ASSIGNMENT - 1

1) An incomplete distribution of families according to their expenditure per week is given below. The median and mode for the distribution is ₹ 25 & ₹ 24 respectively. Calculate the missing frequencies.

Expenditure	0-10	10-20	$\boxed{20-30}$	30-40	40-50
No. of Families	14	? x	27	? y	15

Given : Median = 25 , Mode = 24

Modal Class = 20-30

$$\text{Median} = l + \frac{h}{f}\left[\frac{N}{2} - cf\right]$$

Class	f	cf
0-10	14	14
10-20	x	$14+x$
$\boxed{20-30}$	27	$41+x$
30-40	y	$41+x+y$
40-50	15	$56+x+y$
	$56+x+y$	

$$\Rightarrow \quad 25 = 20 + \frac{10}{27}\left[\frac{56+x+y}{2} - (14+x)\right]$$

$$\cancel{5} = \frac{\cancel{10}^{5}}{27}\left[\frac{56+x+y-28-2x}{\cancel{2}}\right]$$

$$27 = 28 - x + y$$

$$x - y = 1 \quad \text{———} (1)$$

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

$$24 = 20 + \frac{(27 - x)10}{54 - x - y}$$

$$24 = \frac{270 - 10x}{54 - x - y}$$

$$216 - 4x - 4y = 270 - 10x$$

$$6x - 4y = 54$$

$$3x - 2y = 27 \quad \text{---} (2)$$

$$x - y - 1 = 0$$

$$3x - 2y - 27 = 0$$

$$\boxed{\begin{aligned} x &= 25 \\ y &= 24 \end{aligned}}$$

2) Calculate mode of the following distribution:

Wages	50 - 70	70 - 90	90 - 110	110 - 130	130 - 150	150 - 170	170 - 190	190 - 210	210 - 230
No. of Workers	4	44	38	28	6	8	12	2	2

Modal Class - 70 - 90

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

$$= 70 + \frac{44 - 4}{88 - 4 - 38} \times 20$$

$$= 70 + \frac{40(20)}{46}$$

$$= 70 + 17.39$$

$$= 87.39$$

3) The first four moments of a distribution about 2 are 1, 2.5, 5.5 and 16 respectively. Calculate the four moments about mean and about the origin.

Given: $A = 2$

$\mu_1' = 1, \quad \mu_2' = 2.5, \quad \mu_3' = 5.5, \quad \mu_4' = 16$

$\mu_1' = \bar{x} - A$

$\therefore \bar{x} = \mu_1' + A$

$= 1 + 2$

$\boxed{\bar{x} = 3}$

Moments about Mean —

$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'^2$

$= 2.5 - (1)^2$

$= 1.5$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= 5.5 - 3(2.5)(1) + 2(1)^3$

$= 5.5 - 7.5 + 2$

$= 0$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 16 - 4(5.5)(1) + 6(2.5)(1)^2 - 3(1)^4$

$= 16 - 22.0 + 15.0 - 3$

$= 31 - 25$

$= 6$

Moments about Origin —

$\nu_1 = \bar{x} = 3$

$\nu_2 = \mu_2 + \bar{x}^2$

$= 1.5 + (3)^2 \Rightarrow 1.5 + 9$

$= 10.5$

Page No. ______
Date

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$
$$= 0 + 3(1\cdot5)(3) + (3)^3$$
$$= 13\cdot5 + 27$$
$$= 40\cdot5$$

$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$
$$= 6 + 4(0)(3) + 6(1\cdot5)(3)^2 + (3)^4$$
$$= 6 + 81 + 81$$
$$= 168$$

4) Find the moment coefficient of skewness and kurtosis of the following data.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	10	20	40	20	10

x_i	CI	f_i	u_i	f_iu_i	$f_iu_i^2$	$f_iu_i^3$	$f_iu_i^4$
5	0-10	10	-2	-20	40	-80	160
15	10-20	20	-1	-20	20	-20	20
25	20-30	40	0	0	0	0	0
35	30-40	20	1	20	20	20	20
55	40-50	10	2	20	40	80	160
		100		0	120	0	360

$\mu_1' = 0$, $\mu_2' = \frac{120}{\cancel{100}} \times \cancel{100}$

$= 120$

$\mu_3' = 0$

$\mu_4' = \frac{360}{\cancel{100}} \times 10000 = 36000$

$\mu_1 = 0$, $\mu_2 = \mu_2' - \mu_1'^2$

$= 120 - 0$

$= 120$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= 0 - 3(120)(0) + 2(0)$

$= 0$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 36000 - 4(0)(0) + 6(120)(0) - 3(0)$

$= 36000$

$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{0}{(120)^{3/2}} = 0$

Page No. ______
Date

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{36000}{(120)^2} = \frac{36000}{14400}$$

$$= 2.5$$

$\boxed{\beta_2 < 3}$ → Platykurtic //

Q) ~~For two~~ By method of least square fit a curve $y = ax^b$ to the following data :

x : 1 2 3 4 5

y : 7.1 27.8 62.1 110 161

$$y = ax^b$$

$$\log y = \log a + b \log x$$

$$Y = A + bX$$

where, $Y = \log y$, $A = \log a$

$X = \log x$

Normal Equations :

$$\Sigma Y = nA + b\Sigma X$$

$$\Sigma XY = A\Sigma X + b\Sigma X^2$$

x	y	$X = \log x$	$Y = \log y$	XY	X^2
1	7.1	0	0.8513	0	0
2	27.8	0.3010	1.4440	0.4346	0.0906
3	62.1	0.4771	1.7931	0.8555	0.2276
4	110	0.6021	2.0413	1.2291	0.3625
5	161	0.6990	2.2068	1.5426	0.48860
$n = 5$		2.0792	8.3365	4.0618	1.1693

Page No. ____
Date ____

$8.3365 = 5A + 2.0792b$

$4.0618 = 2.0792A + 1.1693b$

$A = 0.8550$

$b = 1.9533$

$A = \log a \Rightarrow a = \text{Antilog}(A)$

$= \text{Antilog}(0.8550)$

$= 7.1614$

$\Rightarrow y = 7.1614x^{1.9533}$

5) Calculate the correlation coefficient b/w X & Y from the following data :

X :	60	34	40	50	45	41	22	43
Y :	75	32	34	40	45	33	12	30

X	Y	XY	X^2	Y^2
60	75	4500	3600	5625
34	32	1088	1156	1024
40	34	1360	1600	1156
50	40	2000	2500	1600
45	45	2025	2025	2025
41	33	1353	1681	1089
22	12	264	484	144
43	30	1290	1849	900
335	301	13880	14895	13563

$n = 8$

$(\Sigma x)^2 = (335)^2 = 112225$

$(\Sigma y)^2 = (301)^2 = 90601$

Page No.
Date

$$r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{8(13880) - (335)(301)}{\sqrt{8(14895) - 112225}\sqrt{8(13563) - 90601}}$$

$$r_{xy} = 0.9159$$

6) Calculate the rank correlation coefficient between X and Y from the following data:

X: 15 20 27 13 45 60 20 75

Y: 50 30 55 30 25 10 30 70

X	Y	$R_1(X)$	$R_2(Y)$	D_i	D_i^2
15	50	7	3	4	16
20	30	5.5	5	0.5	0.25
27	55	4	2	2	4
13	30	8	5	3	9
45	25	3	7	-4	16
60	10	2	8	-6	36
20	30	5.5	5	0.5	0.25
75	70	1	1	0	0
					81.5

$$r_{xy} = 1 - \frac{6\Sigma D_i^2}{n(n^2-1)}$$

$$= 1 - \frac{6(81.5)}{8(64-1)}$$

$$= 0.0298$$

Page No.	
Date	

7) If the coefficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}(3/5)$. Show that $\sigma_x = \sigma_y/2$.

$$\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \cdot \frac{1 - r^2}{r} = \tan\theta$$

$$\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \cdot \frac{0.75}{0.5} = \frac{3}{5}$$

$$5\sigma_x \sigma_y = 2\sigma_x^2 + 2\sigma_y^2$$

$$2\sigma_x^2 - 5\sigma_x \sigma_y + 2\sigma_y^2 = 0$$

$$(2\sigma_x - \sigma_y)(\sigma_x - 2\sigma_y) = 0$$

$$\boxed{\sigma_x = \frac{\sigma_y}{2}}$$

Hence proved.

Page No.
Date

8) For two random variables, x and y with same mean, the two regression equations are $y = ax + b$ & $x = \alpha y + \beta$. Show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$. Also find Common Mean.

$$\bar{x} = \bar{y}$$

$$\bar{y} = a\bar{x} + b$$

$$\bar{x} = \alpha\bar{y} + \beta$$

$$\bar{y} = \frac{b}{1-a}$$

$$\bar{x} = \frac{\beta}{1-\alpha}$$

On equating —

$$\bar{x} = \frac{b}{1-a} = \bar{y} \Rightarrow \text{Common Mean}$$

$$\bar{y} = \bar{x}$$

$$\frac{b}{1-a} = \frac{\beta}{1-\alpha}$$

$$\boxed{\frac{b}{\beta} = \frac{1-a}{1-\alpha}}$$

Hence proved.

Page No.	
Date	

10) Two lines of regression are given by $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$ and $var(x) = 16$. Calcule —

(i) The mean of x and y.

(ii) Varience of y

(iii) The correlation coefficient.

(i)
$$3\bar{x} + 2\bar{y} - 26 = 0$$
$$6\bar{x} + \bar{y} - 31 = 0$$
$$\bar{x} = 4$$
$$\bar{y} = 7$$

(iii)
$$y = \frac{26}{2} - \frac{3}{2}x \Rightarrow y = 13 - \frac{3}{2}x$$
$$b_{yx} = -\frac{3}{2}$$

$$6x + y - 31 = 0$$
$$\Rightarrow x = \frac{31}{6} - \frac{y}{6} \Rightarrow b_{xy} = -\frac{1}{6}$$

$$r = \pm\sqrt{b_{yx} \cdot b_{xy}}$$
$$= \pm\sqrt{\frac{-\cancel{3}}{2} \cdot \frac{-1}{\cancel{6}\,2}} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$
$$= \pm 0.5$$

∵ Both b_{xy} & b_{yx} is $-ve$.

So, $\boxed{r = -0.5}$

(ii) $Var(x) = 16$, $\sigma_x = 4$

$$b_{xy} = -0.5 \times \frac{4}{\sigma_y} \Rightarrow -\frac{1}{6} = -0.5 \times \frac{4}{\sigma_y}$$

Page No.
Date

$$\sigma_y = 0.5 \times 24$$
$$\sigma_y = 12$$

10) Find the multiple linear regressions of x on y and z from the data relating to three variables :

x	4	6	7	9	13	15
y	15	12	8	6	4	3
z	30	24	20	14	10	4

x	y	z	x^2	y^2	xy	yz	xz
4	15	30	16	225	60	450	120
6	12	24	36	144	72	288	144
7	8	20	49	64	56	160	140
9	6	14	81	36	54	84	126
13	4	10	169	16	52	40	130
15	3	4	225	9	45	12	60
54	48	102	576	494	339	1034	720

$$z = a + bx + cy$$

$\Rightarrow \Sigma z = na + b\Sigma x + c\Sigma y$

$102 = 6a + 54b + 48c$

$\Rightarrow \Sigma xz = a\Sigma x + b\Sigma x^2 + c\Sigma xy$

$720 = 54a + 576b + 339c$

$\Sigma zy = a\Sigma y + b\Sigma xy + c\Sigma y^2$

$1034 = 48a + 339b + 494c$

On solving —

$a = 685.44$

$b = -43.4100$

$c = -34.7194$

$\Rightarrow Z = 685.44 - 43.41x - 34.7194y$