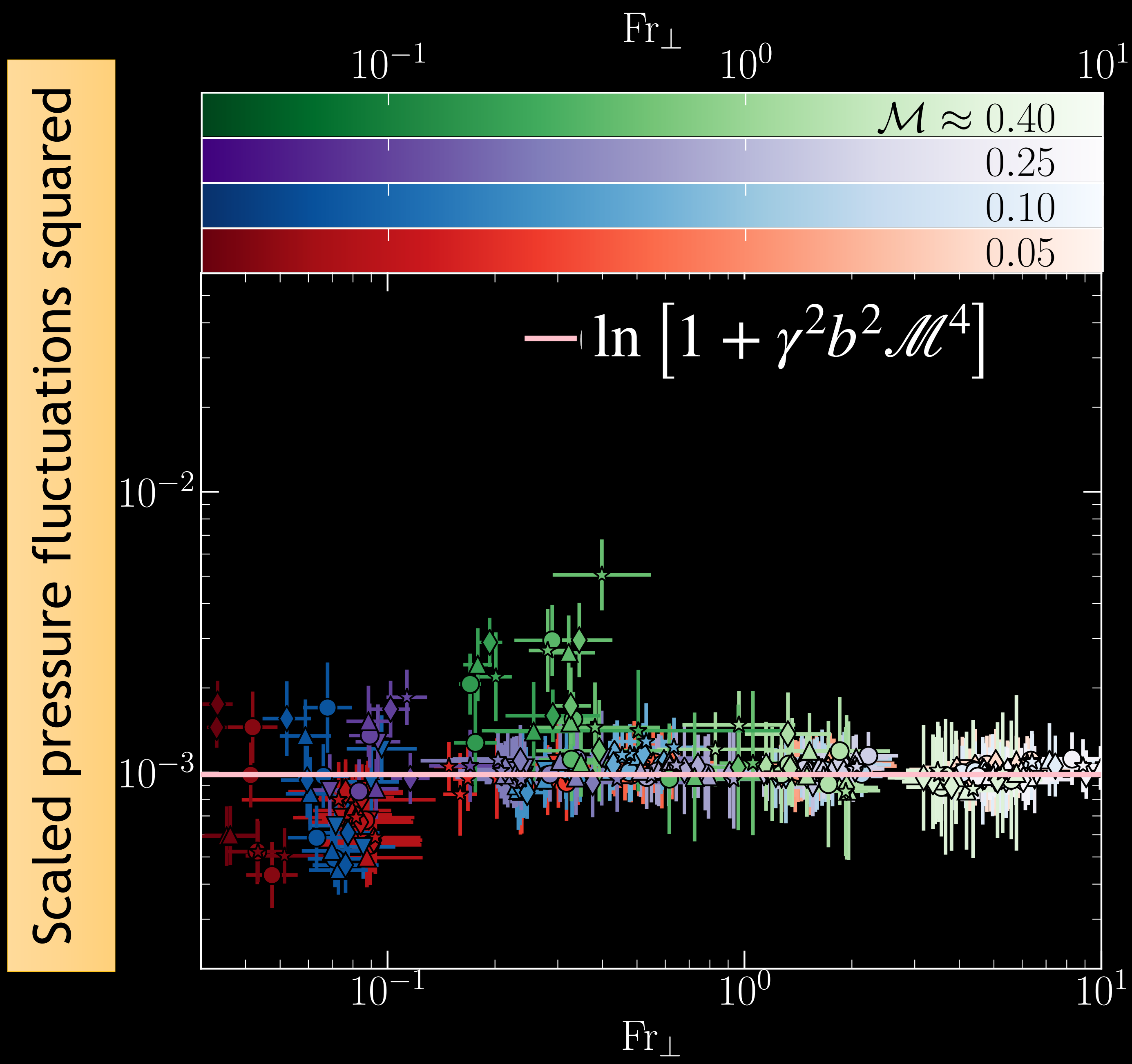
$$\delta\bar{\rho}^2_{\mathrm{buoy}} = \frac{N^4}{g^2}\delta z^2 = \frac{N^4}{g^2}\zeta_2^2\frac{v_z^2}{N^2}$$

$$= \zeta_2^2\frac{N^2}{g}v_\perp^2\mathrm{Fr}_\perp \approx \zeta'^2\mathscr{M}^2\mathrm{Fr}_\perp\frac{H_P}{H_S}$$

- Try to write a functional form now that takes this asymptotic behaviour into account

Mohapatra+, Submitted

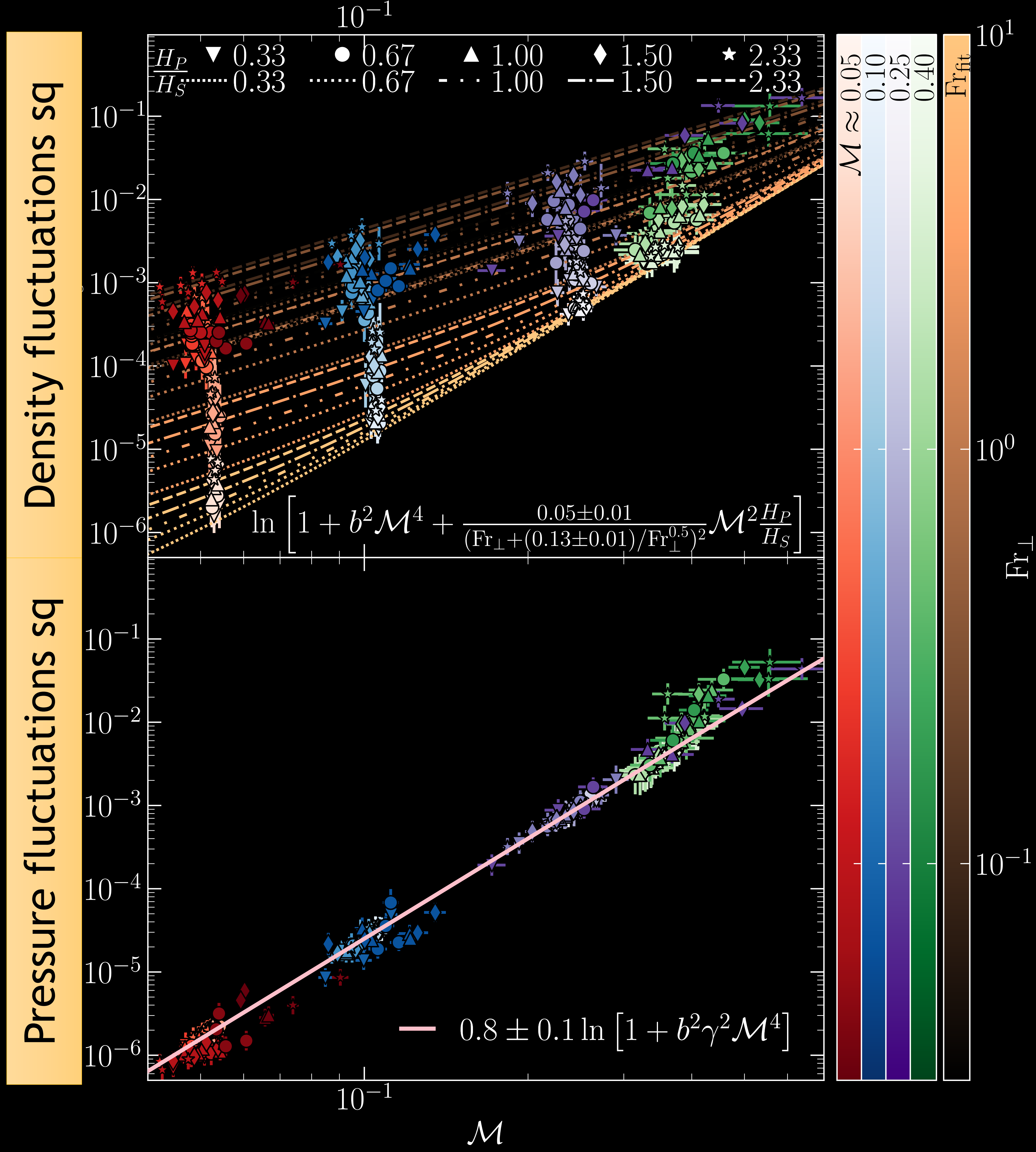
Pressure fluctuations - strong stratification



- Pressure fluctuations are still independent of stratification.
- Rms pressure fluctuations can be obtained from thermal SZ effect observations.
- Then use this to measure velocities.
- Since this is independent of stratification, relation doesn't change for each kind of cluster.

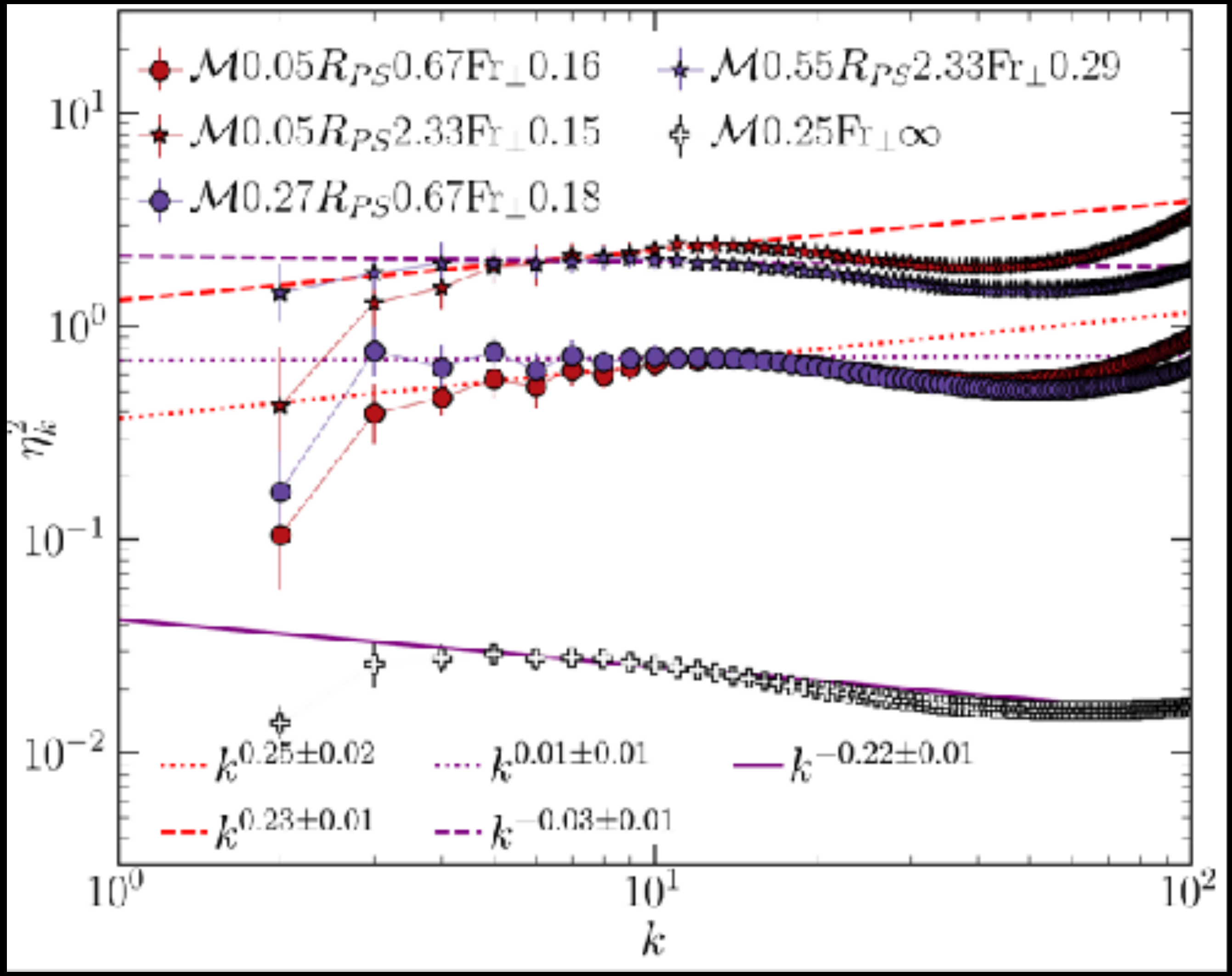
Mohapatra+, Submitted

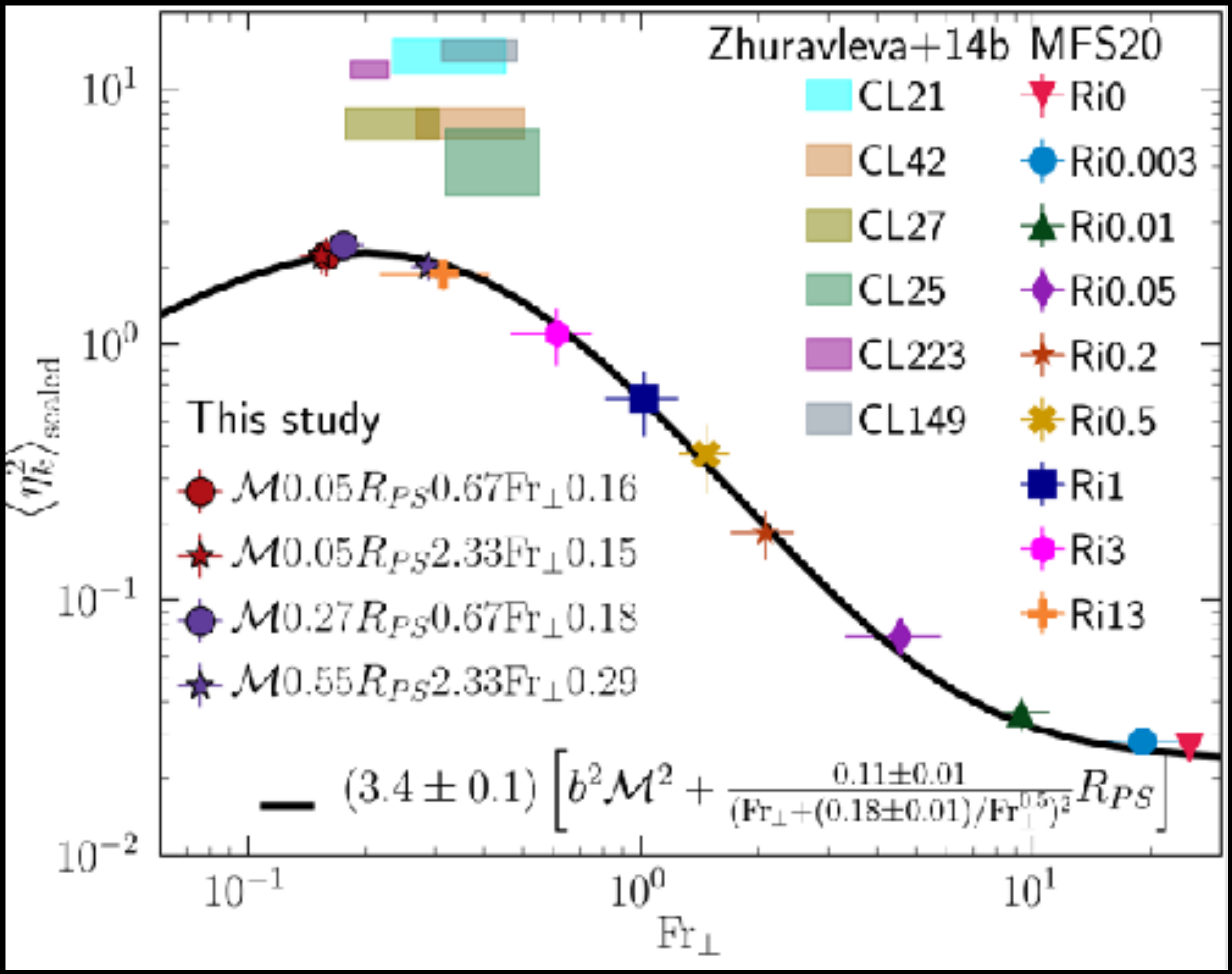
Density and pressure fluctuations vs $\mathscr{M}$



Mohapatra+, Submitted

Power spectra for strong stratification and scaling of eta





Conclusions

We derived a scaling relation density fluctuations, that holds across three orders of magnitude of the Froude number:

- $\sigma_s^2 = \ln\left[1 + b^2\mathcal{M}^4 + \frac{0.05}{(\mathrm{Fr}_\perp + 0.13/\sqrt{\mathrm{Fr}_\perp})^2}\mathcal{M}^2\frac{H_P}{H_S}\right]$
- $\sigma_{\ln\bar{P}}^2 = \ln(1 + b^2\gamma^2\mathcal{M}^4)$
- Density fluctuations become more isobaric as Fr decreases - and match more with observations of ICM gas.
- Velocity power spectrum slope stays same, but density power spectrum increases in amplitude but becomes shallower

For Fr<1:

- Velocity and density fields show anisotropy
- Scaling relations become flatter.

Future work:

- Include magnetic fields and cooling
- Vary the driving parameter b, driving length scale l