## Άσκηση 1

$$I(x,y,t) = 4x^2 + yt^3 + xy + 5$$

$$I(x+\delta x, y+\delta y, t+\delta t) = I(x,y,t) - I(x,y,t) = 0 \Rightarrow$$

Taylor $\Rightarrow$ $$I(x,y,t) + \frac{dI}{dx}(x+\delta x - x) + \frac{dI}{dy}(y+\delta y - y) +$$

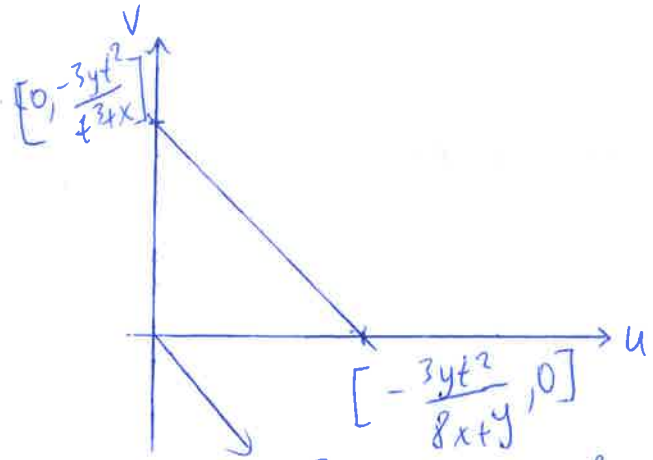
$$\frac{dI}{dt}(t+\delta t - t) - I(x,y,t) = 0 \Rightarrow$$

$$\Rightarrow \frac{dI}{dx}\delta x + \frac{dI}{dy}\cdot\delta y + \frac{dI}{dt}\cdot\delta t = 0 \overset{/dt}{\Rightarrow}$$

$$\Rightarrow \frac{dI}{dx}\cdot u + \frac{dI}{dy}\cdot v + \frac{dI}{dt} = 0$$

$$\frac{dI}{dx} = 8x + y \;,\; \frac{dI}{dy} = t^3 + x \;,\; \frac{dI}{dt} = 3yt^2$$

$$(8x+y)\cdot u + (t^3+x)\cdot v + 3yt^2 = 0 \Leftrightarrow v = -\frac{3yt^2 + (8x+y)\cdot u}{t^3+x}$$




$$\left[-\frac{It}{Ix}, \frac{It}{Iy}\right] = \left[\frac{-3yt^2}{8x+y}, \frac{3yt^2}{t^3+x}\right]$$

$$I(x+\delta x, y+\delta y, t+\delta t) = I(x,y,t) + (x-\delta x) I_x + (y-\delta y) I_y + (t-\delta t) I_t + \frac{1}{2!}\left((x-\delta x)^2 I_{xx} + 2(x-\delta x)(y-\delta y) I_{xy} + (y-\delta y)^2 I_{yy}\right) + \frac{1}{2!}\left((y-\delta y)^2 I_{yy} + 2(y-\delta y)(t-\delta t) I_{yt} + (t-\delta t)^2 I_{tt}\right) + \frac{1}{2!}\left((x-\delta x)^2 I_{xx} + 2(x-\delta x)(t-\delta t) I_{xt} + (t-\delta t)^2 I_{tt}\right) = 0$$

$$I_{xx}=8,\quad I_{yy}=0,\quad I_{tt}=6yt,\quad I_{yt}=3t^2,\quad I_{xy}=1,\quad I_{xt}=0$$

$$I(x+\delta x, y+\delta y, t+\delta t) = (4x^2+yt^3+xy+5) + (x-\delta x)(8x+y) + (y-\delta y)(t^3+x) + (t-\delta t)3yt^2 + \frac{1}{2}\left((x-\delta x)^2 8 + 2(x-\delta x)(y-\delta y) + \frac{1}{2}\left(2(y-\delta y)(t-\delta t)3t^2 + (t-\delta t)^2 6yt\right) + \frac{1}{2}\left((x-\delta x)^2 8_t + (t-\delta t)^2 6yt\right)$$

$$\begin{bmatrix} I_{xx} & I_{xy} & I_{xt} \\ I_{yx} & I_{yy} & I_{yt} \\ I_{tx} & I_{ty} & I_{tt} \end{bmatrix} = \begin{bmatrix} 8 & 1 & 0 \\ t^3 & 0 & 3t^2 \\ 0 & 3t^2 & 6yt \end{bmatrix}$$