

Assignment 1-2: Formulating Linear Programming Model

Problem Description:

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of \$32. Each Mini requires 40 minutes of labor and generates a unit profit of \$24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

- Clearly define the decision variables
- What is the objective function?
- What are the constraints?
- Write down the full mathematical formulation for this LP problem.

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Summary of Problem:

Backpack Type	Fabric	Material Required (ft ²)	Max Sales Per Week	Labor Per Unit (min)	Profit Per Unit (\$)
Collegiate	Nylon	3	1,000	45	32
Mini	Nylon	2	1,200	40	24

Additional Notes:

5,000 ft² of nylon fabric is received weekly

35 laborers

Each laborer provides 40 hours per week

a) Decision Variables:

The decision variables in this example are the weekly production of each backpack type:

X_i = Units of i backpacks produced per week, $i = 1, 2$

1 = Collegiate Backpack, 2 = Mini Backpack

b) Objective Function:

The objective function is this problem is to maximize profits:

$$\text{Maximize } Z = 32 \cdot X_1 + 24 \cdot X_2$$

c) Constraints:

The constraints in problem are the fixed weekly amount of fabric received in, amount of fabric used per backpack, maximum sales per week, labor per unit, and amount of labor per week.

Fabric Constraint (5,000 ft² / week):

$$3 \cdot X_1 + 2 \cdot X_2 \leq 5000$$

Sales Constraint (1,000 Collegiates/wk and 1,200 Minis/wk and neither can be negative)

$$X_1 \leq 1000$$

$$X_2 \leq 1200$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Labor Constraint (40 hours/week and 35 laborers) (40 hours = 2,400 minutes)

40 hours = 2,400 minutes

$$45 \cdot X_1 + 40 \cdot X_2 \leq (2400 \cdot 35)$$

d) Full Mathematical Formulation:

Mathematical Formulation Written in Standard Form:

$$\text{Maximize: } Z = 32 \cdot X_1 + 24 \cdot X_2$$

Subject to the Restrictions:

$$3 \cdot X_1 + 2 \cdot X_2 \leq 5000$$

$$45 \cdot X_1 + 40 \cdot X_2 \leq 84000$$

And:

$$0 \leq X_1 \leq 1000$$

$$0 \leq X_2 \leq 1200$$