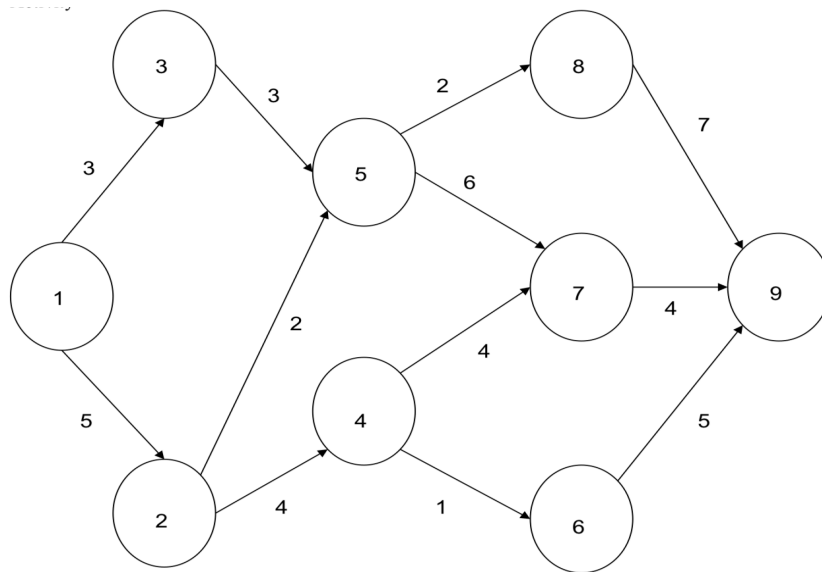


### Assignment 5: Integer Programming Problem

#### Question #1:



See the R Markdown file “sspenc12\_5 RMD” for details on the RStudio details for the solution. The critical path (longest path) found via the lpSolveAPI package is shown below (where a “1” means that the path is chosen):

```
arc.names
[1,] "X12"      "1"
[2,] "X13"      "0"
[3,] "X24"      "0"
[4,] "X25"      "1"
[5,] "X35"      "0"
[6,] "X46"      "0"
[7,] "X47"      "0"
[8,] "X57"      "1"
[9,] "X58"      "0"
[10,] "X69"     "0"
[11,] "X79"     "1"
[12,] "X89"     "0"
```

The total time for the objective function using this path is 17 time units.

## **Problem #2:**

### **Decision Variables:**

$X_i$  – Where  $X$  is the number (in 1000's) of stock  $i$  that is purchased for the portfolio ( $i = 1:8$ )  
(1 = S1, 2 = S2, 3 = S3, 4 = H1, 5 = H2, 6 = H3, 7 = C1, 8 = C2)

### **Objective Function:**

Maximize the return,  $Z$ , from both growth and dividends.

Return for each stock is based off of the following calculation:

$$X_i * (\text{Growth\_Rate} * \text{Price\_Per\_Share} + \text{Expected\_Dividend})$$

$$\text{Maximize } Z = 4 * X_1 + 6.5 * X_2 + 5.9 * X_3 + 5.4 * X_4 + 5.15 * X_5 + 10 * X_6 + 8.4 * X_7 + 6.25 * X_8$$

### **Subject to Constraints:**

Total Investment:

$$40 * X_1 + 50 * X_2 + 80 * X_3 + 60 * X_4 + 45 * X_5 + 60 * X_6 + 30 * X_7 + 25 * X_8 \leq 2,500$$

Diversification:

$$40 * X_1 + 50 * X_2 + 80 * X_3 \leq 1,000$$

$$60 * X_4 + 45 * X_5 + 60 * X_6 \leq 1,000$$

$$30 * X_7 + 25 * X_8 \leq 1,000$$

Minimum Investment:

$$40 * X_1 \geq 100$$

$$50 * X_2 \geq 100$$

$$80 * X_3 \geq 100$$

$$60 * X_4 \geq 100$$

$$45 * X_5 \geq 100$$

$$60 * X_6 \geq 100$$

$$30 * X_7 \geq 100$$

$$25 * X_8 \geq 100$$

And:

$X_i$  are integers ( $i = 1:8$ )

- 1) The maximum return on the portfolio would be \$477,400 (which correlates to a 19.10% ROI) The investment strategy would be as follows:

Stock S1: 3,000 shares (\$120,000 total)  
Stock S2: 5,000 shares (\$250,000 total)  
Stock S3: 2,000 shares (\$160,000 total)  
Stock H1: 2,000 shares (\$120,000 total)  
Stock H2: 3,000 shares (\$135,000 total)  
Stock H3: 12,000 shares (\$720,000 total)  
Stock C1: 29,000 shares (\$870,000 total)  
Stock C2: 5,000 shares (\$125,000 total)

Screenshot of the integer programming model is shown below (see R Markdown file for additional details of the code).

Model name:									
	C1	C2	C3	C4	C5	C6	C7	C8	
Maximize	4	6.5	5.9	5.4	5.15	10	8.4	6.25	
R1	40	50	80	60	45	60	30	25	<= 2500
R2	40	50	80	0	0	0	0	0	<= 1000
R3	0	0	0	60	45	60	0	0	<= 1000
R4	0	0	0	0	0	0	30	25	<= 1000
R5	40	0	0	0	0	0	0	0	>= 100
R6	0	50	0	0	0	0	0	0	>= 100
R7	0	0	80	0	0	0	0	0	>= 100
R8	0	0	0	60	0	0	0	0	>= 100
R9	0	0	0	0	45	0	0	0	>= 100
R10	0	0	0	0	0	60	0	0	>= 100
R11	0	0	0	0	0	0	30	0	>= 100
R12	0	0	0	0	0	0	0	25	>= 100
Kind	Std	Std	Std	Std	Std	Std	Std	Std	
Type	Int	Int	Int	Int	Int	Int	Int	Int	
Upper	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	
Lower	0	0	0	0	0	0	0	0	

```
[1] 0
[1] 477.4
[1] 3 5 2 2 3 12 29 5
```

- 2) The maximum return on the portfolio with no integer restriction would be \$487,152.80 (which correlates to a 19.50% ROI) This means there is a 0.4% increase in ROI by removing the integer restriction. The investment strategy would be as follows:

Stock S1: 2,500 shares (\$100,000 total) – (This is a -16.7% change from Part 1)  
 Stock S2: 6,000 shares (\$300,000 total) – (This is a 20% change from Part 1)  
 Stock S3: 1,250 shares (\$100,000 total) – (This is a -37.5% change from Part 1)  
 Stock H1: 1,667 shares (\$100,020 total) – (This is a -16.7% change from Part 1)  
 Stock H2: 2,222 shares (\$99,990 total) – (This is a -25.9% change from Part 1)  
 Stock H3: 13,333 shares (\$799,980 total) – (This is a 11.1% change from Part 1)  
 Stock C1: 30,000 shares (\$900,000 total) – (This is a 3.5% change from Part 1)  
 Stock C2: 4,000 shares (\$100,000 total) – (This is a -20% change from Part 1)

Screenshot of the updated linear model is shown below (see R Markdown file for additional details on the coding).

Model name:										
	C1	C2	C3	C4	C5	C6	C7	C8		
Maximize	4	6.5	5.9	5.4	5.15	10	8.4	6.25		
R1	40	50	80	60	45	60	30	25	<=	2500
R2	40	50	80	0	0	0	0	0	<=	1000
R3	0	0	0	60	45	60	0	0	<=	1000
R4	0	0	0	0	0	0	30	25	<=	1000
R5	40	0	0	0	0	0	0	0	>=	100
R6	0	50	0	0	0	0	0	0	>=	100
R7	0	0	80	0	0	0	0	0	>=	100
R8	0	0	0	60	0	0	0	0	>=	100
R9	0	0	0	0	45	0	0	0	>=	100
R10	0	0	0	0	0	60	0	0	>=	100
R11	0	0	0	0	0	0	30	0	>=	100
R12	0	0	0	0	0	0	0	25	>=	100
Kind	Std	Std	Std	Std	Std	Std	Std	Std		
Type	Real	Real	Real	Real	Real	Real	Real	Real		
Upper	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf		
Lower	0	0	0	0	0	0	0	0		

```
[1] 0
[1] 487.1528
[1] 2.500000 6.000000 1.250000 1.666667 2.222222 13.333333 30.000000 4.000000
```