Noida Institute of Engineering and Technology, Greater Noida

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Estd.2001
AN AUTONOMOUS INSTITUTE

Numerical Techniques

Unit: IV

Subject Name & Sub.Code
Eng. Maths III (AAS0301A)

B.Tech CSE 3rd Sem

Dr. Ritika Saini
Department of Mathematics

• Faculty Introduction

Name: Dr. Ritika Saini
Qualification: M. Sc., M. Phill., Ph.D.(Mathematics)
Experience: More than 5 years of experience
Subject Taught: Statistics and Probability, Engineering Mathematics, Linear Algebra, Operations Research

Evaluation Scheme

NOIDA INSTITUTE OF ENGINEERING & TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute)

B. TECH (CSE)
EVALUATION SCHEME
SEMESTER-III

Sl. No.	Subject Codes	Subject Name	Periods			Evaluation Schemes				End Semester		Total	Credit
			L	T	P	CT	TA	TOTAL	PS	TE	PE		
WEEKS COMPULSORY INDUCTION PROGRAM													
1	AAS0301A	Engineering Mathematics III	3	1	0	30	20	50		100		150	4
2	ACSE0304	Discrete Structures	3	0	0	30	20	50		100		150	3
3	ACSE0306	Digital Logic & Circuit Design	3	0	0	30	20	50		100		150	3
4	ACSE0301	Data Structures	3	1	0	30	20	50		100		150	4
5	ACSE0302	Object Oriented Techniques using Java	3	0	0	30	20	50		100		150	3
6	ACSE0305	Computer Organization & Architecture	3	0	0	30	20	50		100		150	3
7	ACSE0353	Digital Logic & Circuit Design Lab	0	0	2				25		25	50	1
8	ACSE0351	Data Structures Lab	0	0	2				25		25	50	1
9	ACSE0352	Object Oriented Techniques using Java Lab	0	0	2				25		25	50	1
10	ACSE0354	Internship Assessment-I	0	0	2				50			50	1
11	ANC0301 / ANC0302	Cyber Security*/ Environmental Science*(Non Credit)	2	0	0	30	20	50		50		100	0
12		MOOCs (For B.Tech. Hons. Degree)											
		GRAND TOTAL										**1100**	**24**

Syllabus

Unit-1 (Complex Variable: Differentiation)

Limit, Continuity and differentiability, Functions of complex variable, Analytic functions, Cauchy- Riemann equations (Cartesian and Polar form), Harmonic function, Method to find Analytic functions, Conformal mapping, Mobius transformation and their properties.

Unit-2 (Complex Variable: Integration)

Complex integrals, Contour integrals, Cauchy- Goursat theorem, Cauchy integral formula, Taylor's Series, Laurent series, Liouville's Theorem, Singularities, zero of analytic function, Residues, Method of finding residues, Cauchy Residue's theorem, Evaluation of real integral of the type $\int_0^{2\pi} f(\sin\theta, \cos\theta) d\theta$ and $\int_{-\infty}^{\infty} f(x)\, dx$

Syllabus

Unit-3 (Partial Differential Equation and its Applications)

Introduction of partial differential equations, Second order linear partial differential equations with constant coefficients. Classification of second order partial differential equations, Method of separation of variables for solving partial differential equations, Solution of one and two dimensional wave and heat conduction equations.

Unit-4 (Numerical Techniques)

Error analysis, Zeroes of transcendental and polynomial equations using Bisection method, Regula-falsi method and Newton-Raphson method, Interpolation: Finite differences, Newton's forward and backward interpolation, Lagrange's and Newton's divided difference formula for unequal intervals. Solution of system of linear equations, Crout's method, Gauss- Seidel method. Numerical integration: Trapezoidal rule, Simpson's one third and three-eight rules, Solution of 1^{st} order ordinary differential equations by fourth-order Runge- Kutta methods.

Syllabus

Unit-5 (Aptitude-III)

Time & Work, Pipe & Cistern, Time, Speed & Distance, Boat & Stream, Sitting Arrangement, Clock & Calendar.

Branch wise Application

- Digital signal processing
- Computer graphics

Course Objective

The objective of this course is to familiarize the engineers with concept of function of complex variables, Partial differential equations & their applications, Numerical techniques for various mathematical tasks and numerical aptitude. It aims to show case the students with standard concepts and tools from B. Tech to deal with advanced level of mathematics and applications that would be essential for their disciplines. The students will learn:

- The idea of function of complex variables and analytic functions.
- The idea of concepts of complex functions for evaluation of definite integrals
- The concept of partial differential equation to solve partial differential and its applications.
- The concept of finding roots by numerical method, interpolation and numerical methods for system of linear equations, definite integral and 1st order ordinary differential equations.
- The concept of problems based on Time & Work, Pipe & Cistern, Time, Speed & Distance, Boat & Stream, Sitting Arrangement, Clock & Calendar.

Course Outcome(COs)

CO1: Apply the working methods of complex functions for finding analytic functions.

CO2: Apply the concepts of complex functions for finding Taylor's series, Laurent's series and evaluation of definite integrals.

CO3: Apply the concept of partial differential equation to solve partial differential Equations and problems concerned with partial differential equations

CO4: Apply the concept of numerical techniques to evaluate the zeroes of the Equation, concept of interpolation and numerical methods for various mathematical operations and tasks, such as integration, the solution of linear system of equations and the solution of differential equation.

CO5: Solve the problems of Time & Work, Pipe & Cistern, Time, Speed & Distance, Boat & Stream, Sitting Arrangement , Clock & Calendar.

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Program Outcomes(POs)

S.No	Program Outcomes (POs)
PO 1	Engineering Knowledge
PO 2	Problem Analysis
PO 3	Design/Development of Solutions
PO 4	Conduct Investigations of Complex Problems
PO 5	Modern Tool Usage
PO 6	The Engineer & Society
PO 7	Environment and Sustainability
PO 8	Ethics
PO 9	Individual & Team Work
PO 10	Communication
PO 11	Project Management & Finance
PO 12	Lifelong Learning

CO-PO Mapping(CO4)

Sr. No	Course Outcome	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
1	CO1	H	H	H	H	L	L	L	L	L	L	L	M
2	CO2	H	H	H	H	L	L	L	L	L	L	M	M
3	CO3	H	H	H	H	L	L	L	L	L	L	M	M
4	CO4	H	H	H	H	L	L	L	L	L	L	L	M
5	CO5	H	H	H	H	L	L	L	L	L	L	M	M

*L= Low *M= Medium *H= High

PSO

PSO	Program Specific Outcomes (PSOs)
PSO 1	The ability to identify, analyze real world problems and design their ethical solutions using artificial intelligence, robotics, virtual/augmented reality, data analytics, block chain technology, and cloud computing.
PSO 2	The ability to design and develop the hardware sensor devices and related interfacing software systems for solving complex engineering problems.
PSO 3	:The ability to understand inter disciplinary computing techniques and to apply them in the design of advanced computing.

CO-PSO Mapping(CO4)

CO	PSO 1	PSO 2	PSO 3
CO1	H	L	M
CO2	L	M	L
CO3	M	M	M
CO4	H	M	M
CO5	H	M	M

*L= Low *M= Medium *H= High

Program Educational Objectives(PEOs)

PEO-1: To have an excellent scientific and engineering breadth so as to comprehend, analyze, design and provide sustainable solutions for real-life problems using state-of-the-art technologies.

PEO-2: To have a successful career in industries, to pursue higher studies or to support entrepreneurial endeavors and to face the global challenges.

PEO-3: To have an effective communication skills, professional attitude, ethical values and a desire to learn specific knowledge in emerging trends, technologies for research, innovation and product development and contribution to society.

PEO-4: To have life-long learning for up-skilling and re-skilling for successful professional career as engineer, scientist, entrepreneur and bureaucrat for betterment of society.

Result Analysis

End Semester Question Paper

Prerequisite and Recap(CO4)

- Knowledge of Maths 1 B.Tech.
- Knowledge of Maths 2 B.Tech

Brief Introduction about the subject with videos

- We will discuss properties of complex function (limits, continuity, differentiability, Analytic, integration)
- In 3rd module we will discuss application of partial differential equations
- In 4th module we will discuss numerical methods for solving algebraic equations, system of linear equations, definite integral and 1st order ordinary differential equation.
- In 5th module we will discuss aptitude part.

Unit Content

- Zeroes of transcendental and algebraic equations: Bisection method, Regula-Falsi method, Newton-Raphson method,
- Interpolation: Finite differences
- Newton’s forward and backward interpolation
- Lagrange’s and Newton’s divided difference formula for unequal intervals.
- Solution of system of linear equations: Crout’s Method, Gauss Seidel Method
- Numerical integration: Trapezoidal Rule, Simpson’s one third and Three-eight rule
- Solution of first order ordinary differential equations by fourth-order Runge- Kutta methods.

Module Objective(CO4)

- The objective of this module is to familiarize the engineers with numerical techniques and its applications.

Prerequisite and Recap

- Roots of an equation
- Polynomials
- Non-linear equations

Objective of Numerical solution of Algebraic & Transcendental equations[CO4]

There are lots of problems in science & engineering which we cannot solve directly using method which gives the exact solutions. To find approximate solution we solve these problems numerically by numerical methods.

Here we will learn how we predict a root of an equation and how can we find it numerically using iterative methods.

Zeroes of transcendental and algebraic equations[CO4]

Algebraic equation: An equation $f(x) = 0$ is called algebraic equation if $f(x)$ is polynomial in x. i.e.

$a_0 + a_1x + a_2x^2 + \cdots\cdots + a_nx^n + \cdots\cdots = 0$ Here $a_i \in \mathbb{R}$.

For e.g-

1. $x^3 - 3x + 1 = 0$.

Transcendental equation: An equation $f(x) = 0$ is called transcendental equation if $f(x)$ is some combination of algebraic function $(\mathrm{e.g} - x, x^2)$,trigonometric function, logarithmic function, exponential function.

For e.g-

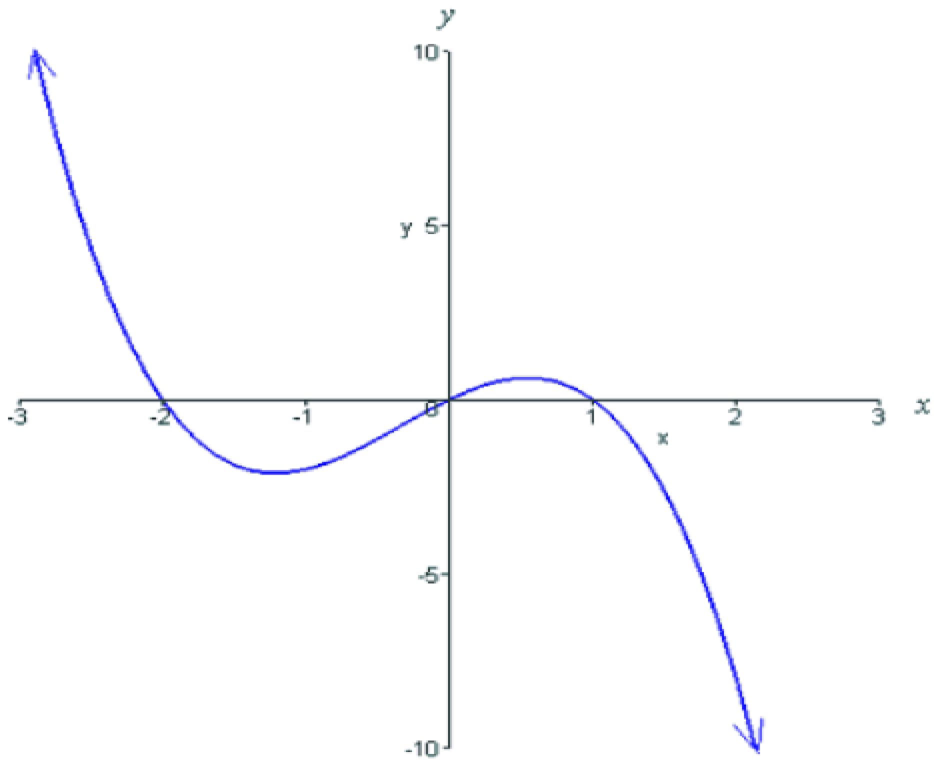
1.$xsinx - x = 0$

2. $xe^x - 1 = 0$

3. $xlog_{10}x - 1.2 = 0$

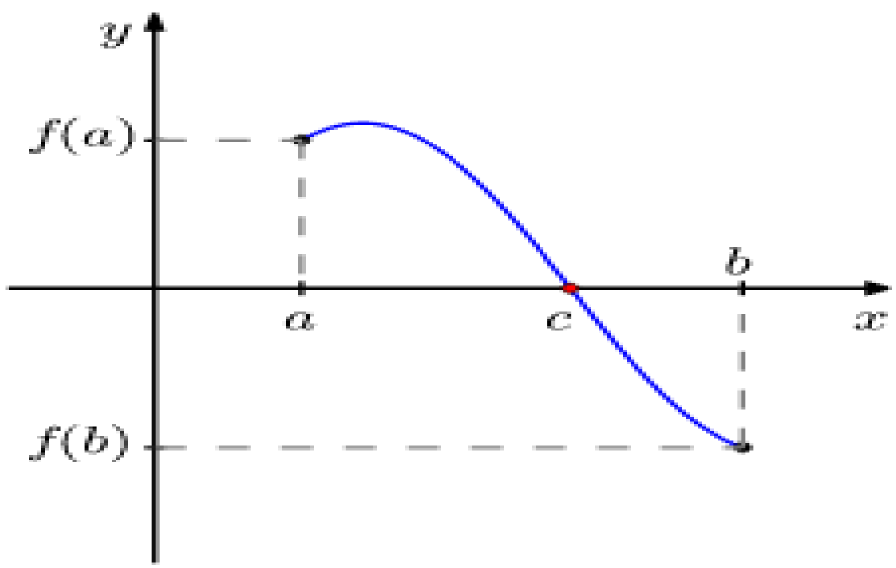
Zeroes of transcendental and polynomial equations[CO4]

- **Root/Zero**- A point $x = a$ will be called a root of an equation $f(x) = 0$ if $f(a) = 0$. Graphically, we can say where it cuts the x-axis.



Zeroes of transcendental and polynomial equations[CO4]

- To find the root of a equation numerically we find initial approximation by intermediate value theorem
- **Intermediate value theorem-**If $f(x)$ is continuous on $[a, b]$ and $f(a)$ and $f(b)$ are of opposite sign i.e. $f(a).f(b) < 0$.Then $f(x)$ has at least one root or odd number of roots on that interval.



Iterative method for finding root of an equation[CO4]

- Bisection method
- Regula-falsi method/method of false position
- Newton Raphson method

Bisection method[CO4]

Bisection method:

This method is used to find the roots of an equation $f(x) = 0$.

In this method we use the following steps-

Step-1 Find the interval $[a, b]$ using intermediate value theorem i.e. $f(a).f(b) < 0$.

Step-2 Find first approximation as $x_1 = \frac{a+b}{2}$ then$f(x_1) = +$ive/$-$ive.

If $f(x_1) = 0$ then x_1 is root of an equation.

Step-3 Now root lie either $[a, x_1]$ or $[x_1, b]$.To check this we use step-1.

If root lie in $[a, x_1]$ Then we find second approximation as- $x_2 = \frac{a+x_1}{2}$

If root lie in $[x_1, b]$ Then we find second approximation as- $x_2 = \frac{x_1+b}{2}$

Similarly we can find third aprroximation,fourth approximation and so on using step-1 and step-2.

Step-4 We stop iteration when the last two consecutive iterations are nearly equal or same.

Bisection method[CO4]

Example-1: Find a positive real root of $x^3 - x - 1 = 0$ in [1,2] by bisection method, correct to four decimal places.

Sol: $f(x) = x^3 - x - 1,$

$f(1) = -\text{ive}$

$f(2) = +\text{ive}$

$f(1.324) = -\text{ive}$

$f(1.325) = +\text{ive}$

Hence root lie in [1.324, 1.325].

First approximation $x_1 = \dfrac{1.324 + 1.325}{2} = 1.3245, f(1.3245) = -\text{ive}$

Now root lie in [1.3245, 1.325].

Second approximation x_2=1.32475 $f(1.32475) = +\text{ive}$

Now root lie in [1.3245, 1.32475].

Third approximation $x_3 = 1.324625$

Now we stop iteration because second and third approximation are same upto 3 decimal places.

So root is 1.324

Regula Falsi method/Method of false position[CO4]

Regula Falsi method: This method is used to find the roots of an equation $f(x) = 0$. This method has following steps-

Step-1 Find the interval $[a, b]$ using intermediate value theorem i.e. $f(a).f(b) < 0$.

Step-2 Find first approximation as : $x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)}$ then

$f(x_1) = +\text{ive}/-\text{ive}$. If $f(x_1) = 0$ then x_1 is root of an equation.

Step-3 Now root lie either $[a, x_1]$ or $[x_1, b]$.To check this we use step-1.If root lie in $[a, x_1]$.Then we find second approximation as:

$$x_2 = \frac{af(x_1) - x_1 f(a)}{f(x_1) - f(a)}$$

If root lie in $[x_1, b]$. Then we find second approximation as-

$$x_2 = \frac{x_1 f(b) - bf(x_1)}{f(b) - f(x_1)}$$

Similarly we can find third aprroximation,fourth approximation and so on.

Step-4 We stop iteration when the last two consecutive iterations are nearly equal or same.

Regula Falsi method[CO4]

Example-1: Find the root of $xe^x - cosx = 0$ in [0,1] by Regula falsi method, correct to four decimal places.

Sol: $f(x) = xe^x - cosx$

$f(0) = -\text{ive}$

$f(1) = +\text{ive}$

$f(0.51) = -0.02344$

$f(0.52) = 0.00683$

Hence root lie in [0.51, 0.52]

First approximation $x_1 = \frac{0.51f(0.52)-0.52f(0.51)}{f(0.52)-f(0.51)} = 0.517744$

$f(0.517744) = -0.000041$

Using IVT, root lie in [0.517744, 0.52]

Second approximation $x_2 = 0.517757$

Now we stop iteration because first and second approximation are same upto 4 decimal places.

So root is 0.5177

Bisection method and Regula Falsi method[CO4]

Q1: Find a positive real root of $xlog_{10}x - 1.2 = 0$ by bisection method, correct to three decimal places.

Q2: Find a positive real root of $xe^{x} - 1 = 0$ by Regula Falsi method, correct to three decimal places.

Prerequisite and Recap

In previous lectures we have discussed-

- Algebraic & transcendental equation
- Bisection method
- Regula falsi method

Newton Raphson method[CO4]

Newton Raphson method: Suppose we have an equation $f(x) = 0$.

To start this method we require an initial approximation x_0. Equation of tangent Passing through the pt.$\left(x_0, f(x_0)\right)$.

$y - f(x_0) = f'(x_0)(x - x_0)$

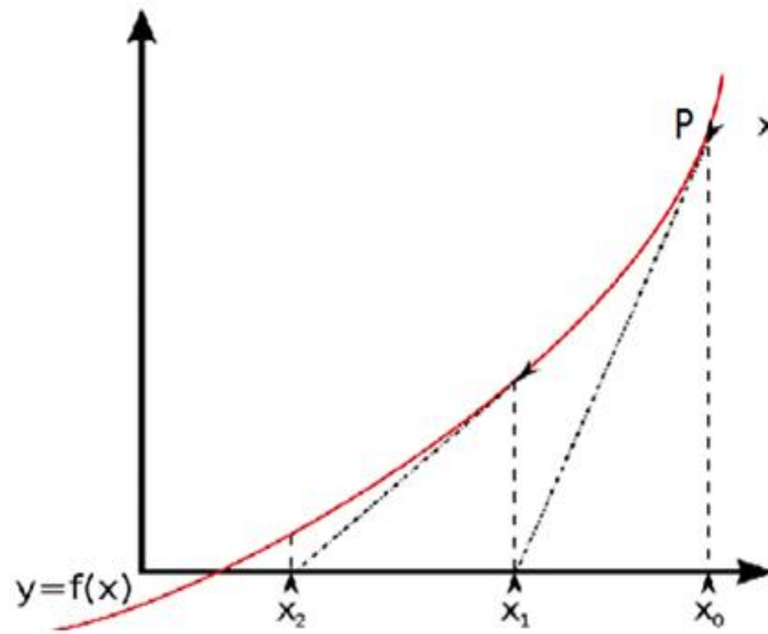
Since it cuts the x-axis so $y = 0$.

So $x = x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

Similarly we can find second, third approximation and so on.

Then general formula is given by-

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$



Newton Raphson method[CO4]

Example-1: Using Newton Raphson method. Find the real root of equation $3x - cosx - 1 = 0$ correct to 4 decimal places.

Sol: $f(x) = 3x - cosx - 1$

$f'(x) = 3 + sinx$

Using Newton Raphson Formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{n+1} = \frac{x_n sinx_n + cosx_n + 1}{3 + sinx_n}$$

Taking $x_0 = 0.6$

First approximation $x_1 = 0.6071$

Second approximation $x_2 = 0.6071$

Now we stop iteration because first and second approximation are same upto 4 decimal places.

So root is 0.6071

Newton Raphson method[CO4]

Example-1: Using Newton Raphson method. Find the real root of equation $x\, log_{10}x - 1.2 = 0$ correct to 4 decimal places.

Sol: $f(x) = x\, log_{10}x - 1.2$

$f'(x) = 0.4342 + log_{10}x$

Using Newton Raphson Formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{n+1} = \frac{0.4342x_n + 1.2}{0.4342 + log_{10}x}$$

Taking $x_0 = 2.5$

First approximation $x_1 = 2.746533007$

Second approximation $x_2 = 2.7406486$

Third approximation $x_3 = 2.740646096$

Now we stop iteration because second and third approximation are same upto 4 decimal places.

So root is 2.7406

Newton Raphson method[CO4]

QUIZ!

Q1 Find a positive real root of $xsinx + cosx = 0$ by Newton Raphson method, correct to three decimal places.

Q2 Find a square root of 12 by Newton Raphson method, correct to 4 decimal places.

Prerequisite and Recap

In previous lectures we have discussed-

- Algebraic and transcendental equation
- Bisection method
- Regula falsi method
- Newton raphson method
- Order of convergence

Objective of Interpolation

- We will learn the process of deriving a simple function from a set of discrete data points so that the function passes through all the given data points (i.e. reproduces the data points exactly) and can be used to estimate data points in-between the given ones.

Interpolation[CO4]

Interpolation: According to Theile, "Interpolation is the art of reading between the lines of table. "It also means insertion or filling up intermediate terms of the series.

Suppose we are giving the following values of $y = f(x)$ for a set of values of x.

x:	x_0	x_1	x_2	..............x_n
y:	y_0	y_1	y_2	..............y_n

Thus, the process of finding the values of y corresponding to any value of $x = x_i$ between x_0 and x_n is called Interpolation.

Here x is called argument and y is called entry.

Finite differences[CO4]

Finite differences:

Suppose we are giving the following values of $y = f(x)$ for a set of values of x.

x: $\quad x_0 \quad x_0 + h \quad x_0 + 2h \ldots\ldots\ldots\ldots\ x_0 + nh$

y: $\quad y_0 \quad y_1 \quad y_2 \quad \ldots\ldots\ldots\ldots \quad y_n$

To determine y for any intermediate value of x, two types of differences are useful-

1. Forward difference- It is denoted by Δ and operate on the value of y.

First forward difference $\Delta y_0 = y_1 - y_0$

$\Delta y_1 = y_2 - y_1, \ldots\ldots\ldots\ldots\ldots\ \Delta y_n = y_n - y_{n-1}$

Second forward difference $\Delta^2 y_0 = \Delta y_1 - \Delta y_0$

$\Delta^2 y_1 = \Delta y_2 - \Delta y_1, \ldots\ldots\ldots\ldots\ldots\ \Delta^2 y_{n-1} = \Delta y_n - \Delta y_{n-1}$

Similarly we can find 3rd ,4th and n^{th} forward differences.

Finite differences[CO4]

2. Backward difference- It is denoted by ∇ and operate on the value of y.

First backward difference $\nabla y_n = y_n - y_{n-1}$

$\nabla y_{n-1} = y_{n-1} - y_{n-2}, \ldots\ldots\ldots\ldots\ldots\ldots \nabla y_1 = y_1 - y_0$

Second forward difference $\nabla^2 y_n = \nabla y_n - \nabla y_{n-1}$

$\nabla^2 y_{n-1} = \nabla y_{n-1} - \nabla y_{n-2}, \ldots\ldots\ldots\ldots\ldots\ldots \nabla^2 y_2 = \nabla y_2 - \nabla y_1$

Similarly we can find 3^{rd} ,4^{th} and n^{th} backward differences.

Newton forward interpolation formula[CO4]

Newton forward interpolation formula: Suppose we are giving the following values of $y = f(x)$ for a set of values of x.

x:	x_0	$x_0 + h$	$x_0 + 2h$	....	x_0+nh
y:	y_0	y_1	y_2	....	y_n

Suppose we have to find y at $x = x_0 + ph$

From here $p = \frac{x-x_0}{h}$

Then y is given by

$$\boxed{y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \ldots\ldots + \frac{p(p-1)\ldots\ldots(p-n+1)}{n!}\Delta^n y_0}$$

Newton forward interpolation formula[CO4]

Example-1: The population of a town was as given below. Estimate the population for the year 1895

Year x:	1891	1901	1911	1921	1931
Population y:	46	66	81	93	101

Sol: Here $x_0 = 1891,$

$$h = 10,$$

$$x = 1895,$$

$$p = \frac{x - x_0}{h} = 0.4$$

Construct the forward difference table-

Newton forward interpolation formula[CO4]

(Forward Difference Table)

x	$y=f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

Newton forward interpolation formula[CO4]

Using Newton forward interpolation formula-

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$y = 54.85$$

Newton forward interpolation formula[CO4]

Example-2: From the table, estimate the number of students who obtained marks between 40 and 45.

Marks :	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

Sol:

Less than marks(x) :	40	50	60	70	80
No. of students(y):	31	73	124	159	190

Now we calculate y at $x = 45$

Constructing forward difference table-

Newton forward interpolation formula[CO4]

(Forward Difference Table)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

Newton forward interpolation formula[CO4]

Here $x_0 = 40,$
$h = 10,$
$x = 45,$
$p = \dfrac{x - x_0}{h} = 0.5$

Using Newton forward interpolation formula-

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$y = 47.86$

Hence no. of students getting less than 45$= 48$

No. of students getting less than 40$= 31$

No. of students getting marks between 40 and 45$= 48 - 31 = 17$

Newton forward interpolation formula[CO4]

Example-3 find the missing term in the following table-

x	y
1	7
2	?
3	13
4	21
5	37

Sol: Constructing forward difference table-

Newton forward interpolation formula[CO4]

(Forward Difference Table)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	7				
		$x-7$			
2	x		$20-2x$		
		$13-x$		$3x-25$	
3	13		$x-5$		$38-4x$
		8		$13-x$	
4	21		8		
		16			
5	37				

Newton forward interpolation formula[CO4]

Since we have given known value of y at 4 points so degree of the polynomial is ≤ 3.

So $\Delta^4 y = 0$

$\Rightarrow 38 - 4x = 0$

$\Rightarrow x = 9.5$

Newton Backward interpolation formula[CO4]

Newton Backward interpolation formula: Suppose we are giving the following values of $y = f(x)$ for a set of values of x.

x:	x_0	$x_0 + h$	$x_0 + 2h$	....	x_0+nh
y:	y_0	y_1	y_2	....	y_n

Suppose we have to find y at $x = x_n + ph$

Here $p = \frac{x-x_n}{h}$

Then y is given by

$$\boxed{y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \ldots\ldots + \frac{p(p+1)\ldots\ldots(p+n-1)}{n!}\nabla^2 y_n}$$

Newton Backward interpolation formula[CO4]

Example-1: The population of a town was as given below. Estimate the population for the year 1925

Year x:	1891	1901	1911	1921	1931
Population y:	46	66	81	93	101

Sol: Here $x_n = 1931$

$$h = 10$$

$$x = 1925$$

$$p = \frac{x - x_n}{h} = -0.6$$

Construct the backward difference table-

Newton Backward interpolation formula[CO4]

(Backward Difference Table)

x	$y = f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

Newton Backward interpolation formula[CO4]

Using Newton Backward interpolation formula-

$$y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$$

$$y = 96.83$$

Newton Forward & Backward interpolation formula[CO4]

Q1 : From the table, estimate the number of students who obtained marks between 36 and 45.

Marks :	30-40	40-50	50-60	60-70	70-80
No. of students:	25	35	22	11	7

Q2 : find the cubic polynomial which takes the following values.

x:	0	1	2	3
y:	1	2	1	10

Prerequisite and Recap

In previous lectures we have discussed-

- Algebraic and transcendental equation
- Bisection method
- Regula falsi method
- Newton raphson method
- Order of convergence

❖ Interpolation for equal intervals

- Newton Forward interpolation formula
- Newton Backward interpolation formula

Interpolation for unequal intervals[CO4]

- Lagrange’s interpolation formula
- Newton divided interpolation formula

Lagrange's interpolation formula[CO4]

Lagrange's interpolation formula: Suppose we are giving the following values of $y = f(x)$ for a set of values of x.

x:	x_0	x_1	x_2	..............x_n
y:	y_0	y_1	y_2	..............y_n

Here argument x may or may not be equispaced.

Suppose we have to find polynomial y which fits the data exactly is given by-

$$y = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)}y_1 + \cdots\cdots\cdots\cdots\cdots + \frac{(x-x_0)(x-x_1)(x-x_2)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\dots(x_n-x_{n-1})}y_n$$

This is known as Lagrange's interpolation formula.

Lagrange's interpolation formula[CO4]

Example-1: Using Lagrange's interpolation formula, find the cubic polynomial from the following data-

x:	0	1	2	5
y:	2	3	12	147

Sol: Here arguments are not equispaced, Using Lagrange's interpolation formula-

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_2)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

Put all the values we get

$$y = x^3 + x^2 - x + 2$$

Divided Difference[CO4]

Divided Difference: If $(x_0, y_0), (x_1, y_1), (x_2, y_2)$........ are given points.
Then First divided difference for argument x_0, x_1 is define as-

$$[x_0x_1] = \frac{y_1 - y_0}{x_1 - x_0}$$

$$[x_1x_2] = \frac{y_2 - y_1}{x_2 - x_1}$$ and so on

Second divided difference for argument x_0, x_1, x_2 is define as-

$$[x_0x_1x_2] = \frac{[x_1x_2] - [x_0x_1]}{x_2 - x_0}$$

$$[x_1x_2x_3] = \frac{[x_2x_3] - [x_1x_2]}{x_3 - x_1}$$

and so on

Third divided difference for argument x_0, x_1, x_2, x_3 is define as-

$$[x_0x_1x_2x_3] = \frac{[x_1x_2x_3] - [x_0x_1x_2]}{x_3 - x_0}$$

$$[x_1x_2x_3x_4] = \frac{[x_2x_3x_4] - [x_1x_2x_3]}{x_4 - x_1}$$ and so on.

Similarly we can find 4^{th} ,5^{th} divided difference and so on.

Newton divided difference formula[CO4]

Newton divided difference formula: Suppose we are giving the following values of $y = f(x)$ for a set of values of x.

x:	x_0	x_1	x_2	..............x_n
y:	y_0	y_1	y_2	..............y_n

Here arguments x may or may not be equi-spaced.T hen

$$\boxed{\begin{aligned} y = y_0 + (x - x_0)[x_0 x_1] + (x - x_0)(x - x_1)[x_0 x_1 x_2] + \cdots\cdots\cdots \\ +(x - x_0)\cdots(x - x_n)[x_0 x_1 \ldots x_n] \end{aligned}}$$

This is known as Newton divided difference interpolation formula.

Newton divided difference formula[CO4]

Example-1: Using Newton Divided Difference interpolation formula, find the polynomial function from the following data-

x:	-4	-1	0	2	5
y:	1245	33	5	9	1335

Sol:

Constructing divided difference table-

Newton divided difference formula[CO4]

(Divided Difference Table)

x	$y=f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		-404			
-1	33		94		
		-28		-14	
0	5		10		3
		2		13	
2	9		88		
		442			
5	1335				

Newton divided difference formula[CO4]

Applying newton divided difference formula-

$$y = y_0 + (x - x_0)[x_0x_1] + (x - x_0)(x - x_1)[x_0x_1x_2] + (x - x_0)(x - x_1)(x - x_2)[x_0x_1x_2x_3] + (x - x_0)(x - x_1)(x - x_2)(x - x_3)[x_0x_1x_2x_3x_4]$$

We get

$$y = 3x^4 - 5x^3 + 6x^2 - 14x + 5$$

Newton divided difference formula[CO4]

QUIZ!

Q1: Evaluate $y(10)$ using Lagrange's interpolation formula.

x:	5	6	9	11
y:	12	13	14	16

Q2 : Using Newton Divided Difference interpolation formula, find $f(8)$ and $f(15)$ from the following data-

x:	4	5	7	10	11	13
y:	48	100	294	900	1210	2028

Summary

In previous lectures we have discussed-

- Algebraic and transcendental equation
- Bisection method
- Regula falsi method
- Newton raphson method
- Order of convergence

❖ Interpolation for equal intervals

- Newton Forward interpolation formula
- Newton Backward interpolation formula

❖ Interpolation for unequal intervals

- Lagrange's interpolation formula
- Newton divided difference interpolation formula

Weekly Assignment

Q1. Using Newton Raphson Method-find a positive root of equation

$x\log_{10}x = 4.77$ correct to 3 decimal places. **Ans: 6.083**

Q2. Using Regula Falsi Method- find a positive root of equation

$x^4 - x - 10 = 0$ correct to 3 decimal places. **Ans: 1.855**

Q3. Using Bisection Method- find a positive root of equation

$xe^x = cosx$ correct to 4 decimal places. **Ans: 0.5177**

Q4. Determine the missing values in given table-

x	10	15	20	25	30	35
$f(x)$	43	?	29	32	?	77

Ans: 33.933, 46.733

Weekly Assignment

Q5. Find the number of students from the following data who secured marks not more than 45

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	35	48	70	40	22

Ans: 51

Q6. From the following table find the value of y at $x = 8$

x	1	3	5	7	9
y	9	21	81	237	537

Ans: 366

Weekly Assignment

Q7. Obtain Lagrange's interpolatory polynomial for the following data-

x	-1	1	2	3
y	-21	15	12	3

Also find the value of y at $x = 1.5$.

Ans: $\boldsymbol{x^3 - 9x^2 + 17x + 6,\ \ 14.625}$

Q8. Find the interpolating polynomial and hence compute $f(1)$ from the following table-

x	-3	-1	0	3	5
$f(x)$	-30	-22	-12	330	3458

Ans: $\boldsymbol{5x^4 + 9x^3 - 27x^2 - 21x - 12,\ \ -46}$

Objective of solving system of equations by numerical method[CO4]

Applying numerical methods we can solve large system of linear equations.

Solution of system of linear equations[CO4]

Consider a non-homogeneous system of three simultaneous linear algebraic equations in three unknowns as

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \quad \ldots\ldots\ldots\ldots\ldots..(1) \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

where all a_{ij}'s and b_j's are constants.

Using matrix notation, the above system (1) can be written as

$AX = B$ ....(2)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

By finding a solution of the system, we mean to obtain the values of three unknowns x_1, x_2 and x_3 such that they satisfy the given equations

Matrix Decomposition Method [CO4]

- This method is also known as Triangularization method or method of factorization.
- The given system of equations can be written in matrix form as AX=B.......(1)
- Factor A into LU, where L is lower triangular matrix and U is upper triangular matrix.
- Put A=LU in (1), we have LUX=B......(2)
- Let UX=Y....(3) then (2) becomes LY=B.....(4)
- Solve (4) for Y using forward substitution

Matrix Decomposition Method[CO4]

- Putting the value of Y in (3) and solve it for X using back substitution method.

Some important points:

- If we take all the diagonal elements of L=1 then the method is called Doolittle's method.
- If we take all the diagonal elements of U=1 then the method is called Crout's method.
- The method fails if any of the diagonal elements of L or U is zero.

Crout's method [CO4]

Example: Solve, by Crout's method, the following system of equations:

$$\boldsymbol{x+y+z=3,2x-y+3z=16,3x+y-z=-3}$$

Solution:

The above system of equations can be written in matrix form as AX=B...........(1)

Where $A=\begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 3\\ 3 & 1 & -1\end{bmatrix}$ $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}3\\ 16\\ -3\end{bmatrix}$

In Crout's method $A=LU$............(2)

$$\begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 3\\ 3 & 1 & -1\end{bmatrix}=\begin{bmatrix}l_{11} & 0 & 0\\ l_{21} & l_{22} & 0\\ l_{31} & l_{32} & l_{33}\end{bmatrix}\begin{bmatrix}1 & u_{12} & u_{13}\\ 0 & 1 & u_{23}\\ 0 & 0 & 1\end{bmatrix}$$

Crout's method [CO4]

On simplifying, we get

$$L=\begin{bmatrix}1 & 0 & 0\\ 2 & -3 & 0\\ 3 & -2 & \frac{-14}{3}\end{bmatrix}\quad U=\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & \frac{-1}{3}\\ 0 & 0 & 1\end{bmatrix}$$

Put A=LU in (1), we have LUX=B........................(3)

Let UX=Y......(4) so that (3) becomes LY=B...........(5)

So by (5), we have $\begin{bmatrix}1 & 0 & 0\\ 2 & -3 & 0\\ 3 & -2 & \frac{-14}{3}\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}=\begin{bmatrix}3\\ 16\\ -3\end{bmatrix}$.........(6)

Crout's method [CO4]

After solving (6), using forward substitution method, we have

$$Y=\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ \frac{-10}{3} \\ 4 \end{bmatrix}$$

Now, UX=Y

So $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -\frac{10}{3} \\ 4 \end{bmatrix}$.........(7)

After solving (7), using backward substitution method,

we have $x = 1, y = -2$ and $z = 4$.

QUIZ!

Q.1 Solve by Crout's Method

$10x + y + z = 12$

$2x + 10y + z = 13$

$2x + 2y + 10z = 14$

Diagonally Dominant Matrix [CO4]

In mathematics, a matrix is said to be diagonally dominant if for every row of the matrix, the magnitude of the diagonal entry in a row is larger than or equal to the sum of the magnitudes of all the other (non-diagonal) entries in that row.

Ex: The matrix A= $\begin{bmatrix} 4 & 1 & 1 \\ 2 & -6 & 3 \\ 3 & 1 & -4 \end{bmatrix}$ is diagonally dominant matrix because

$|4|>|1|+|1|$

$|-6|>|2|+|3|$

$|-4|=|3|+|1|$.

Gauss-Seidel Method [CO4]

The Gauss-Seidel method is a method for determining the solution of a diagonally dominant system of linear equations.

Consider a non-homogeneous system of three simultaneous linear algebraic equations in three unknowns as-

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \quad \ldots\ldots\ldots\ldots\ldots..(1)$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

where a_{ii}'s is the largest coefficient in the i^{th} equation $(i = 1,2,3)$

System (1) of equations can be written as

$$x = \frac{b_1}{a_{11}} - \frac{a_{12}y}{a_{11}} - \frac{a_{13}z}{a_{11}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$
$$y = \frac{b_2}{a_{22}} - \frac{a_{21}x}{a_{22}} - \frac{a_{23}z}{a_{22}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$
$$z = \frac{b_3}{a_{33}} - \frac{a_{31}x}{a_{33}} - \frac{a_{32}y}{a_{33}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(4)$$

Gauss-Seidel Method [CO4]

Taking the initial approximation $y^{(0)} = z^{(0)} = 0.$

Put the initial approximation in R.H.S. of (2) to get the next approximation for x i.e.

$\frac{b_1}{a_{11}}$

Now put $x^{(1)} = \frac{b_1}{a_{11}}, z^{(0)} = 0$ in (3)

we have $y^{(1)} = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}\frac{b_1}{a_{11}}$

Now put $x^{(1)} = \frac{b_1}{a_{11}}, y^{(1)} = \frac{b_2}{a_{22}}$ - $\frac{a_{21}}{a_{22}}\frac{b_1}{a_{11}}$ in(4)

we have $z^{(0)} = \frac{b_3}{a_{33}} - \frac{a_{31}}{a_{33}}\frac{b_1}{a_{11}} - \frac{a_{32}}{a_{33}}(\frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}\frac{b_1}{a_{11}})$

Repeat this process until we get the desired valued of unknowns (variables x, y, z) upto required accuracy.

Gauss-Seidel Method [CO4]

Example: Solve the following system of equations by Gauss-Seidel method

$$\mathbf{2x + 10y + z = 51}$$

$$\mathbf{10x + y + 2z = 44}$$

$$\mathbf{x + 2y + 10z = 61}$$

Solution: The given system is not diagonally dominant. So, rearrange the equations as follows to make them diagonally dominant:

$$10x + y + 2z = 44$$

$$2x + 10y + z = 51 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

$$x + 2y + 10z = 61$$

$System\ (1)\ of$ equations can be written as

$$x = \frac{1}{10}(44 - y - 2z) \quad \ldots\ldots\ldots\ldots.(2)$$

$$y = \frac{1}{10}(51 - 2x - z) \quad \ldots\ldots\ldots\ldots..(3)$$

$$z = \frac{1}{10}(61 - x - 2y) \quad \ldots\ldots...(4)$$

Gauss-Seidel Method [CO4]

The first approximation is obtained by putting $y = z = 0$ in (2)

$x^{(1)} = 4.4$ Now, putting $x = 4.4$ and $z = 0$ in (3), we get $y^{(1)} = 4.2$

Again putting $x = 4.4, y = 4.2$ in (4), we get $z^{(1)} = -2.9753$

Proceeding in the same way , we get

$x^{(2)}$	3.02
$y^{(2)}$	4.02
$z^{(2)}$	4.99
$x^{(3)}$	2.99
$y^{(3)}$	4.00
$z^{(3)}$	4.99
$x^{(4)}$	2.99~3
$y^{(4)}$	4.00
$z^{(4)}$	4.99~5

Gauss-Seidel Method [CO4]

Proceeding in the same way, we get

Hence, after four iterations, we get

$x = 3, y = 4\ , z = 5$

Gauss-Seidel Method [CO4]

QUIZ!

Q.1 Solve the following system of equations by Gauss-Seidel method(Perform four iteration)

$9x + 4y + z = -17$

$x - 2y - 6z = 14$

$x + 6y = 4$

Q.2 Solve the following system of equations by Gauss-Seidel method.

$20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

Objective of Numerical Integration [CO4]

Using numerical techniques, we can solve definite integral.

Numerical Integration [CO4]

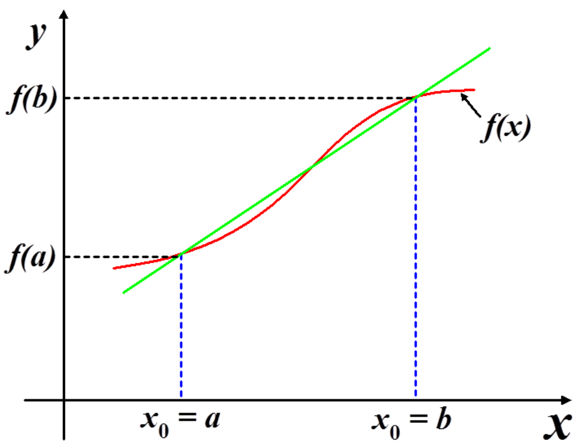
The process of evaluating a definite integral from a set of tabulated values of the integrand $f(x)$ is called numerical integration.

Numerical integration means the numeric evaluation of integral

$$I = \int_a^b f(x)dx \quad ....(1)$$

where a and b are given and f is a function given analytically by a formula or empirically by a table of values.

Geometrically, (1) gives the area under the curve of f between a and b.



Trapezoidal Rule [CO4]

Let $y_0, y_1, y_2, \ldots \ldots . y_n$ be the values of $y = f(x)$ corresponding to $x = x_0 = a, x_1, x_2, \ldots \ldots . x_n = b$, which are equally spaced with interval as h. Then the Trapezoidal rule is given by

$$\mathrm{I} = \int_a^b f(x)dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + \cdots \ldots + y_{n-1}) + y_n] \ldots\ldots(1)$$

Where $h = \frac{(b-a)}{n}$, $n =$ number of subintervals

Trapezoidal Rule [CO4]

Example: Evaluate $\mathbf{I} = \int_0^1 \frac{1}{1+x} dx$ **by Trapezoidal rule (Take** $n = 6$**).**

Solution:

x	0	1/6	2/6	3/6	4/6	5/6	1
$f(x) = \frac{1}{1+x}$	1	0.8571	0.75	0.6666	0.6	0.5454	0.5

Here $h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

Now, by (1)

$$\int_0^1 \frac{1}{1+x} dx = \frac{1}{6 \times 2} [1$$
$$+ 2(0.8571 + 0.75 + 0.6666 + 0.6 + 0.5454) + 0.5]$$
$$= 0.69485$$

Simpson's $\frac{1}{3}$ Rule [CO4]

Let $y_0, y_1, y_2, \dots \dots . y_n$ be the values of $y = f(x)$ corresponding to $x = x_0 = a, x_1, x_2, \dots \dots . x_n = b$, which are equally spaced with interval as h.

Then the Simpson's $\frac{1}{3}$ rule is given by

$$\mathrm{I} = \int_a^b f(x)dx = \frac{h}{3}[y_0 + 2(y_2 + y_4 + \cdots \dots + y_{n-2}) + 4(y_1 + y_3 + \cdots . + y_{n-1}) + y_n]$$

Where $h = \frac{(b-a)}{n}$, $n =$ number of subintervals

To apply this rule, the number of subintervals should be even i.e. multiple of 2.

Simpson's $\frac{1}{3}$ rule may be written as

$$\mathrm{I} = \int_a^b f(x)dx = \frac{h}{3}[(sum\ of\ first\ and\ last\ ordinates) + 2(sum\ of\ remaining\ even\ ordinates) + 4(sum\ of\ remaining\ odd\ ordinates)]$$

Simpson's $\frac{1}{3}$ Rule [CO4]

Example: Evaluate $\mathbf{I = \int_0^1 \frac{1}{1+x} dx}$ **by Simpson's** $\mathbf{\frac{1}{3}}$ **rule (take** $\mathbf{n = 6}$**).**

Solution:

Simpson's $\frac{1}{3}$ rule is given by

$$I = \int_a^b f(x)dx = \frac{h}{3}[y_0 + 2(y_2 + \cdots + y_{n-2}) + 4(y_1 + y_3 + \cdots + y_{n-1}) + y_n]$$

Where $h = \frac{(b-a)}{n}$, n = number of subintervals

x	0	$1/6$	$2/6$	$3/6$	$4/6$	$5/6$	1
$F(x) = 1/(1+x)$	1	0.857	0.75	0.666	0.6	0.545	0.5

Then $I = 0.69$

Simpson's 3/8 Rule [CO4]

Let $y_0, y_1, y_2, \ldots \ldots . y_n$ be the values of $y = f(x)$ corresponding to $x = x_0 = a, x_1, x_2, \ldots \ldots . x_n = b$, which are equally spaced with interval as h.

Then the Simpson's $\frac{3}{8}$ rule is given by

$\mathrm{I} = \int_a^b f(x)dx = \frac{3h}{8}[y_0 + 2(y_3 + y_6 + \cdots + y_{n-3}) + 3(y_1 + y_2 + y_4 + \cdots + y_{n-1}) + y_n]$ ..(1)

where $h = \frac{(b-a)}{n}$, $n =$ number of subintervals

To apply this rule, the number of subintervals should be multiple of 3.

Simpson's $\frac{3}{8}$ rule may be writeen as

$$\mathrm{I} = \int_a^b f(x)dx = \frac{h}{3}[(sum\ of\ first\ and\ last\ ordinates) + 2(sum\ of\ multiple\ of\ 3\ ordinates) + 3(sum\ of\ remaining\ \ ordinates)]$$

Simpson's 3/8 Rule [CO4]

Example: Evaluate $\mathbf{I} = \int_0^1 \frac{1}{1+x} dx$ **by Simpson's** $\frac{3}{8}$ **rule (take** $n = 6$**).**

Solution:

Here $h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

x	0	1/6	2/6	3/6	4/6	5/6	1
$f(x)$	1	0.857	0.75	0.666	0.6	0.545	0.5

$$I = \int_0^1 \frac{1}{1+x} dx = \frac{3}{6\times 8}[1 + 2(0.666) + 3(0.857 + 0.75 + 0.6 + 0.545) + 0.5]$$

$$= 0.6932$$

Numerical Integration [CO4]

QUIZ!

Q.1 Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$

By Trapezoidal, Simpson’s 1/3 and Simpson 3/8 rule.

Objective of Numerical solution of ODE [CO4]

Using numerical method, we can find the numerical solution of 1st order ordinary differential equation.

Fourth Order Runge-Kutta Method[CO4]

Let us consider the first order initial value problem

$$\frac{dy}{dx} = f(x, y)$$

and $y = y_0$ for $x = x_0$. Let h be the interval between equidistant values of x then the $(n+1)^{th}$ approximation to y is given by

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), n = 0,1,2, \ldots \ldots ..\text{etc.}$$

Where $k_1 = hf(x_n, y_n)$

$$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$

$$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

Fourth Order Runge-Kutta Method[CO4]

Example: Given $\frac{dy}{dx} = y - x, y(0) = 2$.Find $y(0.1)$ and $y(0.2)$ correct to four decimal places using fourth order Runge-Kutta method.

Solution:

Given $f(x, y) = y - x, x_0 = 0, y_0 = 2$

Choose $h = 0.1$

Now by fourth order Runge-Kutta method

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$..........(1)

Where $k_1 = hf(x_0, y_0)$...............(2)

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$...............(3)

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$.................(4)

$k_4 = hf(x_0 + h, y_0 + k_3)$...............(5)

Fourth Order Runge-Kutta Method [CO4]

Now putting the given values in (1),(2),(3),(4) & (5) we have

$k_1 = 0.2$

$k_2 = 0.205$

$k_3 = 0.20525$

$k_4 = 0.210525$

$y_1 = y(0.1) = 2.2052$

Now, $x_1 = x_0 + h = 0 + 0.1 = 0.1$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$..........(6)

Where $k_1 = hf(x_1, y_1)$..............................(7)

$k_2 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2})$..........................(8)

$k_3 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2})$..........................(9)

$k_4 = hf(x_1 + h, y_1 + k_3)$.......................(10)

Fourth Order Runge-Kutta Method [CO4]

After simplification equations (6),(7),(8),(9) and (10). We have

$k_1 = 0.21052$

$k_2 = 0.21605$

$k_3 = 0.216323$

$k_4 = 0.221523$

$y_2 = y(0.2) = 2.4213$

Fourth Order Runge-Kutta Method [CO4]

QUIZ!

Q.1 Find the value of $y(1.1)$ using Runge Kutta Method of fourth order given that $\frac{dy}{dx} = y^2 + xy$, $y(1) = 1$, take $h = 0.05$.

Q.2 Find the value of $y(0.1), y(0.2), y(0.3)$ using Runge Kutta Method of fourth order given that $\frac{dy}{dx} = y^2 + xy$, $y(0) = 1$, take $h = 0.1$.

Weekly Assignment

1. Solve the following system of equations by Crout's Method:

$4x + y + z = 4\,, x + 4y - 2z = 4, 3x + 2y - 4z = 6$

Ans: $\boldsymbol{x = 1, y = 0.5, z = -0.5}$

2. Solve the following system of equations by Gauss Seidel Method:

$7x + 52y + 13z = 104\,, 83x + 11y - 4z = 95, 3x + 8y + 29z = 71$

Ans: $\boldsymbol{x = 1.057, y = 1.367, z = 1.961}$

3. Evaluate the integral $\int_0^{2\pi} e^{-x} sin10x\, dx$ using

(i) Simpson's $3/8$ rule

(ii) Simpson's rule with 8 intervals **Ans:** $\boldsymbol{-0.24245, 0.39466}$

4. Given the IVP $\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}, y(1) = 3$. find the numerical solution at $x = 1.2$ with step size $h = 0.1$by using Fourth order Runge-Kutta method. **Ans: 2.8233**

Faculty Video Links, YouTube & NPTEL Video Links and Online Courses Details

- **Self Made Video Link:**

- **YouTube/other Video Links**
- https://www.youtube.com/channel/UCqpVOOZS6-OFQaPKWBZLKJQ/videos

MCQs

Q1. If a continuous function $f(x)$ on $[a\ b]$satisfy $f(a).f(b) < 0$ then $f(x)$ has

(a) has one root in $[a\ b]$.

(b) has two root in $[a\ b]$.

(c) has odd no. of root in $[a\ b]$.

(d) None of these

Q2. Which method is quadratic convergent

(a) Bisection method

(b) Regula falsi method

(c) Newton Raphson method

(d) all of above

MCQs

Q3. Iterative method is used to

(a) Finding the solution of ordinary differential equation

(b) Finding the root of transcendental equation

(c) Finding the root of algebraic equation

(d) All of the above

Q4. Which method is used to obtained complex root

(a) Bisection method

(b) Newton Raphson method

(c) Regula falsi method

(d) None of the above.

MCQs

Q5. Interpolation formula used for unevenly spaced points-

(a) Newton Gregory forward interpolation formula

(b) Newton Gregory backward interpolation formula

(c) Newton divided difference interpolation formula

(d) Lagrange's interpolation formula

Q6. The equation $f(x) = (x-1)^2(x-3)^2$ has roots at $x = 1$ and $x = 3$. Which of the following method can be applied to find all the roots?

(a) Bisection method

(b) False position method

(c) Newton Raphson method

(d) None of these

Glossary Questions

1. Pick out the correction option from Glossary-

I. Bisection method

II. Regula falsi method

III. Newton Raphson method

IV. Gauss seidel method

A. May fail for solving an algebraic equation

B. Is used for solving System of linear equations

C. Is also knowns as interval halving method

D. Requires two initial approximation to start iterations

Glossary Questions

1. Pick out the correction option from Glossary-

I. Simpson's 1/3 rule

II. Trapezoidal rule

III. Runge kutta 4^{th} order method

IV. Interpolation

A. Gives exact solution of polynomial of degree≤ 1

B. Gives exact solution of polynomial of degree≤ 3

C. Is process of finding the value of y in intermediate value of x in tabulated form of $y = f(x)$

D. Is used for solving an IVP

First Sessional Paper

Printed page:2 **Subject Code: AAS0301A**

Roll No: | | | | | | | | | | | | | | |

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech (CSE/CS/IT)

(SEM. III SESSIONAL EXAMINATION –I) (2021-2022)

Subject Name: Eng. Mathematics III

Time: 1.15 Hours **Max. Marks:30**

General Instructions:

- All questions are compulsory. Answers should be brief and to the point.
- This Question paper consists of 2 pages & 5 questions.
- It comprises of three Sections, A, B, and C. You are to attempt all the sections.
- Section A -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- Section B - Question No-3 is short answer type questions carrying 5 marks each. You need to attempt any two out of three questions given.
- Section C -Question No. 4 & 5 Long answer type (within unit choice) questions carrying 6 marks each. You need to attempt any one-part *a* or *b*.
- Students are instructed to cross the blank sheets before handing over the answer sheet to the invigilator.

First Sessional Paper

- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.
- **Blooms Level:** *K1: Remember, K2: Understand, K3: Apply, K4: Analyze, K5: Evaluate, K6: Create*

		SECTION – A	**[8]**	**CO**	**Blooms level**
1.	**Attempt all parts**		**(4×1=4)**	**CO**	
	a.	$\lim_{z\to 0}\frac{z}{\bar{z}}$ (i) Limit exists (ii) Limit does not exist (iii) Limit exists and equal to 1 (iv) None of these	**(1)**	**1**	**K5**
	b.	If $f(z)=\frac{z}{z^2+9}$ then (i) $f(z)$ is continuous (ii) $f(z)$ is discontinuous at $z=\pm 3i$ (iii) $\lim_{z\to i}\frac{z}{z^2+9}=-\frac{i}{8}$ (iv) Both B & C	**(1)**	**1**	**K2**
	c.	Function $f(z)=z\lvert z\rvert$ is (i) Analytic anywhere (ii) Not analytic anywhere (ii) Harmonic (iv) None of these	**(1)**	**1**	**K3**

First Sessional Paper

	d.	There exists no analytic function $f(z)$ if (i) $real\ f(z) = y - 2x$ (ii) $real\ f(z) = y^2 - 2x$ (ii) $real\ f(z) = y^2 - x^2$ (iv) $real\ f(z) = y - x$	**(1)**	**1**	**K2**
2.	**Attempt all parts**		**(2×2=4)**	**CO**	
	a.	Show that if $f(z)$ is analytic and $Imf(z) = constant$ then $f(z)$ is constant.	**(2)**	**1**	**K3**
	b.	Find the bilinear transformation which maps the points $z = 0,1,\infty$ into the points $w = i,-1,-i$ respectively.	**(2)**	**1**	**K5**
		SECTION – B			
3.	**Answer any two of the following-**		**[2×5=10]**	**CO**	
	a.	Examine the nature of the function $f(z) = \frac{x^3y(y-ix)}{x^6+y^2}$, $z \neq 0, f(0) = 0$, prove that $\frac{f(z)-f(0)}{z} \to 0$ as $z \to 0$ along any radius vector but not as $z \to 0$ in any manner and also that $f(z)$ is not analytic at $z = 0$.	**(5)**	**1**	**K4**
	b.	Find the image of $\lvert z-1\rvert = 1$ under the transformation $w = \frac{1}{z}$.	**(5)**	**1**	**K5**
	c.	Show that $f(z) = \cos z$ is analytic in entire complex plane.	**(5)**	**1**	**K3**

First Sessional Paper

		SECTION – C			
4	**Answer any one of the following-**		**[2×6=12]**	**CO**	
	a.	Determine an analytic function $f(z)$ in terms of z whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$.	**(6)**	**1**	**K5**
	b.	If $w = \varphi + i\psi$ represent the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$. Determine the function φ.	**(6)**	**1**	**K5**
5.	**Answer any one of the following-**				
	a.	Determine an analytic function $f(z)$ in terms of z if $3u + v = 3\sin x\, cos\, hy + \cos x . \sin hy$.	**(6)**	**1**	**K5**
	b.	Find an analytic function $f(z)$ in terms of z if $Re[f'(z)] = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0$ & $f'(0) = 0$.	**(6)**	**1**	**K5**

Expected Questions for University Exam

Q1. Using Newton Raphson Method-find a positive root of equation $x\ log_{10}x = 1.2$ correct to 4 decimal places.

Q2. Using Regula Falsi Method- find a positive root of equation $x^3 - 5x + 3 = 0$ correct to 3 decimal places.

Q3. From the table, estimate the number of students who obtained marks between 40 and 45.

Marks :	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

Q4. Using Lagrange's interpolating formula, compute $f(3)$from the following table-

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

Expected Questions for University Exam

Q5. Find the missing terms in the following table-

x	0	5	10	15	20	25
$f(x)$	6	10	?	17	?	31

Q6. Using Newton forward interpolation formula, find the interpolating polynomial to the following data-

x	4	6	8	10
y	1	3	8	16

Expected Questions for University Exam

Q7. Solve the following system of equations by Crout's Method:

$4x + y + z = 4\,, x + 4y - 2z = 4, 3x + 2y - 4z = 6$

Q8. Solve the following system of equations by Gauss Seidel Method:

$7x + 52y + 13z = 104\,, 83x + 11y - 4z = 95, 3x + 8y + 29z = 71$

Q9. Given the IVP $\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}, y(1) = 3$. find the numerical solution at $x = 1.2$ with step size $h = 0.1$by using Fourth order Runge-Kutta method.

Summary

In previous lectures we have discussed-

- Algebraic and transcendental equation
- Bisection method
- Regula falsi method
- Newton raphson method
- Order of convergence

❖ Interpolation for equal intervals

- Newton Forward interpolation formula
- Newton Backward interpolation formula

❖ Interpolation for unequal intervals

- Lagrange's interpolation formula
- Newton divided difference interpolation formula

Summary

- Solution of system of equations
  - Crout’s method
  - Gauss seidel method
- Numerical integration
  - Trapezoidal rule
  - Simpson’s 1/3 rule
  - Simpson’s 3/8 rule
- Numerical solution of ODE
  - Runge Kutta 4^{th} order method

References

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Thank You