CMPSC/Math 451, Numerical Computation

Wen Shen

Department of Mathematics, Penn State University

Polynomial interpolation: Van der Monde matrix

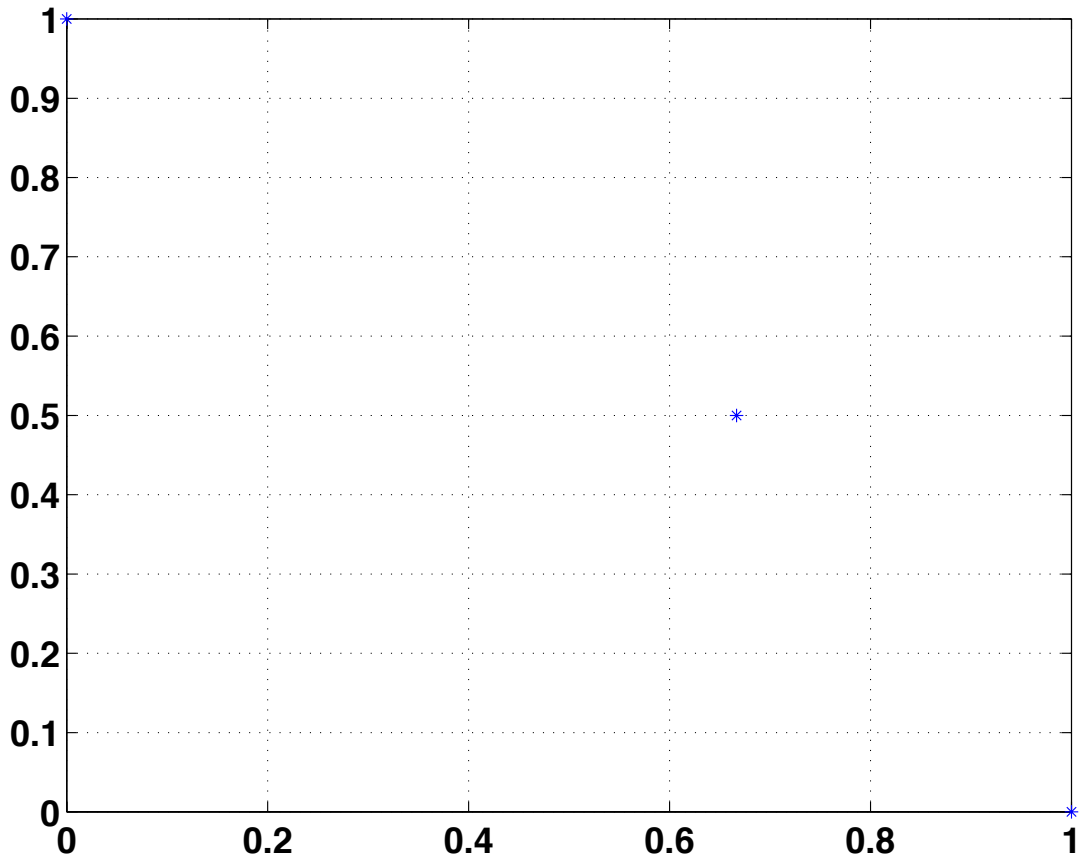
```
>> X = [1,0,0;1,1,1;1,2/3,4/9]   % van der Monde matrix
X =
    1.0000         0         0
    1.0000    1.0000    1.0000
    1.0000    0.6667    0.4444

>> y = [1;0;1/2]
y =
    1.0000
         0
    0.5000

>> a = X\y
a =
    1.0000
   -0.2500
   -0.7500
```

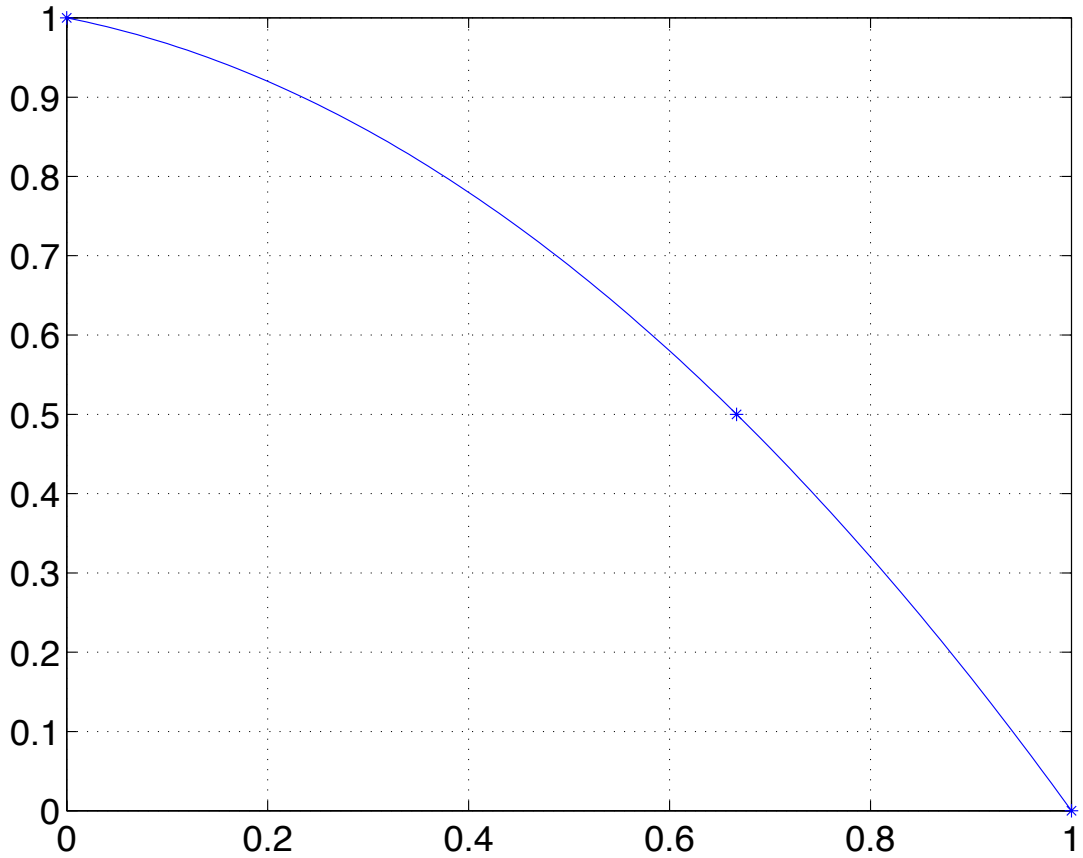
Plot the interpolating points:

```
>> x = [0;1;2/3]; y = [1;0;1/2];
>> plot(x,y,'*')
>> grid
```



Plot the interpolating polynomial:

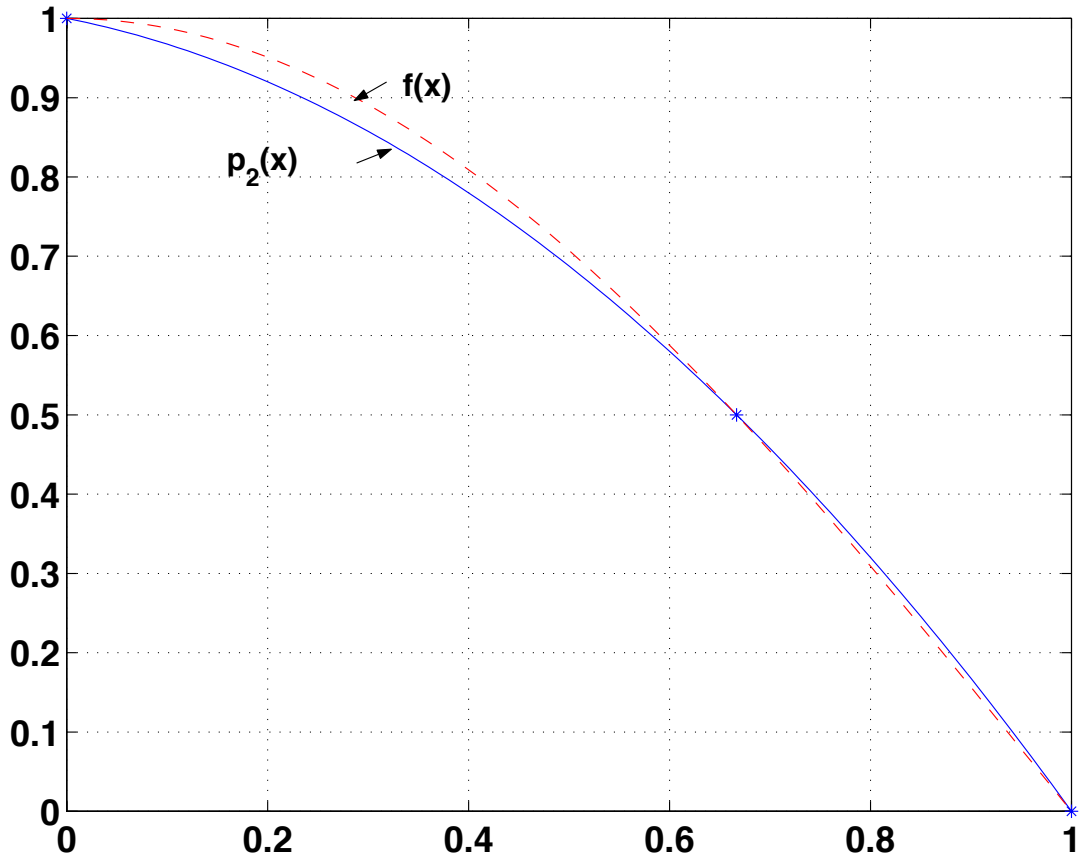
```
>> hold on
>> t = [0:0.01:1];
>> p2 = a(1)+a(2)*t+a(3)*t.^2;
>> plot(t,p2)
```



With $f(x) = \cos(\frac{\pi}{2}x)$, we get:

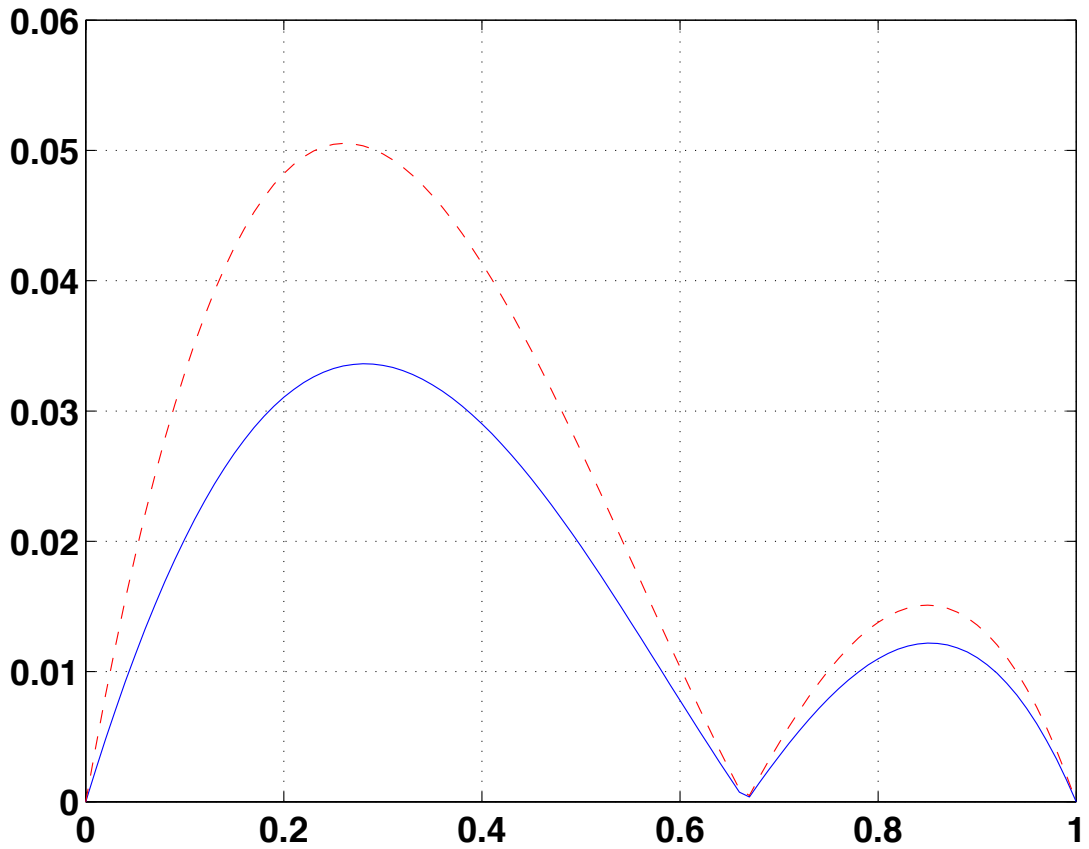
x_i	0	1	2/3
$f(x_i)$	1	0	1/2

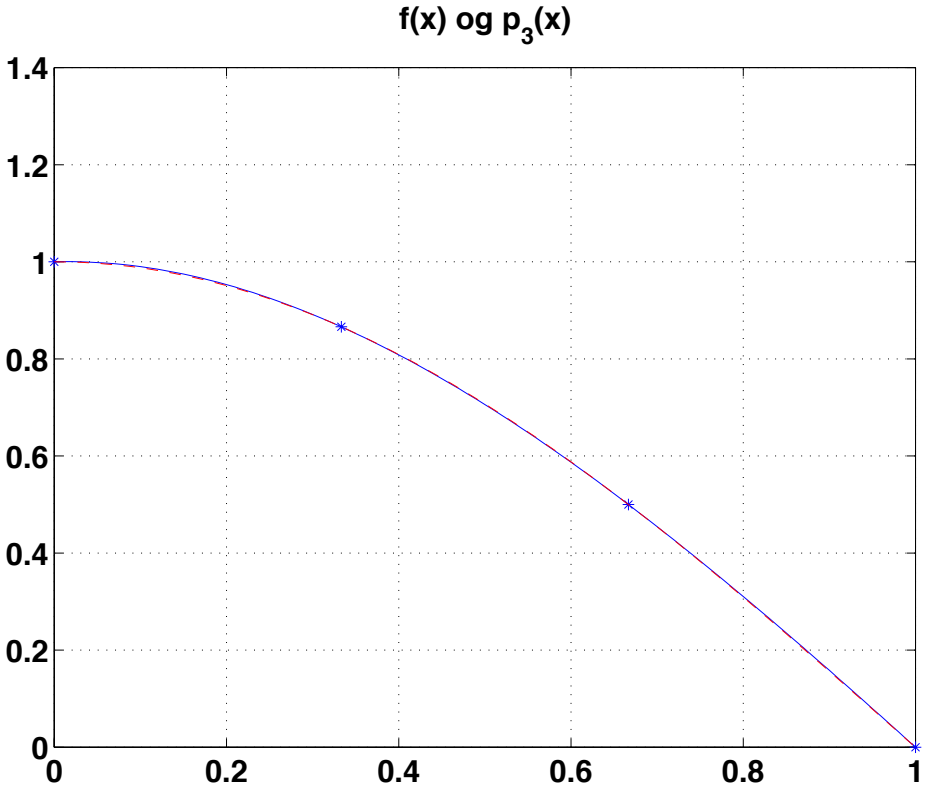
```
>> plot(t,cos(pi/2*t),'--r')
```

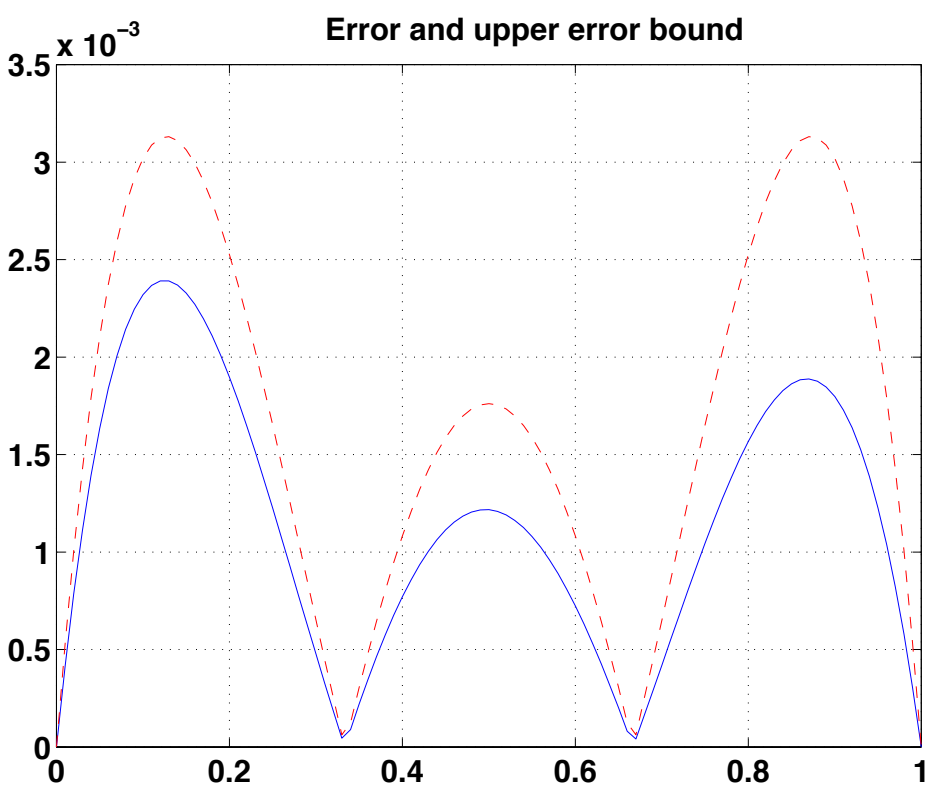


Now let us plot the error $e(x) = f(x) - p_2(x)$ (—) and upper error bound (- - -)

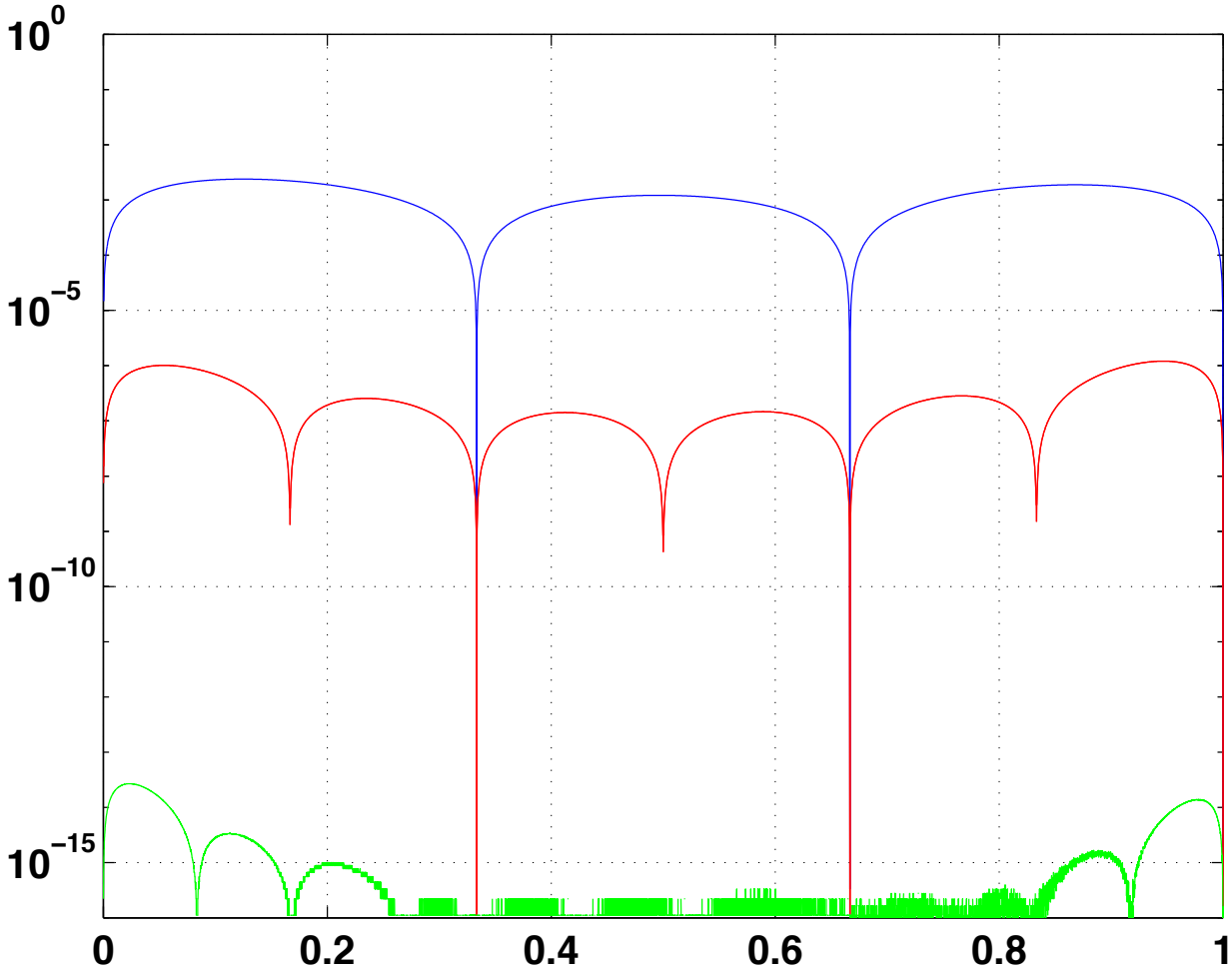
```
>> hold off
>> errorbound = abs(pi^3/48*t.*(t-1).*(t-2/3));
>> error = abs(cos(pi/2*t)-p2);
>> plot(t,error,t,errorbound,'--r')
```



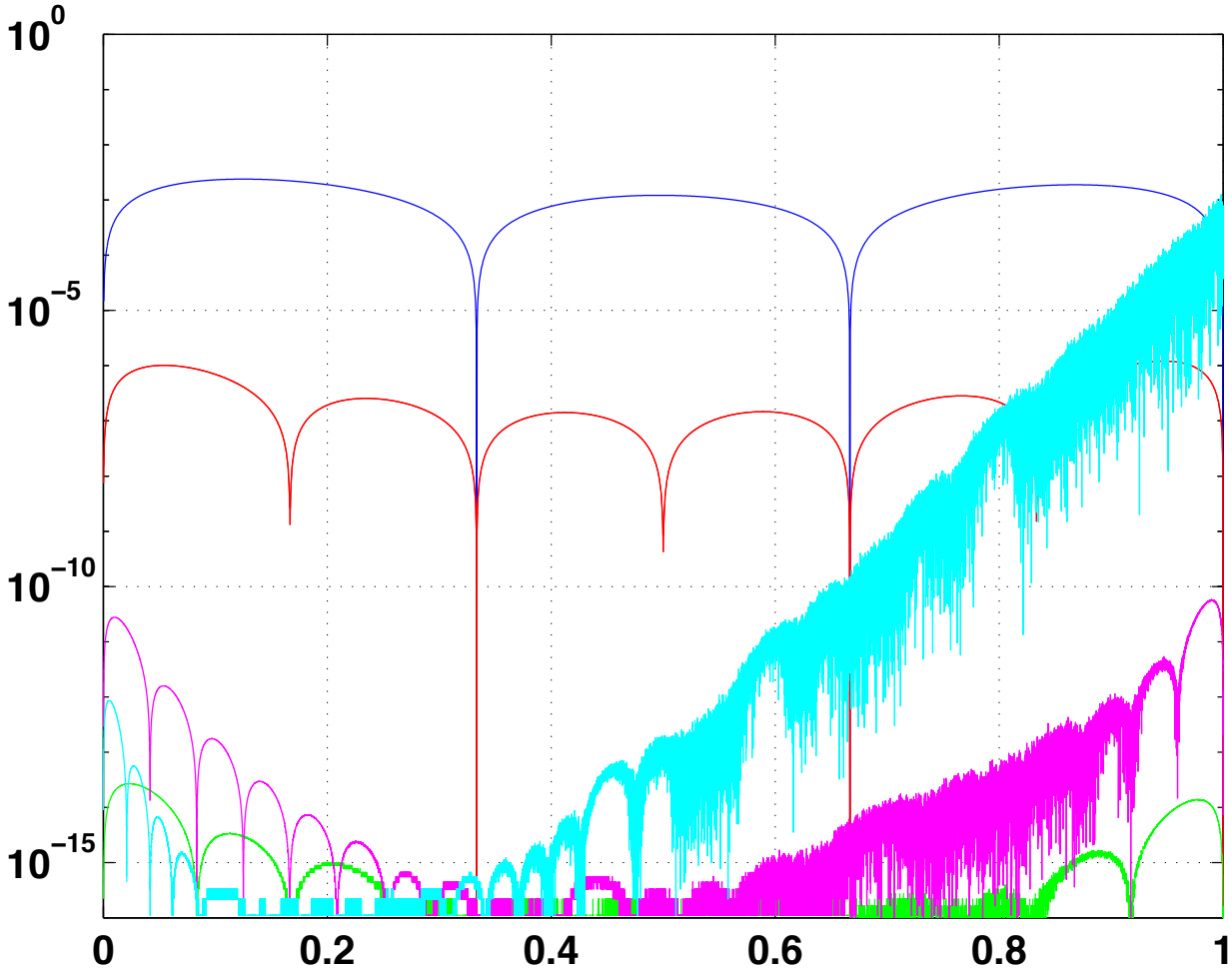

f(x) og p3(x)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1


Error and upper error bound
x 10−3
3.5
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1

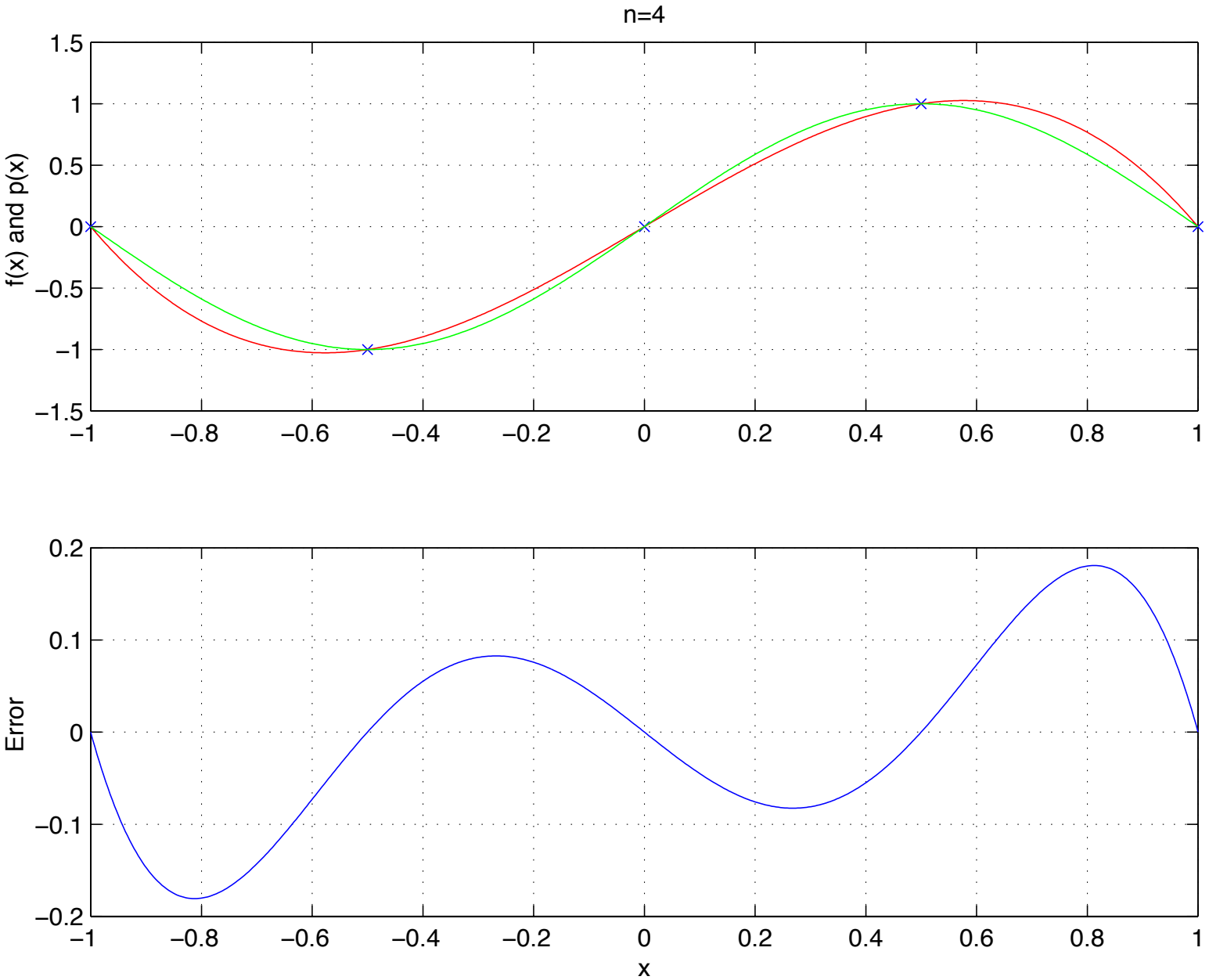
Error when interpolating number increases:



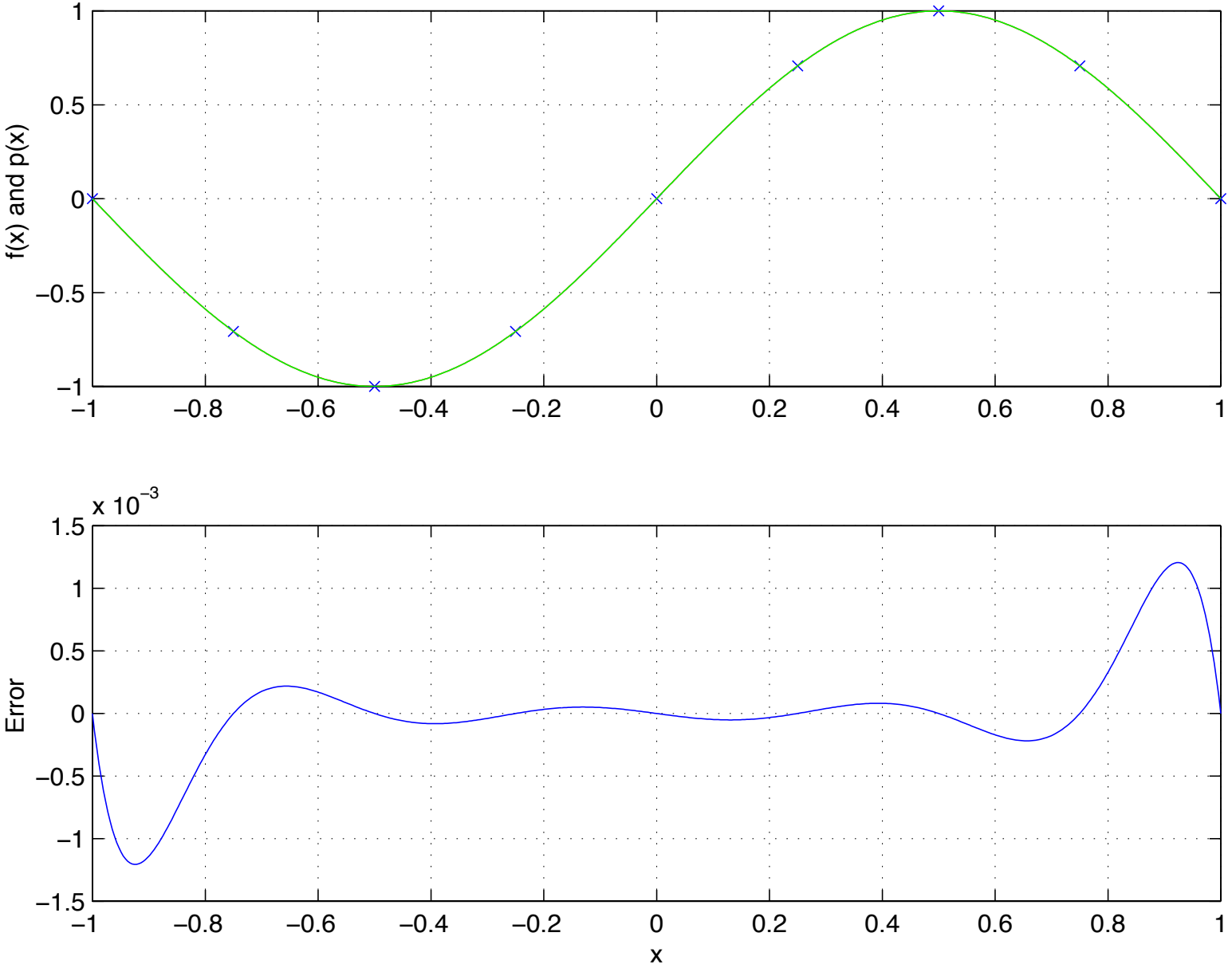
Error when interpolating number increases even more:



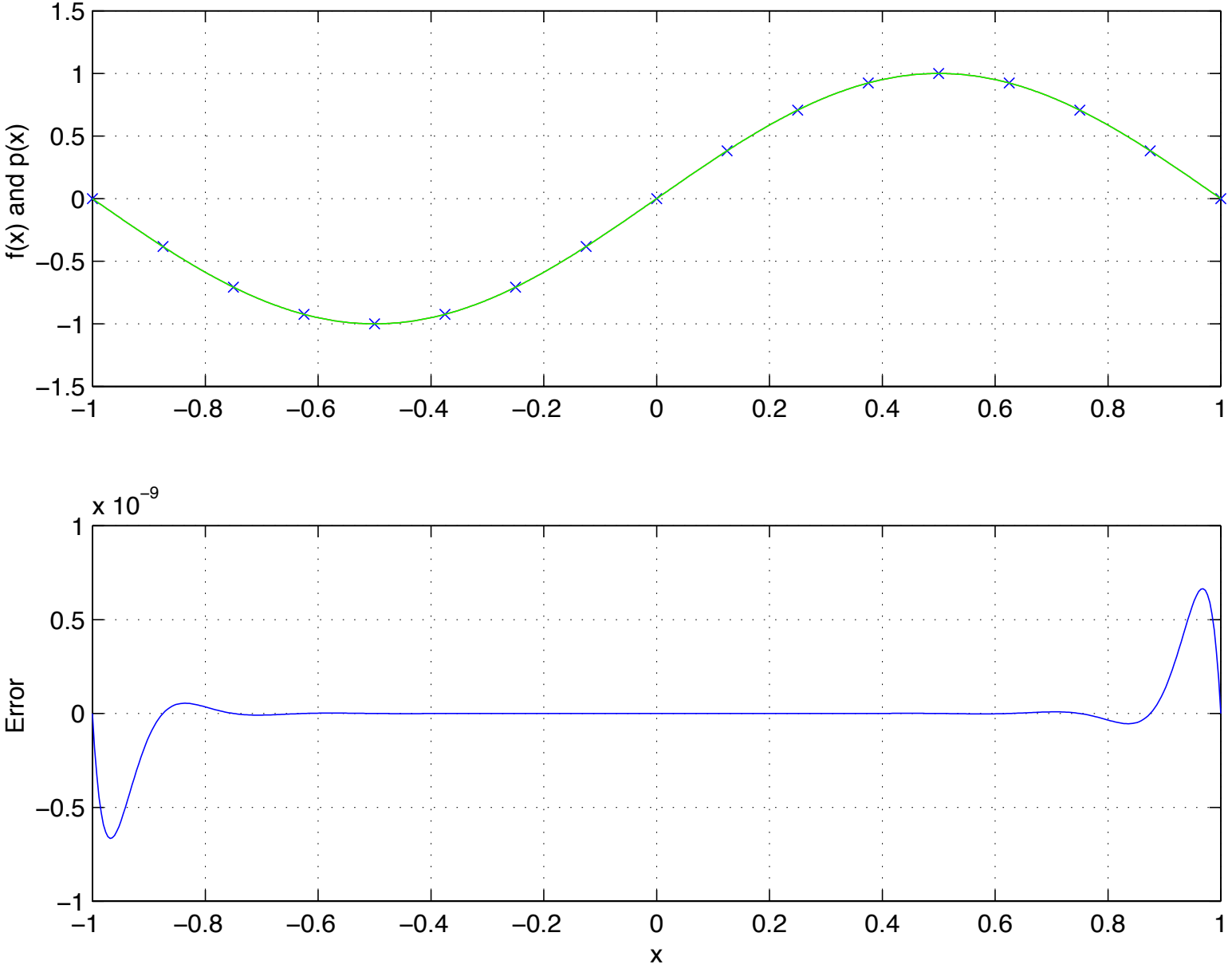
Error with uniform nodes: **n=4**



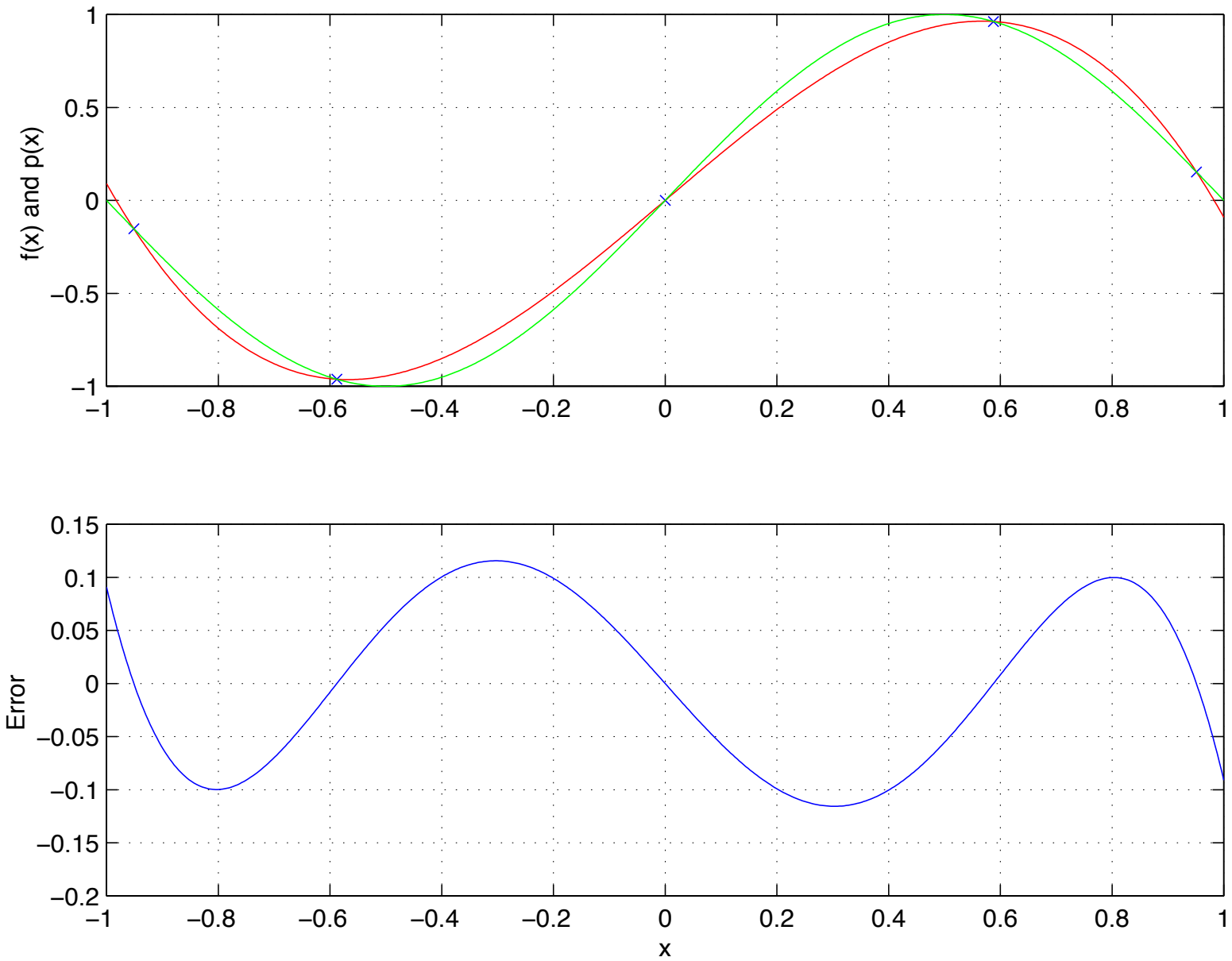
Error with uniform nodes: **n=8**



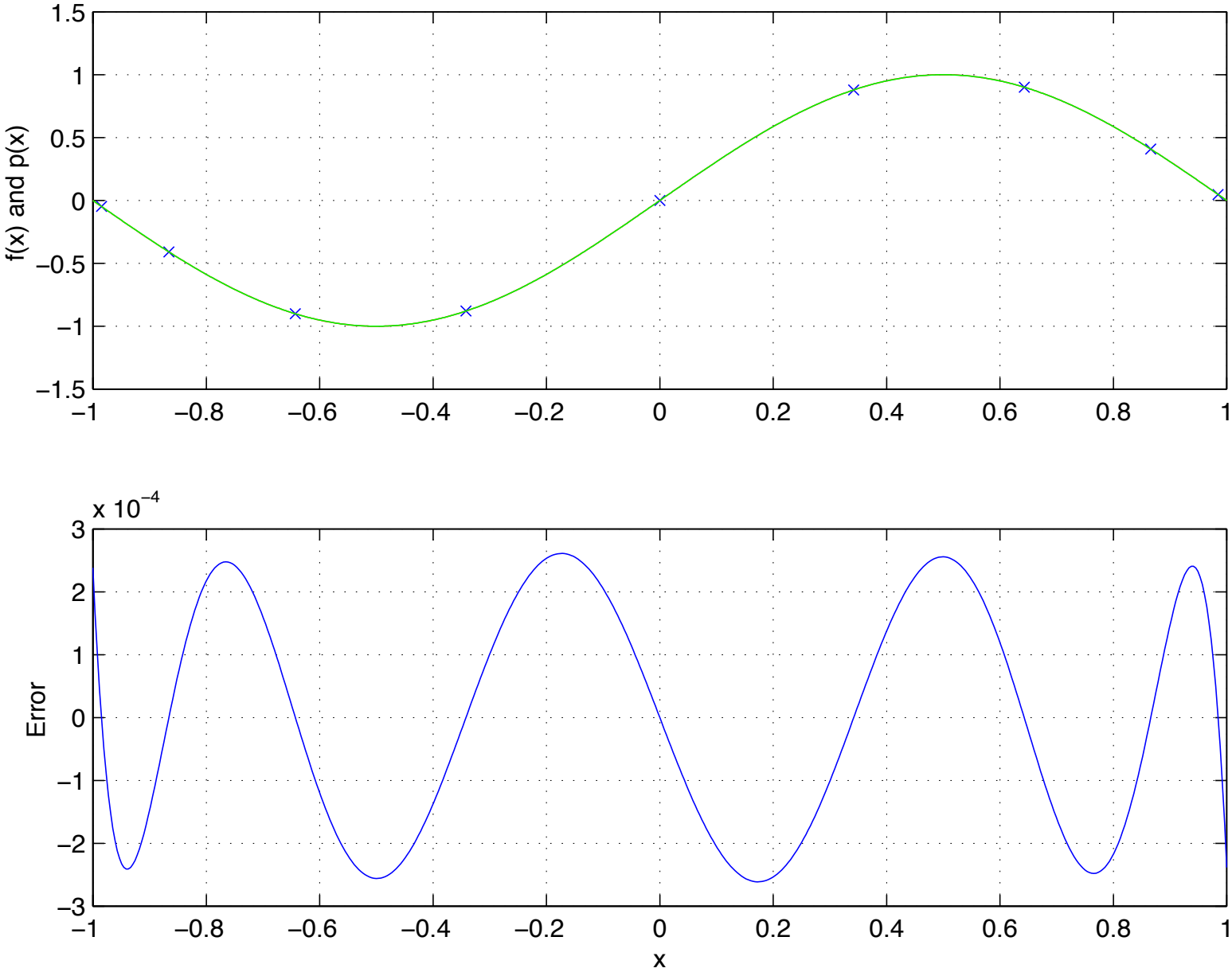
Error with uniform nodes: **n=16**



Chebyshev-nodes, $n = 4$



Error with Chebyshev nodes: **n=8**



Error with Chebyshev nodes: **n=16**