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Assignment - 1

Ans 1. $T(n) = T(n-1) + n^4$

$T(n-1) = T(n-2) + (n-1)^4$

$T(n-2) = T(n-3) + (n-2)^4$

$T(n) = T(n-2) + (n-1)^4 + n^4$

$= T(n-3) + (n-2)^4 + (n-1)^4 + n^4$

$n-k = 1$

$k = n-1$

$T = T(n-k) + (n-(k-1))^4 + (n-(k-2))^4 + \dots$

$= T(1) + 2^4 + 3^4 + \dots + n^4$

$= 1^4 + 2^4 + 3^4 + \dots + n^4$

$= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

$T(n) = \Theta(n^5)$

An 2. $\left(\frac{1}{3}\right)^n < n^{1/\log n} < \log^2 n = \log(\log n) <$

$(\sqrt{2})^{\log n} < 2^{\log n} < \log n! < \left(\frac{3}{2}\right)^n$

An 3. $T(n) = 3T(n/2) + n$

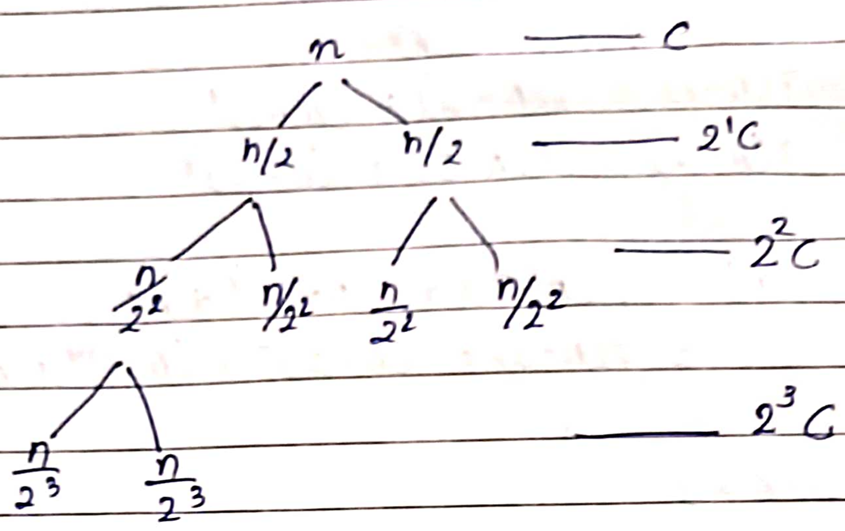
$a = 3, \quad b = 2 \quad k = 1 \quad P = 0$

$b^k = 2^1 = 2$

$a > b^k$ as $3 > 2$

$T(n) = \Theta(n^{\log_2 3})$

Ans4- $T(n) = 2T(n/2) + c$



$$\frac{n}{2^k} \quad \frac{n}{2^k} = 1$$

$$2^0c + 2^1c + 2^2c + 2^3c + \cdots + 2^kc$$

$$c(2^0 + 2^1 + 2^2 + 2^3 + \cdots + 2^k)$$

$$c \times 1 \left[\frac{2^{k+1} - 1}{2 - 1}\right]$$

$$= 2^{k+1} - 1 = 2^{k+1} = 2^k$$

$$\theta(2^k) = \theta(2^{\log_2 n}) = \theta(n)$$

Ans5- $T(n) = 4T(n/2) + n$

$T(n) = 4T(n/2) + n$

$a = 4, \quad b = 2, \quad k = 1, \quad p = 0$

$b^k = 2^1$ as $4 > 2$

$T(n) = \theta(n^{\log_2 4}) = \theta(n^{2 \log_2 2})$

$= \theta(n^2)$

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An6- $T(n) = 2T(\sqrt{n}) + 1$

Let $n = 2^m$ $\quad \log n = m$

$T(2^m) = 2T(2^{m/2}) + 1$

$S(m) = T(2^m)$

$= 2T(2^{m/2}) + 1$

$= 2S(m/2) + 1$

$a = 2 \quad b = 2 \quad k = 0 \quad p = 0$

$b^k = 2^0 = 1$

$a > b^k$

$S(m) = \theta(m^{\log_2 2}) = \theta(m)$

$T(n) = T(2^m) = S(m) = \theta(m) = \theta(\log n)$

An7- $T(n) = T(n-2) + 2\log n$

$T(n-1) = T(n-3) + 2\log(n-1)$

$T(n-2) = T(n-4) + 2\log(n-2)$

$T(n-4) = T(n-6) + 2\log(n-4)$

$\rightarrow T(n) = T(n-4) + 2\log(n-2) + 2\log n$

$= T(n-6) + 2\log(n-4) + 2\log(n-2) + 2\log n$

$= T(n-2\times3) + 2(\log(n-2\times2 + \log(n-2n) + \log(n-2\times0)$

$\vdots$

$= T(n-2k) + 2[\log(n-2(k-1)) + \log(n-2(k-2)) + \log(n-2\times0)]$

$$= 1 + 2[\log 3 + \log 5 + \cdots + \log n]$$

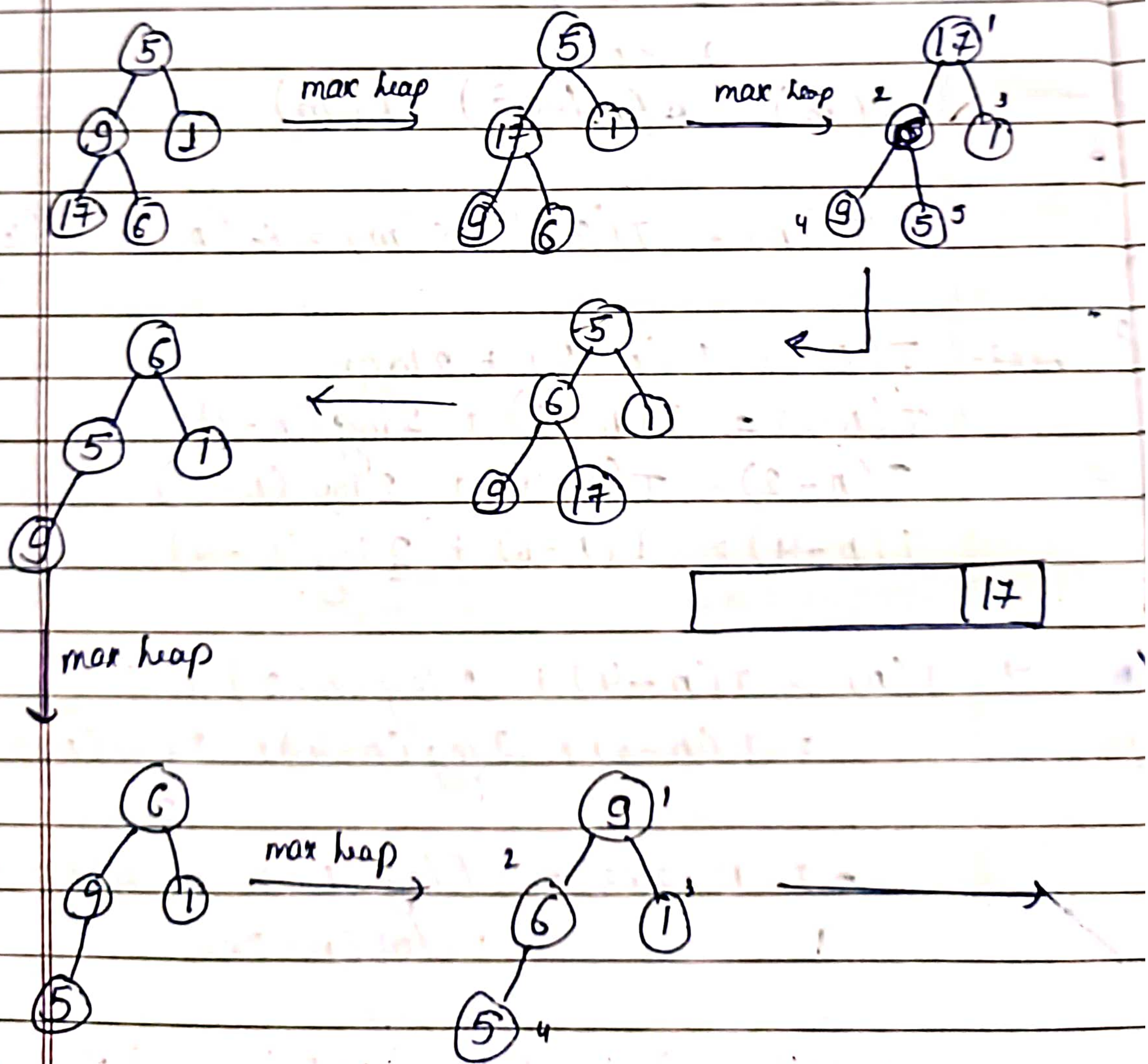
$$= 1 + 2[\log(3 \times 5 \times 7 \times \cdots \times n)]$$

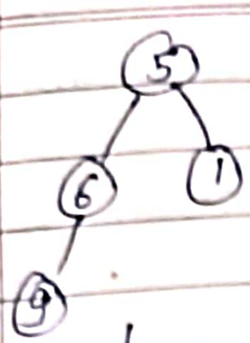
$$= 1 + 2[\log(1 \times 3 \times 5 \times 7 \cdots \times n)]$$

$$= 1 + 2\left[\frac{n!}{2^{n/2}(n/2)!}\right]$$

$$= \Theta(n!)$$

Ans 8 - 5, 9, 1, 17, 6





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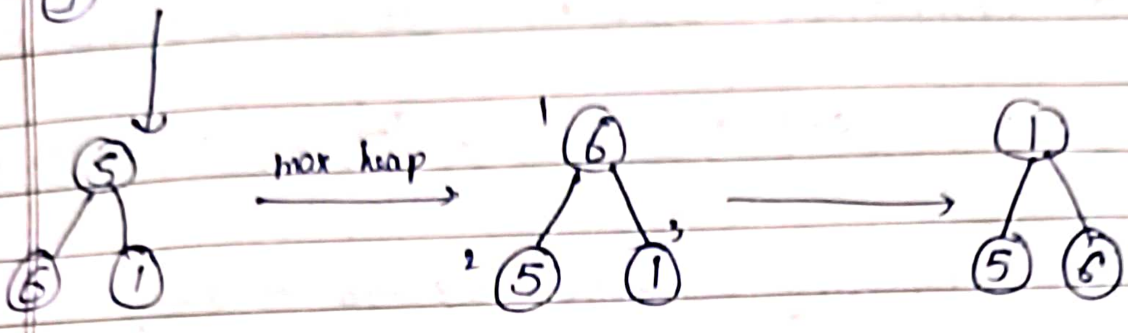

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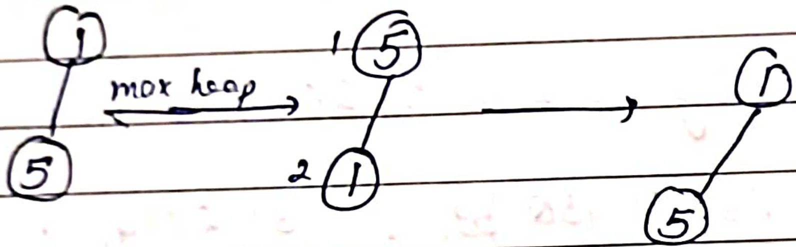

	5	6	9	17

1 →

1	5	6	9	17

Ans. A

31	41	59	26	41	58
1	2	3	4	5	6

$i \leftarrow 1$ $\qquad j \leftarrow 2$

2) $A[i+1] \leftarrow A[i]$

$i \leftarrow i-1$

$A[i+1] \leftarrow key$

31	41	59	26	41	58
1	2	3	4	5	6

* Comparing 59 & 26 ∵ 59 > 26 ⇒ swap 59 & 26

| 31 | 41 | 59 | 26 | 41 | 58 | ⇒ | 31 | 41 | 26 | 59 | 41 | 58 |

* Comparing 59 & 41 ∵ 59 > 41 ⇒ swap 59 & 41

| 31 | 41 | 26 | 59 | 41 | 58 | ⇒ | 31 | 41 | 26 | 41 | 59 | 58 |

* Comparing 59 & 58 ∵ 59 > 58 ⇒ swap 59 & 58

| 31 | 41 | 26 | 41 | 59 | 58 | ⇒ | 31 | 41 | 26 | 41 | 58 | 59 |

* Comparing 26 & 41 ∵ 26 < 41 ⇒ swap 26 & 41

| 31 | 41 | 26 | 41 | 58 | 59 | ⇒ | 31 | 26 | 41 | 41 | 58 | 59 |

* Comparing 31 & 26 ∵ 31 < 26 ⇒ swap 31 & 26

| 31 | 26 | 41 | 41 | 58 | 59 | ⇒ | 26 | 31 | 41 | 41 | 58 | 59 |

<u>Ans</u> - * Time complexity of Counting sort -

$O(n + k)$

where

n → no. of element in array

k → rang of i/p

* It has linear time complexity

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A

1	2	3	4	5	6	7	8	9	10
1	8	3	3	4	5	6	7	7	8

(i) in A;

max element = 8

∴ $\boxed{8}$

1	2	3	4	5	6	7	8	9	10
1	8	3	3	4	5	6	7	7	8

(ii) max element = 8

=) we initialize an array C of length (max + 1) with all elements '0'

C

0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0

(iii) Store the count of each element as their respective index in count array.

A

1	2	3	4	5	6	7	8	9	10
1	8	3	3	4	5	6	7	7	8

C

0	1	2	3	4	5	6	7	8	9
0	1	0	2	1	1	1	2	1	1

(iv) ~~Store the sum.~~

C

0	1	2	3	4	5	6	7	8	9
0	1	1	3	4	5	6	8	9	10

(v) Now create an output array of same size of input array.

B

0	1	2	3	4	5	6	7	8	9

(vi) ~~Aloto~~ $B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

A

1	2	3	4	5	6	7	8	9	10
1	9	3	3	4	5	6	7	7	8

C

0	1	2	3	4	5	6	7	8	9
0	1	1	3	4	5	6	8	9	10

B

0	1	2	3	4	5	6	7	8	9
1	3	3	4	5	6	7	7	8	9