CMPSC/Math 451, Numerical Computation

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Iterative methods for systems of linear equations in Matlab

Example: We wish to solve a system: $Ax = b$ where A is a 6×6 matrix

```
A =
     4    -1    -1     0     0     0
    -1     4     0    -1     0     0
    -1     0     4    -1    -1     0
     0    -1    -1     4     0    -1
     0     0    -1     0     4    -1
     0     0     0    -1    -1     4
```

and the rhs vector is: $b = [1; 5; 0; 3; 1; 5]$.
The exact solution becomes: $x = [1; 2; 1; 2; 1; 2]$.
We solve the system with iterative methods, with the initial value:

$$x^{(0)} = [0.25; 1.25; 0; 0.75; 0.25; 1.25].$$

Jacobi iterations: tolerance $\varepsilon = 10^{-5}$, with $\|\cdot\|_\infty$

k	x1	x2	x3	x4	x5	x6
1	0.2500	1.2500	0	0.7500	0.2500	1.2500
2	0.5625	1.5000	0.3125	1.3750	0.5625	1.5000
3	0.7031	1.7344	0.6250	1.5781	0.7031	1.7344
4	0.8398	1.8203	0.7461	1.7734	0.8398	1.8203
5	0.8916	1.9033	0.8633	1.8467	0.8916	1.9033
6	0.9417	1.9346	0.9075	1.9175	0.9417	1.9346
7	0.9605	1.9648	0.9502	1.9442	0.9605	1.9648
8	0.9787	1.9762	0.9663	1.9699	0.9787	1.9762
9	0.9856	1.9872	0.9819	1.9797	0.9856	1.9872
10	0.9923	1.9913	0.9877	1.9890	0.9923	1.9913
...						
18	0.9999	1.9998	0.9998	1.9998	0.9999	1.9998
19	0.9999	1.9999	0.9999	1.9999	0.9999	1.9999
20	1.0000	1.9999	0.9999	1.9999	1.0000	1.9999

One needs more iterations to get the convergence.

Gauss-Seidal iterations:

k	x1	x2	x3	x4	x5	x6
1	0.2500	1.2500	0	0.7500	0.2500	1.2500
2	0.5625	1.5781	0.3906	1.5547	0.6602	1.8037
3	0.7422	1.8242	0.7393	1.8418	0.8857	1.9319
4	0.8909	1.9332	0.9046	1.9424	0.9591	1.9754
5	0.9594	1.9755	0.9652	1.9790	0.9852	1.9910
6	0.9852	1.9911	0.9873	1.9924	0.9946	1.9967
7	0.9946	1.9967	0.9954	1.9972	0.9980	1.9988
8	0.9980	1.9988	0.9983	1.9990	0.9993	1.9996
9	0.9993	1.9996	0.9994	1.9996	0.9997	1.9998
10	0.9997	1.9998	0.9998	1.9999	0.9999	1.9999
11	0.9999	1.9999	0.9999	2.0000	1.0000	2.0000
12	1.0000	2.0000	1.0000	2.0000	1.0000	2.0000
13	1.0000	2.0000	1.0000	2.0000	1.0000	2.0000

We see that after 12 iterations the method converges.

SOR iterations with $\omega = 1.12$:

k	x1	x2	x3	x4	x5	x6
1	0.2500	1.2500	0	0.7500	0.2500	1.2500
2	0.6000	1.6280	0.4480	1.6813	0.7254	1.9239
3	0.7893	1.8964	0.8411	1.9434	0.9671	1.9841
4	0.9518	1.9831	0.9805	1.9922	0.9940	1.9980
5	0.9956	1.9986	0.9972	1.9992	0.9994	1.9998
6	0.9994	1.9998	0.9998	1.9999	1.0000	2.0000
7	0.9999	2.0000	1.0000	2.0000	1.0000	2.0000
8	1.0000	2.0000	1.0000	2.0000	1.0000	2.0000
9	1.0000	2.0000	1.0000	2.0000	1.0000	2.0000

We see that after 8 iterations the method converges.

Error against number of iterations