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MIS-64018
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Assignment 2: Formulating and Solving Linear Programming Model

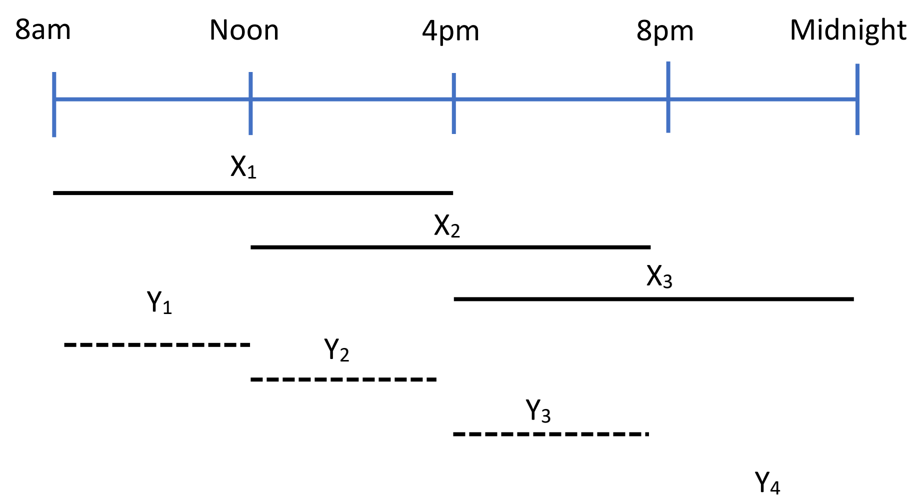
Problem #1:

Part A)

Decision Variables:

The decision variables in this problem are the number of full-time and part-time consultants per shift:

Shift Timeline:



X_i = Number of full-time consultants to work shift (i = 1, 2, 3)
1 = 8am to 4pm, 2 = Noon to 8pm, 3 = 4pm to Midnight
Y_i = Number of part-time consultants to work shift (i = 1, 2, 3, 4)
1= 8am to Noon, 2 = Noon to 4pm, 3 = 4pm to 8pm, 4 = 8pm to Midnight

Objective Function:

The objective function is to minimize the cost function:

Minimize: $Z = 112*X_1 + 112*X_2 + 112*X_3 + 48*Y_1 + 48*Y_2 + 48*Y_3 + 48*Y_4$

Subject To (Constraints):

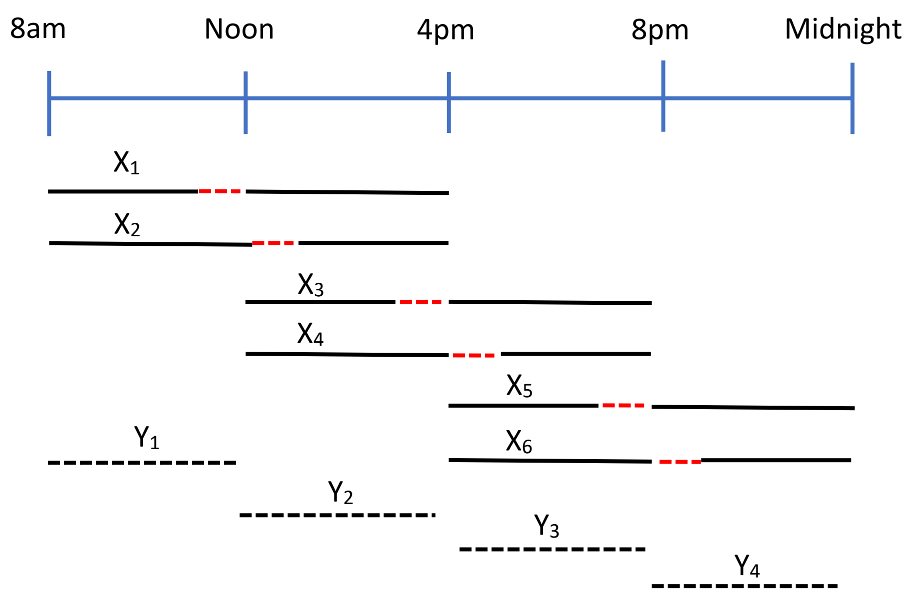
$X_1 + Y_1 \geq 4$
$X_1 + X_2 + Y_2 \geq 8$
$X_2 + X_3 + Y_3 \geq 10$
$X_3 + Y_4 \geq 6$
$X_1, X_2, X_3 \geq 0$
$Y_1, Y_2, Y_3, Y_4 \geq 0$
$X_1 \geq Y_1$
$X_1 + X_2 \geq Y_2$
$X_2 + X_3 \geq Y_3$
$X_3 \geq Y_4$

Part B)

Decision Variables:

The decision variables in this problem are the number of full-time (with lunch breaks) and part-time (no lunch breaks) consultants per shift:

Shift Timeline:



X_i = Number of full-time consultants to work shift (i = 1, 2, 3, 4, 5, 6)
1 = 8am to 4pm (Lunch after 3 hrs), 2 = 8am to 4pm (Lunch after 4 hrs)
3 = Noon to 8pm (Lunch after 3 hrs), 4 = Noon to 8pm (Lunch after 4 hrs)
5 = 4pm to Midnight (Lunch after 3 hrs), 6 = 4pm to Midnight (Lunch after 4 hrs)
Y_i = Number of part-time consultants to work shift (i = 1, 2, 3, 4)
1= 8am to Noon, 2 = Noon to 4pm, 3 = 4pm to 8pm, 4 = 8pm to Midnight

Objective Function:

The objective function is still to minimize the cost function, but with added shift options:

Minimize: $Z = 112*X_1 + 112*X_2 + 112*X_3 + 112*X_4 + 112*X_5 + 112*X_6 + 48*Y_1 + 48*Y_2 + 48*Y_3 + 48*Y_4$

Subject To (Constraints):

$X_1 + X_2 + Y_1 \geq 4$
$X_1 + X_2 + X_3 + X_4 + Y_2 \geq 8$
$X_3 + X_4 + X_5 + X_6 + Y_3 \geq 10$
$X_5 + X_6 + Y_4 \geq 6$
$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$
$Y_1, Y_2, Y_3, Y_4 \geq 0$
$X_1 + X_2 \geq Y_1$
$X_1 + X_2 + X_3 + X_4 \geq Y_2$
$X_3 + X_4 + X_5 + X_6 \geq Y_3$
$X_5 + X_6 \geq Y_4$

Problem #2:

Solve Problem from Assignment #1 Graphically:

X_i = Units of *i* backpacks produced per week, i = 1,2
1 = Collegiate Backpack, 2 = Mini Backpack

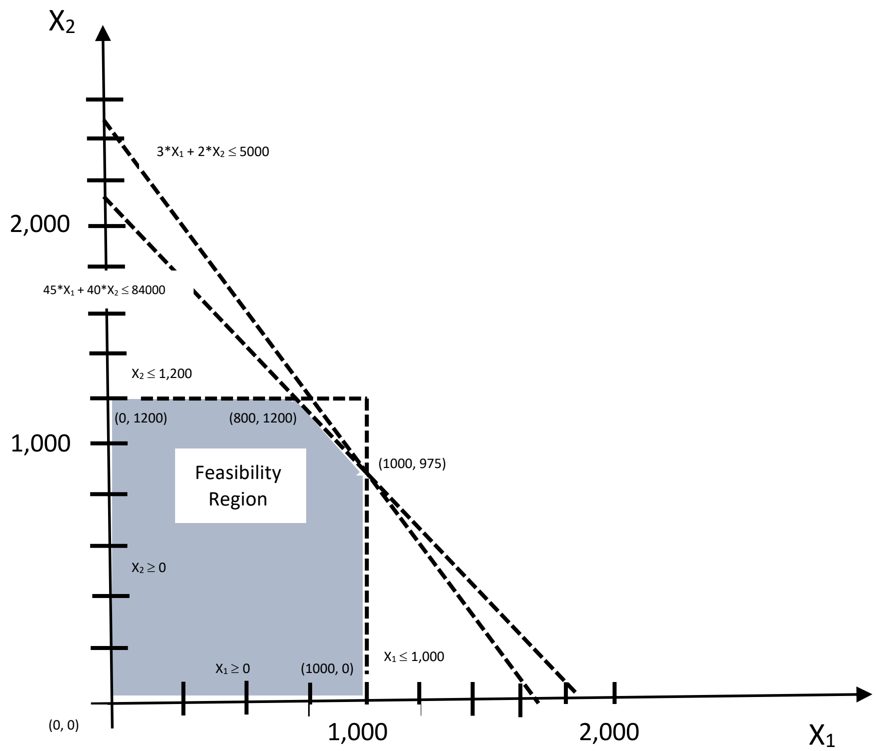
Maximize: $Z = 32*X_1 + 24*X_2$

Subject to:

$3*X_1 + 2*X_2 \leq 5000$
$45*X_1 + 40*X_2 \leq 84000$

And:

$0 \leq X_1 \leq 1000$
$0 \leq X_2 \leq 1200$



Corner Points of the Feasibility Region Are:

- (0,0)
- (1000, 0)
- (0, 1200)
- (800, 1200)
- (1000, 975)

Plugging the Values into the Objective Function ($Z = 32*X_1 + 24*X_2$) We Get:

- (0,0) --- Z = 0
- (1000, 0) --- Z = 32,000
- (0, 1200) --- Z = 28,800
- (800, 1200) --- Z = 54,400
- (1000, 975) --- Z = 55,400

Therefore, the optimal solution for this problem is to produce:

1,000 Collegiate backpacks per week

975 Mini backpacks per week

Problem #3:

A) **Decision Variables:**

$X_{i,j}$ = Number of units of product *j* produced at Plant *i* (i = 1,2,3) (j = 1,2,3)

i = 1 (Plant 1), i = 2 (Plant 2), i = 3 (Plant 3)

j = 1 (Large), j = 2 (Medium), j = 3 (Small)

B) **Linear Programming Model:**

Objective Function:

Maximize Profits: $Z = 420*(X_{1,1} + X_{2,1} + X_{3,1}) + 360*(X_{1,2} + X_{2,2} + X_{3,2}) + 300*(X_{1,3} + X_{2,3} + X_{3,3})$

Subject to (Constraints):

Production:

$$\left.\begin{aligned} X_{1,1} + X_{1,2} + X_{1,3} &\leq 750 \\ X_{2,1} + X_{2,2} + X_{2,3} &\leq 900 \\ X_{3,1} + X_{3,2} + X_{3,3} &\leq 450 \end{aligned}\right\} \text{Capacity Constraints}$$

Storage:

$$\left.\begin{aligned} 20*X_{1,1} + 15*X_{1,2} + 12*X_{1,3} &\leq 13{,}000 \\ 20*X_{2,1} + 15*X_{2,2} + 12*X_{2,3} &\leq 12{,}000 \\ 20*X_{3,1} + 15*X_{3,2} + 12*X_{3,3} &\leq 5{,}000 \end{aligned}\right\} \text{Storage Constraints}$$

Sales:

$$\left.\begin{aligned} X_{1,1} + X_{2,1} + X_{3,1} &\leq 900 \\ X_{1,2} + X_{2,2} + X_{3,2} &\leq 1{,}200 \\ X_{1,3} + X_{2,3} + X_{3,3} &\leq 750 \end{aligned}\right\} \text{Sales Constraints}$$

Equal Capacity Usage:

$$\frac{(X_{1,1} + X_{1,2} + X_{1,3})}{750} = \frac{(X_{2,1} + X_{2,2} + X_{2,3})}{900}$$

$$\frac{(X_{2,1} + X_{2,2} + X_{2,3})}{900} = \frac{(X_{3,1} + X_{3,2} + X_{3,3})}{450}$$

$$\left.\begin{aligned} 900*(X_{1,1} + X_{1,2} + X_{1,3}) - 750*(X_{2,1} + X_{2,2} + X_{2,3}) &= 0 \\ 450*(X_{2,1} + X_{2,2} + X_{2,3}) - 900*(X_{3,1} + X_{3,2} + X_{3,3}) &= 0 \\ 450*(X_{1,1} + X_{1,2} + X_{1,3}) - 450*(X_{3,1} + X_{3,2} + X_{3,3}) &= 0 \end{aligned}\right\} \text{Equal Capacity Usage Constraints}$$

Non-Negative Boundary:

$X_{1,1}, X_{1,2}, X_{1,3}, X_{2,1}, X_{2,2}, X_{2,3}, X_{3,1}, X_{3,2}, X_{3,3} \geq 0$

C) **Solve Using *lpsolve* in RStudio:**

Based on the *lpsolve* output, the optimal solution to this problem is the following (assuming fractional amounts are allowed):

Maximum profits would be $696,000 per day, operating at 92.5% excess capacity at all three plants with the following production amounts per day:

- Plant 1, Large: 516.67 units/day
- Plant 2, Large: 0 units/day
- Plant 3, Large: 0 units/day
- Plant 1, Medium: 177.78 units/day
- Plant 2, Medium: 666.67 units/day
- Plant 3, Medium: 0 units/day
- Plant 1, Small: 0 units/day
- Plant 2, Small: 166.67 units/day
- Plant 3, Small: 416.67 units/day

See the solution steps in the R Markdown File “Assignment 2 – Problem 3C” or the knitted Word Document with the same file name.