CMPSC/Math 451, Numerical Computation

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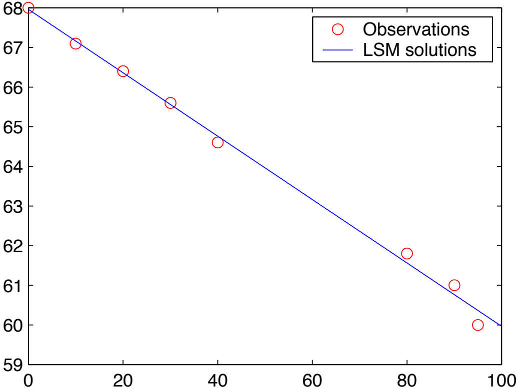
Matlab Simulations for Least Squares Method

Example 1: Linear regression $S = aT + b$ for the data:

T_k	0	10	20	30	40	80	90	95
S_k	68.0	67.1	66.4	65.6	64.6	61.8	61.0	60.0



Example 2:

The water tide in the North Sea can be characterized by the following formulae for the height of water $H(t)$, a periodic function of time t with the period equals to 12 hours:

$$H(t) = a_0 + a_1 \sin \frac{2\pi t}{12} + a_2 \cos \frac{2\pi t}{12}$$

We have the following observation data of the height of the water:

t_k	0.0	2.0	4.0	6.0	8.0	10.0	(hours)
H_k	1.0	1.6	1.4	0.6	0.2	0.8	(meters)

What is a_0, a_1 and a_2?

Answer. The normal equations:

$$
\begin{aligned}
\sum_k a_0 + a_1 \sin\frac{\pi t_k}{6} + a_2 \cos\frac{\pi t_k}{6} - H_k &= 0 \\
\sum_k \left[a_0 + a_1 \sin\frac{\pi t_k}{6} + a_2 \cos\frac{2\pi t_k}{12} - H_k\right] \sin\frac{\pi t_k}{6} &= 0 \\
\sum_k \left[a_0 + a_1 \sin\frac{\pi t_k}{6} + a_2 \cos\frac{2\pi t_k}{12} - H_k\right] \cos\frac{\pi t_k}{6} &= 0
\end{aligned}
$$

i.e.,

$$
\begin{aligned}
& a_0(n+1) + a_1 \textstyle\sum_k \sin\frac{\pi t_k}{6} + a_2 \sum_k \cos\frac{\pi t_k}{6} \\
& \qquad = \textstyle\sum_k H_k \\
& a_0 \textstyle\sum_k \sin\frac{\pi t_k}{6} + a_1 \sum_k \sin^2\frac{\pi t_k}{6} + a_2 \sum_k \cos\frac{\pi t_k}{6} \sin\frac{\pi t_k}{6} \\
& \qquad = \textstyle\sum_k H_k \sin\frac{\pi t_k}{6} \\
& a_0 \textstyle\sum_k \cos\frac{\pi t_k}{6} + a_1 \sum_k \sin\frac{\pi t_k}{6} \cos\frac{\pi t_k}{6} + a_2 \sum_k \cos^2\frac{\pi t_k}{6} \\
& \qquad = \textstyle\sum_k H_k \cos\frac{\pi t_k}{6}
\end{aligned}
$$

Simple Matlab codes:

```
t=[0 2 4 6 8 10];
H=[1 1.6 1.4 0.6 0.2 0.8];
n=length(t); va=pi/6;

s1=sum(sin(va*t));
s2=sum(cos(va*t));
s3=sum(sin(va*t).^2);
s4=sum(cos(va*t).*sin(va*t));
s5=sum(cos(va*t).^2);

A=[n,s1,s2; s1, s3, s4; s2, s4, s5];
h=[sum(H);sum(H.*sin(va*t));sum(H.*cos(va*t))];
a=A\h;

x=[0:0.05:12];
fx=a(1)+a(2)*sin(va*x)+a(3)*cos(va*x);
plot(x,fx,'b',t,H,'ro')
```

The code gives:

$$a_0 = 0.9333, a_1 = 0.5774, a_2 = 0.2667$$

The plot: