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Assignment 3: Post-Optimality and Sensitivity Analysis for Linear Programming

Part #1 - Solve Using *lpsolve* in RStudio:

Based on the *lpsolve* output, the optimal solution to this problem is the following (assuming fractional amounts are allowed):

Maximum profits would be $696,000 per day, operating at 92.5% excess capacity at all three plants with the following production amounts per day:

- Plant 1, Large: 516.67 units/day
- Plant 2, Large: 0 units/day
- Plant 3, Large: 0 units/day
- Plant 1, Medium: 177.78 units/day
- Plant 2, Medium: 666.67 units/day
- Plant 3, Medium: 0 units/day
- Plant 1, Small: 0 units/day
- Plant 2, Small: 166.67 units/day
- Plant 3, Small: 416.67 units/day

See the solution steps in the R Markdown File "Assignment 2 – Problem 3C" or the knitted Word Document with the same file name.

Part #2 – Identify the Shadow Prices, Dual Solution, and Reduced Costs:

Based on the output from the "get.sensitivity.rhs()" and "get.sensitivity.obj()" (shown in the "Assignment 3 – RMD" file) function output, we get the following values for the shadow prices, dual solution, and reduced costs:

```{r}

get.sensitivity.rhs(lprec)

```

```
$duals
 [1]    0.00    0.00    0.00   12.00   20.00   60.00    0.00    0.00    0.00   -0.08    0.00    0.56
[13]    0.00  -40.00 -360.00    0.00    0.00 -120.00  -24.00    0.00    0.00

$dualsfrom
 [1] -1.000000e+30 -1.000000e+30 -1.000000e+30  1.122222e+04  1.150000e+04  4.800000e+03 -1.000000e+30
 [8] -1.000000e+30 -1.000000e+30  0.000000e+00 -1.000000e+30  0.000000e+00 -1.000000e+30 -1.000000e+02
[15] -2.000000e+01 -1.000000e+30 -1.000000e+30 -4.444444e+01 -2.222222e+02 -1.000000e+30 -1.000000e+30

$dualstill
 [1] 1.000000e+30 1.000000e+30 1.000000e+30 1.388889e+04 1.250000e+04 5.181818e+03 1.000000e+30
 [8] 1.000000e+30 1.000000e+30 0.000000e+00 1.000000e+30 0.000000e+00 1.000000e+30 1.000000e+02
[15] 2.500000e+01 1.000000e+30 1.000000e+30 6.666667e+01 1.111111e+02 1.000000e+30 1.000000e+30
```

Shadow Prices:

Constraint #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Shadow Price	0.00	0.00	0.00	12.00	20.00	60.00	0.00	0.00	0.00	-0.08	0.00	0.56

Dual Solution (shadow prices are also the solution to the dual problem):

Constraint #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Shadow Price	0.00	0.00	0.00	12.00	20.00	60.00	0.00	0.00	0.00	-0.08	0.00	0.56

Reduced Costs:

Product	**1L**	**2L**	**3L**	**1M**	**2M**	**3M**	**1S**	**2S**	**3S**
Reduced Cost	0.00	-40.00	-360.00	0.00	0.00	-120.00	-24.00	0.00	0.00

Part #3 – Further Identify the Sensitivity of the Prices and Costs:

```{r}

get.sensitivity.rhs(lprec)

```

```
$duals
 [1]    0.00    0.00    0.00   12.00   20.00   60.00    0.00    0.00    0.00   -0.08    0.00    0.56
[13]    0.00  -40.00 -360.00    0.00    0.00 -120.00  -24.00    0.00    0.00

$dualsfrom
 [1] -1.000000e+30 -1.000000e+30 -1.000000e+30  1.122222e+04  1.150000e+04  4.800000e+03 -1.000000e+30
 [8] -1.000000e+30 -1.000000e+30  0.000000e+00 -1.000000e+30  0.000000e+00 -1.000000e+30 -1.000000e+02
[15] -2.000000e+01 -1.000000e+30 -1.000000e+30 -4.444444e+01 -2.222222e+02 -1.000000e+30 -1.000000e+30

$dualstill
 [1] 1.000000e+30 1.000000e+30 1.000000e+30 1.388889e+04 1.250000e+04 5.181818e+03 1.000000e+30
 [8] 1.000000e+30 1.000000e+30 0.000000e+00 1.000000e+30 0.000000e+00 1.000000e+30 1.000000e+02
[15] 2.500000e+01 1.000000e+30 1.000000e+30 6.666667e+01 1.111111e+02 1.000000e+30 1.000000e+30
```

Shadow Price Sensitivity:

Duals	From	Till
0.00	-1.00e+30	1.00e+30
0.00	-1.00e+30	1.00e+30
0.00	-1.00e+30	1.00e+30
12.00	1.12e+04	1.38e+04
20.00	1.15e+04	1.25e+04
60.00	4.80e+03	5.18e+03
0.00	-1.00e+30	1.00e+30
0.00	-1.00e+30	1.00e+30
0.00	-1.00e+30	1.00e+30
-0.08	0.00e+00	0.00e+00
0.00	-1.00e+30	1.00e+30
0.56	0.00e+00	0.00e+00

Reduced Costs Sensitivity:

Reduced Costs	From	Till
0	-1.00e+30	1.00e+30
-40.00	-1.00e+02	1.00e+02
-360.00	-2.00e+01	2.50e+01
0.00	-1.00e+30	1.00e+30
0.00	-1.00e+30	1.00e+30
-120.00	-4.44e+01	6.67e+01
-24.00	-2.22e+02	1.11e+02
0.00	-1.00e+30	1.00e+30
0.00	-1.00e+30	1.00e+30

Part #4 – Formulate the Dual of the Above Problem and Solve It:

The formulation of the dual problem:

Minimize: $W = 750*Y_1 + 900*Y_2 + 450*Y_3 + 13000*Y_4 + 12000*Y_5 + 5000*Y_6 + 900*Y_7 + 1200*Y_8 + 750*Y_9 + 0*Y_{10} + 0*Y_{11} + 0*Y_{12}$

Subject To:

$Y_1 + 20*Y_4 + Y_7 + 900*Y_{10} + 450*Y_{12} \geq 420$
$Y_1 + 15*Y_4 + Y_8 + 900*Y_{10} + 450*Y_{12} \geq 420$
$Y_1 + 12*Y_4 + Y_9 + 900*Y_{10} + 450*Y_{12} \geq 420$
$Y_2 + 20*Y_5 + Y_7 - 750*Y_{10} + 450*Y_{11} \geq 360$
$Y_2 + 15*Y_5 + Y_8 - 750*Y_{10} + 450*Y_{11} \geq 360$
$Y_2 + 12*Y_5 + Y_9 - 750*Y_{10} + 450*Y_{11} \geq 360$
$Y_3 + 20*Y_6 + Y_7 - 900*Y_{11} - 450*Y_{12} \geq 300$
$Y_3 + 15*Y_6 + Y_8 - 900*Y_{11} - 450*Y_{12} \geq 300$
$Y_3 + 12*Y_6 + Y_9 - 900*Y_{11} - 450*Y_{12} \geq 300$
$Y_{1:9} \geq 0$, $Y_{10:12}$ = unrestricted

Solution for the dual should be the same as the shadow price in the primal problem. See attached RMD file “Assignment 3 – RMD” for walkthrough of problem in RStudio.